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On Entropy and Similarity Measure of Interval Valued Neutrosophic Sets

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Abstract. In this study, we generalize the similarity measure of intuitionistic fuzzy set, which was defined by Hung and Yang, to interval valued neutrosophic sets. Then we propose an entropy measure for interval valued neutrosophic sets which generalizes the entropy measures defined Wei, Wang and Zhang, for interval valued intuitionistic fuzzy sets.

Keywords: Interval valued neutrosophic set; Entropy; Similarity measure.

1 Introduction

Neutrosophy is a branch of philosophy which studies the origin, nature and scope of neutralities. Smarandache [6] introduced neutrosophic set by adding an inde-terminancy membership on the basis of intuitionistic fuzzy set. Neutrosophic set generalizes the concept of the classic set, fuzzy set [12], interval valued fuzzy set [7], intuitionistic fuzzy set [1], interval valued intuitionistic fuzzy sets [2], etc. A neutrosophic set consider truth-membership, indeterminacy-membership and falsity-membership which are completely independent. Wang et al. [9] introduced single valued neutrosophic sets (SVNS) which is an instance of the neutrosophic set. However, in many applications, the decision information may be provided with intervals, instead of real numbers. Thus interval neutrosophic sets, as a useful generation of neutrosophic set, was introduced by Wang et al. [8]. Interval neutrosophic set described by a truth membership interval, an indeterminacy membership interval and false membership interval.

Many methods have been introduced for measuring the degree of similarity between neutrosophic set. Broumi and Smarandache [3] gave some similarity measures of neutrosophic sets. Ye [11] introduced similarity measures based on the distances of interval neutrosophic sets. Further entropy describes the degree of fuzziness in fuzzy set. Similarity measures and entropy of single valued neutrosophic sets was introduced by Majumder and Samanta [5] for the first time.

The rest of paper is organized as it follows. Some preliminary definitions and notations interval neutrosophic sets in the following section. In section 3, similarity measure between the two interval neutrosophic sets has been introduced. The notation entropy of interval neutrosophic sets has been given in section 4. In section 5 presents our conclusion.

2 Preliminaries

In this section, we give some basic definition re-lated interval valued neutrosophic sets (IVNS) from [8].

Definition 2.1. Let X be a universal set, with generic element of X denoted by x. An interval valued neutrosophic set (IVNS) A in X is characterized by a truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A, with for each $x \in X$, $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.

When the universal set X is continuous, an IVNS A can be written as

$$A = \int_X \frac{\langle T_A(x), I_A(x), F_A(x)\rangle}{x}, x \in X.$$

When the universal set X is discrete, an IVNS A can be written as

$$A = \sum_{i=1}^{n} \langle T_A(x_i), I_A(x_i), F_A(x_i)\rangle / x_i\,, x \in X.$$

Definition 2.2. An IVNS A is empty if and only if $\inf T_A(x) = \sup T_A(x) = 0$, $\inf I_A(x) = \sup I_A(x) = 1$ and $\inf F_A(x) = \sup F_A(x) = 0$, for all $x \in X$.

Definition 2.3. An IVNS A is contained in the other IVNS B, $A \subseteq B$, if and only if

$$\inf T_A(x) \le \inf T_B(x)\,, \quad \sup T_A(x) \le \sup T_B(x)$$
$$\inf I_A(x) \ge \inf I_B(x)\,, \quad \sup I_A(x) \ge \sup I_B(x)$$
$$\inf F_A(x) \ge \inf F_B(x)\,, \quad \sup F_A(x) \ge \sup F_B(x)$$

for all $x \in X$.

Definition 2.4. The complement of an IVNS A is denoted by A^c and is defined by

$$T_{A^c}(x) = F_A(x)$$
$$\inf I_{A^c}(x) = 1 - \sup I_A(x)$$
$$\sup I_{A^c}(x) = 1 - \inf I_A(x)$$
$$F_{A^c}(x) = T_A(x)$$
for all $x \in X$.

Definition 2.5. Two IVNSs A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.

Definition 2.6. The union of two IVNSs A and B is an IVNS C, written as $C = A \cup B$, is defined as follow
$$\inf T_C(x) = \max\{\inf T_A(x), \inf T_B(x)\}$$
$$\sup T_C(x) = \max\{\sup T_A(x), \sup T_B(x)\}$$
$$\inf I_C(x) = \min\{\inf I_A(x), \inf I_B(x)\}$$
$$\sup I_C(x) = \min\{\sup I_A(x), \sup I_B(x)\}$$
$$\inf F_C(x) = \min\{\inf F_A(x), \inf F_B(x)\}$$
$$\sup F_C(x) = \min\{\sup F_A(x), \sup F_B(x)\}$$
for all $x \in X$.

Definition 2.7. The intersection of two IVNSs A and B, written as $C = A \cap B$, is defined as follow
$$\inf T_C(x) = \min\{\inf T_A(x), \inf T_B(x)\}$$
$$\sup T_C(x) = \min\{\sup T_A(x), \sup T_B(x)\}$$
$$\inf I_C(x) = \max\{\inf I_A(x), \inf I_B(x)\}$$
$$\sup I_C(x) = \max\{\sup I_A(x), \sup I_B(x)\}$$
$$\inf F_C(x) = \max\{\inf F_A(x), \inf F_B(x)\}$$
$$\sup F_C(x) = \max\{\sup F_A(x), \sup F_B(x)\}$$
for all $x \in X$.

3 Similarity measure between interval valued neutrosophic sets

Similarity measure between Interval Neutrosophic sets is defined by Ye [11] as follow.

Definition 3.1. Let $N(X)$ be all IVNSs on X and $A, B \in N(X)$. A similarity measure between two IVNSs is a function $S: N(X) \times N(X) \to [0,1]$ which is satisfies the following conditions:

i. $0 \le S(A,B) \le 1$
ii. $S(A,B) = 1$ iff $A = B$
iii. $S(A,B) = S(B,A)$
iv. If $A \subset B \subset C$ then $S(A,C) \le S(A,B)$ and $S(A,C) \le S(B,C)$ for all $A, B, C \in N(X)$.

We generalize the similarity measure of intuitionistic fuzzy set, which defined in [4], to interval valued neutrosophic sets as follow.

Definition 3.2. Let A, B be two IVN sets in X. The similarity measure between the IVN sets A and B can be evaluated by the function,

$$\begin{aligned} S(A,B) = 1 - 1/2 \Big[& \max_i(|\inf T_A(x_i) - \inf T_B(x_i)|) \\ & + \max_i(|\sup T_A(x_i) - \sup T_B(x_i)|) \\ & + \max_i(|\inf I_A(x_i) - \inf I_B(x_i)|) \\ & + \max_i(|\sup I_A(x_i) - \sup I_B(x_i)|) \\ & + \max_i(|\inf F_A(x_i) - \inf F_B(x_i)|) \\ & + \max_i(|\sup F_A(x_i) - \sup F_B(x_i)|) \Big] \end{aligned}$$

for all $x \in X$.

We shall prove this similarity measure satisfies the properties of the Definition 3.1.

Proof: We show that $S(A,B)$ satisfies all properties 1-4 as above. It is obvious, the properties 1-3 is satisfied of definition 3.1. In the following we only prove 4.

Let $A \subset B \subset C$, the we have
$$\inf T_A(x_i) \le \inf T_B(x_i) \le \inf T_C(x_i)$$
$$\sup T_A(x_i) \le \sup T_B(x_i) \le \sup T_C(x_i)$$
$$\inf I_A(x_i) \ge \inf I_B(x_i) \ge \inf I_C(x_i)$$
$$\sup I_A(x_i) \ge \sup I_B(x_i) \ge \sup I_C(x_i)$$
$$\inf F_A(x_i) \ge \inf F_B(x_i) \ge \inf F_C(x_i)$$
$$\sup F_A(x_i) \ge \sup F_B(x_i) \ge \sup F_C(x_i)$$
for all $x \in X$. It follows that
$$|\inf T_A(x_i) - \inf T_B(x_i)| \le |\inf T_A(x_i) - \inf T_C(x_i)|$$
$$|\sup T_A(x_i) - \sup T_B(x_i)| \le |\sup T_A(x_i) - \sup T_C(x_i)|$$
$$|\inf I_A(x_i) - \inf I_B(x_i)| \le |\inf I_A(x_i) - \inf I_C(x_i)|$$
$$|\sup I_A(x_i) - \sup I_B(x_i)| \le |\sup I_A(x_i) - \sup I_C(x_i)|$$
$$|\inf F_A(x_i) - \inf F_B(x_i)| \le |\inf F_A(x_i) - \inf F_C(x_i)|$$
$$|\sup F_A(x_i) - \sup F_B(x_i)| \le |\sup F_A(x_i) - \sup F_C(x_i)|.$$

It means that $S(A,C) \le S(A,B)$. Similarly, it seems that $S(A,C) \le S(B,C)$.

The proof is completed

4 Entropy of an interval valued neutrosophic set

The entropy measure on IVIF sets is given by Wei [10]. We extend the entropy measure on IVIF set to interval valued neutrosophic set.

Definition 4.1. Let $N(X)$ be all IVNSs on X and $A \in N(X)$. An entropy on IVNSs is a function $E_N: N(X) \to [0,1]$ which is satisfies the following axioms:

i. $E_N(A) = 0$ if A is crisp set
ii. $E_N(A) = 1$ if $[\inf T_A(x), \sup T_A(x)] = [\inf F_A(x), \sup F_A(x)]$ and $\inf I_A(x) = \sup I_A(x)$ for all $x \in X$
iii. $E_N(A) = E_N(A^c)$ for all $A \in N(X)$
iv. $E_N(A) \ge E_N(B)$ if $A \subseteq B$ when $\sup I_A(x) - \sup I_B(x) < \inf I_A(x) - \inf I_B(x)$ for all $x \in X$.

Definition 4.2. The entropy of IVNS set A is,

$$E(A) = \frac{1}{n}\sum_{i=1}^{n} \left[\frac{2 - |inf\ T_A(x) - inf\ F_A(x)| - |sup\ T_A(x) - sup\ F_A(x)| - |inf\ I_A(x) - sup\ I_A(x)|}{2 + |inf\ T_A(x) - inf\ F_A(x)| + |sup\ T_A(x) - sup\ F_A(x)| + |inf\ I_A(x) - sup\ I_A(x)|} \right]$$

for all $x \in X$.

Theorem: The IVN entropy of $E_N(A)$ is an entropy measure for IVN sets.

Proof: We show that the $E_N(A)$ satisfies the all properties given in Definition 4.1.

i. When A is a crisp set, i.e.,
$$[\inf T_A(x), \sup T_A(x)] = [0,0]$$
$$[\inf I_A(x), \sup I_A(x)] = [0,0]$$
$$[\inf F_A(x), \sup F_A(x)] = [1,1]$$
or
$$[\inf T_A(x), \sup T_A(x)] = [1,1]$$
$$[\inf I_A(x), \sup I_A(x)] = [0,0]$$
$$[\inf F_A(x), \sup F_A(x)] = [0,0]$$
for all $x_i \in X$. It is clear that $E_N(A) = 0$.

ii. Let $[\inf T_A(x), \sup T_A(x)] = [\inf F_A(x), \sup F_A(x)]$ and $\inf I_A(x) = \sup I_A(x)$ for all $x \in X$. Then

$$E_N(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{2-0-0-0}{2+0+0+0} = \frac{1}{n}\sum_{i=1}^{n} 1 = 1.$$

iii. Since $T_{A^c}(x) = F_A(x)$, $\inf I_{A^c}(x) = 1 - \sup I_A(x)$, $\sup I_{A^c}(x) = 1 - \inf I_A(x)$ and $F_{A^c}(x) = T_A(x)$, it is clear that $E_N(A) = E_N(A^c)$.

iv. If $A \subseteq B$, then $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$ and $\sup F_A(x) \geq \sup F_B(x)$. So

$$|\inf T_A(x) - \inf F_A(x)| \leq |\inf T_B(x) - \inf F_B(x)|$$

and

$$|\sup T_A(x) - \sup F_A(x)| \leq |\sup T_B(x) - \sup F_B(x)|$$

for all $x \in X$. And

$$|\inf I_A(x) - \sup I_A(x)| \leq |\inf I_B(x) - \sup I_B(x)|$$

when

$$\sup I_A(x) - \sup I_B(x) \leq \inf I_A(x) - \inf I_B(x)$$

for all $x \in X$. Therefore it is clear that $E(A) \geq E(B)$.

The proof is completed

Conclusion

In this paper we introduced similarity measure of interval valued neutrosophic sets. These measures are consistent with similar considerations for other sets. Then we give entropy of an interval valued neutrosophic set. This entropy was generalized the entropy measure on interval valued intuitionistic sets in [10].

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