Neutrosophic Bipolar Vague Soft Set and Its Application to Decision Making Problems

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Abstract: In this paper we study the concept of neutrosophic bipolar vague soft sets and some of its operations. It is the combination of neutrosophic bipolar vague sets and soft sets. Further we develop a decision making method based on neutrosophic bipolar vague soft set. A numerical example has been shown. Some new operations on neutrosophic bipolar vague soft set have also been designed.

Keywords: Neutrosophic set, Neutrosophic bipolar vague set, Soft set, vague set, Neutrosophic bipolar vague soft set.

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1. Introduction

Most real life problems involve data with a high level of uncertainty and imprecision. Traditionally, classical mathematical theories such as fuzzy mathematics, probability theories and interval mathematics are used to deal with uncertain and fuzziness. However all these theories have their difficulties and weakness as pointed out by Molodtsov. This led to the introduction of the theory of soft sets by Molodtsov [18] in 1999. Among the significant milestones in the development of the theory of soft sets and its generalizations in the introduction of the possibility value which indicates the degree of possibility of belongingness of the elements in the universal set as well as the elements of each sets which enables the users to know the opinion of the experts in one model without the need for any operation. However, in order to handle the indeterminate and inconsistent information, neutrosophic set is defined [23,24] as a new mathematical tool for dealing with problems involving incomplete, indeterminacy and inconsistent knowledge. The theory of vague set was first proposed by Gau and Buehrer [13] as an extension of fuzzy set theory[29] and vague sets are regarded as a special case of content-dependent fuzzy sets.

In, [23] ,Samerandeche talked about neutrosophic set theory, one of the most important new mathematical tools for handling problems involving imprecise, indeterminacy and inconsistent data. Neutrosophic vague set was defined by S. Allehezaleh [2] in 2015. Lee [17] introduced bipolar-valued fuzzy sets and their operations in 2000. It an extension of fuzzy set [29]. Ali et al.[1] introduced the notion of bipolar neutrosophic soft set in 2017. Hassain et al. [16] introduced the concept of neutrosophic bipolar vague set and its application to neutrosophic bipolar vague graphs. For real life problems see the following ([3] to [12],[14],[15],[19]to[22],[25]to[28],[30]to[32]).

In this paper, we first introduce the concept of neutrosophic bipolar vague soft set and some of its operations. It is the combination of neutrosophic bipolar vague set and soft set. We develop a decision making method based on neutrosophic bipolar vague soft set. A numerical example has been shown. Some new operations on neutrosophic bipolar vague soft set have been designed. Finally we present an application of this concept in solving a decision making problem.

2. Materials and Methods (proposed work with more details)

In this section we recall some definitions and results for our future work.

Definition2.1:[8] Let *U* be an initial universal set and let *E* be a set of parameters. Let P(*U*) denote the power set of all subsets of *U* and let $A \subseteq E$. *A* collection of pairs (*f*, *A*) is called a soft set over *U*, where f is a mapping given by $f: A \rightarrow P(U)$.

Definition2.2:[17] Let *U* be the universe. Then a bipolar fuzzy set *A* on *U* is defined by

$$A = \{< x, \mu_A^+(x), \mu_A^-(x) >; x \in U\}.$$

Here $\mu_A^+ : U \rightarrow [0,1]$ the positive membership function.

$\mu_A^- : U \rightarrow [-1,0]$the negative membership function.

Definition2.3:[17] If *A* and *B* be two bipolar fuzzy sets then their union, intersection and complement are defined as follows:

(i) $\mu_{A\cup B}^+(x) = \max\{\mu_A^+(x), \mu_B^+(x)\}$

(ii) $\mu_{A\cup B}^-(x) = \max\{\mu_A^-(x), \mu_B^-(x)\}$

(iii) $\mu_{A\cap B}^+(x) = \min\{\mu_A^+(x), \mu_B^+(x)\}$

(iv) $\mu_{A\cup B}^-(x) = \max\ \{\mu_A^-(x), \mu_B^-(x)\}$

(v) $\mu_{A^c}^+(x) = 1 - \mu_A^+(x)$ and $\mu_{A^c}^-(x) = -1 - \mu_A^-(x), \forall x \in U$.

Definition2.4: [13] A vague set *A* in the universe of discourse *U* is a pair

(t_A, f_A) where t_A, F_A:$U \rightarrow [0, 1]$ such that $t_A + f_A \leq 1$ for all U. The function t_A and f_A are called the true membership function and the false membership function respectively. The interval $[t_A, 1 - f_A]$ is called the value of *u* in *A* and is denoted by $V_A = [t_A, 1 - f_A]$.

Definition2.5:[13] Let *X* be a non-empty set. Let *A* and *B* be two vague sets in the form $A = \{< x, t_A, 1 - f_A >: x \in X\}$, $B = \{< x, t_B, 1 - f_B >: x \in X\}$. Then

(i) $A \subseteq B\ if\ and\ only\ if\ t_A \leq t_B$ and $1 - f_A \leq 1 - f_B$.

(ii) $A \cup B = \{< x, \max(t_A(x), t_B(x)), \max(1 - f_A(x), 1 - f_B(x)) >: x \in X\}$

(iii) $A \cap B = \{< x, \min(t_A(x), t_B(x)), \min(1 - f_A(x), 1 - f_B(x)) >: x \in X\}$

(iv) $A^c = \{< x, f_A, 1 - t_A >: x \in X\}$.

Definition2.6: [23,24] A neutrosophic set A on the universe of discourse U is defined as $A = \{\langle x, \mu_A(x), \gamma_A(x), \delta_A(x)\rangle : x \in U\}$, where $\mu_A, \gamma_A, \delta_A : U \to]^-0, 1^+[$ are functions such that the condition: $\forall x \in U$, $^-0 \le \mu_A(x) + \gamma_A(x) + \delta_A(x) \le 3^+$ is satisfied.

Here $\mu_A(x), \gamma_A(x), \delta_A(x)$ represent the truth-membership, indeterminacy-membership and falsity-membership respectively of the element $x \in U$ .From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0,1]$.

Definition2.7: [2] A neutrosophic vague set A_{NV} on the universe of discourse U written as $A_{NV} = \{<x;$ $\hat{T}A_{NV}(x)$; $\hat{I}A_{NV}(x)$; $\hat{F}A_{NV}(x)>; x \in U\}$ whose truth-membership, indeterminacy-membership, and falsity-membership functions is defined as $\hat{T}A_{NV}(x) = [T^-, T^+]$, $\hat{I}A_{NV}(x) = [I^-, I^+]$ and $\hat{F}A_{NV}(x) = [F^-, F^+]$, where (1) $T^+ = 1 - F^-$, (2) $F^+ = 1 - T^-$ and (3) $^-0 \le T^- + I^- + F^- \le 2^+$.

Definition2.8:[16] Let U be the universe of discourse. The neutrosophic bipolar vague set defined as A_{NBV} where

$$A_{NBV} = \{< x, \tilde{T}^p_{A_{NBV}}(x), \tilde{I}^p_{A_{NBV}}(x), \tilde{F}^p_{A_{NBV}}(x), \tilde{T}^n_{A_{NBV}}(x), \tilde{I}^n_{A_{NBV}}(x), \tilde{F}^n_{A_{NBV}}(x) >: x \in V\}$$

Here

$\tilde{T}^p_{A_{NBV}}(x) = [(T^-)^p(x),\ (T^+)^p(x)]$,

$\tilde{I}^p_{A_{NBV}}(x) = [(I^-)^p(x),\ (I^+)^p(x)]$,

$\tilde{F}^p_{A_{NBV}}(x) = [(F^-)^p(x),\ (F^+)^p(x)]$, Where $(T^+)^p(x) = 1 - (F^-)^p(x)$,

$(F^+)^p(x) = 1 - (T^-)^p(x)$

The condition is $0 \le (T^-)^p(x) + (I^-)^p(x) + (F^-)^p(x) \le 2$.

Also, $\tilde{T}^n_{A_{NBV}}(x) = [(T^-)^n(x),\ (T^+)^n(x)]$,

$\tilde{I}^n_{A_{NBV}}(x) = [(I^-)^n(x),\ (I^+)^n(x)]$,

$\tilde{F}^n_{A_{NBV}}(x) = [(F^-)^n(x),\ (F^+)^n(x)]$,

where $(T^+)^n(x) = -1 - (F^-)^n(x)$, $(F^+)^n(x) = -1 - (T^-)^n(x)$

The condition is $0 \geq (T^-)^n(x) + (I^-)^n(x) + (F^-)^n(x) \geq -2$.

Example 2.9: Let $U = \{u_1, u_2, u_3\}$ be a set of universe then the NBV set *ANBV* as follows:

ANBV = $\Big(\frac{u_1}{[0.3,0.5]^p,[0.5,0.5]^p,[0.5,0.7]^p,[-0.3,-0.4]^n,[-0.4,-0.4]^n,[-0.6,-0.7]^n},$

$\frac{u_2}{[0.3,0.7]^p,[0.4,0.6]^p,[0.3,0.7]^p,[-0.3,-0.3]^n,[-0.4,-0.4]^n,[-0.7,-0.7]^n},$

$\frac{u_3}{[0.4,0.7]^p,[0.4,0.6]^p,[0.3,0.6]^p,[-0.2,-0.6]^n,[-0.5,-0.6]^n,[-0.4,-0.8]^n}\Big).$

Definition 2.10: [16] The compliment A^c_{NBV} of A_{NBV} is as

$(\tilde{T}^c_{A_{NBV}}(x))^p = \{(1 - T^+(x))^p, (1 - T^-(x))^p\}$,

$(\tilde{T}^c_{A_{NBV}}(x))^n = \{(-1 - T^+(x))^n, (-1 - T^-(x))^n\}$,

$(\tilde{I}^c_{A_{NBV}}(x))^p = \{(1 - I^+(x))^p, (1 - I^-(x))^p\}$,

$(\tilde{I}^c_{A_{NBV}}(x))^n = \{(-1 - I^+(x))^n, (-1 - I^-(x))^n\}$,

$(\tilde{F}^c_{A_{NBV}}(x))^p = \{(1 - F^+(x))^p, (1 - F^-(x))^p\}$,

$(\tilde{F}^c_{A_{NBV}}(x))^n = \{(-1 - F^+(x))^n, (-1 - F^-(x))^n\}$.

Example 2.11: Considering the example 2.9, we have

$A^c_{NBV} = \Big(\frac{u_1}{[0.7,0.5]^p,[0.5,0.5]^p,[0.5,0.3]^p,[-0.7,-0.6]^n,[-0.6,-0.6]^n,[-0.4,-0.3]^n},$

$\frac{u_2}{[0.7,0.3]^p,[0.6,0.4]^p,[0.7,0.3]^p,[-0.7,-0.7]^n,[-0.6,-0.6]^n,[-0.3,-0.3]^n},$

$\frac{u_3}{[0.6,0.3]^p,[0.6,0.4]^p,[0.7,0.4]^p,[-0.8,-0.4]^n,[-0.5,-0.4]^n,[-0.6,-0.2]^n}\Big).$

Definition 2.12: [16] Two NBV sets A_{NBV} and B_{NBV} of the universe U are said to be equal if for all $u_i \in U$,

$\left(\tilde{T}_{A_{NBV}}\right)^p(u_i) = \left(\tilde{T}_{B_{NBV}}\right)^p(u_i)$, $\left(\tilde{I}_{A_{NBV}}\right)^p(u_i) = \left(\tilde{I}_{B_{NBV}}\right)^p(u_i)$,

$\left(\tilde{F}_{A_{NBV}}\right)^p(u_i) = \left(\tilde{F}_{B_{NBV}}\right)^p(u_i)$ and $\left(\tilde{T}_{A_{NBV}}\right)^n(u_i) = \left(\tilde{T}_{B_{NBV}}\right)^n(u_i)$,

$\left(\tilde{I}_{A_{NBV}}\right)^n(u_i) = \left(\tilde{I}_{B_{NBV}}\right)^n(u_i)$, $\left(\tilde{F}_{A_{NBV}}\right)^n(u_i) = \left(\tilde{F}_{B_{NBV}}\right)^n(u_i)$.

Where $1 \leq i \leq m$(say).

Definition 2.13 [16] If in the universe *U*, two *NBV* sets A_{NBV} and B_{NBV} be given as

$\left(\tilde{T}_{A_{NBV}}\right)^p(u_i) \leq \left(\tilde{T}_{B_{NBV}}\right)^p(u_i)$, $\left(\tilde{I}_{A_{NBV}}\right)^p(u_i) \geq \left(\tilde{I}_{B_{NBV}}\right)^p(u_i)$,

$\left(\tilde{F}_{A_{NBV}}\right)^p(u_i) \geq \left(\tilde{F}_{B_{NBV}}\right)^p(u_i)$ and $\left(\tilde{T}_{A_{NBV}}\right)^n(u_i) \geq \left(\tilde{T}_{B_{NBV}}\right)^n(u_i)$,

$\left(\tilde{I}_{A_{NBV}}\right)^n(u_i) \leq \left(\tilde{I}_{B_{NBV}}\right)^n(u_i)$, $\left(\tilde{F}_{A_{NBV}}\right)^n(u_i) \leq \left(\tilde{F}_{B_{NBV}}\right)^n(u_i)$

Then $(A_{NBV})^p \subseteq (B_{NBV})^p$, and $(A_{NBV})^n \subseteq (B_{NBV})^n$ for $1 \leq i \leq m$

Definition 2.14: [16] The union and intersection of two *NBV* sets A_{NBV} and B_{NBV} are given as

(i) $A_{NBV} \cup B_{NBV} = C_{NBV}$ where

$$\left(\tilde{T}_{C_{NBV}}\right)^p(x) = \{\left(\left(\tilde{T}^-_{A_{NBV}}\right)^p(x) \vee \left(\tilde{T}^-_{B_{NBV}}\right)^p(x), \left(\left(\tilde{T}^+_{A_{NBV}}\right)^p(x) \vee \left(\tilde{T}^+_{B_{NBV}}\right)^p(x)\right)\right)\}$$

$$\left(\tilde{I}_{C_{NBV}}\right)^p(x) = \{\left(\left(\tilde{I}^-_{A_{NBV}}\right)^p(x) \wedge \left(\tilde{I}^-_{B_{NBV}}\right)^p(x), \left(\left(\tilde{I}^+_{A_{NBV}}\right)^p(x) \wedge \left(\tilde{I}^+_{B_{NBV}}\right)^p(x)\right)\right)\}$$

$$\left(\tilde{F}_{C_{NBV}}\right)^p(x) = \{\left(\left(\tilde{F}^-_{A_{NBV}}\right)^p(x) \wedge \left(\tilde{F}^-_{B_{NBV}}\right)^p(x), \left(\left(\tilde{F}^+_{A_{NBV}}\right)^p(x) \wedge \left(\tilde{F}^+_{B_{NBV}}\right)^p(x)\right)\right)\}, \text{ and}$$

$$\left(\tilde{T}_{C_{NBV}}\right)^n(x) = \{\left(\left(\tilde{T}^-_{A_{NBV}}\right)^n(x) \wedge \left(\tilde{T}^-_{B_{NBV}}\right)^n(x), \left(\left(\tilde{T}^+_{A_{NBV}}\right)^n(x) \wedge \left(\tilde{T}^+_{B_{NBV}}\right)^n(x)\right)\right)\}$$

$$\left(\tilde{I}_{C_{NBV}}\right)^n(x) = \{\left(\left(\tilde{I}^-_{A_{NBV}}\right)^n(x) \vee \left(\tilde{I}^-_{B_{NBV}}\right)^n(x), \left(\left(\tilde{I}^+_{A_{NBV}}\right)^n(x) \vee \left(\tilde{I}^+_{B_{NBV}}\right)^n(x)\right)\right)\}$$

$$\left(\tilde{F}_{C_{NBV}}\right)^n(x) = \{\left(\left(\tilde{F}^-_{A_{NBV}}\right)^n(x) \vee \left(\tilde{F}^-_{B_{NBV}}\right)^n(x), \left(\left(\tilde{F}^+_{A_{NBV}}\right)^n(x) \vee \left(\tilde{F}^+_{B_{NBV}}\right)^n(x)\right)\right)\}.$$

(ii) $A_{NBV} \cap B_{NBV} = D_{NBV}$ is given by

$$\left(\tilde{T}_{D_{NBV}}\right)^p(x) = \{\left(\left(\tilde{T}^-_{A_{NBV}}\right)^p(x) \wedge \left(\tilde{T}^-_{B_{NBV}}\right)^p(x), \left(\left(\tilde{T}^+_{A_{NBV}}\right)^p(x) \wedge \left(\tilde{T}^+_{B_{NBV}}\right)^p(x)\right)\right)\}$$

$$\left(\tilde{I}_{D_{NBV}}\right)^p(x) = \{\left(\left(\tilde{I}^-_{A_{NBV}}\right)^p(x) \vee \left(\tilde{I}^-_{B_{NBV}}\right)^p(x), \left(\left(\tilde{I}^+_{A_{NBV}}\right)^p(x) \vee \left(\tilde{I}^+_{B_{NBV}}\right)^p(x)\right)\right)\}$$

$$\left(\tilde{F}_{D_{NBV}}\right)^p(x) = \{\left(\left(\tilde{F}^-_{A_{NBV}}\right)^p(x) \vee \left(\tilde{F}^-_{B_{NBV}}\right)^p(x), \left(\left(\tilde{F}^+_{A_{NBV}}\right)^p(x) \vee \left(\tilde{F}^+_{B_{NBV}}\right)^p(x)\right)\right)\}, \text{ and}$$

$$\left(\tilde{T}_{D_{NBV}}\right)^n(x) = \{\left(\left(\tilde{T}^-_{A_{NBV}}\right)^n(x) \vee \left(\tilde{T}^-_{B_{NBV}}\right)^n(x), \left(\left(\tilde{T}^+_{A_{NBV}}\right)^n(x) \vee \left(\tilde{T}^+_{B_{NBV}}\right)^n(x)\right)\right)\}$$

$$\left(\tilde{I}_{D_{NBV}}\right)^n(x) = \{\left(\left(\tilde{I}^-_{A_{NBV}}\right)^n(x) \wedge \left(\tilde{I}^-_{B_{NBV}}\right)^n(x), \left(\left(\tilde{I}^+_{A_{NBV}}\right)^n(x) \wedge \left(\tilde{I}^+_{B_{NBV}}\right)^n(x)\right)\right)\}$$

$$\left(\tilde{F}_{D_{NBV}}\right)^n(x) = \{\left(\left(\tilde{F}^-_{A_{NBV}}\right)^n(x) \wedge \left(\tilde{F}^-_{B_{NBV}}\right)^n(x), \left(\left(\tilde{F}^+_{A_{NBV}}\right)^n(x) \wedge \left(\tilde{F}^+_{B_{NBV}}\right)^n(x)\right)\right)\}.$$

3. Neutrosophic Bipolar Vague Soft Set.

In this section we study the concept of Neutrosophic bipolar vague soft set. It is a combination of neutrosophic vague set & the soft set. Further we study some of its operation and properties.

Definition 3.1: Let U be a universal set. E be a set of parameters and $A \subseteq E$. Let NBVset(U) denotes the set of all neutrosophic bipolar vague set of U. Then the pair (f, A) is called an neutrosophic bipolar vague soft set (*NBVS* set in short) over U. Here f is a mapping $f: A \to NBV\ set(u)$. The collection of all neutrosophic bipolar vague soft sets over U is denoted by $NBVS\ set(U)$.

Example 3.2: Let $U = \{u_1, u_2, u_3\}$, $E = \{e_1, e_2\}$. Then neutrosophic bipolar vague soft sets A_1 and A_2 over U are as follows:

A_1
$= [(e_1, \{(u_1, [0.3, 0.5]^p, [0.5, 0.5]^p, [0.5, 0.7]^p, [-0.3, -0.4]^n, [-0.4, -0.4]^n, [-0.6, -0.7]^n),$

$(u_2, [0.2, 0.6]^p, [0.6, 0.7]^p, [0.4, 0.8]^p, [-0.2, -0.5]^n, [-0.3, -0.5]^n, [-0.5, -0.8]^n),$

$(u_3, [0.4, 0.6]^p, [0.3, 0.4]^p, [0.4, 0.6]^p, [-0.3, -0.5]^n, [-0.4, -0.5]^n, [-0.5, -0.7]^n)\ \}),$

$(e_2, \{(u_1, [0.5, 0.6]^p, [0.3, 0.4]^p, [0.4, 0.5]^p, [-0.4, -0.5]^n, [-0.6, -0.7]^n, [-0.5, -0.6]^n),$

$(u_2, [0.3, 0.4]^p, [0.6, 0.8]^p, [0.6, 0.7]^p, [-0.4, -0.7]^n, [-0.6, -0.8]^n, [-0.3, -0.6]^n),$

$(u_3, [0.5, 0.6]^p, [0.7, 0.8]^p, [0.4, 0.5]^p, [-0.2, -0.4]^n, [-0.5, -0.6]^n, [-0.6, -0.8]^n)\ ]$

A_2
$= [(e_1, \{(u_1, [0.4, 0.5]^p, [0.3, 0.4]^p, [0.5, 0.6]^p, [-0.4, -0.5]^n, [-0.3, -0.4]^n, [-0.5, -0.6]^n),$

$(u_2, [0.3, 0.7]^p, [0.5, 0.6]^p, [0.3, 0.7]^p, [-0.3, -0.6]^n, [-0.2, -0.4]^n, [-0.4, -0.7]^n),$

$(u_3, [0.5, 0.7]^p, [0.2, 0.3]^p, [0.3, 0.5]^p, [-0.4, -0.6]^n, [-0.3, -0.4]^n, [-0.4, -0.6]^n)\ \}),$

$(e_2, \{(u_1, [0.6, 0.7]^p, [0.2, 0.4]^p, [0.3, 0.4]^p, [-0.5, -0.6]^n, [-0.5, -0.6]^n, [-0.4, -0.5]^n),$

$(u_2, [0.4, 0.5]^p, [0.5, 0.7]^p, [0.5, 0.6]^p, [-0.5, -0.8]^n, [-0.5, -0.7]^n, [-0.2, -0.5]^n),$

$(u_3, [0.6, 0.7]^p, [0.5, 0.7]^p, [0.3, 0.4]^p, [-0.3, -0.5]^n, [-0.4, -0.5]^n, [-0.5, -0.7]^n)\ ].$

Definition 3.3: An empty neutrosophic bipolar vague soft set $\emptyset$ in U is defined as

$$\emptyset = \{(e, \{(u, [0,0]^p, [0,0]^p, [1,1]^p, [-1,-1]^n, [0,0]^n, [0,0]^n)\}: e \in E \text{ and } u \in U\}.$$

Definition 3.4: An absolute neutrosophic bipolar vague soft set I in U is defined as

$$I = \{(e, \{(u, [1,1]^p, [1,1]^p, [0,0]^p, [0,0]^n, [-1,-1]^n, [-1,-1]^n)\}: e \in E \text{ and } u \in U\}.$$

Example 3.5: Let $U = \{u_1, u_2, u_3\}, E = \{e_1, e_2\}$ then

$\emptyset = \{(e_1, (u_1, [0,0]^p, [0,0]^p, [1,1]^p, [-1,-1]^n, [0,0]^n, [0,0]^n),$

$(u_2, [0,0]^p, [0,0]^p, [1,1]^p, [-1,-1]^n, [0,0]^n, [0,0]^n),$

$(u_3, [0,0]^p, [0,0]^p, [1,1]^p, [-1,-1]^n, [0,0]^n, [0,0]^n)$,

$(e_2, (u_1, [0,0]^p, [0,0]^p, [1,1]^p, [-1,-1]^n, [0,0]^n, [0,0]^n),$

$(u_2, [0,0]^p, [0,0]^p, [1,1]^p, [-1,-1]^n, [0,0]^n, [0,0]^n),$

$(u_3, [0,0]^p, [0,0]^p, [1,1]^p, [-1,-1]^n, [0,0]^n, [0,0]^n)$

(a) Absolute neutrosophic bipolar vague soft set I in U is defined as

$I = \{(e_1, (u_1, [1,1]^p, [1,1]^p, [0,0]^p, [0,0]^n, [-1,-1]^n, [-1,-1]^n),$

$(u_2, [1,1]^p, [1,1]^p, [0,0]^p, [0,0]^n, [-1,-1]^n, [-1,-1]^n)$,

$(u_3, [1,1]^p, [1,1]^p, [0,0]^p, [0,0]^n, [-1,-1]^n, [-1,-1]^n),$

$(e_2, (u_1, [1,1]^p, [1,1]^p, [0,0]^p, [0,0]^n, [-1,-1]^n, [-1,-1]^n),$

$(u_2, [1,1]^p, [1,1]^p, [0,0]^p, [0,0]^n, [-1,-1]^n, [-1,-1]^n)$,

$(u_3, [1,1]^p, [1,1]^p, [0,0]^p, [0,0]^n, [-1,-1]^n, [-1,-1]^n)\}$

Definition 3.6: $C^i = \{(e, (u, (\widetilde{T}^{\xi}_{C^i_{NBVS}})^p(u), (\widetilde{I}^{\xi}_{C^i_{NBVS}})^p(u), (\widetilde{F}^{\xi}_{C^i_{NBVS}})^p(u),$

$(\widetilde{T}^{\xi}_{C^i_{NBVS}})^n(u), (\widetilde{I}^{\xi}_{C^i_{NBVS}})^n(u), (\widetilde{F}^{\xi}_{C^i_{NBVS}})^n(u) >: u \in U, e \in E\}$. Where $i = 1, 2$ be two neutrosophic bipolar vague soft set over U. then C^1 is neutrosophic bipolar vague soft subset of C^2 is denoted by $C^1 \subseteq C^2$ if

$(\tilde{T}_{C^1_{NBVS}})^p(u) \leq (\tilde{T}_{C^2_{NBVS}})^p(u)$, And $(\tilde{T}_{C^1_{NBVS}})^n(u) \geq (\tilde{T}_{C^2_{NBVS}})^n(u)$,

$(\tilde{I}_{C^1_{NBVS}})^p(u) \geq (\tilde{I}_{C^2_{NBVS}})^p(u)$, $(\tilde{I}_{C^1_{NBVS}})^n(u) \leq (\tilde{I}_{C^2_{NBVS}})^n(u)$,

$(\tilde{F}_{C^1_{NBVS}})^p(u) \geq (\tilde{F}_{C^2_{NBVS}})^p(u)$ $(\tilde{F}_{C^1_{NBVS}})^n(u) \leq (\tilde{F}_{C^2_{NBVS}})^n(u)$.

Example 3.7: Consider the example 3.2. In this case $A_1 \subseteq A_2$ as per our definition 3.6.

Definition 3.8: Let *A* be a neutrosophic bipolar vague soft set over$U$. Then the complement of a neutrosophic bipolar vague soft set *A* is denoted by A^c is defined as

$A^c = \{(e, (u, (\tilde{T}_{A^c_{NBVS}})^p(u), (\tilde{I}_{A^c_{NBVS}})^p(u), (\tilde{F}_{A^c_{NBVS}})^p(u),$

$(\tilde{T}_{A^c_{NBVS}})^n(u), (\tilde{I}_{A^c_{NBVS}})^n(u), (\tilde{F}_{A^c_{NBVS}})^n(u)\}$.

$(\tilde{T}_{A^c_{NBVS}})^p(u) = \{(1 - T^+(u))^p, (1 - T^-(u))^p\}$,

$(\tilde{I}_{A^c_{NBVS}})^p(u) = \{(1 - I^+(u))^p, (1 - I^-(u))^p\}$,

$(\tilde{F}_{A^c_{NBVS}})^p(u) = \{(1 - F^+(u))^p, (1 - F^-(u))^p\}$ and

$(\tilde{T}_{A^c_{NBVS}})^n(u) = \{(-1 - T^+(u))^n, (-1 - T^-(u))^n\}$,

$(\tilde{I}_{A^c_{NBVS}})^n(u) = \{(-1 - I^+(u))^n, (-1 - I^-(u))^n\}$,

$(\tilde{F}_{A^c_{NBVS}})^n(u) = \{(-1 - F^+(u))^n, (-1 - F^-(u))^n\}$.

Example 3.9: Let $U = \{u_1, u_2\}, E = \{e_1, e_2\}$ then the neutrosophic bipolar vague soft set *A* is

$$A = [(e_1, \{(u_1, [0.1,0.3]^p, [0.2,0.4]^p, [0.7,0.9]^p, [-0.5,-0.2]^n, [-0.6,-0.4]^n, [-0.8,-0.5]^n)\},$$

$$\{(u_2, [0.7,0.9]^p, [0.2,0.4]^p, [0.1,0.3]^p, [-0.6,-0.2]^n, [-0.7,-0.3]^n, [-0.8,-0.4]^n),$$

$$(e_2, \{(u_1, [0.7,0.9]^p, [0.2,0.5]^p, [0.1,0.3]^p, [-0.9,-0.8]^n, [-0.4,-0.2]^n, [-0.2,-0.1]^n)\},$$

$$\{(u_2, [0.8,0.9]^p, [0.5,0.6]^p, [0.1,0.2]^p, [-0.7,-0.5]^n, [-0.7,-0.4]^n, [-0.5,-0.3]^n)$$

Then the complement of *A* is A^c is as

$$A^c = [(e_1, \{(u_1, [0.7,0.9]^p, [0.6,0.8]^p, [0.1,0.3]^p, [-0.8,-0.5]^n, [-0.6,-0.4]^n, [-0.5,-0.2]^n)\},$$

$$\{(u_2, [0.1,0.3]^p, [0.6,0.8]^p, [0.7,0.9]^p, [-0.8,-0.4]^n, [-0.7,-0.3]^n, [-0.6,-0.2]^n),$$

$$(e_2, \{(u_1, [0.1,0.3]^p, [0.5,0.8]^p, [0.7,0.9]^p, [-0.2,-0.1]^n, [-0.8,-0.6]^n, [-0.9,-0.8]^n)\},$$

$$\{(u_2, [0.1,0.2]^p, [0.4,0.5]^p, [0.8,0.9]^p, [-0.5,-0.3]^n, [-0.6,-0.3]^n, [-0.7,-0.5]^n)$$

Definition 3.10: Let

$A^i = \{(e, (u, (\tilde{T}_{A^i_{NBVS}})^p(u), (\tilde{I}_{A^i_{NBVS}})^p(u), (\tilde{F}_{A^i_{NBVS}})^p(u),$

$(\tilde{T}_{A^i_{NBVS}})^n(u), (\tilde{I}_{A^i_{NBVS}})^n(u), (\tilde{F}_{A^i_{NBVS}})^n(u)\}\ e \in E, u \in U, i = 1,2$. Then the union and intersection of A^1 and A^2 of two neutrosophic bipolar vague soft set are defined as follows:

(a) $A^1 \cup A^2 = A^3$

$$= \{(e, (u, (\tilde{T}_{A^3_{NBVS}})^p(u), (\tilde{I}_{A^3_{NBVS}})^p(u), (\tilde{F}_{A^3_{NBVS}})^p(u),$$

$$(\tilde{T}_{A^3_{NBVS}})^n(u), (\tilde{I}_{A^3_{NBVS}})^n(u), (\tilde{F}_{A^3_{NBVS}})^n(u)\}$$

Where

$$(\tilde{T}_{A^3_{NBVS}})^p(u) =$$

$$\{((\tilde{T}^-_{A^1_{NBVS}})^p(u) \vee \left(\tilde{T}^-_{A^2_{NBVS}}\right)^p(u)), \left(\tilde{T}^+_{A^1_{NBVS}}\right)^p(u) \vee \left(\tilde{T}^+_{A^2_{NBVS}}\right)^p(u))\}$$

$$(\tilde{I}_{A^3_{NBVS}})^p(u) = \{((\tilde{I}^-_{A^1_{NBVS}})^p(u) \wedge \left(\tilde{I}^-_{A^2_{NBVS}}\right)^p(u)),\ \left(\tilde{I}^+_{A^1_{NBVS}}\right)^p(u) \wedge \left(\tilde{I}^+_{A^2_{NBVS}}\right)^p(u))\}$$

$$(\tilde{F}_{A^3_{NBVS}})^p(u) = \{((\tilde{F}^-_{A^1_{NBVS}})^p(u) \wedge \left(\tilde{F}^-_{A^2_{NBVS}}\right)^p(u)),\ \left(\tilde{F}^+_{A^1_{NBVS}}\right)^p(u) \wedge \left(\tilde{F}^+_{A^2_{NBVS}}\right)^p(u))\}$$

And

$$(\tilde{T}_{A^3_{NBVS}})^n(u) = \{((\tilde{T}^-_{A^1_{NBVS}})^n(u) \wedge \left(\tilde{T}^-_{A^2_{NBVS}}\right)^n(u)),\ \left(\tilde{T}^+_{A^1_{NBVS}}\right)^n(u) \wedge \left(\tilde{T}^+_{A^2_{NBVS}}\right)^n(u))\}$$

$$(\tilde{I}_{A^3_{NBVS}})^n(u) = \{((\tilde{I}^-_{A^1_{NBVS}})^n(u) \vee \left(\tilde{I}^-_{A^2_{NBVS}}\right)^n(u)),\ \left(\tilde{I}^+_{A^1_{NBVS}}\right)^n(u) \vee \left(\tilde{I}^+_{A^2_{NBVS}}\right)^n(u))\}$$

$$(\tilde{F}_{A^3_{NBVS}})^n(u) = \{((\tilde{F}^-_{A^1_{NBVS}})^n(u) \vee \left(\tilde{F}^-_{A^2_{NBVS}}\right)^n(u)),\ \left(\tilde{F}^+_{A^1_{NBVS}}\right)^n(u) \vee \left(\tilde{F}^+_{A^2_{NBVS}}\right)^n(u))\}$$

(b) $A^1 \cap A^2 = A^4$

$$= \{(e, (u, (\tilde{T}_{A^4_{NBVS}})^p(u), (\tilde{I}_{A^4_{NBVS}})^p(u), (\tilde{F}_{A^4_{NBVS}})^p(u),$$

$$(\tilde{T}_{A^4_{NBVS}})^n(u), (\tilde{I}_{A^4_{NBVS}})^n(u), (\tilde{F}_{A^4_{NBVS}})^n(u)\}$$

Where

$$(\tilde{T}_{A^4_{NBVS}})^p(u) = \{((\tilde{T}^-_{A^1_{NBVS}})^p(u) \wedge \left(\tilde{T}^-_{A^2_{NBVS}}\right)^p(u)),\ \left(\tilde{T}^+_{A^1_{NBVS}}\right)^p(u) \wedge \left(\tilde{T}^+_{A^2_{NBVS}}\right)^p(u))\}$$

$$(\tilde{I}_{A^4_{NBVS}})^p(u) = \{((\tilde{I}^-_{A^1_{NBVS}})^p(u) \vee \left(\tilde{I}^-_{A^2_{NBVS}}\right)^p(u)),\ \left(\tilde{I}^+_{A^1_{NBVS}}\right)^p(u) \vee \left(\tilde{I}^+_{A^2_{NBVS}}\right)^p(u))\}$$

$$(\tilde{F}_{A^4_{NBVS}})^p(u) = \{((\tilde{F}^-_{A^1_{NBVS}})^p(u) \vee \left(\tilde{F}^-_{A^2_{NBVS}}\right)^p(u)),\ \left(\tilde{F}^+_{A^1_{NBVS}}\right)^p(u) \vee \left(\tilde{F}^+_{A^2_{NBVS}}\right)^p(u))\}$$

And

$$(\tilde{T}_{A^4_{NBVS}})^n(u) = \{((\tilde{T}^-_{A^1_{NBVS}})^n(u) \vee \left(\tilde{T}^-_{A^2_{NBVS}}\right)^n(u)),\ \left(\tilde{T}^+_{A^1_{NBVS}}\right)^n(u) \vee \left(\tilde{T}^+_{A^2_{NBVS}}\right)^n(u))\}$$

$$(\tilde{I}_{A^4_{NBVS}})^n(u) = \{((\tilde{I}^-_{A^1_{NBVS}})^n(u) \wedge \left(\tilde{I}^-_{A^2_{NBVS}}\right)^n(u)),\ \left(\tilde{I}^+_{A^1_{NBVS}}\right)^n(u) \wedge \left(\tilde{I}^+_{A^2_{NBVS}}\right)^n(u))\}$$

$$(\tilde{F}_{A^4_{NBVS}})^n(u) = \{((\tilde{F}^-_{A^1_{NBVS}})^n(u) \wedge \left(\tilde{F}^-_{A^2_{NBVS}}\right)^n(u)),\ \left(\tilde{F}^+_{A^1_{NBVS}}\right)^n(u) \wedge \left(\tilde{F}^+_{A^2_{NBVS}}\right)^n(u))\}$$

Example 3.11. Consider the example 3.2 then

$A_1 \cup A_2 =$
$[(e_1, \{(u_1, [0.4, 0.5]^p, [0.3, 0.4]^p, [0.5, 0.6]^p, [-0.5, -0.4]^n, [-0.4, -0.3]^n, [-0.6, -0.5]^n),$

$(u_2, [0.3, 0.7]^p, [0.5, 0.6]^p, [0.3, 0.7]^p, [-0.6, -0.3]^n, [-0.1, -0.2]^n, [-0.7, -0.4]^n),$

$(u_3, [0.5, 0.7]^p, [0.2, 0.3]^p, [0.3, 0.5]^p, [-0.6, -0.4]^n, [-0.4, -0.3]^n, [-0.6, -0.4]^n)\}),$

$(e_2, \{(u_1, [0.6, 0.7]^p, [0.2, 0.4]^p, [0.3, 0.4]^p, [-0.6, -0.5]^n, [-0.6, -0.5]^n, [-0.6, -0.4]^n),$

$(u_2, [0.4, 0.5]^p, [0.5, 0.7]^p, [0.5, 0.6]^p, [-0.8, -0.5]^n, [-0.7, -0.5]^n, [-0.5, -0.2]^n),$

$(u_3, [0.6, 0.7]^p, [0.5, 0.7]^p, [0.3, 0.4]^p, [-0.5, -0.3]^n, [-0.5, -0.4]^n, [-0.7, -0.5]^n)] = A_2$

Similarly $A_1 \cap A_2 = A_1$.

Definition 3.12 Let $A = \{(e, \{((\tilde{T}_{C_{NBVS}})^p(u), (\tilde{I}_{C_{NBVS}})^p(u), (\tilde{F}_{C_{NBVS}})^p(u), (\tilde{T}_{C_{NBVS}})^n(u),$ $(\tilde{I}_{C_{NBVS}})^n(u), (\tilde{F}_{C_{NBVS}})^n(u)\}: : u \in U \text{ and } e \in E\}$ be a neutrosophic bipolar vague soft set over U. then aggregation neutrosophic bipolar vague soft operator denoted by A_{agg} is defined as

$$A_{agg} = \left\{\frac{[\mu_A^+(u), \mu_A^-(u)]}{u} : u \in U\right\}.$$

Where $[\mu_A^+(u), \mu_A^-(u)]$

$$= \frac{1}{2|E \times U|}\left[\sum_{e \in E}([1,1] - (\tilde{I}_e)^p(u), [(\tilde{T}_e)^p(u) - (\tilde{F}_e)^p(u)]\right.$$

$$\left. + ((\tilde{I}_e)^n(u)[(\tilde{T}_e)^p(u) - (\tilde{F}_e)^p(u)]\right]$$

Where

$$(\tilde{I}_e)^p(u) = [(I_e^+)^p(u) - (I_e^-)^p(u)]$$

$$(\tilde{T}_e)^p(u) = [(T_e^+)^p(u) - (T_e^-)^p(u)]$$

$$(\tilde{F}_e)^p(u) = [(F_e^+)^p(u) - (F_e^-)^p(u)]$$

$$(\tilde{I}_e)^n(u) = [(I_e^+)^n(u) - (I_e^-)^n(u)]$$

$$(\tilde{T}_e)^n(u) = [(T_e^+)^n(u) - (T_e^-)^n(u)]$$

$$(\tilde{F}_e)^n(u) = [(F_e^+)^n(u) - (F_e^-)^n(u)]$$

Where $|E \times U|$ is the cardinality of $E \times U$.

4. Application of neutrosophic bipolar vague soft set.

We develop an algorithm based on neutrosophic bipolar vague soft sets and give numerical example to show the fossibility and effectiveness of the approaches in definition 3.12.

Algorithm

1. First we construct the neutrosophic bipolar vague soft set on U.
2. Then we compute the neutrosophic bipolar vague soft set aggregation operator.
3. Average of each intervals and find $|A_{agg}|$.
4. Find the optimum value on U.

Assume that a farm wants to fill a position in the office. There are three candidates for the post. The selection committee use the neutrosophic bipolar vague soft decision making method. Assume that the set of candidate $U = \{u_1, u_2, u_3\}$ which may be characterized by a set of parameters $E = \{e_1, e_2, e_3\}$. Where e_1= "experience", e_2 = "technical knowledge", e_3= "age".

(a) The selection committee construct a neutrosophic bipolar vague soft set *A* over the set U as

A_2
$= \{(e_1, \{(u_1, [0.8, 0.9]^p, [0.5, 0.7]^p, [0.1, 0.2]^p, [-0.5, -0.3]^n, [-0.7, -0.5]^n, [-0.7, -0.5]^n),$

$(u_2, [0.5, 0.7]^p, [0.4, 0.6]^p, [0.3, 0.5]^p, [-0.5, -0.4]^n, [-0.7, -0.5]^n, [-0.6, -0.5]^n),$

$(u_3, [0.5, 0.7]^p, [0.5, 0.6]^p, [0.3, 0.5]^p, [-0.8, -0.6]^n, [-0.5, -0.3]^n, [-0.4, -0.2]^n) \}),$

$(e_2, \{(u_1, [0.5, 0.7]^p, [0.4, 0.6]^p, [0.3, 0.5]^p, [-0.4, -0.2]^n, [-0.2, -0.1]^n, [-0.8, -0.6]^n),$

$(u_2, [0.7, 0.9]^p, [0.4, 0.6]^p, [0.1, 0.3]^p, [-0.6, -0.4]^n, [-0.3, -0.2]^n, [-0.6, -0.4]^n),$

$(u_3, [0.2, 0.4]^p, [0.8, 0.9]^p, [0.6, 0.8]^p, [-0.3, -0.1]^n, [-0.5, -0.3]^n, [-0.9, -0.7]^n) \},$

$(e_3, \{(u_1, [0.7, 0.9]^p, [0.2, 0.4]^p, [0.1, 0.3]^p, [-0.3, -0.2]^n, [-0.5, -0.3]^n, [-0.8, -0.7]^n),$

$(u_2, [0.6, 0.8]^p, [0.4, 0.6]^p, [0.2, 0.4]^p, [-0.4, -0.2]^n, [-0.5, -0.4]^n, [-0.8, -0.6]^n),$

$(u_3, [0.3, 0.5]^p, [0.5, 0.7]^p, [0.5, 0.7]^p, [-0.3, -0.2]^n, [-0.6, -0.4]^n, [-0.8, -0.7]^n)$ }.

(b) Then we find the neutrosophic vague soft set aggregation operator A_{agg} of A_2 as

For u_1 ,

$$\frac{1}{18}[[1,1] - [0.5, 0.7]([0.8, 0.9] - [0.1, 0.2]) + [-0.7, -0.5]([-0.5, -0.3] - [-0.7, -0.5]) + [1,1] - [0.4, 0.6]([0.5, 0.7] - [0.3, 0.5]) + [-0.2, -0.1]([-0.4, -0.2] - [-0.8, -0.6])$$

$$+[1,1] - [0.2, 0.4]([0.7, 0.9] - [0.1, 0.3]) +$$

$$[-0.5, -0.3]([-0.3, -0.2] - [-0.8, -0.7])]$$

For u_2 ,

$$\frac{1}{18}[[1,1] - [0.4, 0.6]([0.5, 0.7] - [0.3, 0.5]) + [-0.7, -0.5]([-0.5, -0.4] - [-0.6, -0.5]) + [1,1] - [0.4, 0.6]([0.7, 0.9] - [0.1, 0.3]) + [-0.3, -0.2]([-0.6, -0.4] - [-0.6, -0.4])$$

$$+[1,1] - [0.4, 0.6]([0.6, 0.8] - [0.2, 0.4]) +$$

$$[-0.5, -0.4]([-0.4, -0.2] - [-0.8, -0.6])]$$

For u_3 ,

$$\frac{1}{18}[[1,1] - [0.5, 0.6]([0.5, 0.7] - [0.3, 0.5]) + [-0.7, -0.5]([-0.8, -0.6] - [-0.4, -0.2]) + [1,1] - [0.8, 0.9]([0.2, 0.4] - [0.6, 0.8]) + [-0.5, -0.3]([-0.3, -0.1] - [-0.9, -0.7])$$

$$+[1,1] - [0.5, 0.7]([0.3, 0.5] - [0.5, 0.7]) +$$

$$[-0.6, -0.4]([-0.3, -0.2] - [-0.8, -0.7])]$$

(c) We take the average of each interval.
i.e. [1,1]=1,

$$(\tilde{T}_e)^p(x) = [(T_e^-)^p(x), (T_e^+)^p(x)]$$

$$(\tilde{I}_e)^p(x) = [(I_e^-)^p(x), (I_e^+)^p(x)]$$

$$(\tilde{F}_e)^p(x) = [(F_e^-)^p(x), (F_e^+)^p(x)]$$

$$(\tilde{T}_e)^n(x) = [(T_e^-)^n(x), (T_e^+)^n(x)]$$

$$(\tilde{I}_e)^n(x) = [(I_e^-)^n(x), (I_e^+)^n(x)]$$

$$(\tilde{F}_e)^n(x) = [(F_e^-)^n(x), (F_e^+)^n(x)]$$

(d) The $|A_{agg}| = \frac{0.1006}{u_1}, \frac{0.1311}{u_2}, \frac{0.1455}{u_3}$

(e) Finally the selection committee choose u_3 for the post since $|A_{agg}|$ has the maximum degree 0.1455 among them.

5. Conclusion

In this paper, we introduce the neutrosophic bipolar vague soft set. It is a combination of soft set and the neutrosophic bipolar vague set. We develop a decision making method based on neutrosophic bipolar vague soft set. A numerical example has beengiven. Some new operations on neutrosophic bipolar vague soft set have been designed. For further study, it may be applied to real world problems with realistic data and extend proposed algorithm to other decision making problem with vagueness and uncertainty. Here we require less calculations and few steps to get our result.

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Conflicts of Interest

The authors declare no conflict of interest.

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