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On The Symbolic 2-Plithogenic Weak Fuzzy Complex Numbers

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Abstract:

The goal of this paper is to define for the first time the concept of symbolic 2-plithogenic weak fuzzy complex number as new generalization generated by combining real numbers with symbolic 2-plithogenic numbers.

We study the elementary properties of this new class such as Invertibility and nilpotency, with many related examples that explain its novelty.

Keywords: symbolic 2-plithogenic number, weak fuzzy complex number, real number.

Introduction and preliminaries.

The concept of weak fuzzy complex numbers was defined firstly in [7] by the following form: $C_w = \{a + bJ;\ J^2 = t \in]0,1[, a, b \in R\}$.

It is clear that C_w contains the real field R.

Weak fuzzy complex numbers were used to study vector space theory in [10], and programmed with Python [3].

Weak fuzzy complex numbers and their similar real extensions [8-9,15] are very useful in algebraic studies and computer science, especially split-complex numbers. The concept of symbolic 2-plithogenic numbers was presented in [4] as a direct application of symbolic n-plithogenic sets in algebraic structures [1-3]. Also, many

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generalizations of symbolic 2-plithogenic algebraic structures and 3-plithogenic structures were defined by many authors, see [5-6,11-14].

In this paper, we combine symbolic 2-plithogenic real ring $2-SP_R$ with weak fuzzy complex ring C_w, to get a novel generalization of real numbers.

We discuss some of their elementary algebraic properties in terms of theorems with many easy and clear illustrated examples.

Main concepts.

Definition.

We define the set of symbolic 2-plithogenic weak complex numbers as follow:

$2-SP_w=\{(x_0+x_1P_1+x_2P_2)+J(y_0+y_1P_1+y_2P_2); x_i, y_i, \in R, J^2=t\in]0,1[\}$

Addition on $2-SP_w$ is defined as follows:

For $X=(a_0+a_1P_1+a_2P_2)+J(b_0+b_1P_1+b_2P_2)$,

$Y=(c_0+c_1P_1+c_2P_2)+J(d_0+d_1P_1+d_2P_2)$.

$X+Y=[(a_0+c_0)+(a_1+c_1)P_1+(a_2+c_2)P_2]+J[(b_0+d_0)+(b_1+d_1)P_1+(b_2+d_2)P_2]$.

Multiplication on $2-SP_w$ is defined as follows:

$X.Y=(a_0+a_1P_1+a_2P_2)(c_0+c_1P_1+c_2P_2)+t(b_0+b_1P_1+b_2P_2)(d_0+d_1P_1+d_2P_2)+J[(a_0+a_1P_1+a_2P_2)(d_0+d_1P_1+d_2P_2)+(b_0+b_1P_1+b_2P_2)(c_0+c_1P_1+c_2P_2)]=(a_0c_0+tb_0d_0)+P_1(a_0c_1+a_1c_0+a_1c_1+tb_0d_1+tb_1d_0+tb_1d_1)+P_2(a_0c_2+a_1c_2+a_2c_0+a_2c_1+a_2c_2+tb_0d_2+tb_1d_2+tb_2d_0+tb_2d_1+tb_2d_2)+J[(a_0d_0+b_0c_0)+P_1(a_0d_1+a_1d_0+a_1d_1+b_0c_1+b_1c_0+b_1c_1)+P_2(a_0d_2+a_1d_2+a_2d_0+a_2d_1+a_1d_2+a_2d_2+b_0c_2+b_1c_2+b_2c_0+b_2c_1+b_2c_2)]$.

Example.

Take $X=(P_1-P_2)+J(3-P_2), Y=(1+P_2)+J(P_2); J^2=t=\frac{1}{2}$.

$X+Y=(1+P_1)+J(3)=(1+P_1)+3J$.

$X.Y=P_1+P_1-P_2-P_2+\frac{1}{2}(3P_2-P_2)+J[P_2-P_2+3+3P_2-P_2-P_2]=(2P_1-2P_2+P_2)+J(3-P_2)=(2P_1-P_2)+J(3+P_2)$.

Remark.

$(2-SP_w,+,.)$ Is a commutative ring.

Invertibility

It is known that $A+BJ$ is invertible if and only if $A+B\sqrt{t}, A-B\sqrt{t}; J^2=t\in]0,1[$ are invertible.

This means that $X=(a_0+a_1P_1+a_2P_2)+J(b_0+b_1P_1+b_2P_2)$ is invertible if and only if

$A+B\sqrt{t}=(a_0+b_0\sqrt{t})+(a_1+b_1\sqrt{t})P_1+(a_2+b_2\sqrt{t})P_2$

$A-B\sqrt{t}=(a_0-b_0\sqrt{t})+(a_1-b_1\sqrt{t})P_1+(a_2-b_2\sqrt{t})P_2$

Are invertible in $2-SP_R$.

It is known from the invertibility of symbolic 2-plithogenic real numbers that:

$A+B\sqrt{t}$ is invertible if and only if:

$a_0+b_0\sqrt{t}\neq 0, (a_0+a_1)+(b_0+b_1)\sqrt{t}\neq 0, (a_0+a_1+a_2)+(b_0+b_1+b_2)\sqrt{t}\neq 0$

which is equivalent to:

$$\begin{cases}\sqrt{t}\neq -\frac{a_0}{b_0} \ or\\ \sqrt{t}\neq -\frac{(a_0+a_1)}{b_0+b_1} \ or\\ \sqrt{t}\neq -\frac{(a_0+a_1+a_2)}{b_0+b_1+b_2}\end{cases} \text{for } b_0, b_0+b_1, b_0+b_1+b_2\neq 0.$$

Or $\begin{cases}b_0\neq 0\\ b_0+b_1\neq 0\\ b_0+b_1+b_2\neq 0\end{cases}$

$A-B\sqrt{t}$ is invertible if and only if:

$a_0-b_0\sqrt{t}\neq 0, (a_0+a_1)-(b_0+b_1)\sqrt{t}\neq 0, (a_0+a_1+a_2)-(b_0+b_1+b_2)\sqrt{t}\neq 0$

which is equivalent to:

$b_0\neq 0, b_0+b_1\neq 0, b_0+b_1+b_2\neq 0.$

Or $\begin{cases}\sqrt{t}\neq \frac{a_0}{b_0} \ or\\ \sqrt{t}\neq \frac{(a_0+a_1)}{b_0+b_1} \ or\\ \sqrt{t}\neq \frac{(a_0+a_1+a_2)}{b_0+b_1+b_2}\end{cases}$

Example.

We try to find all non-invertible elements in $2-SP_w$.

Case1.

For $b_0 = 0,\ X = (a_0 + a_1P_1 + a_2P_2) + J(b_0 + b_1P_1 + b_2P_2);\ a_i, b_i \in R.$

Case2.

For $b_0 \neq 0, b_0 + b_1 = 0, X = (a_0 + a_1P_1 + a_2P_2) + J(b_0 - b_1P_1 + b_2P_2);\ a_i, b_i \in R.$

Case3.

For $b_0 \neq 0, b_0 + b_1 \neq 0, b_0 + b_1 + b_2 = 0, X = (a_0 + a_1P_1 + a_2P_2) + J(b_0 + b_1P_1 + (-b_0 - b_1)P_2);\ a_i, b_i \in R.$

Case4.

$\sqrt{t} = \frac{a_0}{b_0}$ or $\sqrt{t} = -\frac{a_0}{b_0}, X = \left(\sqrt{t}b_0 + a_1P_1 + a_2P_2\right) + J(b_0 + b_1P_1 + b_2P_2);\ a_i, b_i \in R.$

Case5.

$\sqrt{t} = \frac{(a_0+a_1)}{b_0+b_1}$ or $\sqrt{t} = -\frac{(a_0+a_1)}{b_0+b_1}$, then:

$X = a_0 + P_1\left(\sqrt{t}(b_0 + b_1) - a_0\right) + a_2P_2 + J(b_0 + b_1P_1 + b_2P_2);\ a_i, b_i \in R.$

Case6.

$\sqrt{t} = \frac{(a_0+a_1+a_2)}{b_0+b_1+b_2}$ or $\sqrt{t} = -\frac{(a_0+a_1+a_2)}{b_0+b_1+b_2}$, then:

$X = a_0 + a_1P_1 + P_2\left(-\sqrt{t}(b_0 + b_1 + b_2) - a_0 - a_1\right) + J(b_0 + b_1P_1 + b_2P_2);\ a_i, b_i \in R$

Example.

For $J^2 = t = \frac{1}{9}$, take $X = (2 + P_1 - 5P_2) + J(5 + 6P_1 + 12P_2)$, X is invertible that is because:

$b_0 \neq 0, b_0 + b_1 \neq 0, b_0 + b_1 + b_2 \neq 0$, and

$$\begin{cases} \sqrt{t} = \frac{1}{3} \neq \frac{a_0}{b_0} = \frac{2}{5} \\ \sqrt{t} = \frac{1}{3} \neq -\frac{a_0}{b_0} = -\frac{2}{5} \\ \sqrt{t} = \frac{1}{3} \neq \frac{(a_0 + a_1)}{b_0 + b_1} = \frac{3}{11} \\ \sqrt{t} = \frac{1}{3} \neq -\frac{(a_0 + a_1)}{b_0 + b_1} = -\frac{3}{11} \\ \sqrt{t} = \frac{1}{3} \neq \frac{(a_0 + a_1 + a_2)}{b_0 + b_1 + b_2} = -\frac{2}{23} \\ \sqrt{t} = \frac{1}{3} \neq -\frac{(a_0 + a_1 + a_2)}{b_0 + b_1 + b_2} = \frac{2}{23} \end{cases}$$

Theorem.

Let $X = A + BJ \in 2 - SP_w, A, B \in 2 - SP_R$, then if X is invertible, we get:

$$X^{-1}=\frac{1}{2}\left[\left(A+B\sqrt{t}\right)^{-1}+\left(A-B\sqrt{t}\right)^{-1}\right]+\frac{1}{2\sqrt{t}}J\left[\left(A+B\sqrt{t}\right)^{-1}-\left(A-B\sqrt{t}\right)^{-1}\right]$$

Proof.

Put $Y=\frac{1}{2}\left[\left(A+B\sqrt{t}\right)^{-1}+\left(A-B\sqrt{t}\right)^{-1}\right]+\frac{1}{2\sqrt{t}}J\left[\left(A+B\sqrt{t}\right)^{-1}-\left(A-B\sqrt{t}\right)^{-1}\right]$.

$X.Y=\frac{1}{2}\left[A\left(A+B\sqrt{t}\right)^{-1}+A\left(A-B\sqrt{t}\right)^{-1}\right]+\frac{\sqrt{t}}{2}B\left(A+B\sqrt{t}\right)^{-1}-\frac{\sqrt{t}}{2}B\left(A-B\sqrt{t}\right)^{-1}+\frac{1}{2\sqrt{t}}J\left[\frac{1}{2}B\left(A+B\sqrt{t}\right)^{-1}+\frac{1}{2}B\left(A-B\sqrt{t}\right)^{-1}+\frac{1}{2\sqrt{t}}A\left(A+B\sqrt{t}\right)^{-1}-\frac{1}{2\sqrt{t}}A\left(A-B\sqrt{t}\right)^{-1}\right]=\frac{1}{2}\left(A+B\sqrt{t}\right)\left(A+B\sqrt{t}\right)^{-1}+\frac{1}{2}\left(A-B\sqrt{t}\right)\left(A-B\sqrt{t}\right)^{-1}+J\left[\frac{1}{2}\left(B+\frac{A}{\sqrt{t}}\right)\left(A+B\sqrt{t}\right)^{-1}+\frac{1}{2}\left(B-\frac{A}{\sqrt{t}}\right)\left(A-B\sqrt{t}\right)^{-1}\right]=1+J\left[\frac{1}{2}\left(B+\frac{A}{\sqrt{t}}\right)\left(A+B\sqrt{t}\right)^{-1}-\frac{1}{2}\left(\frac{A-B\sqrt{t}}{\sqrt{t}}\right)\left(A-B\sqrt{t}\right)^{-1}\right]=1+J(0)=1$, thus $X^{-1}=Y$.

Remark.

$A+B\sqrt{t}=\left(a_0+b_0\sqrt{t}\right)+\left(a_1+b_1\sqrt{t}\right)P_1+\left(a_2+b_2\sqrt{t}\right)P_2$.

$$\left(A+B\sqrt{t}\right)^{-1}=\frac{1}{a_0+b_0\sqrt{t}}+\left[\frac{1}{(a_0+a_1)+(b_0+b_1)\sqrt{t}}-\frac{1}{a_0+b_0\sqrt{t}}\right]P_1+\left[\frac{1}{(a_0+a_1+a_2)+(b_0+b_1+b_2)\sqrt{t}}-\frac{1}{(a_0+a_1)+(b_0+b_1)\sqrt{t}}\right]P_2$$

$A-B\sqrt{t}=\left(a_0-b_0\sqrt{t}\right)+\left(a_1-b_1\sqrt{t}\right)P_1+\left(a_2-b_2\sqrt{t}\right)P_2$.

$\left(A+B\sqrt{t}\right)^{-1}=\frac{1}{a_0-b_0\sqrt{t}}+\left[\frac{1}{(a_0+a_1)-(b_0+b_1)\sqrt{t}}-\frac{1}{a_0-b_0\sqrt{t}}\right]P_1+\left[\frac{1}{(a_0+a_1+a_2)-(b_0+b_1+b_2)\sqrt{t}}-\frac{1}{(a_0+a_1)-(b_0+b_1)\sqrt{t}}\right]P_2$.

On the other hand, we have:

$$\frac{1}{a_0+b_0\sqrt{t}}+\frac{1}{a_0-b_0\sqrt{t}}=\frac{a_0-b_0\sqrt{t}+a_0+b_0\sqrt{t}}{{a_0}^2-{b_0}^2t}=\frac{2a_0}{{a_0}^2-{b_0}^2t}\ldots\ (1)$$

$$\frac{1}{a_0+b_0\sqrt{t}}-\frac{1}{a_0-b_0\sqrt{t}}=\frac{a_0-b_0\sqrt{t}-a_0-b_0\sqrt{t}}{{a_0}^2-{b_0}^2t}=\frac{-2b_0\sqrt{t}}{{a_0}^2-{b_0}^2t}\ldots\ (\acute{1})$$

$$\frac{1}{(a_0+a_1)+(b_0+b_1)\sqrt{t}}+\frac{1}{(a_0+a_1)-(b_0+b_1)\sqrt{t}}=\frac{2(a_0+a_1)}{(a_0+a_1)^2-(b_0+b_1)^2t}\ldots\ (2)$$

$$\frac{1}{(a_0+a_1)+(b_0+b_1)\sqrt{t}}-\frac{1}{(a_0+a_1)-(b_0+b_1)\sqrt{t}}=\frac{-2(b_0+b_1)\sqrt{t}}{(a_0+a_1)^2-(b_0+b_1)^2t}\ldots\ (\acute{2})$$

$$\frac{1}{(a_0+a_1+a_2)+(b_0+b_1+b_2)\sqrt{t}}+\frac{1}{(a_0+a_1+a_2)-(b_0+b_1+b_2)\sqrt{t}}$$
$$=\frac{2(a_0+a_1+a_2)}{(a_0+a_1+a_2)^2-(b_0+b_1+b_2)^2t}\ldots(3)$$
$$\frac{1}{(a_0+a_1+a_2)+(b_0+b_1+b_2)\sqrt{t}}-\frac{1}{(a_0+a_1+a_2)-(b_0+b_1+b_2)\sqrt{t}}$$
$$=\frac{2(b_0+b_1+b_2)\sqrt{t}}{(a_0+a_1+a_2)^2-(b_0+b_1+b_2)^2t}\ldots(\acute{3})$$

This implies that:

$$X^{-1}=\frac{a_0}{{a_0}^2-{b_0}^2t}+\left(\frac{a_0+a_1}{(a_0+a_1)^2-(b_0+b_1)^2t}-\frac{a_0}{{a_0}^2-{b_0}^2t}\right)P_1+\left(\frac{a_0+a_1+a_2}{(a_0+a_1+a_2)^2-(b_0+b_1+b_2)^2t}-\frac{a_0+a_1}{(a_0+a_1)^2-(b_0+b_1)^2t}\right)P_2+J\left[-\frac{b_0}{{a_0}^2-{b_0}^2t}+\left(\frac{-(b_0+b_1)}{(a_0+a_1)^2-(b_0+b_1)^2t}+\frac{b_0}{{a_0}^2-{b_0}^2t}\right)P_1+\left(\frac{-(b_0+b_1+b_2)}{(a_0+a_1+a_2)^2-(b_0+b_1+b_2)^2t}+\frac{(b_0+b_1)}{(a_0+a_1)^2-(b_0+b_1)^2t}\right)P_2\right].$$

Examples.

Take $J^2=t=\frac{1}{100}$ $X=(1+P_1+P_2)+J(5-P_1+P_2)$, then $\sqrt{t}=\frac{1}{10}, a_0=a_1=a_2=1, b_0=5, b_1=-1, b_2=1.$

$$X^{-1}=\frac{1}{1-\frac{25}{10}}+\left(\frac{2}{4-\frac{16}{10}}-\frac{1}{1-\frac{25}{10}}\right)P_1+\left(\frac{3}{9-\frac{25}{10}}-\frac{2}{4-\frac{16}{10}}\right)P_2$$
$$+J\left[\frac{-5}{1-\frac{25}{10}}+\left(\frac{-4}{4-\frac{16}{10}}+\frac{5}{1-\frac{25}{10}}\right)P_1+\left(\frac{-5}{1-\frac{25}{10}}+\frac{4}{4-\frac{16}{10}}\right)P_2\right]$$
$$=\frac{-10}{15}+\left(\frac{20}{24}+\frac{10}{15}\right)P_1+\left(\frac{30}{65}-\frac{20}{24}\right)P_2$$
$$+J\left[\frac{50}{15}+\left(\frac{-40}{24}-\frac{50}{15}\right)P_1+\left(\frac{-50}{65}+\frac{40}{24}\right)P_2\right]$$
$$=\frac{-2}{3}+\left(\frac{5}{6}+\frac{2}{3}\right)P_1+\left(\frac{6}{13}-\frac{5}{6}\right)P_2$$
$$+J\left[\frac{10}{3}+\left(\frac{-5}{3}-\frac{10}{3}\right)P_1+\left(\frac{-10}{3}+\frac{5}{3}\right)P_2\right]$$
$$=\left(\frac{-2}{3}+\frac{3}{2}P_1-\frac{29}{79}P_2\right)+J\left[\left(\frac{10}{3}-5P_1+\frac{35}{39}P_2\right)\right]$$

Natural power.

Let $X=A+BJ; A,B\in 2-SP_R$, then:

$$X^n = \frac{1}{2}\left[\left(A + B\sqrt{t}\right)^n + \left(A - B\sqrt{t}\right)^n\right] + \frac{1}{2\sqrt{t}}J\left[\left(A + B\sqrt{t}\right)^n - \left(A - B\sqrt{t}\right)^n\right]$$

The previous result can be proven easily by induction.

We have:

$$A + B\sqrt{t} = \left(a_0 + b_0\sqrt{t}\right) + \left(a_1 + b_1\sqrt{t}\right)P_1 + \left(a_2 + b_2\sqrt{t}\right)P_2$$

$$\left(A + B\sqrt{t}\right)^n = \left(a_0 + b_0\sqrt{t}\right)^n + \left[\left(a_0 + a_1 + (b_0 + b_1)\sqrt{t}\right)^n - \left(a_0 + b_0\sqrt{t}\right)^n\right]P_1$$
$$+ \left[\left(a_0 + a_1 + a_2 + (b_0 + b_1 + b_2)\sqrt{t}\right)^n - \left(a_0 + a_1 + (b_0 + b_1)\sqrt{t}\right)^n\right]P_2$$

$$A - B\sqrt{t} = \left(a_0 - b_0\sqrt{t}\right) + \left(a_1 - b_1\sqrt{t}\right)P_1 + \left(a_2 - b_2\sqrt{t}\right)P_2$$

$$\left(A + B\sqrt{t}\right)^n = \left(a_0 - b_0\sqrt{t}\right)^n + \left[\left(a_0 + a_1 - (b_0 + b_1)\sqrt{t}\right)^n - \left(a_0 - b_0\sqrt{t}\right)^n\right]P_1$$
$$+ \left[\left((a_0 + a_1 + a_2) - (b_0 + b_1 + b_2)\sqrt{t}\right)^n - \left(a_0 + a_1 - (b_0 + b_1)\sqrt{t}\right)^n\right]P_2$$

This implies that:

$$X^n = \tfrac{1}{2}\Big[\left(a_0 + b_0\sqrt{t}\right)^n + \left(a_0 - b_0\sqrt{t}\right)^n + \Big(\left(a_0 + a_1 + (b_0 + b_1)\sqrt{t}\right)^n + \big(a_0 + a_1 - (b_0 + b_1)\sqrt{t}\big)^n - \left(a_0 + b_0\sqrt{t}\right)^n - \left(a_0 - b_0\sqrt{t}\right)^n\Big)P_1 + \Big(\left((a_0 + a_1 + a_2) + (b_0 + b_1 + b_2)\sqrt{t}\right)^n + \left((a_0 + a_1 + a_2) - (b_0 + b_1 + b_2)\sqrt{t}\right)^n - \left(a_0 + a_1 + (b_0 + b_1)\sqrt{t}\right)^n - \left(a_0 + a_1 - (b_0 + b_1)\sqrt{t}\right)^n\Big)P_2\Big] + \tfrac{1}{2\sqrt{t}}J\tfrac{1}{2}\Big[\left(a_0 + b_0\sqrt{t}\right)^n + \left(a_0 - b_0\sqrt{t}\right)^n + \Big(\big(a_0 + a_1 + (b_0 + b_1)\sqrt{t}\big)^n - \left(a_0 + a_1 - (b_0 + b_1)\sqrt{t}\right)^n - \left(a_0 + b_0\sqrt{t}\right)^n + \left(a_0 - b_0\sqrt{t}\right)^n\Big)P_1 + \Big(\left((a_0 + a_1 + a_2) + (b_0 + b_1 + b_2)\sqrt{t}\right)^n + \left((a_0 + a_1 + a_2) - (b_0 + b_1 + b_2)\sqrt{t}\right)^n - \left(a_0 + a_1 + (b_0 + b_1)\sqrt{t}\right)^n + \left(a_0 + a_1 - (b_0 + b_1)\sqrt{t}\right)^n\Big)P_2\Big].$$

Definition.

Let $X = A + BJ \in 2 - SP_w; A, B \in 2 - SP_R$, we say that:

1). X is 2-nilpotent if and only if $X^2 = 0$.

2). X is 3-nilpotent if and only if $X^3 = 0$.

The equation $X^2 = 0$ is equivalent to:

$$\begin{cases} A^2 + B^2 t = 0 \dots (1) \\ 2AB = 0 \dots (2) \end{cases}$$

We multiply (1) by A to get $A^3 = 0 \Rightarrow A = 0$.

We multiply (2) by B to get $B^3 = 0 \Rightarrow B = 0$

So that the only 2-nilpotent element in $2 - SP_w$ is 0.

By a similar discussion, we get that only m-nilpotent element in $2-SP_w$ is 0.

Conclusion:

In this paper, we have defined for the first time the class of symbolic 2-plithogenic weak fuzzy complex numbers by combining two algebraic classes (symbolic 2-plithogenic numbers and weak fuzzy complex numbers). Also, we have studied some of their elementary properties such as Invertibility and nilpotency, where a formula to compute the invers of a symbolic 2-plithogenic weak fuzzy complex number is obtained.

In the future, we encourage other researchers to study matrices with symbolic 2-plithogenic weak fuzzy complex numbers.

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