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Neutrosophic Bi-LA-Semigroup and Neutrosophic N-LA-Semigroup

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Abstract. In this paper we define neutrosophic bi-LA-semigroup and neutrosophic N-LA-semigroup. Infact this paper is an extension of our previous paper neutrosophic left almost semigroup shortly neutrosophic LA-semigroup. We also extend the neutrosophic ideal to neutrosophic biideal and neutrosophic N-ideal. We also find some new type of neutrosophic ideal which is related to the strong or pure part of neutrosophy. We have given sufficient amount of examples to illustrate the theory of neutrosophic bi-LA-semigroup, neutrosophic N-LA-semigroup and display many properties of them this paper.

Keywords: Neutrosophic LA-semigroup, neutrosophic ideal, neutrosophic bi-LA-semigroup, neutrosophic biideal, neutrosophic N-LA-semigroup, neutrosophic N-ideal.

1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set $[1]$, intuitionistic fuzzy set $[2]$ and interval valued fuzzy set $[3]$. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures in $[11]$. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

A left almost semigroup abbreviated as LA-semigroup is an algebraic structure which was introduced by M .A. Kazim and M. Naseeruddin $[3]$ in 1972. This structure is basically a midway structure between a groupoid and a commutative semigroup. This structure is also termed as Able-Grassmann's groupoid abbreviated as AG-groupoid $[6]$. This is a non associative and non commutative algebraic structure which closely resemble to commutative semigroup. The generalization of semigroup theory is an LA-semigroup and this structure has wide applications in collaboration with semigroup.

We have tried to develop the ideal theory of LA-semigroups in a logical manner. Firstly, preliminaries and basic concepts are given for neutrosophic LA-semigroup. Then we presented the newly defined notions and results in neutrosophic bi-LA-semigroups and neutrosophic N-LA-semigroups. Various types of neutrosophic biideals and neutrosophic N-ideal are defined and elaborated with the help of examples.

2 Preliminaries

Definition 1. Let $(S,*)$ be an LA-semigroup and let $\langle S\cup I\rangle=\{a+bI : a,b\in S\}$. The neutrosophic LA-semigroup is generated by S and I under $*$ denoted as $N(S)=\{\langle S\cup I\rangle,*\}$, where I is called the neutrosophic element with property $I^2=I$. For an integer n, $n+I$ and nI are neutrosophic elements and

$0.I = 0.I^{-1}$, the inverse of I is not defined and hence does not exist.
Similarly we can define neutrosophic RA-semigroup on the same lines.

Definition 2. Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is called a neutrosophic sub LA-semigroup if $N(H)$ itself is a neutrosophic LA-semigroup under the operation of $N(S)$.

Definition 3. A neutrosophic sub LA-semigroup $N(H)$ is called strong neutrosophic sub LA-semigroup or pure neutrosophic sub LA-semigroup if all the elements of $N(H)$ are neutrosophic elements.

Definition 4. Let $N(S)$ be a neutrosophic LA-semigroup and $N(K)$ be a subset of $N(S)$. Then $N(K)$ is called Left (right) neutrosophic ideal of $N(S)$ if $N(S)N(K) \subseteq N(K)$
{ $N(K)N(S) \subseteq N(K)$ }.
If $N(K)$ is both left and right neutrosophic ideal, then $N(K)$ is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

Definition 5. A neutrosophic ideal $N(K)$ is called strong neutrosophic ideal or pure neutrosophic ideal if all of its elements are neutrosophic elements.

3 Neutrosophic Bi-LA-Semigroup

Definition 6. Let $(BN(S),*,\circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(S),*,\circ)$ is said to be a neutrosophic bi-LA-semigroup if $BN(S) = P_1 \cup P_2$ where atleast one of $(P_1,*)$ or $(P_2,\circ)$ is a neutrosophic LA-semigroup and other is just an LA- semigroup. P_1 and P_2 are proper subsets of $BN(S)$.
Similarly we can define neutrosophic bi-RA-semigroup on the same lines.

Theorem 1. All neutrosophic bi-LA-semigroups contains the corresponding bi-LA-semigroups.

Example 1. Let $BN(S) = \{\langle S_1 \cup I\rangle \cup \langle S_2 \cup I\rangle\}$ be a neutrosophic bi-LA-semigroup where

$\langle S_1 \cup I\rangle = \{1,2,3,4,1I,2I,3I,4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

$\langle S_2 \cup I\rangle = \{1,2,3,1I,2I,3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Definition 7. Let $(BN(S) = P_1 \cup P;:*,\circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T,\circ,*)$ is said to be a neutrosophic sub bi-LA-semigroup of $BN(S)$ if

1. $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and
2. At least one of $(T_1,\circ)$ or $(T_2,*)$ is a neutrosophic LA-semigroup.

Example 2: $BN(S)$ be a neutrosophic bi-LA-semigroup in Example 1. Then $P = \{1,1I\} \cup \{3,3I\}$ and $Q = \{2,2I\} \cup \{1,1I\}$ are neutrosophic sub bi-LA-semigroups of $BN(S)$.

Theorem 2. Let $BN(S)$ be a neutrosophic bi-LA-semigroup and $N(H)$ be a proper subset of $BN(S)$. Then $N(H)$ is a neutrosophic sub bi-LA-semigroup of $BN(S)$ if $N(H).N(H) \subseteq N(H)$.

Definition 8. Let $(BN(S) = P_1 \cup P, *, \circ)$ be any neutrosophic bi-LA-semigroup. Let J be a proper subset of $BN(S)$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic biideal of $BN(S)$.

Example 3. Let $BN(S) = \{\langle S_1 \cup I\rangle \cup \langle S_2 \cup I\rangle\}$ be a neutrosophic bi-LA-semigroup, where

$\langle S_1 \cup I\rangle = \{1,2,3,1I,2I,3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

And $\langle S_2 \cup I\rangle = \{1,2,3,I,2I,3I\}$ be another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Then $P = \{1,1\mathrm{I},3,3I\} \cup \{2,2I\}$, $Q = \{1,3,1I,3I\} \cup \{2,3,2\mathrm{I},3\mathrm{I}\}$ are neutrosophic biideals of $BN(S)$.

Proposition 1. Every neutrosophic biideal of a neutrosophic bi-LA-semigroup is trivially a Neutrosophic sub bi-LA-semigroup but the conver is not true in general.
One can easily see the converse by the help of example.

3 Neutrosophic Strong Bi-LA-Semigroup

Definition 9: If both $(P_1,*)$ and $(P_2,\circ)$ in the Definition 6. are neutrosophic strong LA-semigroups then we call $(BN(S),*,\circ)$ is a neutrosophic strong bi-LA-semigroup.

Definition 10. Let $(BN(S) = P_1 \cup P, *, \circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T,\circ,*)$ is said to be a neutrosophic strong sub bi-LA-semigroup of $BN(S)$ if

1. $T = T_1 \cup T_2$ where $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and
2. $(T_1,\circ)$ and $(T_2,*)$ are neutrosophic strong LA-semigroups.

Example 4. Let $BN(S)$ be a neutrosophic bi-LA-semigroup in Example 3.
Then $P = \{1\mathrm{I},3I\} \cup \{2I\}$, and $Q = \{1I,3I\} \cup \{2\mathrm{I},3\mathrm{I}\}$ are neutrosophic strong sub bi-LA-semigroup of $BN(S)$.

Theorem 4: Every neutrosophic strong sub bi-LA-semigroup is a neutrosophic sub bi-LA-semigroup.

Definition 11. Let $(BN(S),*,\circ)$ be a strong neutrosophic bi-LA-semigroup where $BN(S) = P_1 \cup P_2$ with $(P_1,*)$ and $(P_2,\circ)$ be any two neutrosophic LA-semigroups. Let J be a proper subset of $BN(S)$ where $I = I_1 \cup I_2$ with $I_1 = I \cap P_1$ and $I_2 = I \cap P_2$ are neutrosophic ideals of the neutrosophic LA-semigroups P_1 and P_2 respectively. Then I is called or defined as the neutrosophic strong biideal of $BN(S)$.

Theorem 5: Every neutrosophic strong biideal is trivially a neutrosophic sub bi-LA-semigroup.

Theorem 6: Every neutrosophic strong biideal is a neutrosophic strong sub bi-LA-semigroup.

Theorem 7: Every neutrosophic strong biideal is a neutrosophic biideal.

Example 5. Let $BN(S)$ be a neutrosophic bi-LA semigroup in Example (*) .Then $P=\{1I,3I\}\cup\{2I\}$, and $Q=\{1I,3I\}\cup\{2I,3I\}$ are neutrosophic strong biideal of $BN(S)$.

4 Neutrosophic N-LA-Semigroup

Definition 12. Let $\{S(N),*_1,...,*_2\}$ be a non-empty set with N -binary operations defined on it. We call $S(N)$ a neutrosophic N -LA-semigroup (N a positive integer) if the following conditions are satisfied.

1) $S(N)=S_1\cup...S_N$ where each S_i is a proper subset of $S(N)$ i.e. $S_i\subset S_j$ or $S_j\subset S_i$ if $i\neq j$.
2) $(S_i,*_i)$ is either a neutrosophic LA-semigroup or an LA-semigroup for $i=1,2,3,...,N$.

Example 6. Let $S(N)=\{S_1\cup S_2\cup S_3,*_1,*_2,*_3\}$ be a neutrosophic 3-LA-semigroup where $S_1=\{1,2,3,4,1I,2I,3I,4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

$S_2=\{1,2,3,1I,2I,3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

And $S_3=\{1,2,3,I,2I,3I\}$ is another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Theorem 8 All neutrosophic N-LA-semigroups contains the corresponding N-LA-semigroups.

Definition 13. Let $S(N)=\{S_1\cup S_2\cup....S_N,*_1,*_2,...,*_N\}$ be a neutrosophic N -LA-semigroup. A proper subset $P=\{P_1\cup P_2\cup....P_N,*_1,*_2,...,*_N\}$ of $S(N)$ is said to be a neutrosophic sub N -LA-semigroup if $P_i=P\cap S_i, i=1,2,...,N$ are sub LA-semigroups of S_i in which atleast some of the sub LA-semigroups are neutrosophic sub LA-semigroups.

Example 7: Let $S(N)=\{S_1\cup S_2\cup S_3,*_1,*_2,*_3\}$ be a neutrosophic 3-LA-semigroup in above Example 6. Then clearly $P=\{1,1I\}\cup\{2,3,3I\}\cup\{2,2I\}$, $Q=\{2,2I\}\cup\{1,3,1I,3I\}\cup\{2,3,2I,3I\}$, and

$R=\{4,4I\}\cup\{1I,3I\}\cup\{2\text{I},3\text{I}\}$ are neutrosophic sub 3-LA-semigroups of S(N).

Theorem 19. Let $N(S)$ be a neutrosophic N-LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is a neutrosophic sub N-LA-semigroup of $N(S)$ if $N(H).N(H)\subseteq N(H)$.

Definition 14. Let $S(N)=\{S_1\cup S_2\cup ....S_N,*_1,*_2,...,*_N\}$ be a neutrosophic N -LA-semigroup. A proper subset $P=\{\text{P}_1\cup P_2\cup ....\cup P_N,*_1,*_2,...,*_N\}$ of $S(N)$ is said to be a neutrosophic N -ideal, if the following conditions are true,

1. P is a neutrosophic sub N -LA-semigroup of $S(N)$.
2. Each $P_i=S\cap P_i, i=1,2,...,N$ is an ideal of S_i.

Example 8. Consider Example 6.
Then $I_1=\{1,1I\}\cup\{3,3I\}\cup\{2,2I\}$, and $I_2=\{2,2I\}\cup\{1I,3I\}\cup\{2,3,3\text{I}\}$ are neutrosophic 3-ideals of $S(N)$.

Theorem 10: Every neutrosophic N-ideal is trivially a neutrosophic sub N-LA-semigroup but the converse is not true in general.

One can easily see the converse by the help of example.

5 Neutrosophic Strong N-LA-Semigroup

Definition 15: If all the N -LA-semigroups $(S_i,*_i)$ in Definition ( ) are neutrosophic strong LA-semigroups (i.e. for $i=1,2,3,...,N$) then we call $S(N)$ to be a neutrosophic strong N -LA-semigroup.

Definition 16. Let $S(N)=\{S_1\cup S_2\cup ....S_N,*_1,*_2,...,*_N\}$ be a neutrosophic strong N -LA-semigroup. A proper subset $T=\{\text{T}_1\cup T_2\cup ....\cup \text{T}_N,*_1,*_2,...,*_N\}$ of $S(N)$ is said to be a neutrosophic strong sub N -LA-semigroup if each $(T_i,*_i)$ is a neutrosophic strong sub LA-semigroup of $(S_i,*_i)$ for $i=1,2,...,N$ where $T_i=S_i\cap T$.

Theorem 11: Every neutrosophic strong sub N-LA-semigroup is a neutrosophic sub N-LA-semigroup.

Definition 17. Let $S(N)=\{S_1\cup S_2\cup ....S_N,*_1,*_2,...,*_N\}$ be a neutrosophic strong N -LA-semigroup. A proper subset $J=\{\text{J}_1\cup J_2\cup ....\text{J}_N,*_1,*_2,...,*_N\}$ where $J_t=J\cap S_t$ for $t=1,2,...,N$ is said to be a neutrosophic strong N -ideal of $S(N)$ if the following conditions are satisfied.

1) Each it is a neutrosophic sub LA-semigroup of $S_t, t=1,2,...,N$ i.e. It is a neutrosophic strong N-sub LA-semigroup of $S(N)$.
2) Each it is a two sided ideal of S_t for $t=1,2,...,N$.

Similarly one can define neutrosophic strong N -left ideal or neutrosophic strong right ideal of $S(N)$.
A neutrosophic strong N -ideal is one which is both a neutrosophic strong N -left ideal and N -right ideal of $S(N)$.

Theorem 12: Every neutrosophic strong Nideal is trivially a neutrosophic sub N-LA-semigroup.

Theorem 13: Every neutrosophic strong N-ideal is a neutrosophic strong sub N-LA-semigroup.

Theorem 14: Every neutrosophic strong N-ideal is a N-ideal.

Conclusion

In this paper we extend neutrosophic LA-semigroup to neutrosophic bi-LA-semigroup and neutrosophic N-LA-semigroup. The neutrosophic ideal theory of neutrosophic LA-semigroup is extend to neutrosophic biideal and neutrosophic N-ideal. Some new type of neutrosophic ideals are discovered which is strongly neutrosophic or purely neutrosophic. Related examples are given to illustrate neutrosophic bi-LA-semigroup, neutrosophic N-LA-semigroup and many theorems and properties are discussed.

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