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On Some Types of Neutrosophic Topological Groups with Respect to Neutrosophic Alpha Open Sets

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Abstract: In this article, we presented eight different types of neutrosophic topological groups, each of which depends on the conceptions of neutrosophic α-open sets and neutrosophic α-continuous functions. Also, we found the relation between these types, and we gave some properties on the other side.

Keywords: Neutrosophic α-open sets, neutrosophic α-continuous functions, neutrosophic topological groups, and neutrosophic topological groups of type (R), $R = 1,2,3,\dots,8$.

1. Introduction

Smarandache [1,2] originally handed the theory of "neutrosophic set". Recently, Abdel-Basset et al. discussed a novel neutrosophic approach [3-6]. Salama et al. [7] gave the clue of neutrosophic topological space. Arokiarani et al. [8] added the view of neutrosophic α-open subsets of neutrosophic topological spaces. Dhavaseelan et al. [9] presented the idea of neutrosophic α^m-continuity. Banupriya et al. [10] investigated the notion of neutrosophic αgs continuity and neutrosophic αgs irresolute maps. Nandhini et al. [11] presented Nαg#ψ-open map, Nαg#ψ-closed map, and Nαg#ψ-homomorphism in neutrosophic topological spaces. Sumathi et al. [12] submitted the perception of neutrosophic topological groups. The target of this article is to perform eight different types of neutrosophic topological groups, each of which depends on the notions of neutrosophic α-open sets and neutrosophic α-continuous functions and also we found the relation between these types.

2. Preliminaries

In all this paper, $(\mathcal{G},\tau)$ and $(\mathcal{H},\sigma)$ (or briefly $\mathcal{G}$ and $\mathcal{H}$) frequently refer to neutrosophic topological spaces (or shortly NTSs). Suppose $\mathcal{A}$ be a neutrosophic open subset (or shortly Ne-OS) of $\mathcal{G}$, then its complement $\mathcal{A}^c$ is closed (or shortly Ne-CS). In addition, its interior and closure are denoted by $Nint(\mathcal{A})$and $Ncl(\mathcal{A})$, correspondingly.

Definition 2.1 [8]: Let $\boldsymbol{\mathcal{A}}$ be a Ne-OS in NTS $\boldsymbol{\mathcal{G}}$, then it is said that a neutrosophic **α**-open subset (or briefly Ne-**α**OS) if $\boldsymbol{\mathcal{A} \subseteq Nint(Ncl(Nint(\mathcal{A})))}$. Then $\boldsymbol{\mathcal{A}^c}$ is the so-called a neutrosophic **α**-closed (or briefly Ne-**α**CS). The collection of all such these Ne-**α**OSs (resp. Ne-**α**CSs) of $\boldsymbol{\mathcal{G}}$ is denoted by $\boldsymbol{N\alpha O(\mathcal{G})}$ (resp. $\boldsymbol{N\alpha C(\mathcal{G})}$).

Definition 2.2 [8]: Let $\mathcal{A}$ be a neurrosophic set in NTS $\mathcal{G}$. Then the union of all such these Ne-αOSs involved in $\mathcal{A}$(symbolized by $\alpha Nint(\mathcal{A})$) is said to be the neutrosophic α-interior of $\mathcal{A}$.

Definition 2.3 [8]: Let $\mathcal{A}$ be a neurrosophic set in NTS $\mathcal{G}$. Then the intersection of all such these Ne-αCSs that contain $\mathcal{A}$ (symbolized by $\alpha Ncl(\mathcal{A})$) is said to be the neutrosophic α-closure of $\mathcal{A}$.

Proposition 2.4 [13]: Let $\mathcal{A}$ be a neutrosophic set in NTS $\mathcal{G}$. Then $\mathcal{A} \in N\alpha O(\mathcal{B})$ iff there exists a Ne-αOS $\mathcal{B}$ where $\mathcal{B} \subseteq \mathcal{A} \subseteq Nint(Ncl(\mathcal{B}))$.

Proposition 2.5 [8]: In any NTS, the following claims hold, and not vice versa:

(i) For each, Ne-OS is a Ne-αOS.

(ii) For each, Ne-CS is a Ne-αCS.

Definition 2.6: Let $\hbar: (\mathcal{G},\tau) \longrightarrow (\mathcal{H},\sigma)$ be a function, then $\hbar$ is called:

(i) a neutrosophic continuous (in short Ne-continuous) iff for each $\mathcal{A}$ Ne-OS in $\mathcal{H}$, then $\hbar^{-1}(\mathcal{A})$ is a Ne-OS in $\mathcal{G}$ [14].

(ii) a neutrosophic α-continuous (in short Ne-α-continuous) iff for each $\mathcal{A}$ Ne-OS in $\mathcal{H}$, then $\hbar^{-1}(\mathcal{A})$ is a Ne-αOS in $\mathcal{G}$ [8].

(iii) a neutrosophic α-irresolute (in short Ne-α-irresolute) iff for each $\mathcal{A}$ Ne-αOS in $\mathcal{H}$, then $\hbar^{-1}(\mathcal{A})$ is a Ne-αOS in $\mathcal{G}$.

Proposition 2.7 [8]: Every Ne-continuous function is a Ne-α-continuous, but the opposite is not valid in general.

Proposition 2.8: Every Ne-α-irresolute function is a Ne-α-continuous, but the opposite is not exact in general.

Proof: Let $\hbar: (\mathcal{G},\tau) \longrightarrow (\mathcal{H},\sigma)$ be a Ne-α-irresolute function and let $\mathcal{A}$ be any Ne-OS in $\mathcal{H}$. From proposition 2.5, we get $\mathcal{A}$ is a Ne-αOS in $\mathcal{H}$. Since $\hbar$ is a Ne-α-irresolute, then $\hbar^{-1}(\mathcal{A})$ is a Ne-αOS in $\mathcal{G}$. Therefore $\hbar$ is a Ne-α-continuous. ■

Example 2.9: Let $\mathcal{G} = \{p, q\}$. Suppose the neutrosophic sets $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and $\mathcal{D}$ be in $\mathcal{G}$ as follows:

$\mathcal{A} = \langle x, \left(\frac{p}{0.5}, \frac{q}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.7}\right) \rangle$, $\mathcal{B} = \langle x, \left(\frac{p}{0.5}, \frac{q}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}\right) \rangle$,

$\mathcal{C} = \langle x, \left(\frac{p}{0.6}, \frac{q}{0.3}\right), \left(\frac{p}{0.6}, \frac{q}{0.3}\right), \left(\frac{p}{0.4}, \frac{q}{0.7}\right) \rangle$ and $\mathcal{D} = \langle x, \left(\frac{p}{0.6}, \frac{q}{0.7}\right), \left(\frac{p}{0.6}, \frac{q}{0.7}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}\right) \rangle$.

Then the families $\tau = \{0_N, \mathcal{A}, 1_N\}$ and $\sigma = \{0_N, \mathcal{D}, 1_N\}$ are neutrosophic topologies on $\mathcal{G}$.

Thus, $(\mathcal{G},\tau)$ and $(\mathcal{G},\sigma)$ are NTSs. Define $\hbar: (\mathcal{G},\tau) \longrightarrow (\mathcal{G},\sigma)$ as $(p) = p$, $\hbar(q) = q$. Hence $\hbar$ is a Ne-α-continuous function, but not Ne-α-irresolute.

Definition 2.10: A function $\hbar: (\mathcal{G},\tau) \longrightarrow (\mathcal{H},\sigma)$ is said to be $\mathcal{M}$-function iff $\hbar^{-1}(Nint(Ncl(\mathcal{B}))) \subseteq Nint(Ncl(\hbar^{-1}(\mathcal{B})))$, for every Ne-αOS $\mathcal{B}$ of $\mathcal{H}$.

Theorem 2.11: If $\hbar: (\mathcal{G},\tau) \longrightarrow (\mathcal{H},\sigma)$ is a Ne-α-continuous function and $\mathcal{M}$-function, then $\hbar$ is a Ne-α-irresolute.

Proof: Let $\mathcal{A}$ be any Ne-αOS of $\mathcal{H}$, there exists a Ne-OS $\mathcal{B}$ of $\mathcal{H}$ where $\mathcal{B} \subseteq \mathcal{A} \subseteq Nint(Ncl(\mathcal{B}))$.

Since $\hbar$ is $\mathcal{M}$-function, we have $\hbar^{-1}(\mathcal{B}) \subseteq \hbar^{-1}(\mathcal{A}) \subseteq \hbar^{-1}(Nint(Ncl(\mathcal{B}))) \subseteq Nint(Ncl(\hbar^{-1}(\mathcal{B})))$.

By proposition 2.4, we have $\hbar^{-1}(\mathcal{A})$ is a Ne-αOS. Hence, $\hbar$ is a Ne-α-irresolute. ■

Definition 2.12 [8]: A function $\hbar: (\mathcal{G}, \tau) \longrightarrow (\mathcal{H}, \sigma)$ is called a neutrosophic α-open (resp. neutrosophic α-closed) iff for each $\mathcal{A} \in N\alpha O(\mathcal{G})$ (resp. $\mathcal{A} \in N\alpha C(\mathcal{G})$), $\hbar(\mathcal{A}) \in N\alpha O(\mathcal{H})$ (resp. $\hbar(\mathcal{A}) \in N\alpha C(\mathcal{H})$).

Definition 2.13 [15]: A bijective function $\hbar: (\mathcal{G}, \tau) \longrightarrow (\mathcal{H}, \sigma)$ is called a neutrosophic homeomorphism iff $\hbar$ and $\hbar^{-1}$ are Ne-continuous.

Definition 2.14 [12]: A neutrosophic topological group (briefly NTG) is a set $\mathcal{G}$ which carries a group structure and a neutrosophic topology with the following two postulates:

(i) The operation function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$, given as $\mu(g, h) = g \cdot h$ is a Ne-continuous.

(ii) The inversion function $I: \mathcal{G} \to \mathcal{G}$, given as $I(g) = g^{-1}$ is a Ne-continuous.

Remark 2.15 [12]:

(i) The function $\gamma: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$, given as $\gamma(g, h) = g \cdot h$ is a Ne-continuous iff for each Ne-OS $\mathcal{C}$ and $g \cdot h \in \mathcal{C}$, there exist Ne-OS $\mathcal{A}, \mathcal{B}$ such that $g \in \mathcal{A}, h \in \mathcal{B}$, and $\mathcal{A} \cdot \mathcal{B} \subseteq \mathcal{C}$.

(ii) The function $inv: \mathcal{G} \to \mathcal{G}$ is a Ne-continuous iff for each Ne-OS $\mathcal{A}$ and $g^{-1} \in \mathcal{A}$, there exists a Ne-OS $\mathcal{B}$ and $g \in \mathcal{B}$ where $\mathcal{B}^{-1} \subseteq \mathcal{A}$.

Definition 2.16 [16]: A group $\mathcal{G}$ is nice iff its operation is nice.

3. Different Types of Neutrosophic Topological Groups

In this section, we introduce eight types of neutrosophic topological groups, each of which depends on the notions of neutrosophic α-open sets and neutrosophic α-continuous functions.

Definition 3.1: Let $\mathcal{G}$ be a set that equips with a group structure and a neutrosophic topology. Then $\mathcal{G}$ is called:

(i) NTG of type (1) iff the operation function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ and the inversion function $I: \mathcal{G} \to \mathcal{G}$ are both Ne-α-continuous.

(ii) NTG of type (2) iff the operation function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ and the inversion function $I: \mathcal{G} \to \mathcal{G}$ are both Ne-α-irresolute.

(iii) NTG of type (3) iff the operation function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ is Ne-α-continuous and the inversion function $I: \mathcal{G} \to \mathcal{G}$ is Ne-continuous.

(iv) NTG of type (4) iff the operation function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ is Ne-α-irresolute and the inversion function $I: \mathcal{G} \to \mathcal{G}$ is Ne-continuous.

(v) NTG of type (5) iff the operation function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ is Ne-α-irresolute and the inversion function $I: \mathcal{G} \to \mathcal{G}$ is Ne-α-continuous.

(vi) NTG of type (6) iff the operation function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ is Ne-α-continuous and the inversion function $I: \mathcal{G} \to \mathcal{G}$ is Ne-α-irresolute.

(vii) NTG of type (7) iff the operation function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ is Ne-continuous, and the inversion function $I: \mathcal{G} \to \mathcal{G}$ is Ne-α-continuous.

(viii) NTG of type (8) iff the operation function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$ is Ne-continuous, and the inversion function $I: \mathcal{G} \to \mathcal{G}$ is Ne-α-irresolute.

Proposition 3.2:

(i) Every NTG is a NTG of type (R), where $R = 1,3,7$.

(ii) Every NTG of type (2) is a NTG of type (5).

(iii) Every NTG of type (2) is a NTG of type (6).

(iv) Every NTG of type (4) is a NTG of type (3).

(v) Every NTG of type (4) is a NTG of type (5).

(vi) Every NTG of type (R) is a NTG of type (1), where $R = 2,3,\dots,8$.

Proof:

(i) Let $\mathcal{G}$ be a NTG, then the operation function μ and the inversion function I are both Ne-continuous. By proposition 2.7, we have that the operation function μ and the inversion function I are both Ne-α-continuous. Hence, $\mathcal{G}$ is a NTG of type (R), where $R = 1,3,7$.

(ii) Let $\mathcal{G}$ be a NTG of type (2), then the operation function μ and the inversion function I are both Ne-α-irresolute. By proposition 2.8, we have that the inversion function I is a Ne-α-continuous. Hence, $\mathcal{G}$ is a NTG of type (5).

(iii) Let $\mathcal{G}$ be a NTG of type (2), then the operation function μ and the inversion function I are both Ne- α -irresolute. By proposition 2.8, we have that the operation function μ is a Ne-α-continuous. Hence, $\mathcal{G}$ is a NTG of type (6).

(iv) Let $\mathcal{G}$ be a NTG of type (4), then the operation function μ is a Ne-α-irresolute and the inversion function I is a Ne-continuous. By proposition 2.8, we have that the operation function μ is a Ne-α-continuous. Hence, $\mathcal{G}$ is a NTG of type (3).

(v) Let $\mathcal{G}$ be a NTG of type (4), then the operation function μ is a Ne-α-irresolute and the inversion function I is a Ne-continuous. By proposition 2.7, we have that the inversion function I is a Ne-α-continuous. Hence, $\mathcal{G}$ is a NTG of type (5).

(vi) Let $\mathcal{G}$ be a NTG of type (R), where $R = 2,3,\dots,8$. By proposition 2.7 and proposition 2.8, we have that the operation function μ and the inversion function I are both Ne-α-continuous. Hence, $\mathcal{G}$ is a NTG of type (1).

Proposition 3.3:

(i) A NTG of type (3) with $\mathcal{M}$-function operation μ is a NTG of type (4).

(ii) A NTG of type (1) with $\mathcal{M}$-function inversion I and $\mathcal{M}$-function operation μ is a NTG of type (2).

(iii) A NTG of type (1) with $\mathcal{M}$-function operation μ is a NTG of type (5).

(iv) A NTG of type (1) with $\mathcal{M}$-function inversion I is a NTG of type (6).

(v) A NTG of type (5) with $\mathcal{M}$-function inversion I is a NTG of type (2).

(vi) A NTG of type (6) with $\mathcal{M}$-function operation μ is a NTG of type (2).

(vii) A NTG of type (7) with $\mathcal{M}$-function inversion I is a NTG of type (8).

Proof:

(i) Let $\mathcal{G}$ be a NTG of type (3), then the operation function μ is a Ne-α-continuous and the inversion function I is a Ne-continuous. Since μ is $\mathcal{M}$-function. So by Theorem 2.11, we get that operation μ

is a Ne-α-irresolute. Hence, $\mathcal{G}$ is a NTG of type (4).

(ii) Let $\mathcal{G}$ be a NTG of type (1), then the operation function μ and the inversion function I are both Ne-α-continuous. Since μ, I are $\mathcal{M}$-function. So by Theorem 2.11, we get that the operation function μ and the inversion function I are both Ne-α-irresolute. Hence, $\mathcal{G}$ is a NTG of type (2). The proof is evident for others.

Remark 3.4: The next illustration displays relationship among different kinds of neutrosophic topological groups mentioned in this section and the neutrosophic topological group:

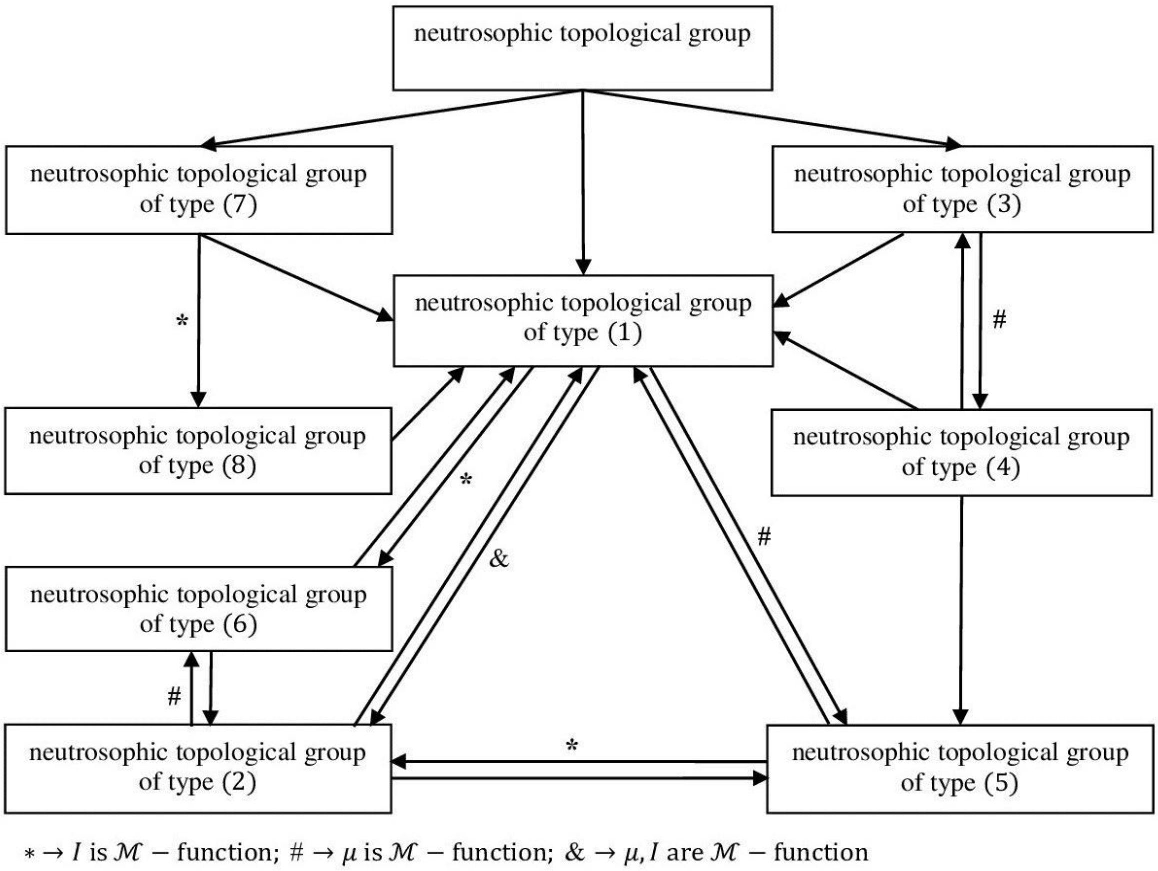


Fig. 3.1

Definition 3.5: A bijective function $\hbar: (\mathcal{G}, \tau) \longrightarrow (\mathcal{H}, \sigma)$ is said to be:

(i) Neutrosophic α-homeomorphism iff $\hbar$ and $\hbar^{-1}$ are Ne-α-continuous.

(ii) Neutrosophic α-irresolute – homeomorphism iff $\hbar$ and $\hbar^{-1}$ are Ne-α-irresolute.

Definition 3.6: Let $(\mathcal{G}, \tau)$ be a NTS, then $\mathcal{G}$ is called neutrosophic α-homogeneous (resp. neutrosophic α-irresolute – homogeneous) iff for any two elements $g, h \in \mathcal{G}$, there exists a neutrosophic α-homeomorphism (resp. neutrosophic α-irresolute – homeomorphism) from $\mathcal{G}$ onto $\mathcal{G}$ which transforms g into h.

Proposition 3.7: The inversion function I in a NTG of type (R), where $R = 1,2, \dots \dots ,8$ is a neutrosophic α-homeomorphism.

Proof: Let $\mathcal{G}$ be a NTG of type (1). Since $\mathcal{G}$ is a group, $I(\mathcal{G}) = \mathcal{G}^{-1} = \mathcal{G}$ which implies I is onto, also for any $g \in \mathcal{G}$, there exists a unique inverse which is equal to $I(g)$ which implies, I is one-to-one. Now; we have I is a Ne-α-continuous and $I^{-1}: \mathcal{G} \to \mathcal{G}$ such that $I^{-1}(g) = g$, i.e $I^{-1}(g) = I(g)$ for each $g \in \mathcal{G}$, so, I^{-1} is a Ne-α-continuous. Thus, I is a neutrosophic α-homeomorphism. In the case of type (R), we have a similar proof, where $R = 2,3, \ldots, 8$.

Corollary 3.8: Let $\mathcal{G}$ be a NTG of type (1) and $\mathcal{A} \subseteq \mathcal{G}$. If $\mathcal{A} \in \tau$, then $\mathcal{A}^{-1} \in N\alpha O(\mathcal{G})$.
Proof: Since the inversion function I is a neutrosophic α-homeomorphism, then $I(\mathcal{A}) = \mathcal{A}^{-1}$ is a Ne-αOS in $\mathcal{G}$ for each $\mathcal{A} \in \tau$.

Proposition 3.9: The inversion function I in a NTG of type (3) [and type (4)] is a neutrosophic homeomorphism.
Proof: Suppose $\mathcal{G}$ be a NTG of type (3). Since $\mathcal{G}$ is a group, $I(\mathcal{G}) = \mathcal{G}^{-1} = \mathcal{G}$ which implies I is onto, also for any $g \in \mathcal{G}$, there exists a unique inverse which is equal to $I(g)$ which implies, I is one-to-one. Now; we have I is a Ne-continuous and $I^{-1}: \mathcal{G} \to \mathcal{G}$ such that $I^{-1}(g) = g$, i.e $I^{-1}(g) = I(g)$ for each $g \in \mathcal{G}$, so, I^{-1} is a Ne-continuous. Thus, I is a neutrosophic homeomorphism. In the case of type (4), we have similar proof.

Proposition 3.10: The inversion function I in a NTG of type (R), where $R = 2,6,8$ is a neutrosophic α-irresolute – homeomorphism.
Proof: Suppose $\mathcal{G}$ be a NTG of type (2). Since $\mathcal{G}$ is a group, $I(\mathcal{G}) = \mathcal{G}^{-1} = \mathcal{G}$ which implies I is onto, also for any $g \in \mathcal{G}$, there exists a unique inverse which is equal to $I(g)$ which implies, I is one-to-one. Now; we have I is a Ne-α-irresolute and $I^{-1}: \mathcal{G} \to \mathcal{G}$ such that $I^{-1}(g) = g$, i.e $I^{-1}(g) = I(g)$ for each $g \in \mathcal{G}$, so, I^{-1} is a Ne-α-irresolute. Thus, I is a neutrosophic α-irresolute – homeomorphism. In the case of type (6) and type (8), we have a similar proof.

Proposition 3.11: Let $\mathcal{G}$ be a set which carries a group structure and a neutrosophic topology, let $k_1, k_2 \in \mathcal{G}$. Then for each $g \in \mathcal{G}$ if one of the following functions:
(i) $l_{k_1}(g) = k_1 \cdot g$
(ii) $r_{k_1}(g) = g \cdot k_1$
(iii) $\hbar_{k_1 k_2}(g) = k_1 \cdot g \cdot k_2$
is a neutrosophic α-homeomorphism (resp. neutrosophic α-irresolute – homeomorphism), then so the others.
Proof: Since $k_1$and k_2 are arbitrary elements in $\mathcal{G}$, clear that $l_{k_1}$and r_{k_1}come from $\hbar_{k_1 k_2}$ by taking $k_2 = e$ or $k_1 = e$ respectively. Hence, when $\hbar_{k_1 k_2}$ is a neutrosophic α-homeomorphism, both l_{k_1} and r_{k_2} are neutrosophic α-homeomorphisms. Now; when l_{k_1} is a neutrosophic α-homeomorphism. Since $\mathcal{G}$ is a group, $\mathcal{G} \cdot k = \mathcal{G}$ for each $k \in \mathcal{G}$ then $\mathcal{G} \cdot k_2 = \mathcal{G}$. Hence, for each $h \in \mathcal{G} \cdot k_2$, $l_{k_1}(h) = k_1 \cdot h$, l_{k_1} is a neutrosophic α-homeomorphism. But $h = g \cdot k_2$ for some $g \in \mathcal{G}$, then for each $g \in \mathcal{G}$, $l_{k_1}(h) = l_{k_1}(g \cdot k_2) = k_1 \cdot g \cdot k_2 = \hbar_{k_1 k_2}(g)$, $\hbar_{k_1 k_2}$ is a neutrosophic α-homeomorphism. Then by the first part of the proof, r_{k_1}. And we have a similar proof if we are beginning with r_{k_1} is a neutrosophic α-homeomorphism. In the case of neutrosophic α-irresolute – homeomorphism, we have a similar proof.

Theorem 3.12: Let $\mathcal{G}$ be a nice NTG of type (R), where $R = 1,2,3,\dots,8$ and let $k_1, k_2 \in \mathcal{G}$. Then for each $g \in \mathcal{G}$ the following functions:

(i) $l_{k_1}(g) = k_1 \cdot g$

(ii) $r_{k_1}(g) = g \cdot k_1$

(iii) $\hbar_{k_1 k_2}(g) = k_1 \cdot g \cdot k_2$

are neutrosophic α-homeomorphisms.

Proof: Let $\mathcal{G}$ be a nice NTG of type (1). It is clear that each of the functions l_{k_1}, r_{k_1} and $\hbar_{k_1 k_2}$ is a bijective function. Let $\hbar$ be the operation of $\mathcal{G}$, then $\hbar$ is a Ne-α-continuous. Since $\mathcal{G}$ is a nice, so $l_{k_1} = \hbar/\{k_1\} \times \mathcal{G}$ is a Ne-α-continuous. Similarly, ${l_{k_1}}^{-1}(g) = {k_1}^{-1} \cdot g$, ${l_{k_1}}^{-1}$ is a Ne-α-continuous. Hence, l_{k_1} is a neutrosophic α-homeomorphism. Thus, because of the preceding proposition, r_{k_1} and $\hbar_{k_1 k_2}$ are neutrosophic α-homeomorphisms. The case of type (R) has a similar proof, where $R = 2,3,\dots,8$.

Theorem 3.13: Let $\mathcal{G}$ be a nice NTG of type (R), where $R = 2,4,5$ and let $k_1, k_2 \in \mathcal{G}$. Then for each $g \in \mathcal{G}$ the following functions:

(i) $l_{k_1}(g) = k_1 \cdot g$

(ii) $r_{k_1}(g) = g \cdot k_1$

(iii) $\hbar_{k_1 k_2}(g) = k_1 \cdot g \cdot k_2$

are neutrosophic α-irresolute – homeomorphisms.

Proof: Let $\mathcal{G}$ be a nice NTG of type (2). It is clear that each of the functions l_{k_1}, r_{k_1} and $\hbar_{k_1 k_2}$ is a bijective function. Let $\hbar$ be the operation of $\mathcal{G}$, then $\hbar$ is a Ne-α-irresolute. Since $\mathcal{G}$ is a nice, so $l_{k_1} = \hbar/\{k_1\} \times \mathcal{G}$ is a Ne-α-irresolute. Similarly, ${l_{k_1}}^{-1}(g) = {k_1}^{-1} \cdot g$, ${l_{k_1}}^{-1}$ is a Ne-α-irresolute. Hence, l_{k_1} is a neutrosophic α-irresolute – homeomorphism. Thus, given the preceding proposition, r_{k_1} and $\hbar_{k_1 k_2}$ are neutrosophic α-irresolute – homeomorphisms. The case of type (R) has a similar proof, where $R = 4,5$.

Corollary 3.14: Let $\mathcal{A}, \mathcal{B}$ and $\mathcal{C}$ be subsets of a nice NTG $\mathcal{G}$ of type (1) (resp. of type (4)) such that $\mathcal{A}$ is a Ne-CS (resp. Ne-αCS), and $\mathcal{B}$ is a Ne-OS (resp. Ne-αOS). Then for each $k \in \mathcal{G}, k \cdot \mathcal{A}$ and $\mathcal{A} \cdot k$ are Ne-α-CSs also $k \cdot \mathcal{B}, \mathcal{B} \cdot k, \mathcal{C} \cdot \mathcal{B}$ and $\mathcal{B} \cdot \mathcal{C}$ are Ne-αOSs.

Proof: Since $\mathcal{A}$ is a Ne-CS so in view of the theorem 3.12, $l_k(\mathcal{A}) = k \cdot \mathcal{A}$ and $r_k(\mathcal{A}) = \mathcal{A} \cdot k$ are Ne-αCSs.

Similarly, since $\mathcal{B}$ is a Ne-OS so in view of the theorem 3.12, $l_k(\mathcal{B}) = k \cdot \mathcal{B}$ and $r_k(\mathcal{B}) = \mathcal{B} \cdot k$ are Ne-αOSs. Also, $\mathcal{C} \cdot \mathcal{B} = \bigcup_{c \in \mathcal{C}} c \cdot \mathcal{B}$ but $c \cdot \mathcal{B}$ is a Ne-αOS for each $c \in \mathcal{C}$. Hence, $\mathcal{C} \cdot \mathcal{B}$ is a Ne-αOS. Similarly, $\mathcal{B} \cdot \mathcal{C}$ is a Ne-αOS. In the case of type (4), we have a similar proof.

Corollary 3.15: A nice NTG of type (R), where $R = 1,2,3,\dots,8$ is neutrosophic α-homogeneous.

Proof: Let $\mathcal{G}$ be a nice NTG of type (1) and $a, b \in \mathcal{G}$. Then for any fixed element $k \in \mathcal{G}, r_k$ is a neutrosophic α-homeomorphism, therefore, it is true when $k = a^{-1} \cdot b$. Thus, $r_{a^{-1}b}(g) = g \cdot a^{-1} \cdot b$ is a neutrosophic α-homeomorphism we need because $r_{a^{-1}b}(a) = b$. Therefore, $\mathcal{G}$ is a neutrosophic α-homogeneous. In the case of type (R), we have a similar proof, where $R = 2,3,\dots,8$.

Corollary 3.16: A nice NTG of type (R), where $R = 2,4,5$ is neutrosophic α-irresolute – homogeneous.

Proof: Let $\mathcal{G}$ be a nice NTG of type (2) and $a, b \in \mathcal{G}$. Then for any fixed element $k \in \mathcal{G}, r_k$ is a neutrosophic α-irresolute – homeomorphism, therefore, it is true when $k = a^{-1} \cdot b$. Thus, $r_{a^{-1}b}(g) = g \cdot a^{-1} \cdot b$ is a neutrosophic α-irresolute – homeomorphism. But $r_{a^{-1}b}(a) = b$, therefore $\mathcal{G}$ is a neutrosophic α-irresolute – homogeneous. In the case of type (R), we have a similar proof, where $R = 4,5$.

Definition 3.17: Let $\mathcal{G}$ be a NTG of type (2), (5), and $\mathcal{F}$ be a fundamental system of neutrosophic α-open nhds of the identity element e. Then for any fixed element $k \in \mathcal{G}, r_k$ is a neutrosophic α-irresolute – homeomorphism. So $\mathcal{F}(k) = \{r_k(\mathcal{A}) = \mathcal{A} \cdot k : \mathcal{A} \in \mathcal{F}\}$ is a fundamental system of neutrosophic α-open nhds of k.

Proposition 3.18: Let $\mathcal{G}$ be a NTG of type (2), (5). Any fundamental system $\mathcal{F}$ of neutrosophic α-open nhds of e in $\mathcal{G}$ has the below postulates:

(i) If $\mathcal{A}, \mathcal{B} \in \mathcal{F}$, then $\exists \mathcal{C} \in \mathcal{F}$ such that $\mathcal{C} \subseteq \mathcal{A} \cap \mathcal{B}$.

(ii) If $g \in \mathcal{A} \in \mathcal{F}$, then $\exists \mathcal{B} \in \mathcal{F}$ such that $\mathcal{B} \cdot g \subseteq \mathcal{A}$.

(iii) If $\mathcal{A} \in \mathcal{F}$, then $\exists \mathcal{B} \in \mathcal{F}$ such that $\mathcal{B}^{-1} \cdot \mathcal{B} \subseteq \mathcal{A}$.

(iv) If $\mathcal{A} \in \mathcal{F}, k \in \mathcal{G}$, then $\exists \mathcal{B} \in \mathcal{F}$ such that $k^{-1} \cdot \mathcal{B} \cdot k \subseteq \mathcal{A}$.

(v) $\forall \mathcal{A} \in \mathcal{F}, \exists \mathcal{B} \in \mathcal{F}$ such that $\mathcal{B}^{-1} \subseteq \mathcal{A}$.

(vi) $\forall \mathcal{A} \in \mathcal{F}, \exists \mathcal{C} \in \mathcal{F}$ such that $\mathcal{C}^2 \subseteq \mathcal{A}$.

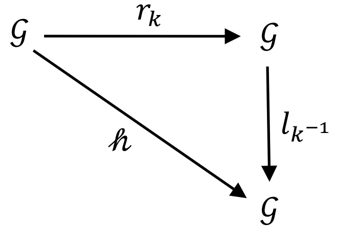
Proof:

(i) Let $\mathcal{A}, \mathcal{B} \in \mathcal{F}$, then $\mathcal{A} \cap \mathcal{B} \in \mathcal{F}$, so $\exists \mathcal{C} \in \mathcal{F}$ such that $\mathcal{C} \subseteq \mathcal{A} \cap \mathcal{B}$.

(ii) Let $\mathcal{A} \in \mathcal{F}$ and $g \in \mathcal{A}$ implies $\mathcal{A} \cdot g^{-1} \in \mathcal{F}$, then $\exists \mathcal{B} \in \mathcal{F}$ such that $\mathcal{B} \subseteq \mathcal{A} \cdot g^{-1}$. Thus, $\mathcal{B} \cdot g \subseteq \mathcal{A}$.

(iii) The function $\mu: \mathcal{G} \times \mathcal{G} \to \mathcal{G}$, given by $\mu(g, h) = g^{-1} \cdot h$ is a Ne-α-irresolute because $\mathcal{G}$ is a NTG of type (2), (5). Thus $\mu^{-1}(\mathcal{A})$ is a neutrosophic α-open nhd in $\mathcal{G} \times \mathcal{G}$ contains (e, e) and hence includes a set of the from $\mathcal{U} \times \mathcal{V}$, where $\mathcal{U}, \mathcal{V}$ are neutrosophic α-open and provide e. But $\mathcal{U} \cap \mathcal{V}$ is a neutrosophic α-open contains e, so $\exists \mathcal{B} \in \mathcal{F}$ such that $\mathcal{B} \subseteq \mathcal{U} \cap \mathcal{V}$ then $\mathcal{B} \subseteq \mathcal{U}$ and $\mathcal{B} \subseteq \mathcal{V}$. Thus $\mathcal{B} \times \mathcal{B} \subseteq \mathcal{U} \times \mathcal{V} \subseteq \mu^{-1}(\mathcal{A})$, then $\mu(\mathcal{B} \times \mathcal{B}) \subseteq \mathcal{A}$ but $\mu(\mathcal{B} \times \mathcal{B}) = \mathcal{B}^{-1} \cdot \mathcal{B} \subseteq \mathcal{A}$.

(iv) The function $\hbar: \mathcal{G} \to \mathcal{G}$ given by $\hbar(g) = k^{-1} \cdot g \cdot k$ is a Ne-α-irresolute. Since $l_{k^{-1}}, r_k$ is Ne-α-irresolute. So $l_{k^{-1}} \circ r_k$ is a Ne-α-irresolute from $\mathcal{G}$ to $\mathcal{G}$ put $\hbar = l_{k^{-1}} \circ r_k, \hbar(g) = (l_{k^{-1}} \circ r_k)(g) = l_{k^{-1}}(r_k(g)) = l_{k^{-1}}(g \cdot k) = k^{-1} \cdot g \cdot k$.



So, $\hbar^{-1}(\mathcal{A})$ is a neutrosophic α-open nhd and contains e, hence $\exists \mathcal{B} \in \mathcal{F}$, $\mathcal{B} \subseteq \hbar^{-1}(\mathcal{A})$ then $\hbar(\mathcal{B}) \subseteq \mathcal{A}$. Thus, $\hbar(\mathcal{B}) = k^{-1} \cdot \mathcal{B} \cdot k \subseteq \mathcal{A}$.

(v) Since I the inverse function in a NTG of type (2) is a Ne-α-irresolute, then $I^{-1}(\mathcal{A})$ is a neutrosophic α-open contains e so $\exists\mathcal{B} \in \mathcal{F}$ such that $\mathcal{B} \subseteq I^{-1}(\mathcal{A})$ then $I(\mathcal{B}) \subseteq \mathcal{A}$. Thus, $I(\mathcal{B}) = \mathcal{B}^{-1} \subseteq \mathcal{A}$.

(vi) Since μ in a NTG of type (5) is a Ne-α-irresolute. So $\mu^{-1}(\mathcal{A})$ is a neutrosophic α-open contains (e, e) and thus contains a neutrosophic set of the from $\mathcal{U} \times \mathcal{V}$, where $\mathcal{U}, \mathcal{V}$ are neutrosophic α-open and contain e then $\mathcal{U} \cap \mathcal{V}$ is a neutrosophic α-open and contain e $\exists\mathcal{C} \in \mathcal{F}$ such that $\mathcal{C} \subseteq \mathcal{U} \cap \mathcal{V}$, then $\mathcal{C} \times \mathcal{C} \subseteq \mathcal{U} \times \mathcal{V} \subseteq \mu^{-1}(\mathcal{A})$. Thus, $\mu(\mathcal{C} \times \mathcal{C}) = \mathcal{C} \cdot \mathcal{C} = \mathcal{C}^2 \subseteq \mathcal{A}$.

Definition 3.19: A neutrosophic α-open nhd $\mathcal{C}$ of g is called symmetric if $\mathcal{C}^{-1} = \mathcal{C}$.

Proposition 3.20: Let $\mathcal{G}$ be a NTG of type (R), where $R = 1,2, \dots ,8$, and let $\mathcal{B}$ be any neutrosophic α-open nhd of a point $g \in \mathcal{G}$. Then $\mathcal{B} \cup \mathcal{B}^{-1}$ is symmetric neutrosophic α-open nhd of g.

Proof: Let $\mathcal{B}$ is a neutrosophic α-open nhd of g, then $\mathcal{B} \cup \mathcal{B}^{-1}$ is a neutrosophic α-open nhd of g;

$\mathcal{B} \cup \mathcal{B}^{-1} = \{b: b \in \mathcal{B} \text{ } or \text{ } b \in \mathcal{B}^{-1}\} = \{b: b^{-1} \in \mathcal{B} \text{ } or \text{ } b^{-1} \in \mathcal{B}^{-1}\}$

$= \{b: b^{-1} \in \mathcal{B} \cup \mathcal{B}^{-1}\} = \{b: b \in (\mathcal{B} \cup \mathcal{B}^{-1})^{-1}\} = (\mathcal{B} \cup \mathcal{B}^{-1})^{-1}$.

That is, $\mathcal{B} \cup \mathcal{B}^{-1}$ is symmetric neutrosophic α-open nhd of g.

Proposition 3.21: Let $\mathcal{B}$ be any neutrosophic α-open nhd of e in a nice NTG of type (R), where $R = 1,2, \dots \dots ,8$. Then $\mathcal{B} \cdot \mathcal{B}^{-1}$ is symmetric neutrosophic α-open nhd of e.

Proof: Let $\mathcal{B}$ be a neutrosophic α-open nhd of e and since $\mathcal{G}$ is a nice, then $\mathcal{B} \cdot \mathcal{B}^{-1}$ is neutrosophic α-open nhd of e;

$\mathcal{B} \cdot \mathcal{B}^{-1} = \{x \cdot y^{-1}: x, y \in \mathcal{B}\} = \{(x^{-1})^{-1} \cdot y^{-1}: x, y \in \mathcal{B}\} = (\mathcal{B}^{-1})^{-1} \cdot \mathcal{B}^{-1} = (\mathcal{B} \cdot \mathcal{B}^{-1})^{-1}$.

That is, $\mathcal{B} \cdot \mathcal{B}^{-1}$ is symmetric neutrosophic α-open nhd of e.

4. Conclusion

In this work, we examined the conceptions of eight different types of neutrosophic topological groups, each of which, depending on the notions of neutrosophic α-open sets and neutrosophic α-continuous function. In the future, we plan to rsearch the ideas of neutrosophic topological subgroups and the neutrosophic topological quotient groups as well as defining the perception of neutrosophic topological product groups with some results.

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