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On Some Algebraic Properties of Symbolic 2-Plithogenic Square Matrices

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Abstract. In this article, the adjoint of symbolic 2-plithogenic square matrices are defined and the inverse of symbolic 2-plithogenic square matrices are studied in terms of symbolic 2-plithogenic determinant and symbolic 2-plithogenic adjoint. We have introduced the concept of symbolic 2-plithogenic characteristic polynomial of symbolic 2-plithogenic square matrices and the symbolic 2-plithogenic version of Cayley-Hamilton theorem. Also, provided enough examples to enhance understanding.

Keywords: Symbolic 2-plithogenic matrix; symbolic 2-plithogenic adjoint; symbolic 2-plithogenic determinant; symbolic 2-plithogenic inverse.

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1. Introduction

The concept of refined neutrosophic structure was studied by many authors in [1–4]. Symbolic plithogenic algebraic structures are introduced by Smarandache, that are very similar to refined neutrosophic structures with some differences in the definition of the multiplication operation [15].

In [12], the algebraic properties of symbolic 2-plithogenic rings generated from the fusion of symbolic plithogenic sets with algebraic rings are studied. In [8], some more algebraic properties of symbolic 2-plithogenic rings are studied. Further, Taffach [17, 18] studied the concepts

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of symbolic 2-plithogenic vector spaces and modules.

Recently, in [7], the concept of symbolic 2-plithogenic matrices with symbolic 2-plithogenic entries, determinants, eigen values and vectors, exponents, and diagonalization are studied. Hamiyet Merkepci et.al [13], studied the the symbolic 2-plithogenic number theory and integers. Ahmad Khaldi et.al [11], studied the different types of algebraic symbolic 2-plithogenic equations and its solutions.

As a continuation of the previous study of symbolic 2-plithogenic matrices, this work discusses the symbolic 2-plithogenic adjoint, where the inverse of symbolic 2-plithogenic matrices will be defined in terms of the symbolic 2-plithogenic adjoint. We present the symbolic 2-plithogenic characteristic polynomials and the symbolic 2-plithogenic version of the Cayley-Hamilton theorem. Also, we illustrate many examples to clarify the validity of our work.

2. **Preliminaries**

Definition 2.1. [12] Let R be a ring, the symbolic 2-plithogenic ring is defined as follows:

$$2-SP_R=\left\{a_0+a_1P_1+a_2P_2; a_i\in R, P_j^2=P_j, P_1\times P_2=P_{max(1,2)}=P_2\right\}$$

Smarandache has defined algebraic operations on $2-SP_R$ as follows:

Addition:

$[a_0+a_1P_1+a_2P_2]+[b_0+b_1P_1+b_2P_2]=(a_0+b_0)+(a_1+b_1)P_1+(a_2+b_2)P_2$

Multiplication:

$[a_0+a_1P_1+a_2P_2].[b_0+b_1P_1+b_2P_2]=a_0b_0+a_0b_1P_1+a_0b_2P_2+a_1b_0P_1^2+a_1b_2P_1P_2+a_2b_0P_2+a_2b_1P_1P_2+a_2b_2P_2^2+a_1b_1P_1P_1=(a_0b_0)+(a_0b_1+a_1b_0+a_1b_1)P_1+(a_0b_2+a_1b_2+a_2b_0+a_2b_1+a_2b_2)P_2$.

It is clear that $2-SP_R$ is a ring. If R is a field, then $2-SP_R$ is called a symbolic 2-plithogenic field. Also, if R is commutative, then $2-SP_R$ is commutative, and if R has a unity (1), than $2-SP_R$ has the same unity (1).

Theorem 2.2. *[12] Let $2-SP_R$ be a 2-plithogenic symbolic ring, with unity (1). Let $X=x_0+x_1P_1+x_2P_2$ be an arbitrary element, then:*

(1) *X is invertible if and only if $x_0, x_0+x_1, x_0+x_1+x_2$ are invertible.*
(2) $X^{-1}=x_0^{-1}+[(x_0+x_1)^{-1}-x_0^{-1}]P_1+[(x_0+x_1+x_2)^{-1}-(x_0+x_1)^{-1}]P_2$

Definition 2.3. [7] A symbolic 2-plithogenic real square matrix is a matrix with symbolic 2-plithogenic real entries.

Theorem 2.4. *[7] Let $S=S_0+S_1P_1+S_2P_2$ be a symbolic 2-plithogenic real square matrix, then*

(1) *S is invertible if and only if $S_0, S_0+S_1, S_0+S_1+S_2$ are invertible.*

(2) *If S is invertible then*

$$S^{-1} = S_0^{-1} + [(S_0+S_1)^{-1} - S_0^{-1}]P_1 + [(S_0+S_1+S_2)^{-1} - (S_0+S_1)^{-1}]$$

(3) $S^m = S_0^m + [(S_0+S_1)^m - S_0^m]P_1 + [(S_0+S_1+S_2)^m - (S_0+S_1)^m]$ *for* $m \in N$.

Definition 2.5. [7] Let $L = L_0 + L_1P_1 + L_2P_2 \in 2 - SP_M$, we define:

$$detL = det(L_0) + [det(L_0+L_1) - detL_0]P_1 + [det(L_0+L_1+L_2) - det(L_0+L_1)]P_2.$$

3. Adjoint of Symbolic 2-Plithogenic Square Matrices

We begin this section with the following definition.

Definition 3.1. Let $L = L_0 + L_1P_1 + L_2P_2$ be a symbolic 2-plithogenic square matrix with real entries. The adjoint matrix of L is defined as

$$adjL = adjL_0 + [adj(L_0+L_1) - adjL_0]P_1 + [adj(L_0+L_1+L_2) - adj(L_0+L_1)]P_2.$$

Example 3.2. Consider the following symbolic 2-plithogenic 2×2 matrix:

$$L = \begin{pmatrix} 2+P_1+3P_2 & 1-P_1-P_2 \\ 3+4P_1 & 1+P_2 \end{pmatrix}$$

Here,

$$L_0 = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}, \; L_0+L_1 = \begin{pmatrix} 3 & 0 \\ 7 & 1 \end{pmatrix} \text{ and } L_0+L_1+L_2 = \begin{pmatrix} 6 & -1 \\ 7 & 2 \end{pmatrix},$$

Then,

$$adjL_0 = \begin{pmatrix} 1 & -1 \\ -3 & 2 \end{pmatrix}, \; adj(L_0+L_1) = \begin{pmatrix} 1 & 0 \\ -7 & 3 \end{pmatrix} \text{ and } adj(L_0+L_1+L_2) = \begin{pmatrix} 2 & 1 \\ -7 & 6 \end{pmatrix}.$$

Therefore,

$$\begin{aligned} adjL &= adjL_0 + [adj(L_0+L_1) - adjL_0]P_1 + [adj(L_0+L_1+L_2) - adj(L_0+L_1)]P_2 \\ &= \begin{pmatrix} 1+P_1 & -1+P_1+P_2 \\ -3-4P_2 & 2+P_1+3P_2 \end{pmatrix} \end{aligned}$$

Example 3.3. Consider the following symbolic 2-plithogenic 3×3 matrix:

$$L = \begin{pmatrix} -3+P_1-P_2 & 1+P_1 & 5 \\ -P_1+P_2 & 3P_1 & 4P_2 \\ -1+2P_1-P_2 & 5+2P_2 & 7+P_1+10P_2 \end{pmatrix}$$

Here,

$$L_0 = \begin{pmatrix} -3 & 1 & 5 \\ 0 & 0 & 0 \\ -1 & 5 & 7 \end{pmatrix}, \; L_0+L_1 = \begin{pmatrix} -2 & 2 & 5 \\ -1 & 3 & 0 \\ -1 & 5 & 8 \end{pmatrix} \text{ and } L_0+L_1+L_2 = \begin{pmatrix} -3 & 2 & 5 \\ 0 & 3 & 4 \\ 0 & 7 & 18 \end{pmatrix},$$

Then,

$$adjL_0 = \begin{pmatrix} 0 & 18 & 0 \\ 0 & -16 & 0 \\ 0 & 14 & 0 \end{pmatrix}, \quad adj(L_0 + L_1) = \begin{pmatrix} 24 & 9 & -15 \\ 8 & -21 & -5 \\ -8 & 12 & -4 \end{pmatrix} \quad \text{and}$$

$$adj(L_0 + L_1 + L_2) = \begin{pmatrix} 26 & -1 & -7 \\ 0 & -54 & 12 \\ 0 & 21 & 9 \end{pmatrix}.$$

Therefore,

$$\begin{aligned} adjL &= adjL_0 + [adj(L_0 + L_1) - adjL_0]P_1 + [adj(L_0 + L_1 + L_2) - adj(L_0 + L_1)]P_2 \\ &= \begin{pmatrix} 24P_1 + 2P_2 & 18 - 9P_1 - 10P_2 & -15P_1 + 8P_2 \\ 8P_1 - 8P_2 & -16 + 5P_1 + 33P_2 & -5P_1 + 17P_2 \\ -8P_1 + 8P_2 & 14 - 2P_1 + 9P_2 & -4P_1 + 13P_2 \end{pmatrix} \end{aligned}$$

Using the definition of adjoint of symbolic 2-plithogenic matrix we can modify the Theorem 2.4 as follows:

Theorem 3.4. *Let $L = L_0 + L_1P_1 + L_2P_2$ be a symbolic 2-plithogenic square matrix, then L is invertible if and only if $detL_0 \neq 0, det(L_0 + L_1) \neq 0$ and $det(L_0 + L_1 + L_2) \neq 0$ and*

$$L^{-1} = \frac{1}{detL}(adjL).$$

Proof. By Theorem 2.4, L is invertible if and only if $detL_0 \neq 0, \quad det(L_0 + L_1) \neq 0$ and $det(L_0 + L_1 + L_2) \neq 0$.

Also,

$$\begin{aligned} \frac{1}{detL}(adjL) &= \left(\frac{1}{detL_0 + [det(L_0 + L_1) - det(L_0)]P_1 + [det(L_0 + L_1 + L_2) - det(L_0 + L_1)]P_2} \right) \\ &\quad (adjL_0 + [adj(L_0 + L_1) - adjL_0]P_1 + [adj(L_0 + L_1 + L_2) - adj(L_0 + L_1)]P_2) \\ &= \frac{adjL_0}{detL_0} + \left[\frac{adj(L_0 + L_1)}{det(L_0 + L_1)} - \frac{adjL_0}{detL_0} \right] P_1 + \left[\frac{adj(L_0 + L_1 + L_2)}{det(L_0 + L_1 + L_2)} - \frac{adj(L_0 + L_1)}{det(L_0 + L_1)} \right] P_2 \\ &= L_0^{-1} + \left[(L_0 + L_1)^{-1} - L_0^{-1} \right] P_1 + \left[(L_0 + L_1 + L_2)^{-1} - (L_0 + L_1)^{-1} \right] P_2 \\ &= L^{-1} \end{aligned}$$

Hence the result holds by Theorem 2.4. □

Example 3.5. Consider the symbolic 2-plithogenic 2×2 matrix

$$L = \begin{pmatrix} 1 + P_1 + P_2 & -1 + P_1 \\ 1 - P_2 & 1 \end{pmatrix}$$

Here, $detL = 2 + P_2$, and $adjL = \begin{pmatrix} 1 & 1 - P_1 \\ -1 + P_2 & 1 + P_1 + P_2 \end{pmatrix}$.

Hence,

$$\begin{aligned} L^{-1} &= \frac{1}{detL}(adjL) \\ &= \frac{1}{2 + P_2}\begin{pmatrix} 1 & 1 - P_1 \\ -1 + P_2 & 1 + P_1 + P_2 \end{pmatrix} \\ &= \left(\frac{1}{2} - \frac{1}{6}P_2\right)\begin{pmatrix} 1 & 1 - P_1 \\ -1 + P_2 & 1 + P_1 + P_2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} - \frac{1}{6}P_2 & \frac{1}{2} - \frac{1}{2}P_1 \\ -\frac{1}{2} + \frac{1}{2}P_2 & \frac{1}{2} + \frac{1}{2}P1 \end{pmatrix} \end{aligned}$$

Example 3.6. Consider the symbolic 2-plithogenic 3×3 matrix

$$L = \begin{pmatrix} 1 + P_1 & 1 - P_1 & 1 + P_1 - P_2 \\ 1 + P_2 & -1 + P_1 + P_2 & 2 + P_1 \\ 1 - P_1 + P_2 & -1 + P_2 & 1 + P_1 \end{pmatrix}$$

Here, $detL = 2 + 2P_1 - P_2$, and

$$adj(L) = \begin{pmatrix} 1 + 2P_1 - P_2 & -2 + 2P_2 & 3 - 3P_1 - P_2 \\ 1 - 3P_1 + P_2 & 4P_1 - P_2 & -1 - 3P_1 \\ -P_1 & 2 - 2P_2 & -2 + 2P_1 + 2P_2 \end{pmatrix}$$

$$\begin{aligned} L^{-1} &= \frac{1}{detL}(adjL) \\ &= \frac{1}{2 + 2P_1 - P_2}\begin{pmatrix} 1 + 2P_1 - P_2 & -2 + 2P_2 & 3 - 3P_1 - P_2 \\ 1 - 3P_1 + P_2 & 4P_1 - P_2 & -1 - 3P_1 \\ -P_1 & 2 - 2P_2 & -2 + 2P_1 + 2P_2 \end{pmatrix} \\ &= \left(\frac{1}{2} - \frac{1}{4}P_1 + \frac{1}{12}P_2\right)\begin{pmatrix} 1 + 2P_1 - P_2 & -2 + 2P_2 & 3 - 3P_1 - P_2 \\ 1 - 3P_1 + P_2 & 4P_1 - P_2 & -1 - 3P_1 \\ -P_1 & 2 - 2P_2 & -2 + 2P_1 + 2P_2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} + \frac{1}{4}P_1 - \frac{1}{12}P_2 & -1 + P_2 & \frac{3}{2} - \frac{3}{2}P_1 - \frac{1}{3}P_2 \\ \frac{1}{2} - P_1 + \frac{1}{6}P_2 & P_1 + \frac{8}{3}P_2 & -\frac{1}{2} - \frac{1}{2}P_1 - \frac{1}{3}P_2 \\ -\frac{1}{4}P_1 - \frac{1}{12}P_2 & 1 - \frac{1}{2}P_1 - \frac{1}{2}P_2 & -1 + P_1 + \frac{2}{3}P_2 \end{pmatrix} \end{aligned}$$

Remark 3.7. If X is a invertible symbolic 2-plithogenic square matrix and X^{-1} is its inverse, then $adjX = detX \cdot X^{-1}$.

Theorem 3.8. *Let $X = A+BP_1+CP_2$ and $Y = M+NP_1+SP_2$ be two symbolic 2-plithogenic invertible square matrices. Then XY is also invertible and $(XY)^{-1} = Y^{-1}X^{-1}$.*

Proof. By Theorem 3.4, if X is invertible then

$$det(A) \neq 0,\ det(A+B) \neq 0 \text{ and } det(A+B+C) \neq 0.$$

Similarly, if Y is invertible then

$$detM \neq 0\ ,\ det(M+N) \neq 0 \text{ and } det(M+N+S) \neq 0.$$

This implies that,

$$det(AM) = detA\ detM \neq 0$$
$$det[(A+B)(M+N)] = det(A+B)\ det(M+N) \neq 0$$
$$det[(A+B+C)(M+N+S)] = det(A+B+C)\ det(M+N+S) \neq 0.$$

Now,

$$det(XY) = det(AM) + [det((A+B)(M+N))]P_1 + [det((A+B+C)(M+N+S))]P_2 \neq 0$$

and hence XY is invertible. Also by associativity of matrix multiplication, we have

$$(XY)(Y^{-1}X^{-1}) = X(YY^{-1})X^{-1} = XX^{-1} = U_{n\times n}$$
$$(Y^{-1}X^{-1})(XY) = Y^{-1}(X^{-1}X)Y = Y^{-1}Y = U_{n\times n}.$$

Thus,$(MN)^{-1} = N^{-1}M^{-1}$. □

Theorem 3.9. *Let X and Y be two $m \times m$ symbolic 2-plithogenic invertible matrices. Then the following properties holds.*

(1) $det(adjX) = (detX)^{m-1}$.
(2) $adj(XY) = adjX\ adjY$.
(3) $adj(X^k) = (adjX)^k$ *for any positive integer* k.
(4) $adj(X^T) = (adjX)^T$.
(5) $adj(adjX) = (detX)^{m-2}X$

Proof. We can prove this results based on the properties adjoint of classical matrices. □

4. Characteristic Polynomial of Symbolic 2-Plithogenic Square Matrices

We begin this section with the following definition.

Definition 4.1. Let $L = L_0 + L_1P_1 + L_2P_2$ be a symbolic 2-plithogenic $n \times n$ square matrix with real entries. The characteristic polynomial of L is defined as

$$\phi(\lambda) = \alpha(\lambda) + [\beta(\lambda) - \alpha(\lambda)]\,P_1 + [\gamma(\lambda) - \beta(\lambda)]\,P_2$$

where,

$$\begin{aligned}\alpha(\lambda) &= det(L_0 - \lambda U_{n\times n})\\ \beta(\lambda) &= det(L_0 + L_1 - \lambda U_{n\times n}) - det(L_0 - \lambda U_{n\times n})\\ \gamma(\lambda) &= det(L_0 + L_1 + L_2 - \lambda U_{n\times n}) - det(L_0 + L_1 - \lambda U_{n\times n}).\end{aligned}$$

Example 4.2. Consider the following symbolic 2-plithogenic 2×2 matrix:

$$L = \begin{pmatrix} 2 + P_1 + 3P_2 & 1 - P_1 - P_2 \\ 3 + 4P_1 & 1 + P_2 \end{pmatrix}$$

with

$$L_0 = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}, L_0 + L_1 = \begin{pmatrix} 3 & 0 \\ 7 & 1 \end{pmatrix} \text{ and } L_0 + L_1 + L_2 = \begin{pmatrix} 6 & -1 \\ 7 & 2 \end{pmatrix}.$$

Here,

$$\begin{aligned}\alpha(\lambda) &= det(L_0 - \lambda U_{n\times n}) = \begin{pmatrix} 2-\lambda & 1 \\ 3 & 1-\lambda \end{pmatrix} = \lambda^2 - 3\lambda - 1.\\ \beta(\lambda) &= det(L_0 + L_1 - \lambda U_{n\times n}) - det(L_0 - \lambda U_{n\times n})\\ &= \begin{pmatrix} 3-\lambda & 0 \\ 7 & 1-\lambda \end{pmatrix} = \lambda^2 - 4\lambda + 3.\\ \gamma(\lambda) &= det(L_0 + L_1 + L_2 - \lambda U_{n\times n}) - det(L_0 + L_1 - \lambda U_{n\times n})\\ &= \begin{pmatrix} 6-\lambda & -1 \\ 7 & 2-\lambda \end{pmatrix} = \lambda^2 - 8\lambda - 19.\end{aligned}$$

Hence the characteristic polynomial of L is

$$\begin{aligned}\phi(\lambda) &= \lambda^2 - 3\lambda - 1 + [(\lambda^2 - 4\lambda + 3) - (\lambda^2 - 3\lambda - 1)]P_1 + [(\lambda^2 - 8\lambda - 19) - (\lambda^2 - 4\lambda + 3)]P_2\\ &= \lambda^2 - 3\lambda - 1 + (-\lambda + 4)P_1 + (-4\lambda + 16)P_2.\end{aligned}$$

Example 4.3. Consider the symbolic 2-plithogenic 3×3 matrix

$$L = \begin{pmatrix} 1 + P_1 & 1 - P_1 & 1 + P_1 - P_2 \\ 1 + P_2 & -1 + P_1 + P_2 & 2 + P_1 \\ 1 - P_1 + P_2 & -1 + P_2 & 1 + P_1 \end{pmatrix}$$

with

$$L_0 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}, L_0 + L_1 = \begin{pmatrix} 2 & 0 & 2 \\ 1 & 0 & 3 \\ 0 & -1 & 2 \end{pmatrix}, \text{ and } L_0 + L_1 + L_2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 0 & 2 \end{pmatrix}.$$

Here,

$$\begin{aligned}
\alpha(\lambda) &= det(L_0 - \lambda U_{n\times n}) = \begin{pmatrix} 1-\lambda & 1 & 1 \\ 1 & -1-\lambda & 2 \\ 1 & -1 & 1-\lambda \end{pmatrix} = -\lambda^3 + \lambda^2 + \lambda + 2. \\
\beta(\lambda) &= det(L_0 + L_1 - \lambda U_{n\times n}) - det(L_0 - \lambda U_{n\times n}) \\
&= \begin{pmatrix} 2-\lambda & 0 & 2 \\ 1 & -\lambda & 3 \\ 0 & -1 & 2-\lambda \end{pmatrix} \\
&= -\lambda^3 + 4\lambda^2 - 7\lambda + 4. \\
\gamma(\lambda) &= det(L_0 + L_1 + L_2 - \lambda U_{n\times n}) - det(L_0 + L_1 - \lambda U_{n\times n}) \\
&= \begin{pmatrix} 2-\lambda & 0 & 1 \\ 2 & 1-\lambda & 3 \\ 1 & 0 & 2-\lambda \end{pmatrix} \\
&= -\lambda^3 + 5\lambda^2 - 7\lambda + 3.
\end{aligned}$$

Hence the characteristic polynomial of L is

$$\begin{aligned}
\phi(\lambda) &= -\lambda^3 + \lambda^2 + \lambda + 2 + [(-\lambda^3 + 4\lambda^2 - 7\lambda + 4) - (-\lambda^3 + \lambda^2 + \lambda + 2)]P_1 \\
&\qquad + [(-\lambda^3 + 5\lambda^2 - 7\lambda + 3) - (-\lambda^3 + 4\lambda^2 - 7\lambda + 4)]P_2 \\
&= -\lambda^3 + \lambda^2 + \lambda + 2 + (3\lambda^2 - 8\lambda + 2)P_1 + (\lambda^2 - 1)P_2.
\end{aligned}$$

Theorem 4.4 (**Symbolic 2-plithogenic Cayely-Hamilton Theorem**). *Every symbolic 2-plithogenic square matrix satisfies its characteristic polynomial.*

Proof. We can prove this result based on the Cayely-Hamilton theorem for classical matrices. □

Example 4.5. Consider the symbolic 2-plithogenic 2×2 matrix given in Example 4.2

$$L = \begin{pmatrix} 1 + P_1 + P_2 & -1 + P_1 \\ 1 - P_2 & 1 \end{pmatrix}$$

The characteristic polynomial of L is $\phi(\lambda) = \lambda^2 - 3\lambda - 1 + (-\lambda + 4)P_1 + (-4\lambda + 16)P_2$. This implies that,

$$\begin{aligned}
\phi(L) &= L^2 - 3L - 1 + (-L + 4)P_1 + (-4L + 16)P_2. \\
&= \begin{pmatrix} -P_1 + 11P_2 & -5P_2 \\ 7P_1 + 28P_2 & -3P_1 - 7P_2 \end{pmatrix} + \begin{pmatrix} P_1 - 3P_2 & P_2 \\ -7P_1 & 3P_1 - P_2 \end{pmatrix} + \begin{pmatrix} -8P_2 & 4P_2 \\ -28P_1 & 8P_2 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}
\end{aligned}$$

Hence, $\phi(L) = 0$.

Remark 4.6. If L is a invertible symbolic 2-plithogenic matrix, then using Cayely-Hamilton theorem we can compute the inverse of L. See the following example.

Example 4.7. Consider the symbolic 2-plithogenic 2×2 matrix

$$L = \begin{pmatrix} 1+P_1+P_2 & -1+P_1 \\ 1-P_2 & 1 \end{pmatrix}$$

with

$$L_0 = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, L_0 + L_1 = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}, \text{ and } L_0 + L_1 + L_2 = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}.$$

Here,

$$\begin{aligned} \alpha(\lambda) &= det(L_0 - \lambda U_{n\times n}) = \begin{pmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{pmatrix} = \lambda^2 - 2\lambda + 2. \\ \beta(\lambda) &= det(L_0 + L_1 - \lambda U_{n\times n}) - det(L_0 - \lambda U_{n\times n}) \\ &= \begin{pmatrix} 2-\lambda & 0 \\ 1 & 1-\lambda \end{pmatrix} = \lambda^2 - 3\lambda + 2. \\ \gamma(\lambda) &= det(L_0 + L_1 + L_2 - \lambda U_{n\times n}) - det(L_0 + L_1 - \lambda U_{n\times n}) \\ &= \begin{pmatrix} 3-\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix} = \lambda^2 - 4\lambda + 3. \end{aligned}$$

Hence the characteristic polynomial of L is

$$\begin{aligned} \phi(\lambda) &= \alpha(\lambda) + [\beta(\lambda) - \alpha(\lambda)]\, P_1 + [\gamma(\lambda) - \beta(\lambda)]\, P_2 \\ &= \lambda^2 - 2\lambda + 2 - \lambda P_1 + (-\lambda + 1)P_2. \end{aligned}$$

Now, by Cayely-Hamilton theorem we have $\phi(\lambda) = 0$, we have,

$$L^2 - 2L + 2 - LP_1 + (-L+1)P_2 = 0$$
$$(2 + P_2)LL^{-1} = -L^2 + 2L + LP_1 + LP_2.$$

This implies that,

$$\begin{aligned} L^{-1} &= \frac{1}{2+P_2}\left[-L + (2 + P_1 + P_2)U_{n\times n}\right] \\ &= \frac{1}{2+P_2}\begin{pmatrix} 1 & 1-P_1 \\ -1+P_2 & 1+P_1+P_2 \end{pmatrix} \\ &= \left(\frac{1}{2} - \frac{1}{6}P_2\right)\begin{pmatrix} 1 & 1-P_1 \\ -1+P_2 & 1+P_1+P_2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} - \frac{1}{6}P_2 & \frac{1}{2} - \frac{1}{2}P_1 \\ -\frac{1}{2} + \frac{1}{2}P_2 & \frac{1}{2} + \frac{1}{2}P1 \end{pmatrix} \end{aligned}$$

5. Conclusion

In this work, the adjoint of symbolic 2-plithogenic square matrices was defined and the inverse of invertible symbolic 2-plithogenic square matrices was studied in terms of symbolic 2-plithogenic adjoint and symbolic 2-plithogenic determinant. Also, we have presented the concept of the characteristic polynomial of symbolic 2-plithogenic matrices and we have proved the symbolic 2-plithogenic version of of Cayley-Hamilton theorem with many examples that clarify the validity of this work.

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References

1. Adeleke, E. O., Agboola, A. A. A., and Smarandache, F., “Refined neutrosophic rings II”, International Journal of Neutrosophic Science, vol. 2, pp. 8994, 2020.
2. Abobala, M., “On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations”, Journal Of Mathematics, Hindawi, 2021.
3. Abobala, M., “On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations”, Mathematical Problems in Engineering, Hindawi, 2021.
4. Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., “The Algebraic Creativity In The Neutrosophic Square Matrices”, Neutrosophic Sets and Systems, Vol. 40, pp.1-11, 2021.
5. Abobala, M., “On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations”, Journal of Mathematics, Hindawi, 2021.
6. Abobala, M., “A Study of Nil Ideals and Kothes Conjecture in Neutrosophic Rings”, International Journal of Mathematics and Mathematical Sciences, Hindawi, 2021.
7. Abuobida Mohammed A. Alfahal, Yaser Ahmad Alhasan, Raja Abdullah Abdulfatah, Arif Mehmood, Mustafa Talal Kadhim, “On Symbolic 2-Plithogenic Real Matrices and Their Algebraic Properties”, International Journal of Neutrosophic Science, Vol. 21, PP. 96-104, 2023.
8. Albasheer, O., Hajjari., A., and Dalla., R., “On The Symbolic 3-Plithogenic Rings and Their Algebraic Properties”, Neutrosophic Sets and Systems, Vol 54, 2023.
9. Kandasamy, W. B. V., and Smarandache, F., “Some Neutrosophic Algebraic Structures and Neutrosophic n-Algebraic Structures”’, (Arizona: Hexis Phoenix), 2006.
10. Khaldi, A., Ben Othman, K., Von Shtawzen, O., Ali, R., and Mosa, S., “On Some Algorithms for Solving Different Types of Symbolic 2-Plithogenic Algebraic Equations”, Neutrosophic Sets and Systems, Vol 54, 2023.
11. Malath F. Alaswad, “On the Neutrosophic Characteristic Polynomials and Neutrosophic Cayley-Hamilton Theorem”, International Journal of Neutrosophic Science, Vol. 18, 2022, PP. 59-71.
12. Merkepci, H., and Abobala, M., “On The Symbolic 2-Plithogenic Rings”, International Journal of Neutrosophic Science, 2023.
13. Merkepci, H., and Rawashdeh, A., “On The Symbolic 2-Plithogenic Number Theory and Integers ”, Neutrosophic Sets and Systems, Vol 54, 2023.
14. Merkepci, H., “On Novel Results about the Algebraic Properties of Symbolic 3-Plithogenic and 4-Plithogenic Real Square Matrices”, Symmetry, 2023.
15. Smarandache, F., “Introduction to the Symbolic Plithogenic Algebraic Structures (revisited)”, Neutrosophic Sets and Systems, vol. 53, 2023.
16. Smarandache, F., “Introduction to Neutrosophic Statistics”, USA: Sitech and Education Publishing, 2014.

17. Taffach, N., "An Introduction to Symbolic 2-Plithogenic Vector Spaces Generated from The Fusion of Symbolic Plithogenic Sets and Vector Spaces", Neutrosophic Sets and Systems, Vol 54, 2023.
18. Taffach, N., and Ben Othman, K., "An Introduction to Symbolic 2-Plithogenic Modules Over Symbolic 2-Plithogenic Rings", Neutrosophic Sets and Systems, Vol 54, 2023.

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