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The Minimum Spanning Tree Problem on networks with Neutrosophic numbers

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Abstract. The minimum spanning tree problem (MSTP) revolves around creating a spanning tree (ST) within a graph/network that incurs the least cost compared to all other potential STs. This represents a vital and fundamental issue in the realm of combinational optimization problems (COP). Supply chain management, communication, transportation, and routing are a few examples of real-world issues that have been represented using the MSTP. Uncertainties exist in almost every real life application of MSTP due to inconsistency, improperness, incompleteness, vagueness and indeterminacy of the information and It generates really challenging scenarios to determine the arc length precisely. The main motivation behind this research work is to design a method for MST which will be simple enough and effective in real world scenarios. Neutrosophic set (NS) is a well known renowned theory, which one can this type of uncertainty in the edge weights of the ST. In this article, we review trapezoid neutrosophic set/number to describe the arc weight of a neutrosophic network for MSTP. Here, we introduce an algorithm for solving MSTP in neutrosophic environment. In our proposed method, we describe the uncertainties in Prim's algorithm for MSTP using trapezoid neutrosophic set as edge cost. Here examples of numerical sets are used to explain the proposed algorithm. **Keywords:** Neutrosophic set; MSTP; neutrosophic network/graph; Prim's algorithm.

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1. Introduction

The MSTP, a renowned and extensively applied constraint optimization problem, finds its applications in both operation research and graph theory. It has numerous practical applications [1–3], including communications problem, transportation problem, logistics problem, supply chain management problem, image processing, wireless telecommunication networks and cluster analysis.

Consider $G = (X, Y)$ as a connected, undirected, weighted graph, where X represents a collection items/nodes/vertices and Y denotes a finite set of arcs characterized by integral cost or weight. Tree is a connected graph without circuits. Consider t to represent as a ST of connected graph if and only if t is a sub graph of the graph G and t must consist all vertices of graph G. Because it has the most edges surrounded by all feasible trees in graph G, a ST T is alternatively referred to as the largest possible subtree within graph G. The MST is a ST where the aggregate weights of edges is minimized.

The traditional MSTP is built on a graph or network. As a result, the G's nodes are used to represent the things, points, and objects, while the arcs are used to indicate the relationship or specific link between the nodes (for instance, highways connecting communities). The information connected to objects/items and the relationship between two things are assumed to be fully known in the traditional networks/graphs descriptions of simple deterministic scenarios. In practical situations, achieving this may prove challenging due to presence of uncertainties that can exist in any conceivable description of an object or the relationship between two objects, or even both cases. Due to this reason, the crisp graph model is not useful to model those problem.

Zadeh [4] proposed the concept of a fuzzy set (FS), which may deal with the occurrence of uncertainty, ambiguity, and imprecision in everyday life. The main characteristic of a FS, described by a membership grade/function/degree, is a grade/function/degree whose interval is [0,1]. The idea of FSs has been applied to model several COPs in many fields. The FS (type-1/classical FS), whose membership grade is an actual value, is incapable of managing many different kinds of uncertainty that are present in problems in the real world. In [5], Turksen described the concept of FS with interval membership vale membership [6] and they developed the idea of intuitionistic FSs to capture the problem of non membership grade of classical FSs. It has applied in several problems, e.g., decision making, COPs, artificial neural network, medical analysis, and so forth. It is a modified version of classical FS that can consider not only one a membership grade for each element and but also it considers a non-membership grade. It helps to capture more flexibility to work with uncertainties of real problem [7] than the simple FS. It has three different types of membership grade: membership, non-membership and hesitation of all elements in this set. In [8], the author modified the idea of FSs to the interval valued intuitionistic FSs to capture more uncertainties than intuitionistic FSs.

However, the intuitionistic FS and intuitionistic fuzzy logic have been used to find the solutions of many COPs, but it cannot be captured several type of uncertainty properly.

For e.g., the FS cannot work with the uncertainties due to indeterminate information and inconsistent information. When seeking the opinion of an expert regarding a decision, then he might state that the likelihood of the decision being true is 0.5, likelihood of it being false is 0.9, and the chances of uncertainty is 0.4. This kind of real-world issue cannot be modeled with FS. To solve this issue, new thinking is therefore necessary.

In [9], The idea of NS was established by the author to explain information and facts that might be insufficient, unsure, hazy, imprecise, indeterminate, and inconsistent in various real-world settings. Three membership grades are used to define it: a true membership degree/grade/function, an indeterminate membership degree/grade/function, and a false membership degree/grade/function independently. They fall inside the nonstandard or standard unit interval in terms of value. Because it can adeptly deal with information that lacks consistency and completeness , NS is frequently employed by researchers to handle challenges that arise in real-life situations [10], [11], [12], [13], [**?**], [**?**], [14].

MSTP is an COP [15] in graph theory [16], [17] which can determine the minimum cost ST of a graph. The classical MSTP has several real life applications, including cluster analysis, wireless communication, computer networks, speech recognition, social network, etc. In the classical MSTP, the edge lengths are considered to be precise and expert assumes some crisp values (real number) to describe the edge lengths of the graph. However, in our day to day life [18], [19], [20], the edge length may represent a criterion such as cost, time, demand, capacity, etc. that shouldn't have a fixed parameter. We can consider a real life scenarios in a road networks of city. The edge length describing the time it takes for the car journey could vary due to changing weather conditions, strong traffic flow, or other unanticipated factors, however the geometric distance/road distance between two cities is fixed. [21]. For this reason, it is very confusing for an expert to assume a proper edge length in real number, i.e., crisp values. Experts may consider a range of feasible values of edge lengths in form of approximate intervals, linguistic terms, etc. In this road network problem, the edge lengths can represent as, "around 30 to 90 minute", "about 1 hour", "between 5 and 10 hour" and "nearly 2 to 3 hours", etc. Many researchers used classical FS to describe those uncertainties in edge weights. But simple FS is not properly model those vagueness/incomplete information because their membership grades are fully crisp. The idea of neutrosophic network/graph can be considered as a modified version of fuzzy graph to deal this types of uncertain situations.

The idea of MSTP and its several applications have taken lot of attention of researchers throughout the prior decade and numerous successful approaches have been created for finding the MSTP solution in classical graphs. Refs. [22], [23], [24], [25], [26] can be found related to this MSTP. Prim's algorithm [23] is an effective and well-known algorithmic technique to solve the MST of a crisp graph. Expert can determine the MST using Prim's algorithm [23] if and only if the edge weight/length of the graph are real number/crisp number. Almost all MSTP applications in real life include a certain amount of uncertainty. Two different causes of parameter uncertainty are randomness and fuzzy or insufficient information. The probability theory uses to handle the uncertainties due randomness. Due to this reason, many scientist assume the link/edge of a MSTP as a random variables. In [27] presents this problem with random edge weights/costs. In [28], the author introduced a method that can efficiently solve this problem within a polynomial time frame. In this method, the arc length is calculated based on the several parameters of the probability distributions and those parameters are determined by considering a confidence interval from a set of stochastic data. However in real life scenarios, those parameter are unknown and the parameter values are uncertain (fuzzy, vague or incomplete) in nature. In [29], the authors has described first time the MSTP in fuzzy environment. They used the idea of chance constrained programming and necessity measurement to solve this problem. Then Chang [30] presented the fuzzy MSTP whose fuzzy arc length are fuzzy number. They applied three different techniques using the ranking index method [31] for comparing the several fuzzy arc lengths. Combing the idea of probability theory and FS theory, the authors have developed algorithmic technique to solve this problem. They have also described a genetic algorithm for this problem. In [32], the author has described the fuzzy ST problem in which the lengths are denoted by interval fuzzy number. They have used the principles of possibility theory to compare and select the edge of the ST for the fuzzy graph. In [33], the author considered different intuitionistic FS/number to denote the edge weight of a fuzzy graph. They have described a new algorithmic approach to find the solution of this COP. In [34], the authors presented the fuzzy MSTP with hesitant FSs as fuzzy edge weight and introduce an algorithmic approach to find the solution of this problem.

Recently, few scientists have worked MSTP in neutrosophic environment. This MSTP is defined as neutrosophic MSTP (NMSTP) problem. In several real life application of MSTP, a NMSTP may be more logical, reliable and reasonable. Ye [35] developed an algorithmic approach to solve the MSTP of a neutrosophic network/graph where objects/nodes/vertices are described in NSs and link between two different nodes describes the dissimilarity between objects. Kandasamy [36] described a double-valued NMSTP and present a clustering method to classify the cluster of data/information. A novel approach to solving optimum ST issues by assuming inconsistent, inappropriate, partial, ambiguous, and indeterminate data was presented by Mandal and Basu [37]. They represented the arc length with NSs. Neutosophic numbers and fuzzy numbers have essentially identical notations, but their representations couldn't be further apart. No algorithm for MST with interval neutrosophic arc lengths exists as far as we are aware.

Due to its extensive uses in real-world situations, the MSTP is a well known COPs in the field of operation research. It is particularly difficult to calculate the edge weights correctly since the information used in the real world application of MSTP is inconsistent, inappropriate, partial, unclear, and indeterminable. The NS/logic theory is well known for its ability to explain the inconsistent, inappropriate, incomplete, nebulous, and undetermined arc lengths of the ST. Numerous scientists believe that neutrosophicness should be used instead of fuzziness to represent doubt because it is inconsistent, incorrect, incomplete, ambiguous, and indefinite.

Although some research works have been done to develop for MSTP using neutrosophic set and its generalizations, still there are some scope of research works in this field. The key driving force behind this scientific study is to identify an algorithmic method for the MSTP of a neutrosophic network which will be able to efficiently handle the MSTP. In the past few years, few researchers [38–40] developed some algorithms to determine the MST of a neutrosophic network/graph. In those algorithms, they consider the simple NS to describe the MST of the neutrosophic network/graph. We consider the interval neutrosophic number to represent the arc length. The objective of this scientific study is to introduce an algorithm that can determine the MST. In this research paper, a MSTP is considered whose edge weights are described by neutrosophic number. We have described a modified Prim's algorithm to determine the NMSTP of a graph and its weight as a score value.Our focus is on a neutrosophic network [41–44] or graph, with its edge weights expressed through the use of neutrosophic numbers. Opt for the arc with the most minimal score, a ranking mechanism based on scores is utilized.

The structure of the paper is as follows. A few essential definitions and concepts related to neutrosophic graphs, single valued trapezoidal neutrosophic number, ranking and addition operation are reviewed in brief in Section 2. We provide a mathematical model for Neutrosophic Minimal Spanning Tree in Section 3. In Section 4 we provide our proposed algorithm for this problem. Section 5 presents the outcomes of the suggested method and draws comparisons with binary programming. In Section 6, we finally come to an end.

2. Preliminary

Definition 2.1. Let $\mathcal{U}$ and Z represent an universal set and NS (NS). The NS [9] Z consists of 3 membership degree. There are true membership grade $\mathcal{T}_Z(k)$, indeterminate membership grade $\mathcal{I}_Z(k)$ and false membership grade $\mathcal{F}_Z(k)$ respectively.

$$^{-}0 \leq \sup\ \mathcal{T}_Z(k) + \sup\ \mathcal{I}_Z(k) + \sup\ \mathcal{F}_Z(k) \leq 3^{+} \tag{1}$$

Definition 2.2. The single valued NS [45] D on the $\mathcal{U}$ is presented as following

$$A = \{\langle k : \mathcal{T}_Z(k), \mathcal{I}_Z(k), \mathcal{F}_Z(k) | k \in \mathcal{U}\rangle\} \tag{2}$$

The true function $\mathcal{T}_Z(k)$ lies in the interval $[0,1]$, indeterminate membership grade $\mathcal{I}_A(k)$ lies between $[0,1]$ and false membership grade $\mathcal{F}_A(k)$ is in the interval $[0,1]$, satisfy the following condition:

$$^{-}0 \leq \sup\ \mathcal{T}_Z(k) + \sup\ \mathcal{I}_Z(k) + \sup\ \mathcal{F}_Z(k) \leq 3^{+} \tag{3}$$

Definition 2.3. Let $\tilde{D}$ describes a single valued trapezoidal neutrosophic number (SVTNN) [?] where $\tilde{D} = \left\langle (d_r, d_l, d_p, d_s)\,; w_{\tilde{D}}, u_{\tilde{D}}, y_{\tilde{D}} \right\rangle$. The membership values can be calculated as follows

$$\mu_{\tilde{D}}(q) = \begin{cases} \frac{(q-d_r)w_{\tilde{D}}}{(d_l-d_r)} & (d_r \leq q < d_l) \\ w_{\tilde{D}} & (d_l \leq q \leq d_p) \\ \frac{(d_s-p)w_{\tilde{D}}}{(d_s-d_p)} & (d_p < p \leq d_s) \\ 0 & otherwise \end{cases}$$

$$v_{\tilde{D}}(q) = \begin{cases} \frac{\left(d_l-p+u_{\tilde{D}}(q-d_r)\right)}{(d_l-d_r)} & (d_r \leq x < d_l) \\ u_{\tilde{D}} & (d_l \leq q \leq d_p) \\ \frac{\left(q-d_p+u_{\tilde{D}}(d_s-p)\right)}{(d_s-d_p)} & (d_p < x \leq d_s) \\ 0 & otherwise \end{cases}$$

and

$$\lambda_{\tilde{D}}(q) = \begin{cases} \frac{\left(d_l-p+y_{\tilde{D}}(q-d_r)\right)}{(d_l-d_r)} & (d_r \leq q < d_l) \\ y_{\tilde{D}} & (d_l \leq q \leq d_p) \\ \frac{\left(x-d_p+y_{\tilde{D}}(d_s-p)\right)}{(d_s-d_p)} & (d_p < p \leq d_s) \\ 0 & otherwise \end{cases}$$

respectively.

Definition 2.4. Let $\tilde{D}$ is a single valued triangular neutrosophic number (SVTrN-number) [?] where $\tilde{D} = \left\langle (d_r, d_l, d_p,)\,; w_{\tilde{D}}, u_{\tilde{D}}, y_{\tilde{D}} \right\rangle$. We can calculate membership values in the following manner:

$$\mu_{\tilde{D}}(q) = \begin{cases} \frac{(q-d_r)w_{\tilde{D}}}{(d_l-d_r)}, & (d_r \leq q < d_l) \\ \frac{(d_p-p)w_{\tilde{D}}}{(d_p-d_l)}, & (d_l \leq q < d_p) \\ 0, & otherwise \end{cases}$$

$$v_{\tilde{D}}(q) = \begin{cases} \frac{\left(d_l-p+u_{\tilde{D}}(q-d_r)\right)}{(d_l-d_r)}, & (d_r \leq q < d_l) \\ \frac{\left(q-d_l+u_{\tilde{D}}(d_p-p)\right)}{(d_p-d_l)}, & (d_l \leq q \leq d_p) \\ 0, & otherwise \end{cases}$$

$$\lambda_{\tilde{D}}(q) = \begin{cases} \frac{\left(d_l-p+y_{\tilde{D}}(q-d_r)\right)}{(d_l-d_r)}, & (d_r \leq q < d_l) \\ \frac{\left(q-d_l+y_{\tilde{D}}(d_p-p)\right)}{(d_p-d_l)}, & (d_l \leq x \leq d_p) \\ 0, & otherwise \end{cases}$$

If $d_r \geq 0$and at least $c > 0$ then $\tilde{D} = \left\langle (d_r, d_l, d_p, d_s)\,; w_{\tilde{D}}, u_{\tilde{D}}, y_{\tilde{D}} \right\rangle$ it is affirmed to be positive SVTrN-number, denoted by $\tilde{D} > 0$.

Definition 2.5.
Let $\tilde{D} = \langle (d_{r1}, d_{l1}, d_{p1}, d_{s1}) ; w_{\tilde{D}}, u_{\tilde{D}}, y_{\tilde{D}} \rangle$ and $\tilde{B} = \langle (d_{r2}, d_{l2}, d_{p2}, d_{s2}) ; w_{\tilde{D}}, u_{\tilde{D}}, y_{\tilde{D}} \rangle$ be two SVTN-number and $d_s \neq 0$ be any real number. Then, the addition operation between $\tilde{D}$ and $\tilde{B}$

$$\tilde{D} + \tilde{B} = \langle (d_{r1} + d_{r2}, d_{l1} + d_{l2}, d_{p1} + d_{p2}, d_{s1} + d_{s2}) ; w_{\tilde{D}} \wedge w_{\tilde{B}}, u_{\tilde{D}} \vee u_{\tilde{B}}, y_{\tilde{D}} \vee y_{\tilde{B}} \rangle \tag{4}$$

Definition 2.6. The score functions is defined as follows:

$$s(\tilde{D}) = \frac{1}{12} (d_{r1} + d_{r2}\, d_{r3} + d_{r4}) * \left(2 + w_{\tilde{D}} - u_{\tilde{D}} - y_{\tilde{D}}\right) \tag{5}$$

Definition 2.7. Let D_1 and D_2 are two SVNs. Then
$D_1 \succ D_2$ if and only if $s(D_1) > s(D_2)$.
$D_1 \prec D_2$ if and only if $s(D_1) < s(D_2)$.
$D_1 \sim D_2$ if and only if $s(D_1) = s(D_2)$.
Here, $D_1 \succ D_2$ expresses that the cost of the arc/edge represented by D_1 is larger than the cost of the arc/edge represented by D_2. Similarly, $D_1 \prec D_2$ expresses that the cost of the edge described by D_1 is lower than the cost of the arc/edge descried by $D2$. $D_1 \sim D_2$ expresses that the cost of the edge presented by D_1 is equal to the cost of the arc/edge described by D_2.

3. Neutrosophic Minimal Spanning Tree (NMSTP)

When considering a connected graph G, an ST, defined as connected, acyclic and maximum sub graph, comprises all nodes within G. Each ST contains precisely $n-1$ arcs, where n represents the number of nodes in the graph G. A MSTP is to identify a ST such that the total length of its arcs is minimum. The precise weights connected to the graph's arcs are taken into account by the traditional MSTP. However, due to insufficient or absent evidence in real-world settings, the arc lengths might not be exact. Neutosophic graphs are the best solution for dealing with this imprecision.

3.1. *Problem formulation for NMSTP*

Let G represents a neutrosophic network/graph. The graph G consists of p number of vertices $V = \{v_1, v_2, ..., v_p\}$ and q number of arcs $A \subseteq V \times V$. Each edge of G is denoted by r, which is a pair of vertices (n, m), where $n, m \in V$ and $n \neq m$. If the edge e is present in the NMSTP then $x_r = 1$, otherwise $x_r = 0$. The NMSTP is expressed as the following linear programming problem.

$$\min \sum_{r \in A} D_r y_r \tag{6}$$

Subject to

$$\sum_{r \in A} y_r = p - 1 \tag{7}$$

$$\sum_{r \in d_l(s)} y_r \geq 1 \ \forall s \subset V, \ \emptyset \neq s \neq V \tag{8}$$

$$y_r \in \{0, 1\} \ \ \forall r \in A \tag{9}$$

Here, D_r is a NS that describes the edge weight $r \in A$ and $\sum$ in (6) is the addition of all NSs. Next Eq. (7) describes that the total number of arc in the ST is $p - 1$. In (8), $d_l(s) = \{(n, m | n \in s, m \notin s)\}$ is applied for the cut of a subset of node s, i.e., the edges that consists of only one node s and the different node outside the s.

4. **Proposed Algorithm for the NMSTP and its cost**

In this document, we acquaint you with a neutrosophic version of Prim's algorithm for finding the MST in an uncertain environment [46, 47]. For managing the uncertainty in the existences world scenarios, we employ NS. In a neutrosophic environment, we present the MSTP on a neutrosophic network or graph where each edge is given a trapezoid neutrosophic number as its edge weight. In this optimization problem MSTP, since it needs ordering and summation between trapezoid neutrosophic number, is not same from the strand MSTP, which can only consider real numbers/value. The neutrosophic number's scoring function is utilized for comparison, and neutrosophic numbers are combined by applying their designated addition formula. Based on this two concept of neutrosophic number, we propose a neutrosophic edition of the conventional Prim's method to solve the NMSTP. We take into account numerous variables that are crucial to describing in our proposed approach. An undirected connected weighted neutrosophic network/graph $G = (X, Y)$ with neutrosophic edge weight, where X contains a set of nodes and E contains a set of arcs. Let $number = |X|$ and $number_1 = |Y|$, so we have a finite set of nodes $X = \{x_1, x_2, ..., x_{number}\}$ and edges $Y = \{y_1, y_2, ..., y_{number_1}\}$. X_n, Y_n and $C_{\tilde{M}}$ describe the finite set of node, arc and weight of the corresponding neutrosophic MST (NMST).

A random vertex t is selected from G. We calculate the score value for all the arcs in graph G using Eq. (7). Start from the node t and add the node t to its nearest neighbour node, say r. To select the nearest neighbour vertex, first we have to determine all the adjacent edge with p. Then, select the arc, i.e., (p, r) with lowest score value among all the adjacent edges of p. Using the same concept, we have to find the nearest neighbour vertex for all other nodes of the graph. Now, we assume p and r as one simple sub-graph and add this sub graph to its nearest neighbour node. Let us consider, the new node is Q. Next time, the neutrosophic tree with nodes p, q and r as one another sub graph and repeat this method until all $number$ nodes have joined by $number - 1$ edges. Our proposed neutrosophic Prim algorithm is presented in Algorithm 1.

Algorithm 1 Algorithm of the modified neutrosophic Prim's algorithm for NMSTP.

Input: An undirected connected weighted neutrosophic network/graph $G = (X, Y)$ with neutrosophic edge weight.

Output: NMST $\tilde{M} = (X_n, Y_n)$ of G and its cost.

$X_n \leftarrow \{t\}$; ▷ $\tilde{M}$ is a randomly selected source node and S is the vertex set of $\tilde{M}$

$Y_n \leftarrow \emptyset$;

$C_{\tilde{M}} \leftarrow 0$;

Calculate the score value for each arc in G using (7);

while $X \setminus X_n \neq \emptyset$ **do**

  Select an arc (p, r) with minimum score value such that p is in X_n and r is not;

  $C_M \leftarrow C_M \oplus \text{Score}(p, r)$;

  $Y_n \leftarrow Y_n \cup (p, r)$;

  $X_n \leftarrow X_n \cup (\{p, r\} \setminus X_n)$;

end while

5. Numerical illustrations

To give an idea, put forward a suggested algorithm where we have included an example of the NMSTP in this section. A network/graph with undirected connections, weighted edges, and neutrosophic nature where network/graph $G = (X, Y)$, neutrosophic edge weight is considered here. This graph has 6 nodes and 9 arcs. Our propose algorithm can solve this NMSTP and it finds the NMST of a neutrosophic network/graph, whose arc length are expressed by trapezoid neutrosophic set/number. The eight trapezoid neutrosophic number, presented in Table 1 are considered as edge weight of neutrosophic network/graph. For this graph, presented in Figure 1, those trapezoid neutrosophic number are given to the arcs as arc length of this graph randomly.

The source vertex 1 is selected randomly from the set of vertices of the neutrosophic network/graph G. The Prim's algorithm will start from the node 1. Initially, $X_n = \{1\}$, Y_n=$\{\emptyset\}$ and $C_{\tilde{M}}$=0.

Step 1. Find all the connected edges with vertex 1 in the first step. The three edges, $(1, 2)$, $(1, 5)$ and $(1, 3)$, are joined with vertex 1. We employ a ranking approach to determine the numeric value associated with three edges. Among them, the smallest one $(1, 2)$ is picked out along with minimum score value. Now, $X_n = 1, 2$ and Y_n=$(1, 2)$.

Step 2. In this second stage, all the arcs must be chosen so that one end vertex is either in $1 or 2$ while the other is in 5,3, or 6. Three edges, $(1, 5)$, $(2, 3)$ and $(2, 6)$ are determined.

Table 1. Arc length of MSTP

Index	**Edge**	**STVN number**
1	(1,2)	$\langle(4.6, 5.5, 7.6, 8.9), (0.4, 0.7, 0.2)\rangle$
2	(1,5)	$\langle(4.2, 6.9, 7.5, 8.7), (0.7, 0.2, 0.6)\rangle$
3	(1,3)	$\langle(6.0, 7.6, 8.2, 8.4), (0.4, 0.1, 0.3)\rangle$
4	(2,3)	$\langle(6.1, 6.7, 8.3, 8.7), (0.5, 0.2, 0.4)\rangle$
5	(4,5)	$\langle(4.7, 5.9, 7.2, 7.4), (0.7, 0.2, 0.3)\rangle$
6	(2,6)	$\langle(6.6, 8.8, 10, 12), (0.6, 0.2, 0.2)\rangle$
7	(4,6)	$\langle(6.3, 6.5, 8.9, 8.99), (0.7, 0.4, 0.6)\rangle$
8	(3,4)	$\langle(5.2, 7.9, 9.1, 9.4), (0.6, 0.3, 0.5)\rangle$

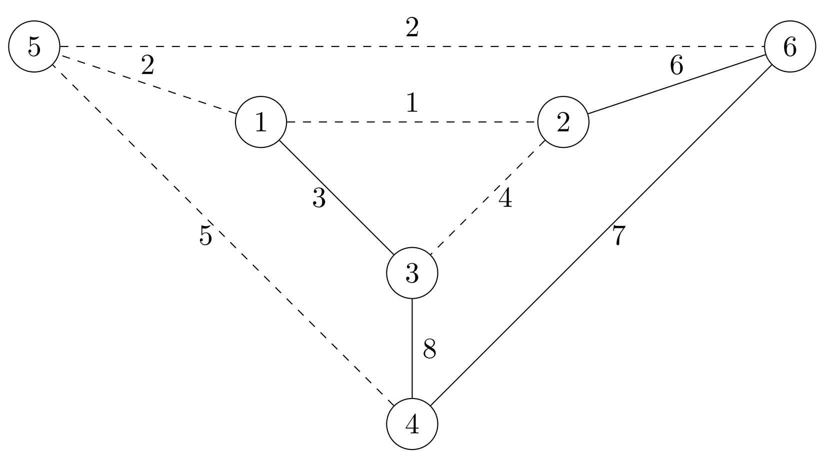

Figure 1. A neutrosophic network/graph

Among this three, the lightest one $(1, 5)$ is selected with their value of score. Now, X_n $= 1, 2, 5$, Y_n=$(1, 2), (1, 5)$.

Step 3. Similarly, we add all other edges of the neutrosophic network/graph. Now, $X_n =$ $1, 2, 3, 4, 5, 6, D$, Y_n=$(1, 2), (1, 5), (5, 6), (2, 3), (4, 5)$.

A LPP model is also considered to solve NMSTP. To find the solution, LINGO software is employed. We describe our obtained result in Table 2 which is determined by software LINGO. We use a variable $x_{i,j}$=1, if any edge i, j is in the MST. Table 2 also provides a description of the Prim's algorithm solution. We get an identical solution of LINGO and our proposed algorithm.

TABLE 2. Result of NMSTP

LINGO using Solution	**Prim's algorithm using Solution**
Min Z= 75.06	Cost = 75.06
$x_{23} = 1$, $x_{12} = 1$, $x_{56} = 1$	MSTP=(23)(12)(51)(45)(56)
$x_{15} = 1$,$x_{45} = 1$	

6. Conclusion

The main objective of this paper is to consider MSTP with neutrosophic set and its generalizations. In this study, we investigate the NMSTP, whose arc lengths are characterized by trapezoid neutrosophic numbers. In addition, we discuss the necessity of using the trapezoid neutrosophic number in MSTP. Using trapezoid neutrosophic number for NMSTP, the classical Prim's algorithm is updated to incorporate uncertainty. In order to demonstrate the efficacy of our algorithm, we have included an illustrative numeric instance for clarification. The propose method is practical and easy to use in scenarios found in the real world. The supply chain management, routing, commutation, and other significant fields will be among the next areas to which we attempt to apply our proposed algorithm. It is important to note that there is more uncertainty in the arc length of a neutrosophic graph in NMSTP than just the geometric distance. For instance, even if the geometric distance is set, the travel time between two cities may be represented as a neutrosophic number because of weather and other unforeseen circumstances.

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