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Foundation of SuperHyperStructure

&

Neutrosophic SuperHyperStructure

(review paper)

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Abstract

In this paper we extend the SuperHyperAlgebra, SuperHyperGraph, SuperHyperTopology, SuperHyperSoft Set, endowed with SuperHyperOperations, SuperHyperAxioms, and SuperHyperFunctions, to the most general form of structure, from our real world, called SuperHyperStructure in any field of knowledge. A practical application of the SuperHyperStructure is presented at the end.

The prefix "Hyper" [Marty, 1934] stand for the codomain of the functions and operations to be *P(H)*, or the powerset of the set *H*. While the prefix "Super" [Smarandache, 2016] stands for using the $P^n(H)$, $n \geq 2$, or the *n-th* PowerSet of the Set *H* {because the *set* (or *system*) *H* (that may be a set of items, a company, institution, country, region, etc.) is organized in *sub-systems*, which in their turn are organized in *sub-sub-systems*, and so on} in the domain and/or codomain of the functions and operations.

Keywords: *n-th* PowerSet of a Set, SuperHyperAlgebra, SuperHyperGraph, SuperHyperTopology, SuperHyperSoft Set, SuperHyperOperations, SuperHyperAxioms, SuperHyperFunctions, HyperStructure, SuperHyperStructure, Neutrosophic SuperHyperStructure

1. From Classical Structure and HyperStructure to SuperHyperStructure

We present below the evolution of structures in all fields of knowledge:
Classical Structure, HyperStructure, SuperHyperStructure (none having indeterminacy);

and

Neutrosophic Classical Structure, Neutrosophic HyperStructure, Neutrosophic SuperHyperStructure (all having some indeterminacy, as in our everyday life).

2. Classical Structure

A **Classical Structure** is built on a non-empty set H, whose **Classical Operations** ($\#_0$) are defined as:

$\#_0 : H^m \rightarrow H$, for integer $m \geq 1$,

and with **Classical Axioms** acting on it.

3. HyperStructure

A **HyperStructure** (Marty [1], 1934) is built on a non-empty set H, whose **HyperOperations** ($\#_{H0}$) are defined as:

$\#_{H0} : H^m \rightarrow P \cdot (H)$, where $P \cdot (H)$ is the set of all non-emtpy subsets of H,

and with **HyperAxioms** acting on it.

4. Neutrosophic HyperStructure

As an extension of the HyperStructure, the **Neutrosophic HyperStructure** (Smarandache [2], 2016) is built on a non-empty set H, whose **Neutrosophic HyperOperations** ($\#_{NS0}$) are defined as:

$\#_{NS0} : H^m \rightarrow P(H)$, where $P(H)$ is the set of all non-emtpy and empty subsets of H,

and the axioms acting on it are called **Neutrosophic HyperAxioms**.

5. Definition of the n[th]-PowerSet $\boldsymbol{P_\star^n(H)}$ *without* Indeterminacy (no empty-set)

The n[th]-PowerSet $\boldsymbol{P_\star^n(H)}$ [2] of the set H, that the SuperHyperStructure is built on, describes a world that does not contain indeterminacy, where similarly the *set* (or *system*) H (that may be a set of items, a company, institution, country, region, etc.) is organized in sub-systems, which in turn are organized in sub-sub-systems, and so on.

The n[th]-PowerSet $\boldsymbol{P_*^n(H)}$ is also defined recursively:

$$\boldsymbol{P_\star^0(H)} \stackrel{\text{def}}{=} H$$

$$\boldsymbol{P_\star^1(H)} = P_\star(\boldsymbol{H})$$

$$\boldsymbol{P_\star^2(H)} = P_\star(P_\star(\boldsymbol{H}))$$

$$\boldsymbol{P_\star^3(H)} = P_\star(\boldsymbol{P_\star^2(H)}) = P_\star\big(P_\star(P_\star(\boldsymbol{H}))\big)$$

… … … ……………………………………….…

$$\boldsymbol{P_\star^n(H)} = P_\star(\boldsymbol{P_\star^{n-1}(H)}) = \underbrace{P_\star\big(P_\star(\dots P_\star(\boldsymbol{H}) \dots)\big)}_{n},$$

where P is repeated n times into the last formula,

and the empty-set $\emptyset$ (that represents indeterminacy, uncertainty) is not allowed in none of the sequence terms:

$\boldsymbol{H}, P_\star(\boldsymbol{H}), \boldsymbol{P_\star^2(H)}, \boldsymbol{P_\star^3(H)}, \dots, \boldsymbol{P_\star^n(H)}$.

6. Definition of the n[th]-PowerSet $P^n(H)$ *with* Indeterminacy (represented by the empty-set)

The n[th]-PowerSet $\boldsymbol{P^n(H)}$ of the set H, that the Neutrosophic SuperHyperStructure is built on, best describes our real world where always the indeterminacy occurs, and a *set* (or *system*) H (that may be a set of items, a company, institution, country, region, etc.) is organized in sub-systems, which in turn are organized in sub-sub-systems, and so on.

The n[th]-PowerSet $\boldsymbol{P^n(H)}$ is defined recursively:

$P^0(\boldsymbol{H}) \stackrel{\text{def}}{=} H$

$P^1(\boldsymbol{H}) = P(\boldsymbol{H})$

$P^2(\boldsymbol{H}) = P(P(\boldsymbol{H}))$

$P^3(\boldsymbol{H}) = P\big(P^2(\boldsymbol{H})\big) = P\big(P(P(\boldsymbol{H}))\big)$

… … … ……………………………………..

$$P^n(\boldsymbol{H}) = P(P^{n-1}(\boldsymbol{H})) = \underbrace{P\big(P(\dots P(\boldsymbol{H})\dots)\big)}_{n},$$

where P is repeated *n* times into the last formula,

and the empty-set $\emptyset$ (that represents indeterminacy, uncertainty) is allowed in all sequence terms:

$H, P(H), P^2(H), P^3(H), \dots, P^n(H)$.

The n[th]-PowerSet $P_\star^n(H)$ and $P^n(H)$ of a non-empty set H were introduced by Smarandache [2] in 2016.

7. SuperHyperStructure

The SuperHyperStructure was founded by Smarandache in 2016 [2], who introduced the SuperHyperAlgebra in 2016 and developed it in 2022 [8], SuperHyperGraph in 2019, 2020, 2022 [3, 4, 5], SuperHyperFunction and SuperHyperTopology in 2022 [6], and respectively the SuperHyperOperations, and SuperHyperAxioms [2016-2022].

A SuperHyperStructure is built on the n-th powerset $P_\star^n(H)$ of a non-empty set H, for integer $n \geq 1$, whose **SuperHyperOperators** ($\#_{SH0}$) are defined as follows:

$\#_{SH0}: (P_\star^r(H))^m \to P_\star^n(H)$,

where $P_\star^r(H)$ is the powerset of H, for integer $r \geq 1$, while similarly $P_\star^n(H)$ is the n-th powerset of H, both without any empty-sets, and the **SuperHyperAxioms** act on it.

8. Neutrosophic SuperHyperStructure

Similarly, a **Neutrosophic SuperHyperStructure** (2016) is built on the n[th]-powerset $P^n(H)$ of a non-empty set H, for $n \geq 1$, whose Neutrosophic SuperHyperOperators ($\#_{NSH0}$) are defined as follows:

$\#_{NSH0}: \left(P^r(H)\right)^m \to P^n(H)$,

where $P^r(H)$ is the r-powerset of H, for integer $r \geq 1$, while $P^n(H)$ is the n[th]-powerset of H, both containing empty-sets.

9. The Triplet of HyperStructure

As an analogy of the neutrosophic triplet [9 – 19] presented between (2016, 2019 – 2023):
<*Algebra, NeutroAlgebra, AntiAlgebra*>,
we propose now the following triplet:
<*HyperStructure, Neutro-HyperStructure, Anti-HyperStructure*>,
that extends Marti's HyperStructure,
where:

— the HyperStructure has all axioms totally (100%) true;

— the Neutro-HyperStructure has at least one axiom which is partially true (*T*), partially indeterminate (*I*), and partially false (*F*); $(T, I, F) \in \{(1,0,0),(0,0,1)\}$, and no axiom is totally (100%) false;

— the Anti-HyperStructure has at least one axiom that is 100% false, or $(T, I, F) = (0,0,1)$, no matter how the other axioms are.

10. The Triplet of SuperHyperStructure

One has, as a further extension of the above, the following triplet:
<*SuperHyperStructure, Neutro-SuperHyperStructure, Anti-SuperHyperStructure*>,
where:

— the SuperHyperStructure has all axioms totally (100%) true;

— the Neutro-SuperHyperStructure has at least one axiom that is partially true (*T*), partially indeterminate (*I*), and partially false (*F*); or $(T, I, F) \in \{(1,0,0),(0,0,1)\}$, and no axiom is totally (100%) false;

— the Anti-SuperHyperStructure has at least one axiom that is totally (100%) false, or $(T, I, F) = (0,0,1)$, no matter how are the other axioms.

11. SuperHyperFunction of Many Variable [7]

$$f : (P_*^r S)^m \to P_*^n(S), \text{ for integers } m \geq 2 \text{ and } r, n \geq 0.$$

It is part of the SuperHyperStructure.

12. Example of SuperHyperFunction of Two Variables

Let's take *m = 2, r = 1,* and *n = 2.*

$$f : (P_*(S))^2 \to P_*^2(S)$$

x / y	{1}	{2}	{1, 2}
{1}	{{1}, {2}}	{1}	{{1}, {1 2}}
{2}	{{2}, {1, 2}}	{{1}, {1, 2}}	{2}
{1, 2}	{1, 2}		{{1}. {2}, {1, 2}}

Table 1 of Values of the above SuperHyperFunction of Two Variable f(x, y)

For example, *f({1}, {1, 2}) = {{1}, {1 2}}*, etc.

13. SuperHyperAlgebra

We recall our 2016 concepts of SuperHyperOperation, SuperHyperAxiom, SuperHyperAlgebra, and their corresponding Neutrosophic SuperHyperOperation Neutrosophic SuperHyperAxiom and Neutrosophic SuperHyperAlgebra [2] and developed later in (2019-2024), especially in [8] in 2022.

Let $P_*^n(H)$ be the n[th]-powerset of the set H such that none of $P(H), P^2(H), \ldots, P^n(H)$ contain the empty set ϕ.

Also, let $P^n(H)$ be the n[th]-powerset of the set H such that at least one of the $P^2(H), \ldots, P^n(H)$ contain the empty set ϕ.

The SuperHyperOperations are operations whose codomain is either $P_*^n(H)$ and in this case one has **classical-type SuperHyperOperations**, or $P^n(H)$ and in this case one has **Neutrosophic SuperHyperOperations**, for integer $n \geq 2$.

14. Classical-type Binary SuperHyperOperation

A classical-type Binary SuperHyperOperation $\circ_{(2,n)}^*$ is defined as follows:

$$\circ_{(2,n)}^* : H^2 \to P_*^n(H)$$

where $P_*^n(H)$ is the n^{th}-powerset of the set H, with no empty-set.

15. Examples of classical-type Binary SuperHyperOperation

1) Let $H = \{a, b\}$ be a finite discrete set; then its power set, without the empty-set ϕ, is:

$$P(H) = \{a, b, \{a, b\}\}, \text{ and:}$$

$$P^2(H) = P(P(H)) = P(\{a, b, \{a, b\}\}) = \{a, b, \{a, b\}, \{a, \{a, b\}\}, \{b, \{a, b\}\}, \{a, b, \{a, b\}\}\}.$$

$$\circ_{(2,2)}^* : H^2 \to P_*^2(H)$$

Table 2. Example 1 of classical-type Binary SuperHyperOperation.

$\circ^*_{(2,2)}$	a	b
a	$\{a,\{a,b\}\}$	$\{b,\{a,b\}\}$
b	a	$\{a,b,\{a,b\}\}$

16. Classical-type m-ary SuperHyperOperation {or more accurate denomination (m, n)-SuperHyperOperation}

Let U be a universe of discourse and a non-empty set H, $H \subset U$. Then:

$$\circ^*_{(m,n)} : H^m \to P_*^n(H)$$

where the integers $m, n \geq 1$,

$$H^m = \underbrace{H \times H \times \ldots \times H}_{m\ times},$$

and $P_*^n(H)$ is the n^{th}-powerset of the set H that includes the empty-set.

This SuperHyperOperation is a m-ary operation defined from the set H to the n^{th}-powerset of the set H.

17. Neutrosophic m-ary SuperHyperOperation {or more accurate denomination Neutrosophic (m, n)-SuperHyperOperation}

Let U be a universe of discourse and a non-empty set H, $H \subset U$. Then:

$$\circ_{(m,n)} : H^m \to P^n(H)$$

where the integers $m, n \geq 1$; $P^n(H)$ - the n-th powerset of the set H that includes the empty-set.

18. SuperHyperAxiom

A **classical-type SuperHyperAxiom** or more accurately a ***(m, n)*-SuperHyperAxiom** is an axiom based on classical-type SuperHyperOperations.

Similarly, a **Neutrosophic SuperHyperAxiom** {or Neutrosphic (m, n)-SuperHyperAxiom} is an axiom based on Neutrosophic SuperHyperOperations.

There are:

- **Strong SuperHyperAxioms**, when the left-hand side is equal to the right-hand side as in non-hyper axioms,
- and **Week SuperHyperAxioms**, when the intersection between the left-hand side and the right-hand side is non-empty.

19. SuperHyperAlgebra and Neutrosophic SuperHyperStructure

A **SuperHyperAlgebra** or more accurately ***(m-n)*-SuperHyperAlgebra** is an algebra dealing with SuperHyperOperations and SuperHyperAxioms.

Again, a **Neutrosophic SuperHyperAlgebra** {or Neutrosphic (m, n)-SuperHyperAlgebra} is an algebra dealing with Neutrosophic SuperHyperOperations and Neutrosophic SuperHyperOperations.

In general, we have **SuperHyperStructures** {or *(m-n)*-SuperHyperStructures}, and corresponding **Neutrosophic SuperHyperStructures**.

For example, there are SuperHyperGrupoid, SuperHyperSemigroup, SuperHyperGroup, SuperHyperRing, SuperHyperVectorSpace, etc.

20. SuperHyperGraph (or n-SuperHyperGraph)

Introduced by F. Smarandache [3, 4, 5] in 2019 and developed in 2020 - 2022.

Let $V = \{v_1, v_2, \dots, v_m\}$, for $1 \leq m \leq \infty$, be a set of finite or infinite number of vertices, that contains <u>Single Vertices</u> (the classical ones), <u>Indeterminate Vertices</u> (unclear, vague, partially known), and <u>Null Vertices</u> (totally unknown, empty).

Let $P(V)$ pe the power of set V, that includes the empty set too.

Then $P^n(V)$ be the n-power set of the set V, defined in a recurrent way, i.e.:

P(V), $P^2(V) = P(P(V))$, $P^3(V) = P(P^2(V)) = P(P(P(V)))$, …, $P^n(V) = P(P^{n-1}(V))$,

for $1 \leq n \leq \infty$, where by definition $P^0(V) \overset{def}{=} V$ and $P^1(V) \overset{def}{=} P(V)$.

Then, the SuperHyperGraph (SHG) [or n-SuperHyperGraph (n-SHG)] is an ordered pair:

$$\text{n-SHG} = (G_n, E_n),$$

where $G_n \subseteq P^n(V)$, and $E_n \subseteq P^n(V)$, for $1 \leq n \leq \infty$.

G_n is the set of vertices, and E_n is the set of edges.

The set of vertices G_n contains all possible types of vertices as in our real world:

- <u>Singles Vertices</u> (the classical ones);
- <u>Indeterminate Vertices</u> (unclear, vague, partially unknown);
- <u>Null Vertices</u> (totally unknown, empty);

and:

- <u>SuperVertex</u> (or <u>SubsetVertex</u>), i.e. two or more (single, indeterminate, or null) vertices put together as a group (organization).
- <u>n-SuperVertex</u> that is a collection of many vertices such that at least one is an $(n-1)$- SuperVertex and all the others into the collection are r-SuperVertices, if any, whose order $r \leq n-1$.

The set of edges E_n contains the following types of edges:

- <u>Singles Edges</u> (the classical ones);
- <u>Indeterminate Edges</u> (unclear, vague, partially unknown);
- <u>Null Edges</u> (totally unknown, empty);

and:

- <u>HyperEdge</u> (connecting three or more single vertices);

- SuperEdge (connecting two vertices, at least one of them being a SuperVertex);
- n-SuperEdge (connecting two vertices, at least one being a n-SuperVertex, and the other of order r-SuperVertex, with $r \le n$);
- SuperHyperEdge (connecting three or more vertices, at least one being a SuperVertex);
- n -SuperHyperEdge (connecting three or more vertices, at least one being a n - SuperVertex, and the other r-SuperVertices with $r \le n$;
- MultiEdges (two or more edges connecting the same two vertices);
- Loop (and edge that connects an element with

itself). and:

- Directed Graph (classical one);
- Undirected Graph (classical one);
- Neutrosophic Directed Graph (partially directed, partially undirected, partially indeterminate direction).

21. SuperHyperTopology [6, 7]

Let consider τ_{SHT} be a family of subsets of $P_*^n(H)$.

Then τ_{SHT} is called a SuperHyperTopology on $P_*^n(H)$, if it satisfies the following axioms:

(SHT-1) ϕ and $P_*^n(H)$ belong to τ_{SHT}.

(SHT-2) The intersection of any finite number of elements in τ_{SHT} is in τ_{SHT}.

(SHT-3) The union of any finite or infinite number of elements in τ_{SHT} is in τ_{SHT}.

Then $(P_*^n(H), \tau_{SHT})$ is called a SuperHyperTopological Space on $P_*^n(H)$.

22. Neutrosophic SuperHyperTopology [6, 7]

Let consider τ_{NSHT} be a family of subsets of $P^n(H)$.

Then τ_{NSHT} is called a Neutrosophic SuperHyperTopology on $P^n(H)$, if it satisfies the following axioms:

(NSHT-1) ϕ and $P^n(H)$ belong to τ_{NSHT}.

(NSHT-2) The intersection of any finite number of elements in τ_{NSHT} is in τ_{NSHT}.

(NSHT-3) The union of any finite or infinite number of elements in τ_{NSHT} is in τ_{NSHT}.

Then $(P^n(H), \tau_{NSHT})$ is called a Neutrosophic SuperHyperTopological Space on $P^n(H)$.

23. SuperHyperSoft Set

The SuperHyperSoft Set [22, 23] is an extension of the HyperSoft Set [21] and Soft Set [20].

Let $\mathcal{U}$ be a universe of discourse, $\mathcal{P}(\mathcal{U})$ the powerset of $\mathcal{U}$.

Let $a_1, a_2, \ldots, a_n$, for $n \ge 1$, be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$,

with $A_i \cap A_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, \dots, n\}$.
Let $\mathcal{P}(A_1)$, $\mathcal{P}(A_2)$, …, $\mathcal{P}(A_n)$ be the powersets of the sets A_1, A_2, …, A_n respectively.

Then the pair $(F, \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n))$, where × meaning Cartesian product, or:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}(\mathcal{U})$

is called a SuperHyperSoft Set.

24. Example of SuperHyperSoft Set

If we define the function:

$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3) \times \mathcal{P}(A_4) \rightarrow \mathcal{P}(\mathcal{U})$.

we get a *SuperHyperSoft Set*.

Let's assume, from the previous example, that:

$F(\{\text{medium}, \text{tall}\}, \{\text{white}, \text{red}, \text{black}\}, \{\text{female}\}, \{\text{American}, \text{Italian}\}) = \{x_1, x_2\}$, which means that:

$F(\{\text{medium or tall}\} \text{ and } \{\text{white or red or black}\} \text{ and } \{\text{female}\} \text{ and } \{\text{American or Italian}\})$ = $\{x_1, x_2\}$.

Therefore, the SuperHyperSoft Set offers a larger variety of selections, so x_1 and x_2 may be:

either medium, or tall (but not small),
either white, or red, or black (but not yellow),
mandatory female (not male),
and either American, or Italian (but not French, Spanish, Chinese).

In this example there are:
Card{medium, tall}· Card{white, red, black}· Card{female}· Card{American, Italian} = 2·3·1·2 =12 possibilities, where Card{ } means cardinal of the set { }.

This is closer to our everyday life, since for example, when selecting something, we have not been too strict, but accepting some variations (for example: medium or tall, white or red or black, etc.).

25. Fuzzy-Extension-SuperHyperSoft Set

$$F: \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \dots \times \mathcal{P}(A_n) \rightarrow \mathcal{P}\left(\mathcal{U}\left(x(d^0)\right)\right)$$

where $x(d^0)$ is the fuzzy or any fuzzy-extension degree of appurtenance of the element x to the set $\mathcal{U}$.

Fuzzy-Extensions mean all types of fuzzy sets [3], such as:
Fuzzy Set, Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), Fermatean Fuzzy Set, q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, n-HyperSpherical Fuzzy Set, Neutrosophic Set, Spherical Neutrosophic Set, Refined Fuzzy/Intuitionistic_Fuzzy/Neutrosophic/other_fuzzy_extension Sets, Plithogenic Set, etc.

26. Example of Fuzzy_Extension SuperHyperSoft Set

In the previous example, taking the degree of a generic element $x(d^0)$ as neutrosophic, one gets the Neutrosophic SuperHyperSoft Set.
Assume, that: $F(\{\text{medium}, \text{tall}\}, \{\text{white}, \text{red}, \text{black}\}, \{\text{female}\}, \{\text{American}, \text{Italian}\}) =$
$= \{x_1(0.7, 0.4, 0.1), x_2(0.9, 0.2, 0.3)\}$.
Which means that: x_1 with respect to the attribute values
$(\{\text{medium or tall}\}$ and $\{\text{white or red or black}\}$ and $\{\text{female}\}$, and $\{\text{American or Italian}\})$
has the degree of appurtenance to the set 0.7, the indeterminate degree of appurtenance 0.4, and the degree of non-appurtenance 0.1.
While x_2 has the degree of appurtenance to the set 0.9, the indeterminate degree of appurtenance 0.2, and the degree of non-appurtenance 0.3.

27. Examples of HyperAlgebra and Neutrosophic HyperAlgebra

27.1. Commutative SemiHyperGroup

The SemiHyperGroup is a particular case of HyperAlgebra.
Let $\mathbb{Z}$ be the set of integers, $\mathbb{Z} = \{-\infty, \dots, -2, -1, 0, 1, 2, \dots, +\infty\}$.
Let's define the HyperLaw * as follows:
$*: Z \times Z \to P(Z)$, $x \star y = \{x, y\} \in P(\mathbb{Z})$,
so the law is *well-defined*.
The law is *associative*, since:
$(x \star y) \star z = x \star (y \star z)$
$\{x, y\} \star z = x \star \{y, z\}$
$(x \star z) \cup (y \star z) = (x \star y) \cup (x \star z)$
$\{x, z\} \cup \{y, z\} = \{x, y\} \cup \{x, z\}$
$\{x, y, z\} = \{x, y, z\}$
The law is *commutative*, since
$x \star y = \{x, y\} = \{y, x\} = y \star x$

27.2. Commutative Neutrosophic SemiHyperGroup

The Neutrosophic SemiHyperGroup is a particular case of Neutrosophic HyperAlgebra.

Let the HyperLaw * be defined as:
$\star: (Z \cup \{\emptyset\}) \times (Z \cup \{\emptyset\}) \to P(Z \cup \{\emptyset\})$
where the empty set $\emptyset$ leaves room for indeterminacy, unknown etc.

$$x \star y = \begin{cases} \{x, y\}, \text{for both } x, y \neq \emptyset \\ \emptyset, \ \text{for } x, \text{or } y, \text{or both} = \emptyset \end{cases}$$

The law is well-defined, associative and commutative (proven as above for the SemiHyperGroup).

28. Examples of SuperHyperAlgebra and Neutrosophic SuperHyperAlgebra

28.1. Commutative SuperHyperGrupoid

Let again $\mathbb{Z}$ be the set of integers, $\mathbb{Z} = \{-\infty, \dots, -2, -1, 0, 1, 2, \dots, +\infty\}$.

For 2-nd powerset $P^2(\mathbb{Z}) = P(P(\mathbb{Z}))$ one has two rows of parentheses, one inside the other, of the form: {… {… } … }.

Let's define the binary SuperHyperLaw

$\star: \mathbb{Z}^2 \to P^2(\mathbb{Z})$

$x \star y = \{x, y, \{x, y\}\} \in P^2(\mathbb{Z})$

Clearly the law is *well-defined*.

The law $\star$ is also *commutative,* but *non-associative,* as proven below.

Commutativity:

$x \star y = \{x, y, \{x, y\}\} = \{y, x, \{y, x\}\} = y \star x.$

Non-Associativity:

$$(x \star y) \star z = \{x, y, \{x, y\} \star z\} = \{x \star z, y \star z, \{x, y\} \star z\} = \{x, z, \{x, z\}, y, z, \{y, z\}, x \star z, y \star z\}$$
$$= \{x, z, \{x, z\}, y, z, \{y, z\}, z, z, \{x, z\}, y, z, \{y, z\}\} = \{x, y, z, \{x, z\}, \{y, z\}\}$$
$$x \star (y \star z) = x \star \{y, z, \{y, z\}\} = \{x \star y, x \star z, x \star \{y, z\}\} = \{x, y, \{x, y\}, x, z, \{x, z\}, x \star y, x \star z\}$$
$$= \{x, y, \{x, y\}, x, z, \{x, z\}, x, y, \{x, y\}, x, z, \{x, z\}\} = \{x, y, z, \{x, y\}, \{x, z\}\}$$

Therefore $(x \star y) \star z \neq x \star (y \star z)$.

28.2. Commutative Neutrosophic SuperHyperGrupoid

Similarly, we define:

Let the Neutrosophic SuperHyperLaw * be defined as:

$*: (Z \cup \{\phi\}) \times (Z \cup \{\phi\}) \to P^2(Z \cup \{\phi\})$

where the empty set $\emptyset$ aslo leaves room for indeterminacy, unknown etc.

$$x \star y = \begin{cases} \{x, y, \{x, y\}\}, \text{for both } x, y \neq \emptyset \\ \emptyset, \text{ for } x, \text{or } y, \text{or both} = \emptyset \end{cases}$$

The law is well-defined, non-associative, and commutative (proven as above for the SuperHyperGroupoid).

29. Practical Application of the SuperHyperStructure

Let H be the set (system) that represent all inhabitants of the US country.

The set H is organized into 50 sub-sets, $H_1, H_2, \dots, H_{50}$

that represent the American <u>states</u>: where $H_1, H_2, ..., H_{50} \in P(H)$.

Each state $H_i, 1 \le i \le 50,$ is organized into <u>counties</u>, $H_{ij}, 1 \le j \le i_M$, where i_M is the maximum number of H_i state's counties, with all $H_{i,1}, H_{i,2}, ..., H_{i,i_M} \in P(H_i) \subset P(P(H)) = P^2(H)$.

(One uses commas in between indexes in order to separate them when the values of some indexes have two or more digits, for example $H_{3,12}$ means the 12th county of the 3rd state; which is different from $H_{31,2}$ that means the 2nd county of the 31st state.)

Further on, each $H_{i,j}$ county, for all *i* and *j* indexes,

is organized into <u>sub-counties</u>, $H_{i,j,k}, 1 \le k \le j_M$, where j_M is the maximum number of sub-counties of the county $H_{i,j}$. Therefore:

all $H_{i,j,1}, H_{i,j,2}, ..., H_{i,j,k_m} \in P(H_{i,j}) \subset P(P(H_i)) \subset P(P(P(H))) = P^3(H)$.

This shows the practical application of the n-th powerset of a set, for *n = 3* in this case (three levels of a SuperHyperStructure): country, states (one index *i*), counties (two indexes *i,j*), and sub-counties (three indexes *i,j,k*).

Surely, if needed, one can go *deeper in* (each <u>sub-county</u> is formed by <u>towns</u>, each town by <u>districts</u>, and so on); or *deeper out* (each <u>country</u> is part of a <u>continent</u>, each continent is part of a <u>planet</u>, each planet is part of a <u>solar system</u>, and so on).

The following SuperHyperStructure (denoted below by *A*), with three levels of structures, has been formed as:

$$A = \{H_i, H_{i,j}, H_{i,j,k}, 1 \le i \le 50, 1 \le j \le i_M, 1 \le k \le j_M\} \subset P^3(H).$$

(i) In the real world, this is a Neutrosophic SuperHyperStructure, because it has a lot of indeterminacy: for example the *population* of the country *H* is dynamic, in a continuous change: new people are born while others die as we speak, there are millions of illegal emigrants that are not counted as citizens, others have dual or triple citizenship so they only partially belong to *H*'s population; other citizen live outside the country.

(ii) Many laws are as well neutrosophic, because they apply to some states H_i (as examples: the law of abortion, or the law of bearing arms, or the law of consuming marijuana, etc.), but not to others. We call them NeutroLaws in the NeutroAlgebraic Structures (we mean: laws that are partially true and partially false in the same space, and sometime also partially indeterminate).

(iii) Let's endow this SuperHyperStructure with some SuperHyperLaw (called SuperHyper because it is built on a 3-rd PowerSet of the Set *H*):

$\#: A \times A \to A$

$x \# y = \{x \wedge (x \subseteq y) \to y\}$

{If x and $x \subseteq y$, then y}

Let $x, y \in A$. If x and $x \subseteq y$, then y.

Consider a cyber company that provides internet connection to people from the sub-county $H_{2,3,5}$, which is included in the county $H_{2,3}$, then the company will provide internet connection to the county $H_{2,3}$ as well.
Let $H_{2,3,5}$ and $H_{2,3} \in A$. If $H_{2,3,5}$ gets internet connection and because $H_{2,3,5} \subset H_{2,3}$, then $H_{2,3}$ also gets internet connection.

30. Conclusion

We have extended the SuperHyperAlgebra and its correspondents (SuperHyperGraph, SuperHyperTopology, etc.) to the SuperHyperStructure in general, for any field of knowledge and on any type of space. The SuperHyperStructures were inspired from, and they perfectly fit, our real world. See the last practical application.

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