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On Clean and Nil-clean Symbolic 2-Plithogenic Rings

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Abstract. A ring is said to be clean if every element of the ring can be written as a sum of an idempotent element and a unit element of the ring and a ring is said to be nil-clean if every element of the ring can be written as a sum of an idempotent element and a nilpotent element of the ring. In this paper, we generalize these arguments to symbolic 2-plithogenic structure. We introduce the structure of clean and nil-clean symbolic 2-plithogenic rings and some of its elementary properties are presented. Also, we have found the equivalence between classical clean(nil-clean) ring R and the corresponding symbolic 2-plithogenic ring $2-SP_R$.

Keywords: Clean ring; nil-clean ring; symbolic 2-plithogenic ring; clean symbolic 2-plithogenic ring; nil-clean symbolic 2-plithogenic ring.

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1. **Introduction**

The concept of refined neutrosophic structure was studied by many authors in [1–5]. Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation [17].

In [14], the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings, and some of the elementary properties and substructures of symbolic 2-plithogenic rings such as AH-ideals, AH-homomorphisms, and AHS-isomorphisms are studied. In [7], some more algebraic properties of symbolic 2-plithogenic

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rings are studied. Further, Taffach [15,16] studied the concepts of symbolic 2-plithogenic vector spaces and modules.

In [8], the concept of symbolic 2-plithogenic matrices with symbolic 2-plithogenic entries, determinants, eigen values and vectors, exponents, and diagonalization are studied. Hamiyet Merkepci et.al [12], studied the the symbolic 2-plithogenic number theory and integers. Ahmad Khaldi et.al [11], studied the different types of algebraic symbolic 2-plithogenic equations and its solutions.

In [18], H. Suryoto and T. Uidjiani studied the concept of neutrosophic clean ring with many elementary interesting properties. Recently, M. Abobala [6], proved that a neutrosophic ring $R(I)$ is clean if and only if R is clean. Motivated by this works, in this paper we have introduced and studied the notion of clean and nil-clean symbolic 2-plithogenic rings. Also, we proved that a symbolic 2-plithogenic $2-SP_R$ is clean(nil-clean) if and only if R is clean(nil-clean).

2. Preliminaries

Definition 2.1. [14] Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2-SP_R = \left\{a_0 + a_1P_1 + a_2P_2; a_i \in R, P_j^2 = P_j, P_1 \times P_2 = P_{max(1,2)} = P_2\right\}$$

Smarandache has defined algebraic operations on $2-SP_R$ as follows:

Addition:

$[a_0 + a_1P_1 + a_2P_2] + [b_0 + b_1P_1 + b_2P_2] = (a_0 + b_0) + (a_1 + b_1)P_1 + (a_2 + b_2)P_2$

Multiplication:

$[a_0 + a_1P_1 + a_2P_2].[b_0 + b_1P_1 + b_2P_2] = a_0b_0 + a_0b_1P_1 + a_0b_2P_2 + a_1b_0P_1^2 + a_1b_2P_1P_2 + a_2b_0P_2 + a_2b_1P_1P_2 + a_2b_2P_2^2 + a_1b_1P_1P_1 = (a_0b_0) + (a_0b_1 + a_1b_0 + a_1b_1)P_1 + (a_0b_2 + a_1b_2 + a_2b_0 + a_2b_1 + a_2b_2)P_2$.

It is clear that $2-SP_R$ is a ring. If R is a field, then $2-SP_R$ is called a symbolic 2-plithogenic field. Also, if R is commutative, then $2-SP_R$ is commutative, and if R has a unity (1), than $2-SP_R$ has the same unity (1).

Example 2.2. [14] Consider the ring $R = Z_4 = \{0, 1, 2, 3, 4\}$, the corresponding $2-SP_R$ is:

$$2-SP_R = \{a + bP_1 + cP_2; a, b, c \in Z_4\}.$$

If $X = 1 + 2P_1 + 3P_2, Y = P_1 + 2P_2$; then, $X + Y = 1 + 3P_1 + P_2$, $X - Y = 1 + P_1 + P_2$, $X.Y = 3P_1 + 3P_2$.

Theorem 2.3. *[14] Let $2-SP_R$ be a 2-plithogenic symbolic ring, with unity (1). Let $X = x_0 + x_1P_1 + x_2P_2$ be an arbitrary element, then:*

(1) *X is invertible if and only if $x_0, x_0 + x_1, x_0 + x_1 + x_2$ are invertible.*

(2) $X^{-1} = x_0^{-1} + [(x_0 + x_1)^{-1} - x_0^{-1}]P_1 + [(x_0 + x_1 + x_2)^{-1} - (x_0 + x_1)^{-1}]P_2$

Definition 2.4. [14] Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, then X is idempotent if and only if $X^2 = X$.

Theorem 2.5. *[14] Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, then X is idempotent if and only if $a, a + b, a + b + c$ are idempotent.*

Theorem 2.6. *[14] Let $2 - SP_R$ be a commutative symbolic 2-plithogenic ring, hence if $X = a + bP_1 + cP_2$, then $X^n = a^n + [(a + b)^n - a^n]P_1 + [(a + b + c)^n - (a + b)^n]P_2$ for every $n \in Z^+$.*

Definition 2.7. [14] X is called nilpotent if there exists $n \in Z^+$ such that $X^n = 0$.

Theorem 2.8. *[14] Let $X = a + bP_1 + cP_2 \in 2 - SP_R$, where R is commutative ring, then X is nilpotent if and only if $a, a + b, a + b + c$ are nilpotent.*

3. Clean Symbolic 2-Plithogenic Rings

We begin with the following definition.

Definition 3.1. Let R be any ring, $2 - SP_R$ be its corresponding symbolic 2-plithogenic ring. An element $x \in 2 - SP_R$ is said to be clean if $x = e + u$, where e is an idempotent and u is a unit element of $2 - SP_R$. If, in addition, the existing idempotent e and the unit u are unique, then x is called uniquely clean element.

In this section, we use the notation $U(2 - SP_R)$ to the set of all units in $2 - SP_R$ and $Id(2 - SR_R)$ to the set of all idempotent elements in $2 - SP_R$.

Example 3.2. Consider the symbolic 2-plithogenic ring

$$\begin{aligned} 2 - SP_{Z_2} &= \{a + bP_1 + cP_2; a, b, c \in Z_2\} \\ &= \Big\{0, 1, P_1, P_2, P_1 + P_2, 1 + P_1, 1 + P_2, 1 + P_1 + P_2\Big\}. \end{aligned}$$

Here, $U(2 - SP_{Z_2}) = 1$ and $Id(2 - SR_{Z_2}) = \{0, 1, P_1, P_2, P_1 + P_2, 1 + P_1, 1 + P_2, 1 + P_1 + P_2\}$. We can easily verify that every element of $2 - SP_{Z_2}$ can be expressed as a sum of an idempotent and a unit in $2 - SP_{Z_2}$. Hence, all the elements in $2 - SP_{Z_2}$ are clean elements. Since 1 is the only unit element in $2 - SP_{Z_2}$, so all the elements in $2 - SP_{Z_2}$ are uniquely clean elements.

Definition 3.3. A symbolic 2-plithogenic ring in which all elements are clean, then the ring is called a clean symbolic 2-plithogenic ring. Furthermore, if each element of the symbolic 2-plithogenic ring is uniquely clean, then the ring is called a uniquely clean symbolic 2-plithogenic ring.

Example 3.4. By the Example 3.2, the ring $2-SP_{Z_2}$ is a uniquely clean symbolic 2-plithogenic ring.

Example 3.5. Consider the symbolic 2-plithogenic ring

$$\begin{aligned} 2-SP_{Z_3} &= \{a+bP_1+cP_2; a,b,c \in Z_3\} \\ &= \left\{ \begin{array}{c} 0,1,2,P_1,P_2,2P_1,2P_2,P_1+P_2,2P_1+2P_2,P_1+2P_2,2P_1+P_2,1+P_1, \\ 1+P_2,1+P_1+P_2,1+2P_1,1+2P_2,1+2P_1+2P_2,1+P_1+2P_2, \\ 1+2P_1+P_2,2+P_1,2+P_2,2+P_1+P_2,2+2P_1,2+2P_2,2+2P_1+2P_2, \\ 2+P_1+2P_2,2+2P_1+P_2 \end{array} \right\}. \end{aligned}$$

Here, $U(2-SP_{Z_3}) = \{1,2,1+P_1,1+P_2,2+2P_1,2+2P_2,1+P_1+2P_2,2+2P_1+P_2\}$ and $Id(2-SR_{Z_3}) = \{0,1,P_1,P_2,P_1+2P_2,1+2P_1,1+2P_2,1+2P_1+P_2\}$. All the elements of $2-SP_{Z_3}$ are clean elements. Hence $2-SP_{Z_3}$ is a clean symbolic 2-plithogenic ring. Take $2+2P_1+P_2 \in 2-SP_{Z_3}$, clearly $2+2P_1+P_2 = (1+2P_1+P_2)+1$ and also we have $2+2P_1+P_2 = 0+(2+2P_1+P_2)$. Therefore $2+2P_1+P_2$ is not a uniquely clean element in $2-SP_{Z_3}$ and hence $2-SP_{Z_3}$ is not a uniquely clean.

Lemma 3.6. *Let R be a ring. Then the class of clean symbolic 2-plithogenic rings is closed under homomorphic images.*

Proof. It is clear since the homomorphic image of an idempotent element in a symbolic 2-plithogenic ring is again an idempotent. □

Theorem 3.7. *Let R be any ring, $2-SP_R$ be its corresponding symbolic 2-plithogenic ring. $2-SP_R$ is clean if and only if R is clean.*

Proof. Assume that $2-SP_R$ is clean. Since R is a homomorphic image of $2-SP_R$, so R is clean by Lemma 3.6.

Conversely, assume that R is clean, we must prove that $2-SP_R$ is clean. Let $x = a+bP_1+cP_2 \in 2-SP_R$ then $a, a+b, a+b+c \in R$. Since R is clean we have $a = e_1+u_1, a+b = e_2+u_2, a+b+c = e_3+u_3$, where e_i are idempotent elements and u_i are unit elements of R. Now,

$$\begin{aligned} x &= a+bP_1+cP_2 \\ &= a+[(a+b)-a]P_1+[(a+b+c)-(a+b)]P_2 \\ &= (e_1+u_1)+[(e_2+u_2)-(e_1+u_1)]P_1+[(e_3+u_3)-(e_2+u_2)]P_2 \\ &= (e_1+u_1)+[(e_2-e_1)+(u_2-u_1)]P_1+[(e_3-e_2)+(u_3-u_2)]P_2 \\ &= [e_1+(e_2-e_1)P_1+(e_3-e_2)P_2]+[u_1+(u_2-u_1)P_1+(u_3-u_2)P_2] \\ &= x_1+x_2. \end{aligned}$$

where, $x_1 = e_1 + (e_2 - e_1)P_1 + (e_3 - e_2)P_2$ and $x_2 = u_1 + (u_2 - u_1)P_1 + (u_3 - u_2)P_2$. By Theorem 2.5, $e_1, e_1 + (e_2 - e_1) = e_2, e_1 + (e_2 - e_1) + (e_3 - e_2) = e_3$ are idempotents in R. Therefore, x_1 is a idempotent element of R. Also, x_2 is a unit element of R by a similar discussion. Hence $2 - SP_R$ is clean. □

Definition 3.8. Let $2 - SP_R$ be a symbolic 2-plithogenic ring. An idempotent element $e \in 2 - SP_R$ is called a central idempotent if $e.x = x.e$ for every $x \in 2 - SP_R$. The set of all central idempotents of $2 - SP_R$ is denoted by $C(2 - SP_R)$.

Example 3.9. In the symbolic 2-plithogenic ring $2 - SP_{Z_3}$, we have

$$Id(2 - SP_{Z_3}) = \{0, 1, P_1, P_2, P_1 + 2P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + P_2\}.$$

As $2 - SP_{Z_3}$ is commutative so all the idempotents of $2 - SP_{Z_3}$ are central. Hence

$$C(2 - SP_{Z_3}) = \{0, 1, P_1, P_2, P_1 + 2P_2, 1 + 2P_1, 1 + 2P_2, 1 + 2P_1 + P_2\} = Id(2 - SP_{Z_3}).$$

Lemma 3.10. *If x is an idempotent element of $2 - SP_R$, then $1 - x$ is also an idempotent element of $2 - SP_R$, where 1 is the unit element of $2 - SP_R$.*

Proof. If x an idempotent element of $2 - SP_R$ then $x^2 = x$. But then, $(1 - x)^2 = 1 - 2x - x^2 = 1 - x$ and so $1 - x$ an idempotent element of $2 - SP_R$. □

Lemma 3.11. *Let $2 - SP_R$ be a symbolic 2-plithogenic ring with the identity 1. If $e \in C(2 - SP_R)$ then $1 - e \in C(2 - SP_R)$, where 1 is the unit element of $2 - SP_R$.*

Proof. Assume that $e \in C(2 - SP_R)$. For any $x \in 2 - SP_R$, we have $(1 - e).x = (1.x) - (e.x) = (x.1) - (x.e) = x(1 - e)$. Hence, $1 - e \in C(2 - SP_R)$. □

Theorem 3.12. *In any symbolic 2-plithogenic ring $2 - SP_R$, every central idempotent is a uniquely clean element.*

Proof. Let $x \in C(2 - SP_R)$. Then we have, $x^2 = x$ and $x = (1 - x) + (2x - 1) = e + u$, where $e = 1 - x$ is an idempotent by Lemma 3.10 and $u = 2x - 1$ is a unit element by Lemma 3.11. Hence x is a clean element. Also, if $x.y = y.x$ we obtain $e + u = (e + u)^2 = e + 2eu + u^2$, so $u = 1 - 2e$. Hence $e = 1 - x$. Thus x is a uniquely clean element. □

Theorem 3.13. *Every idempotent element in a uniquely clean 2-plithogenic ring is a central idempotent.*

Proof. Assume that $2-SP_R$ is a uniquely clean 2-plithogenic ring. Let $e \in 2-SP_R$ be an idempotent element and x be any element of $2-SP_R$. Now, the element $e+(ex-exe)$ is an idempotent and $1+(ex-exe)$ is a unit and $[e+(ex-exe)]+1 = e+[1+(ex-exe)]$. Since, $2-SP_R$ is a uniquely clean 2-plithogenic ring we have $e+(ex-exe) = e$. Hence $ex = exe$ and $xe = exe$, so $ex = ex$ as required. □

Definition 3.14. A symbolic 2-plithogenic ring $2-SP_R$ is called a boolean symbolic 2-plithogenic ring if $x^2 = x$ for all $x \in 2-SP_R$.

Example 3.15. In the symbolic 2-plithogenic ring $2-SR_{Z_2}$, all the elements are idempotent so $2-SR_{Z_2}$ is a boolean symbolic 2-plithogenic ring

For any boolean symbolic 2-plithogenic ring, we have the following result.

Theorem 3.16. *Every boolean symbolic 2-plithogenic ring is uniquely clean.*

Proof. If $2-SP_R$ is a boolean symbolic 2-plithogenic ring, then $2-SP_R = Id(2-SP_R)$. Since boolean rings are abelian, we have $Id(2-SP_R) = C(2-SP_R)$. This implies that, $2-SP_R = C(2-SP_R)$. By Theorem 3.12, every element of the ring $2-SP_R$ are uniquely clean. Hence $2-SP_R$ is uniquely clean ring. □

4. Nil-clean Symbolic 2-Plithogenic Rings

We begin with the following definition.

Definition 4.1. Let R be any ring, $2-SP_R$ be its corresponding symbolic 2-plithogenic ring. An element $x \in 2-SP_R$ is said to be nil-clean if $x = e+n$, where e is an idempotent and n is a nil-potent element of $2-SP_R$. If, in addition, the existing idempotent element and nil-potent elements are unique, then x is called uniquely nil-clean element.

Example 4.2. Consider the symbolic 2-plithogenic ring

$$\begin{aligned} 2-SP_{Z_3} &= \{a+bP_1+cP_2; a,b,c \in Z_3\} \\ &= \left\{ \begin{array}{c} 0,1,2,P_1,P_2,2P_1,2P_2,P_1+P_2,2P_1+2P_2,P_1+2P_2,2P_1+P_2,1+P_1, \\ 1+P_2,1+P_1+P_2,1+2P_1,1+2P_2,1+2P_1+2P_2,1+P_1+2P_2, \\ 1+2P_1+P_2,2+P_1,2+P_2,2+P_1+P_2,2+2P_1,2+2P_2,2+2P_1+2P_2, \\ 2+P_1+2P_2,2+2P_1+P_2 \end{array} \right\}. \end{aligned}$$

Since 0 is a nil-potent element in $2-SP_{Z_3}$, so the idempotent elements $0,1,P_1,P_2,P_1+2P_2,1+2P_1,1+2P_2,1+2P_1+P_2$ are nil-clean elements of $2-SP_{Z_3}$. The only nilpotent elements of $2-SP_{Z_3}$ is 0, so $0,1,P_1,P_2,P_1+2P_2,1+2P_1,1+2P_2,1+2P_1+P_2$ are uniquely nil-clean elements of $2-SP_{Z_3}$.

Definition 4.3. A symbolic 2-plithogenic ring in which all elements are nil-clean, then the ring is called a nil-clean symbolic 2-plithogenic ring. Furthermore, if each element of the symbolic 2-plithogenic ring is uniquely nil-clean, then the ring is called a uniquely nil-clean symbolic 2-plithogenic ring.

Example 4.4. $2 - SP_{Z_2} = \{0, 1, P_1, P_2, P_1 + P_2, 1 + P_1, 1 + P_2, 1 + P_1 + P_2\}$ is a nil-clean symbolic 2-plithogenic ring, that is because all the elements in $2 - SP_{Z_2}$ are idempotents and 0 is a nilpotent element in $2 - SP_{Z_2}$.

Lemma 4.5. *If x is a nilpotent element of $2 - SP_R$, then $1 + x$ is a unit in $2 - SP_R$.*

Proof. If x is a nilpotent element of $2 - SP_R$ then $x^k = 0$ for some $k > 0$. But then, $(1 + x)(1 - x + x^2 - x^3 + ... + (-1)^{k-1}x^{k-1}) = 1$ and so $1 + x$ is unit in $2 - SP_R$. □

Theorem 4.6. *Every nil-clean symbolic 2-plithogenic ring is clean symbolic 2-plithogenic ring.*

Proof. Suppose that $2 - SP_R$ is a nil-clean symbolic 2-plithogenic ring, and let $x \in 2 - SP_R$. Then $x - 1$ is an element of $2 - SP_R$ and hence $x - 1 = e + n$, where e is an idempotent element and n is a nilpotent element of $2 - SP_R$.
This implies that, $x = e + (1 + n)$ is a nil-clean element of $2 - SP_R$ because $1 + n$ is a unit element of $2 - SP_R$ by Lemma 4.5. □

The converse of the Theorem 4.6 is not true. See the following example.

Example 4.7. Consider, the clean symbolic 2-plithogenic ring $2 - SP_{Z_3}$. All the elements of $2 - SP_{Z_3}$ are clean elements. The only nilpotent element of $2 - SP_{Z_3}$ is 0 and $P_1 + P_2$ is not an idempotent element in $2 - SP_{Z_3}$ so it is not nil-clean. Hence $2 - SP_{Z_3}$ is not a nil-clean ring.

Lemma 4.8. *Let R be a ring. Then the class of nil-clean symbolic 2-plithogenic rings is closed under homomorphic images.*

Proof. It is clear since the homomorphic image of a nil-potent element of a symbolic 2-plithogenic rings is again a nil-potent. □

Theorem 4.9. *Let R be any ring, $2 - SP_R$ be its corresponding symbolic 2-plithogenic ring. $2 - SP_R$ is nil-clean if and only if R is nil-clean.*

Proof. Assume that $2-SP_R$ is nil-clean. Since R is a homomorphic image of $2-SP_R$, so R is nil-clean by Lemma 4.8.

Conversely, assume that R is nil-clean, we must prove that $2-SP_R$ is nil-clean. Let $x = a+bP_1+cP_2 \in 2-SP_R$ then $a, a+b, a+b+c \in R$. Since R is nil-clean we have $a = e_1+n_1, a+b = e_2+n_2, a+b+c = e_3+n_3$, where e_i are idempotent elements and n_i are nilpotent elements of R. Now,

$$\begin{aligned}
x &= a+bP_1+cP_2 \\
&= a+[(a+b)-a]P_1+[(a+b+c)-(a+b)]P_2 \\
&= (e_1+n_1)+[(e_2+n_2)-(e_1+n_1)]P_1+[(e_3+n_3)-(e_2+n_2)]P_2 \\
&= (e_1+n_1)+[(e_2-e_1)+(n_2-n_1)]P_1+[(e_3-e_2)+(n_3-n_2)]P_2 \\
&= [e_1+(e_2-e_1)P_1+(e_3-e_2)P_2]+[n_1+(n_2-n_1)P_1+(n_3-n_2)P_2] \\
&= x_1+x_2.
\end{aligned}$$

where, $x_1 = e_1+(e_2-e_1)P_1+(e_3-e_2)P_2$ and $x_2 = n_1+(n_2-n_1)P_1+(n_3-n_2)P_2$. By Theorem 2.5, $e_1, e_1+(e_2-e_1) = e_2, e_1+(e_2-e_1)+(e_3-e_2) = e_3$ are idempotents in R. Therefore, x_1 is a idempotent element of R. Also, x_2 is a nilpotent element of R by a similar discussion and by Theoerem 2.8. Hence $2-SP_R$ is nil-clean. □

Theorem 4.10. *If $2-SP_R$ is a symbolic 2-plithogenic ring, then every central idempotent of $2-SP_R$ is uniquely nil-clean element.*

Proof. We know that, every idempotent element of $2-SP_R$ are nil-clean. Let x be a central idempotent element of $2-SP_R$. Then $x = (1-x)+(2x-1)$. Suppose that $x = e+n$, where e is an idempotent and n is a nilpotent element of $2-SP_R$. Since $nx = xn$, we obtain $e+n = (e+n)^2 = e+2en+n^2$. So, we have $n = 1-2e$ and hence $e = 1-x$, as reuired. □

Lemma 4.11. *Let $2-SP_R$ be uniquely nil-clean symbolic 2-plithogenic ring. Then all idempotents of $2-SP_R$ are central.*

Proof. Let $e \in 2-SP_R$ be an idempotent element and x be any element of $2-SP_R$. Now, the element $e+ex-exe$ can be written as $e+(ex-exe)$ or $(e+(ex-exe))+0$ each time as the sum of an idempotent and a nilpotent element of $2-SP_R$. Since $2-SP_R$ is uniquely nil clean, we have $e = e+(ex-exe)$. This implies that $ex-exe = 0$ and so $ex = exe$. In the similar way, we can show that $xe = exe$. Hence $ex = xe$ as required. □

Theorem 4.12. *Every boolean symbolic 2-plithogenic ring is uniquely nil-clean.*

Proof. If $2 - SP_R$ is a boolean symbolic 2-plithogenic ring, then $2 - SP_R = Id(2 - SP_R)$. Since boolean rings are abelian, we have $Id(2 - SP_R) = C(2 - SP_R)$. This implies that, $2 - SP_R = C(2 - SP_R)$. By Theorem 4.10, every element of the ring $2 - SP_R$ are uniquely nil-clean. Hence $2 - SP_R$ is uniquely nil-clean ring. □

5. **Conclusion**

In this article, we have introduced the the new classes of rings called, clean symbolic 2-plithogenic rings and nil-clean symbolic 2-plithogenic rings and we have studied various properties of clean and nil-clean symbolic 2-plithogenic rings with proper examples. Also, we have determined necessary and sufficient condition for a symbolic 2-plithogenic ring to be clean and nil-clean.

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