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Neutrosophic interval-valued anti fuzzy linear space

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Abstract. In this research paper, we have introduced the concept of Neutrosophic interval-valued anti fuzzy linear space **(NIVAFLS)** and have also examined its various distinct characteristics. A counter example has demonstrated that the intersection of two Neutrosophic interval-valued anti fuzzy linear spaces **(NIVAFLSs)** does not possess the capability to be a Neutrosophic interval-valued anti fuzzy linear space **(NIVAFLS)**. Conversely, the union of two Neutrosophic interval-valued anti fuzzy linear spaces **(NIVAFLSs)** does form a Neutrosophic interval-valued anti fuzzy linear space **(NIVAFLS)**. Additionally, we have defined and provided an explanation for the cartesian product of two (NIVAFLSs). Furthermore, we have performed a study on the homomorphic image and inverse image of Neutrosophic Anti-Fuzzy Linear Space **(NIVAFLS)**, along with investigating some related properties.

Keywords: Fuzzy linear space; Anti fuzzy linear space; Neutrosophic fuzzy set; Neutrosophic interval-valued anti fuzzy linear space.

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1. Introduction

In 1965, Zadeh [1] came up with the concept of "fuzzy set". His pioneering research on coping with uncertainty culminated in a magnificent notion. A membership value has been assigned to each member of a fuzzy set, representing the degree of membership or the degree to which the element belonging to the set. Those membership values range from 0 to 1, with 0 indicating that the element has no membership in the set and 1 indicating full membership. Values between 0 and 1 signify different levels of partial membership. This framework paved the route for an extensive variety of mathematical applications as well as real-world challenges.Later, in 1986, Atanassov [2] developed fuzzy sets theory by introducing the idea of intuitionistic fuzzy sets. Each element in a universe is given a membership value in classic fuzzy sets, ranging from 0 to 1, which indicates how much the element belongs to a specific set. But by introducing the idea of non-membership functions, Atanassov [2] offered a more adaptable framework.. Intuitionistic fuzzy sets give away an expanded framework for decision-making and knowledge portrayal by providing a more complex and powerful tool for resolving ambiguity and uncertainty in real-world scenarios, and Smarandache [3] enacted an entirely novel idea known as Neutrosophic set **(NS)** through the addition of an intermediate membership in 2005. In their research, Arockiarani and Martina [4] delve into the concept of the Neutrosophic set, which is a mathematical paradigm that encompasses values ranging from the subset of $[0, 1]$. This set allows for the representation of uncertain, indeterminate and contradictory information, making it a valuable tool in various fields such as data analysis, pattern recognition and decision-making . Vijayabalaji and Sivaramakrishnan $[5, 6]$ set the standard for the concept of cubic linear space, as well as cubic intuitionistic linear space. The work of Sivaramakrishnann and Vijayabalaji [7] has contributed significantly to the development and understanding of interval-valued anti fuzzy linear space.

Anti fuzzy sets have emerged as a promising alternative to classical fuzzy sets when it comes to dealing with uncertain and imperfect data. In scenarios where the available information is not fully reliable or the data exhibits ambiguity, fuzzy sets may fall short in accurately representing the underlying uncertainty. This is where anti fuzzy sets come into play, providing a different and more robust approach. In the context of interval-valued anti fuzzy sets **(IVAFS)**, the extent of non-membership is expressed through intervals rather than individual values. In this sense, considering a number of alternatives for degrees of non-membership allows for a more adaptive portrayal of uncertainty. **IVAFS** is beneficial in decision-making, risk assessment, pattern detection and other activities that require the successful management of uncertainty and imprecision. Union, intersection and complement operations can be stated for **(IVAFS)** in the same manner as they can for standard fuzzy sets. Because the data is interval-valued, these processes become more complex and may require the inclusion of extra variables.

The amalgamation of Fuzzy Set **(FS)** and Intuitionistic Fuzzy Set **(IFS)** is referred to as Neutrosophic Fuzzy Set (NFS). This mathematical structure that makes it possible to describe inconsistent, ambiguous and incomplete data. Mathematical applications of **NFS** can be found in many fields, including as natural language processing, production planning and scheduling, pattern classification, data mining, data analysis, optimization and decision making. Neutrosophy is concerned with indeterminacy and is made up of three components: truth (T), falsity (F) and indeterminacy (I). Different levels of truth, falsehood and unknowns in a given statement or proposition are indicated by these components: the evidence, the context and the logical reasoning. Neutrosophic fuzzy sets **(NFS)** combine neutrosophy and fuzzy sets to provide a more comprehensive strategy to dealing with unclear and imprecise data. In **(NFS)** theory, In the extension of classical set theory, each element of a set is assigned a membership degree, as well as non-membership and indeterminacy degrees. This enables a more refined depiction of uncertainty and ambiguity in the inclusion of fragments within a set. In **(NIVAFS)**, each element in a set is connected with an interval that indicates the possibility of non-membership (uncertainty) within the neutrosophic context of truth, falsity, and indeterminacy. The **(NIVAFS)** membership function ties universe elements to intervals while taking neutrosophic traits and anti-fuzzy degrees into account. Neutrosophic interval-valued anti fuzzy settings provide an effective way to cope with uncertainty, indeterminacy and imprecision in each of these decision-making challenges, allowing decision-makers to make more complete and informed decisions in difficult real-world scenarios. Because of its adaptability and strength, numerous decision-analysis and problem-solving tasks are ideally suited to this paradigm.

In the realm of algebraic structures, a homomorphism is a function that preserves the operations of two algebraic structures of the same kind. For example, if two groups were homomorphized, the group operation would be maintained. The inverse image of a subset in the codomain is the set of all elements in the domain that map to elements in the given subset, based on a function or mapping between the two sets.

This study presents a methodology for determining the framework of linear space for single-valued Neutrosophic sets. The concepts of Neutrosophic set **(NS)** and interval-valued anti fuzzy setting of linear space **(IVAFLS)** are utilized to define Neutrosophic interval-valued anti fuzzy linear space **(NIVAFLS)**. **(NIVAFLS)** is defined as the union of two Neutrosophic interval-valued anti fuzzy linear spaces **(NIVAFLSs)**. However, the intersection of two **(NIVAFLSs)** may not necessarily be a NIVAFLS. Additionally, the definition and theory for the cartesian product of two **(NIVAFLSs)**, as well as the image and inverse image of a Neutrosophic Anti-Fuzzy Linear Space **(NIVAFLS)**, are established.

2. Preliminaries

Definition 2.1 (3)**.** Assume $\mathbf{X_U}$ is universe of discourse. A **NS** is Ω=$\{v,\ \xi_\Omega(v), \Psi_\Omega(v), \zeta_\Omega(v) | v \in \mathbf{X_U}\}$ or simply denoted by $\Omega = (\xi_\Omega(v), \Psi_\Omega(v), \zeta_\Omega(v))$, where $\xi : \mathbf{X_U} \to [0,1]$, $\Psi : \mathbf{X_U} \to [0,1]$ $\zeta : \mathbf{X_U} \to [0,1]$ the object's degree of truth can be represented through its membership, indeterminacy, and false membership $v \in \mathbf{X_U}$ and $0 \le \xi_\Omega(v) + \Psi_\Omega(v) + \zeta_\Omega(v) \le 3$.

Definition 2.2 (6)**.** Let $\mathbf{V_S}$ represent a crisp linear space over a field $\mathbf{F}$, which is symbolized by $(\mathbf{V_S F})$, a mapping $\delta : \mathbf{V_S} \to [0,1]$ is called as an anti fuzzy linear space (**AFLS**) if $\delta(av_1 * bv_2) \le \mathbf{max}\{\delta(v_1), \delta(v_2)\}, \forall v_1, v_2 \in \mathbf{V_S}$ and $a, b \in \mathbf{F}$ and $*$ is any binary operation on $\mathbf{F}$.

3. Neutrosophic interval-valued anti fuzzy linear space

Definition 3.1. A (**NS**) $\overline{\Omega} = (\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ is known to be a **NIVAFLS** of $\mathbf{V_S}$, if for all $v_1, v_2 \in \mathbf{V_S}$ and $a, b \in \mathbf{F}$, the following holds.

$$(i)\ \overline{\xi}_{\overline{\Omega}}(av_1 * bv_2) \le \max\{\overline{\xi}_{\overline{\Omega}}(v_1), \overline{\xi}_{\overline{\Omega}}(v_2)\},$$

$$(ii)\ \overline{\Psi}_{\overline{\Omega}}(av_1 * bv_2) \le \max\{\overline{\Psi}_{\overline{\Omega}}(v_1), \overline{\Psi}_{\overline{\Omega}}(v_2)\},$$

$$(iii)\ \overline{\zeta}_{\overline{\Omega}}(av_1 * bv_2) \ge \min\{\overline{\zeta}_{\overline{\Omega}}(v_1), \overline{\zeta}_{\overline{\Omega}}(v_2)\}$$

Example 3.2. Let $\mathbf{V_S} = \mathbf{R}^2$ be a crisp linear space over a field $\mathbf{R}$ and let **NS** $\overline{\Omega}$= $(\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ be a **NIVAFLS**. For each $v = (v_1, v_2) \in \mathbf{R}^2$, mappings $\overline{\xi}_{\overline{\Omega}} : \mathbf{V_S} \to \mathbf{D}[0,1]$, $\overline{\Psi}_{\overline{\Omega}} : \mathbf{V_S} \to \mathbf{D}[0,1]$ and $\overline{\zeta}_{\overline{\Omega}} : \mathbf{V_S} \to \mathbf{D}[0,1]$ are defined by

$$\overline{\xi}_{\overline{\Omega}}(v) = \begin{cases} [0.8,\ 0.9], & if\ \ v_1 = 0\ or\ v_2 = 0, \\ [0.28,\ 0.31], & otherwise. \end{cases}$$

$$\overline{\Psi}_{\overline{\Omega}}(v) = \begin{cases} [0.73,\ 0.82], & if\ \ v_1 = 0\ or\ v_2 = 0, \\ [0.32,\ 0.41], & otherwise. \end{cases}$$

and

$$\overline{\zeta}_{\overline{\Omega}}(v) = \begin{cases} [0.51,\ 0.6], & if\ \ v_1 = 0\ or\ v_2 = 0, \\ [0.93,\ 1], & otherwise. \end{cases}$$

Clearly, $\overline{\Omega} = (\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ is a **NIVAFLS** in $\mathbf{V_S}$.

Example 3.3. Consider a Klein 4-group $\mathbf{V_S} = \{\epsilon s_1, \epsilon s_2, \epsilon s_3, \epsilon s_4\}$ with the binary operation $*$.

$*$	ϵs_1	ϵs_2	ϵs_3	ϵs_4
ϵs_1	ϵs_1	ϵs_2	ϵs_3	ϵs_4
ϵs_2	ϵs_2	ϵs_1	ϵs_4	ϵs_3
ϵs_3	ϵs_3	ϵs_4	ϵs_1	ϵs_2
ϵs_4	ϵs_4	ϵs_3	ϵs_2	ϵs_1

Assume $\mathbf{F}$ to be a GF(2). Suppose that $(0)w = e$, $(1)w = w$ for all $w \in \mathbf{V_S}$.

Define the mappings $\overline{\xi}_{\overline{\Omega}} : \mathbf{V_S} \to \mathbf{D}[0,1]$, $\overline{\Psi}_{\overline{\Omega}} : \mathbf{V_S} \to \mathbf{D}[0,1]$ and $\overline{\zeta}_{\overline{\Omega}} : \mathbf{V_S} \to \mathbf{D}[0,1]$ by

$$\overline{\xi}_{\overline{\Omega}}(v) = \begin{cases} [0.4,\ 0.5], & if \ \ v = \epsilon s_1, \\ [0.91,\ 1], & otherwise. \end{cases}$$

$$\overline{\Psi}_{\overline{\Omega}}(v) = \begin{cases} [0.22,\ 0.31], & if \ \ v = \epsilon s_1, \\ [0.72,\ 0.9], & otherwise. \end{cases}$$

and

$$\overline{\zeta}_{\overline{\Omega}}(v) = \begin{cases} [0.8,\ 0.9], & if \ \ v = \epsilon s_1, \\ [0.5,\ 0.42], & otherwise. \end{cases}$$

Note that $\overline{\Omega} = (\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ is a **NIVAFLS** in $\mathbf{V_S}$.

Theorem 3.4. *If $\overline{\Omega}_1$ and $\overline{\Omega}_2$ are **NIVAFLSs** of $\mathbf{V_S}$, then the union $\overline{\Omega}_1 \cup \overline{\Omega}_2$ so is.*

Proof. Let v_1 and $v_2 \in \mathbf{V_S}$ and $a, b \in \mathbf{F}$.

Define $\overline{\Omega}_1 \cup \overline{\Omega}_2 = \{\langle \mathrm{v}, \overline{\xi}_{\overline{\Omega}_1 \cup \overline{\Omega}_2}(v), \Psi_{\overline{\Omega}_1 \cup \overline{\Omega}_2}(v), \zeta_{\overline{\Omega}_1 \cup \overline{\Omega}_2}(v)\rangle : v \in \mathbf{V_S}\}$.

Now $(\overline{\xi}_{\overline{\Omega}_1 \cup \overline{\Omega}_2})(av_1 * bv_2) = \max\{\overline{\xi}_{\overline{\Omega}_1}(av_1 * bv_2), \overline{\xi}_{\overline{\Omega}_2}(av_1 * bv_2)\}$

$$\leq \max\{\max[\overline{\xi}_{\overline{\Omega}_1}(v_1), \overline{\xi}_{\overline{\Omega}_1}(v_2)], \max[\overline{\xi}_{\overline{\Omega}_2}(v_1), \overline{\xi}_{\overline{\Omega}_2}(v_2)]\}$$

$$= \max\{\max[\overline{\xi}_{\overline{\Omega}_1}(v_1), \overline{\xi}_{\overline{\Omega}_2}(v_1)], \max[\overline{\xi}_{\overline{\Omega}_1}(v_2), \overline{\xi}_{\overline{\Omega}_2}(v_2)]\}$$

$$\Rightarrow (\overline{\xi}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(av_1 * bv_2) \leq \max\{(\overline{\xi}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(v_1), (\overline{\xi}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(v_2)\}$$

$$(\overline{\Psi}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(av_1 * bv_2) = \max\{\overline{\Psi}_{\overline{\Omega}_1}(av_1 * bv_2), \overline{\Psi}_{\overline{\Omega}_2}(av_1 * bv_2)\}$$

$$\leq \max\{\max[\overline{\Psi}_{\overline{\Omega}_1}(v_1), \overline{\Psi}_{\overline{\Omega}_1}(v_2)], \max[\overline{\Psi}_{\overline{\Omega}_2}(v_1), \overline{\Psi}_{\overline{\Omega}_2}(v_2)]\}$$

$$= \max\{\max[\overline{\Psi}_{\overline{\Omega}_1}(v_1), \overline{\Psi}_{\overline{\Omega}_2}(v_1)], \max[\overline{\Psi}_{\overline{\Omega}_1}(v_2), \overline{\Psi}_{\overline{\Omega}_2}(v_2)]\}$$

$$\Rightarrow (\overline{\Psi}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(av_1 * bv_2) \leq \max\{(\overline{\Psi}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(v_1), (\overline{\Psi}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(v_2)\}$$

$$(\overline{\zeta}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(av_1 * bv_2) = \min\{\overline{\zeta}_{\overline{\Omega}_1}(av_1 * bv_2), \overline{\zeta}_{\overline{\Omega}_2}(av_1 * bv_2)\}$$

$$\geq \text{mim}\{\min[\overline{\zeta}_{\overline{\Omega}_1}(v_1), \overline{\zeta}_{\overline{\Omega}_1}(v_2)], \min[\overline{\zeta}_{\overline{\Omega}_2}(v_1), \overline{\zeta}_{\overline{\Omega}_2}(v_2)]\}$$

$$= \min\{\min[\overline{\zeta}_{\overline{\Omega}_1}(v_1), \overline{\zeta}_{\overline{\Omega}_2}(v_1)], \min[\overline{\zeta}_{\overline{\Omega}_1}(v_2), \overline{\zeta}_{\overline{\Omega}_2}(v_2)]\}$$

$$\Rightarrow (\overline{\zeta}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(av_1 * bv_2) \geq \min\{(\overline{\zeta}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(v_1), (\overline{\zeta}_{\overline{\Omega}_1\cup\overline{\Omega}_2})(v_2)\}$$

Thus $(\overline{\Omega}_1 \cup \overline{\Omega}_2)$ is a **NIVAFLS** of $\mathbf{V_S}$.□

Remark 3.5. The intersection of two **(NIVAFLSs)** of $\mathbf{V_S}$ need not be a **(NIVAFLS)** of $\mathbf{V_S}$.

Proof. An example will be used to demonstrate the aforementioned claim.

Let $\mathbf{V_S} = \{\epsilon s_1, \epsilon s_2, \epsilon s_3, \epsilon s_4\}$ be the Klein 4-group as in Example 3.3.

Let $\mathbf{F}$ be the field GF(2). Let $(0)w = e$, $(1)w = w$ for all $w \in \mathbf{V_S}$. Then $\mathbf{V_S}$ is a linear space over $\mathbf{F}$.

Define $\overline{\xi}_{\overline{\Omega}_1}$ and $\overline{\xi}_{\overline{\Omega}_2}$ as follows:

$\overline{\xi}_{\overline{\Omega}_1}(\epsilon s_1) = [0.1, 0.2]$, $\overline{\xi}_{\overline{\Omega}_1}(\epsilon s_2) = [0.6, 0.7] = \overline{\xi}_{\overline{\Omega}_1}(\epsilon s_3)$, $\overline{\xi}_{\overline{\Omega}_1}(\epsilon s_4) = [0.4, 0.5]$ and

$\overline{\xi}_{\overline{\Omega}_2}(\epsilon s_1) = [0.2, 0.3]$, $\overline{\xi}_{\overline{\Omega}_2}(\epsilon s_2) = [0.3, 0.4]$, $\overline{\xi}_{\overline{\Omega}_2}(\epsilon s_3) = [0.5, 0.6] = \overline{\xi}_{\overline{\Omega}_2}(\epsilon s_4)$.

Define $\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2}$ by $(\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(v) = \min\{\overline{\xi}_{\overline{\Omega}_1}(v), \overline{\xi}_{\overline{\Omega}_2}(v)\}$ for all $v \in \mathbf{V_S}$.

So, $(\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(\epsilon s_1) = [0.1, 0.2]$, $(\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(\epsilon s_2) = [0.3, 0.4]$,

$(\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(\epsilon s_3) = [0.5, 0.6]$, $(\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(\epsilon s_4) = [0.4, 0.5]$.

When $a = b = 1$, then the Definition 3.1 in (i) becomes

$(\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(\epsilon s_2 * \epsilon s_4) \leq \max\{(\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(\epsilon s_2), (\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(\epsilon s_4)\}$

$\Rightarrow (\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(\epsilon s_3) \leq \max\{[0.3, 0.4], [0.4, 0.5]\}$

But $(\overline{\xi}_{\overline{\Omega}_1 \cap \overline{\Omega}_2})(\epsilon s_3) = [0.5, 0.6] \leq [0.4, 0.5]$

This is absurd.

The other inequalities are similarly proved.

So, the intersection of two **NIVAFLSs** need not be a **NIVAFLS**. □

Definition 3.6. The complement of a Neutrosophic fuzzy subset $\overline{\Omega}$ is denoted by $\overline{\Omega}^c$ and is defined by $\overline{\Omega}^c = \{\langle \text{v}, \overline{\xi}_{\overline{\Omega}^c}(v), \overline{\Psi}_{\overline{\Omega}^c}(v), \overline{\zeta}_{\overline{\Omega}^c}(v)\rangle : v \in \mathbf{V_S}\}$, where $\overline{\xi}_{\overline{\Omega}^c}(v) = \overline{\zeta}_{\overline{\Omega}}(v)$, $\overline{\Psi}_{\overline{\Omega}^c}(v) = 1 - \overline{\Psi}_{\overline{\Omega}}(v)$, $\overline{\zeta}_{\overline{\Omega}^c}(v) = \overline{\xi}_{\overline{\Omega}}(v)$, for all $v \in \mathbf{V_S}$.

Theorem 3.7. *If $\overline{\Omega}$ be a Neutrosophic fuzzy linear space of $\boldsymbol{V_S}$ then its complement $\overline{\Omega}^c$ is a **NIVAFLS** of $\boldsymbol{V_S}$.*

Proof. Let v_1 and $v_2 \in \mathbf{V_S}$ and $a, b \in \mathbf{F}$.

$$\overline{\xi}_{\overline{\Omega}^c}(av_1 * bv_2) = \overline{\zeta}_{\overline{\Omega}}(av_1 * bv_2)$$

$$\leq \max\{\overline{\zeta}_{\overline{\Omega}}(v_1), \overline{\zeta}_{\overline{\Omega}}(v_2)\}$$

$$= \max\{\overline{\xi}_{\overline{\Omega}^c}(v_1), \xi_{\overline{\Omega}^c}(v_2)\}$$

$$\overline{\Psi}_{\overline{\Omega}^c}(av_1 * bv_2) = 1 - \overline{\Psi}_{\overline{\Omega}}(av_1 * bv_2)$$

$$= 1- \leq \min\{\overline{\Psi}_{\overline{\Omega}}(v_1), \overline{\Psi}_{\overline{\Omega}}(v_2)\}$$

$$= \max\{1 - \overline{\Psi}_{\overline{\Omega}}(v_1), 1 - \overline{\Psi}_{\overline{\Omega}}(v_2)\}$$

$$= \max\{\overline{\Psi}_{\overline{\Omega}^c}(v_1), \overline{\Psi}_{\overline{\Omega}^c}(v_2)\}$$

$$\overline{\zeta}_{\overline{\Omega}^c}(av_1 * bv_2) = \overline{\xi}_{\overline{\Omega}}(av_1 * bv_2)$$

$$\geq \min\{\overline{\xi}_{\overline{\Omega}}(v_1), \overline{\xi}_{\overline{\Omega}}(v_2)\}$$

$$\geq \min\{\overline{\zeta}_{\overline{\Omega}^c}(v_1), \overline{\zeta}_{\overline{\Omega}^c}(v_2)\}$$

So, $\overline{\Omega}^c$ is a **NIVAFLS** of $\mathbf{V_S}$. □

Definition 3.8. Let $\overline{\Omega}_1$, $\overline{\Omega}_2$ be Neutrosophic anti fuzzy subsets of $\mathbf{V_{S1}}$ and $\mathbf{V_{S2}}$ respectively. Then the cartesian product of $\overline{\Omega}_1$ and $\overline{\Omega}_2$ denoted by $\overline{\Omega}_1 \times \overline{\Omega}_2$ is defined by
$\overline{\Omega}_1 \times \overline{\Omega}_2 = \{\langle (v_1 \times v_2), \xi_{\overline{\Omega}_1 \times \overline{\Omega}_2}(v_1, v_2), \overline{\Psi}_{\overline{\Omega}_1 \times \overline{\Omega}_2}(v_1, v_2), \overline{\zeta}_{\overline{\Omega}_1 \times \overline{\Omega}_2}(v_1, v_2)\rangle : v_1 \in \mathbf{V_{S1}}, v_2 \in \mathbf{V_{S2}}\}$, where $\xi_{\overline{\Omega}_1 \times \overline{\Omega}_2}(v_1, v_2) = \max\{\overline{\xi}_{\overline{\Omega}_1}(v_1), \overline{\xi}_{\overline{\Omega}_2}(v_2)\}$, $\overline{\Psi}_{\overline{\Omega}_1 \times \overline{\Omega}_2}(v_1, v_2) = \max\{\overline{\Psi}_{\overline{\Omega}_1}(v_1), \overline{\Psi}_{\overline{\Omega}_2}(v_2)\}$ and $\overline{\zeta}_{\overline{\Omega}_1 \times \overline{\Omega}_2}(v_1, v_2) = \min\{\overline{\zeta}_{\overline{\Omega}_1}(v_1), \overline{\zeta}_{\overline{\Omega}_2}(v_2)\}$.

Theorem 3.9. *If $\overline{\Omega}_1$ and $\overline{\Omega}_2$ are **NIVAFLSs** of $\boldsymbol{V_S}$, then $(\overline{\Omega}_1 \times \overline{\Omega}_2)$ is a **NIVAFLS** of $\boldsymbol{V_{S1}} \times \boldsymbol{V_{S2}}$.*

Proof. Let $v = (v_1, v_2)$, $w = (w_1, w_2) \in \mathbf{V_{S1}} \times \mathbf{V_{S2}}$. Then

$$\overline{\xi}_{\overline{\Omega}_1 \times \overline{\Omega}_2}(av * bw) = \overline{\xi}_{\overline{\Omega}_1 \times \overline{\Omega}_2}(a(v_1, v_2) * b(w_1, w_2))$$

$$= \overline{\xi}_{\overline{\Omega}_1 \times \overline{\Omega}_2}((av_1 * bw_1), (av_2 * bw_2))$$

$$= \max\{\overline{\xi}_{\overline{\Omega}_1}(av_1 * bw_1), \overline{\xi}_{\overline{\Omega}_2}(av_2 * bw_2)\}$$

$$\leq \max\{\max[\overline{\xi}_{\overline{\Omega}_1}(v_1), \overline{\xi}_{\overline{\Omega}_1}(w_1)], \max[\overline{\xi}_{\overline{\Omega}_2}(v_2), \overline{\xi}_{\overline{\Omega}_2}(w_2)]\}$$

$$= \max\{\max[\overline{\xi}_{\overline{\Omega}_1}(v_1), \overline{\xi}_{\overline{\Omega}_2}(v_2)], \max[\overline{\xi}_{\overline{\Omega}_1}(w_1), \overline{\xi}_{\overline{\Omega}_2}(w_2)]\}$$

$$= \max\{\overline{\xi}_{\overline{\Omega}_1 \times \overline{\Omega}_2}(v_1, v_2), \overline{\xi}_{\overline{\Omega}_1 \times \overline{\Omega}_2}(w_1, w_2)\}$$

$$= \max\{\overline{\xi}_{\overline{\Omega}_1\times\overline{\Omega}_2}(v), \overline{\xi}_{\overline{\Omega}_1\times\overline{\Omega}_2}(w)\}$$

$$\overline{\Psi}_{\overline{\Omega}_1\times\overline{\Omega}_2}(av*bw) = \overline{\Psi}_{\overline{\Omega}_1\times\overline{\Omega}_2}(a(v_1,v_2)*b(w_1,w_2))$$

$$= \overline{\Psi}_{\overline{\Omega}_1\times\overline{\Omega}_2}((av_1*bw_1),(av_2*bw_2))$$

$$= \max\{\overline{\Psi}_{\overline{\Omega}_1}(av_1*bw_1), \overline{\Psi}_{\overline{\Omega}_2}(av_2*bw_2)\}$$

$$\leq \max\{\max[\overline{\Psi}_{\overline{\Omega}_1}(v_1), \overline{\Psi}_{\overline{\Omega}_1}(w_1)], \max[\overline{\Psi}_{\overline{\Omega}_2}(v_2), \overline{\Psi}_{\overline{\Omega}_2}(w_2)]\}$$

$$= \max\{\max[\overline{\Psi}_{\overline{\Omega}_1}(v_1), \overline{\Psi}_{\overline{\Omega}_2}(v_2)], \max[\overline{\Psi}_{\overline{\Omega}_1}(w_1), \overline{\Psi}_{\overline{\Omega}_2}(w_2)]\}$$

$$= \max\{\overline{\Psi}_{\overline{\Omega}_1\times\overline{\Omega}_2}(v_1,v_2), \overline{\Psi}_{\overline{\Omega}_1\times\overline{\Omega}_2}(w_1,w_2)\}$$

$$= \max\{\overline{\Psi}_{\overline{\Omega}_1\times\overline{\Omega}_2}(v), \overline{\Psi}_{\overline{\Omega}_1\times\overline{\Omega}_2}(w)\}$$

$$\overline{\zeta}_{\overline{\Omega}_1\times\overline{\Omega}_2}(av*bw) = \overline{\zeta}_{\overline{\Omega}_1\times\overline{\Omega}_2}(a(v_1,v_2)*b(w_1,w_2))$$

$$= \overline{\zeta}_{\overline{\Omega}_1\times\overline{\Omega}_2}((av_1*bw_1),(av_2*bw_2))$$

$$= \min\{\overline{\zeta}_{\overline{\Omega}_1}(av_1*bw_1), \overline{\zeta}_{\overline{\Omega}_2}(av_2*bw_2)\}$$

$$\geq \min\{\min[\overline{\zeta}_{\overline{\Omega}_1}(v_1), \overline{\zeta}_{\overline{\Omega}_1}(w_1)], \min[\overline{\zeta}_{\overline{\Omega}_2}(v_2), \overline{\zeta}_{\overline{\Omega}_2}(w_2)]\}$$

$$= \min\{\min[\overline{\zeta}_{\overline{\Omega}_1}(v_1), \overline{\zeta}_{\overline{\Omega}_2}(v_2)], \min[\overline{\zeta}_{\overline{\Omega}_1}(w_1), \overline{\zeta}_{\overline{\Omega}_2}(w_2)]\}$$

$$= \min\{\overline{\zeta}_{\overline{\Omega}_1\times\overline{\Omega}_2}(v_1,v_2), \overline{\zeta}_{\overline{\Omega}_1\times\overline{\Omega}_2}(w_1,w_2)\}$$

$$= \min\{\overline{\zeta}_{\overline{\Omega}_1\times\overline{\Omega}_2}(v), \overline{\zeta}_{\overline{\Omega}_1\times\overline{\Omega}_2}(w)\}$$

So, $(\overline{\Omega}_1 \times \overline{\Omega}_2)$ is a **NIVAFLS** of $\mathbf{V_{S1}} \times \mathbf{V_{S2}}$. □

Definition 3.10. Let $\varpi : \mathbf{V_{S1}} \to \mathbf{V_{S2}}$ be a mapping of linear spaces of $\mathbf{V_S}$ over $\mathbf{F}$. If $\overline{\Omega} = (\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ is a **NIVAFLS** of $\mathbf{V_{S2}}$ over $\mathbf{F}$, then the inverse image of $\overline{\Omega} = (\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ under ϖ, denoted by $\varpi^{-1}(\overline{\Omega}) = (\varpi^{-1}(\overline{\xi}_{\overline{\Omega}}), \varpi^{-1}(\overline{\Psi}_{\overline{\Omega}}), \varpi^{-1}(\overline{\zeta}_{\overline{\Omega}}))$, is a **NIVAFLS** of $\mathbf{V_{S1}}$, defined by $\varpi^{-1}(\overline{\Omega})(x) = \overline{\Omega}(\varpi(x)) = (\overline{\xi}_{\overline{\Omega}}(\varpi(x)), \overline{\Psi}_{\overline{\Omega}}(\varpi(x)), \overline{\zeta}_{\overline{\Omega}}(\varpi(x)))$ for all $x \in \mathbf{V_{S1}}$.

Theorem 3.11. *Consider the homomorphism $\varpi : \boldsymbol{V_S}_1 \to \boldsymbol{V_S}_2$, which represents the mapping between linear spaces $\boldsymbol{V_S F}$. If $\overline{\Omega} = (\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ is a **NIVAFLS** of $\boldsymbol{V_S}_2$, then $\varpi^{-1}(\overline{\Omega})(x) = \overline{\Omega}(\varpi(x)) = (\overline{\xi}_{\overline{\Omega}}(\varpi(x)), \overline{\Psi}_{\overline{\Omega}}(\varpi(x)), \overline{\zeta}_{\overline{\Omega}}(\varpi(x)))$ for every x that belongs to $\boldsymbol{V_S}_1$.*

Proof. Suppose that $\overline{\Omega} = (\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ is a **NIVAFLS** of $\mathbf{V_{S2}}$ and x and y belong to $\mathbf{V_{S1}}$ and a and b belong to $\mathbf{F}$.
Next, we have

$$\begin{aligned}
\text{(i)}\varpi^{-1}(\overline{\xi}_{\overline{\Omega}})(ax * by) &= \overline{\xi}_{\overline{\Omega}}(\varpi(ax * by)) \\
&= \overline{\xi}_{\overline{\Omega}}(\varpi(x)\varpi(y)) \text{ (since } \varpi \text{ is homomorphism)} \\
&\leq \max\{\overline{\xi}_{\overline{\Omega}}(\varpi(x)), \overline{\xi}_{\overline{\Omega}}(\varpi(y))\} \\
&= \max\{\varpi^{-1}(\overline{\xi}_{\overline{\Omega}}(x)),\ \varpi^{-1}(\overline{\xi}_{\overline{\Omega}}(y))\} \\
\Rightarrow \varpi^{-1}(\overline{\xi}_{\overline{\Omega}})(ax * by) &\leq \max\{\varpi^{-1}(\overline{\xi}_{\overline{\Omega}}(x)),\ \varpi^{-1}(\overline{\xi}_{\overline{\Omega}}(y))\}
\end{aligned}$$

Therefore $\varpi^{-1}(\overline{\xi}_{\overline{\Omega}})$ is a **NIVAFLS** of $\mathbf{V_{S1}}$.

$$\begin{aligned}
\text{(ii)}\varpi^{-1}(\overline{\Psi}_{\overline{\Omega}})(ax * by) &= \overline{\Psi}_{\overline{\Omega}}(\varpi(ax * by)) \\
&= \overline{\Psi}_{\overline{\Omega}}(\varpi(x)\varpi(y)) \text{ (since } \varpi \text{ is homomorphism)} \\
&\leq \max\{\overline{\Psi}_{\overline{\Omega}}(\varpi(x)), \overline{\Psi}_{\overline{\Omega}}(\varpi(y))\} \\
&= \max\{\varpi^{-1}(\overline{\Psi}_{\overline{\Omega}}(x)),\ \varpi^{-1}(\overline{\Psi}_{\overline{\Omega}}(y))\} \\
\Rightarrow \varpi^{-1}(\overline{\Psi}_{\overline{\Omega}})(ax * by) &\leq \max\{\varpi^{-1}(\overline{\Psi}_{\overline{\Omega}}(x)),\ \varpi^{-1}(\overline{\Psi}_{\overline{\Omega}}(y))\}
\end{aligned}$$

Therefore $\varpi^{-1}(\overline{\Psi}_{\overline{\Omega}})$ is a **NIVAFLS** of $\mathbf{V_{S1}}$.

$$\begin{aligned}
\text{(iii)}\varpi^{-1}(\overline{\zeta}_{\overline{\Omega}})(ax * by) &= \overline{\zeta}_{\overline{\Omega}}(\varpi(ax * by)) \\
&= \overline{\zeta}_{\overline{\Omega}}(\varpi(x)\varpi(y)) \text{ (since } \varpi \text{ is homomorphism)} \\
&\geq \min\{\overline{\zeta}_{\overline{\Omega}}(\varpi(x)), \overline{\zeta}_{\overline{\Omega}}(\varpi(y))\} \\
&= \min\{\varpi^{-1}(\overline{\zeta}_{\overline{\Omega}}(x)),\ \varpi^{-1}(\overline{\zeta}_{\overline{\Omega}}(y))\} \\
\Rightarrow \varpi^{-1}(\overline{\zeta}_{\overline{\Omega}})(ax * by) &\geq \min\{\varpi^{-1}(\overline{\zeta}_{\overline{\Omega}}(x)),\ \varpi^{-1}(\overline{\zeta}_{\overline{\Omega}}(y))\}
\end{aligned}$$

Therefore $\varpi^{-1}(\overline{\zeta}_{\overline{\Omega}})$ is a **NIVAFLS** of $\mathbf{V_{S1}}$. □

Theorem 3.12. *Let $\overline{\Omega} = (\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ be a **NIVAFLS** of $\boldsymbol{V_S}$ and an onto homomorphism $\varpi : \boldsymbol{V_S} \to \boldsymbol{V_S}$. Subsequently, the mapping $\overline{\Omega}^{\varpi} : \boldsymbol{V_S} \to \boldsymbol{D}[0, 1]$ is defined as follows: for every $x \in \overline{\Omega}^{\varpi}(x) = \overline{\Omega}(\varpi(x))$ for all $x \in \boldsymbol{V_S}$ is a **NIVAFLS** of $\boldsymbol{V_S}$.*

Proof. For any $x, y \in \mathbf{V_S}$ and $a, b \in \mathbf{F}$.

$$\begin{aligned}
\text{(i)}\overline{\xi}^{\varpi}_{\overline{\Omega}}(ax * by) &= \overline{\xi}_{\overline{\Omega}}(\varpi(ax * by)) \\
&= \overline{\xi}_{\overline{\Omega}}(\varpi(x)\varpi(y)) \text{ (since } \varpi \text{ is homomorphism)} \\
&\leq \max\{\overline{\xi}_{\overline{\Omega}}(\varpi(x)), \overline{\xi}_{\overline{\Omega}}(\varpi(y))\} \\
&= \max\{\overline{\xi}^{\varpi}_{\overline{\Omega}}(x), \overline{\xi}^{\varpi}_{\overline{\Omega}}(y)\} \\
\Rightarrow \overline{\xi}^{\varpi}_{\overline{\Omega}}(ax * by) &\leq \max\{\overline{\xi}^{\varpi}_{\overline{\Omega}}(x), \overline{\xi}^{\varpi}_{\overline{\Omega}}(y)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(ii)}\overline{\Psi}^{\varpi}_{\overline{\Omega}}(ax * by) &= \overline{\Psi}_{\overline{\Omega}}(\varpi(ax * by)) \\
&= \overline{\Psi}_{\overline{\Omega}}(\varpi(x)\varpi(y)) \text{ (since } \varpi \text{ is homomorphism)} \\
&\leq \max\{\overline{\Psi}_{\overline{\Omega}}(\varpi(x)), \overline{\Psi}_{\overline{\Omega}}(\varpi(y))\} \\
&= \max\{\overline{\Psi}^{\varpi}_{\overline{\Omega}}(x), \overline{\Psi}^{\varpi}_{\overline{\Omega}}(y)\} \\
\Rightarrow \overline{\Psi}^{\varpi}_{\overline{\Omega}}(ax * by) &\leq \max\{\overline{\Psi}^{\varpi}_{\overline{\Omega}}(x), \overline{\Psi}^{\varpi}_{\overline{\Omega}}(y)\}.
\end{aligned}$$

$$\begin{aligned}
\text{(iii)}\overline{\zeta}^{\varpi}_{\overline{\Omega}}(ax * by) &= \overline{\zeta}_{\overline{\Omega}}(\varpi(ax * by)) \\
&= \overline{\zeta}_{\overline{\Omega}}(\varpi(x)\varpi(y)) \text{ (since } \varpi \text{ is homomorphism)} \\
&\leq \max\{\overline{\zeta}_{\overline{\Omega}}(\varpi(x)), \overline{\zeta}_{\overline{\Omega}}(\varpi(y))\} \\
&= \max\{\overline{\zeta}^{\varpi}_{\overline{\Omega}}(x), \overline{\zeta}^{\varpi}_{\overline{\Omega}}(y)\} \\
\Rightarrow \overline{\zeta}^{\varpi}_{\overline{\Omega}}(ax * by) &\leq \max\{\overline{\zeta}^{\varpi}_{\overline{\Omega}}(x), \overline{\zeta}^{\varpi}_{\overline{\Omega}}(y)\}.
\end{aligned}$$

So, $\overline{\Omega}^{\varpi}$ is a **NIVAFLS** of $\mathbf{V_S}$. □

Theorem 3.13. *Consider an epimorphism $\varpi : \boldsymbol{V_{S1}} \to \boldsymbol{V_{S2}}$ that maps linear spaces $\boldsymbol{V_{S1}}$ and $\boldsymbol{V_{S2}}$ over $\boldsymbol{F}$. Let's assume that$\overline{\Omega} = (\overline{\xi}_{\overline{\Omega}}, \overline{\Psi}_{\overline{\Omega}}, \overline{\zeta}_{\overline{\Omega}})$ be a ϖ-invariant **NIVAFLS** of $\boldsymbol{V_{S1}}$. Consequently, $\varpi(\overline{\Omega})$ is a **NIVAFLS** of $\boldsymbol{V_{S2}}$.*

Proof. For any elements x' and y' belonging to $\mathbf{V_{S2}}$ and a and b belonging to $\mathbf{F}$, there exists x and y belonging to $\mathbf{V_{S1}}$ such that $\varpi(x)$ is equals x' and $\varpi(y)$ equals y'.

Also $ax' * by' = \varpi(ax * by)$. Since $\overline{\Omega}$ is ϖ-invariant,

$$\begin{aligned}
\text{(i)}\varpi(\overline{\xi}_{\overline{\Omega}})(ax * by) &= \overline{\xi}_{\overline{\Omega}}(ax' * by') \leq \max\{\overline{\xi}_{\overline{\Omega}}(x'), \overline{\xi}_{\overline{\Omega}}(y')\} \\
&= \max\{\varpi(\overline{\xi}_{\overline{\Omega}})(x),\ \varpi(\overline{\xi}_{\overline{\Omega}})(y)\} \\
\Rightarrow \varpi(\overline{\xi}_{\overline{\Omega}})(ax * by) &\leq \max\{\varpi(\overline{\xi}_{\overline{\Omega}})(x), \varpi(\overline{\xi}_{\overline{\Omega}})(y)\}
\end{aligned}$$

Therefore $\varpi(\bar{\xi}_{\overline{\Omega}})$ is a **NIVAFLS** of $\mathbf{V_{S2}}$.

$$\begin{aligned}(\text{ii})\varpi(\overline{\Psi}_{\overline{\Omega}})(ax*by) &= \overline{\Psi}_{\overline{\Omega}}(ax^{'}*by^{'}) \leq \max\{\overline{\Psi}_{\overline{\Omega}}(x^{'}),\overline{\Psi}_{\overline{\Omega}}(y^{'})\} \\ &= \max\{\varpi(\overline{\Psi}_{\overline{\Omega}})(x),\ \varpi(\overline{\Psi}_{\overline{\Omega}})(y)\} \\ \Rightarrow \varpi(\overline{\Psi}_{\overline{\Omega}})(ax*by) &\leq \max\{\varpi(\overline{\Psi}_{\overline{\Omega}})(x),\varpi(\overline{\Psi}_{\overline{\Omega}})(y)\}\end{aligned}$$

Therefore $\varpi(\overline{\Psi}_{\overline{\Omega}})$ is a **NIVAFLS** of $\mathbf{V_{S2}}$.

$$\begin{aligned}(iii)\varpi(\bar{\zeta}_{\overline{\Omega}})(ax*by) &= \bar{\zeta}_{\overline{\Omega}}(ax^{'}*by^{'}) \geq \min\{\bar{\zeta}_{\overline{\Omega}}(x^{'}),\bar{\zeta}_{\overline{\Omega}}(y^{'})\} \\ &= \min\{\varpi(\bar{\zeta}_{\overline{\Omega}})(x),\ \varpi(\bar{\zeta}_{\overline{\Omega}})(y)\} \\ \Rightarrow \varpi(\bar{\zeta}_{\overline{\Omega}})(ax*by) &\geq \min\{\varpi(\bar{\zeta}_{\overline{\Omega}})(x),\varpi(\bar{\zeta}_{\overline{\Omega}})(y)\}\end{aligned}$$

Therefore $\varpi(\bar{\zeta}_{\overline{\Omega}})$ is a **NIVAFLS** of $\mathbf{V_{S2}}$. □

4. Conclusion

The present paper introduces a novel concept referred to as a **NIVAFLS**. A counterexample is employed to demonstrate that the intersection of two **NIVAFLSs** need not be a **NIVAFLS**, and we examine into some of the aspects of **NIVAFLS** to show that the union of two **NIVAFLSs** is likewise a **NIVAFLS**.

In the future, we will apply this idea to different algebraic structures like

- semigroup,
- M-semigroup,
- ring,
- rough set,
- soft set together with problems based on decision-makings.

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