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Neutrosophic Decision Making Model of School Choice

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Abstract The purpose of this paper is to present single valued neutrosophic decision making model of school choice. Childhood is a crucial stage in terms of a child's physical, intellectual, emotional and social development i.e. all round development of a child. Mental and physical abilities of children grow at an increasing rate. Children particularly need high quality personal care and learning experiences.

Children begin learning from the moment the child takes his/her birth and continues on throughout his/her life. Babies and toddlers need positive early learning experiences for their mental and physical development and this lays the foundation for later school success. So it is necessary to select the best school for the children among all the feasible alternatives by which all needs of the children are fulfilled.

A large number of parents have an increasing array of options in choosing the best school for their children among all alternatives. Those options vary from place to place. In this paper, neutrosophic multi-attribute decision-making with interval weight information is used to form a decision-making model for choosing the best school for the children. A numerical example is developed based on expert opinions from english medium schools of Nadia districts, West Bengal, India. The problem is solved to show the effectiveness of the proposed single valued neutrosophic decision making model.

Keywords: Neutrosophic multi-attribute decision-making, Grey relational analysis, School choice.

1 Introduction

Decision-making is a challenging act of choosing between two or more possible alternatives. Decision makers have to make decision based on complete or incomplete information. That’s why new scientific strategies must be introduced for improvement the quality of decisions.

Rational choice theory [1, 2, 3] suggests that parents are utility maximizer in decision making who make decisions from clear value preferences based on the costs, benefits, and probabilities of success of various options. The options [1, 2, 3] are namely, ability to fulfil the demand effectively from local schools and teachers, and pursuing the best interests of their children.

Literature review for school choice [4, 5], however, reflects that the context of parental decision-making is more complex than the result of individual rational calculations of the economic return of their investment. Parental choice is a part of social process influenced by salient properties of social class and networks of social relationships [6-9]. Coleman [6], Bauch and Goldring [7], Bosetti [8], Reay and Lucey [9] explain that when an individual comes in close contact with important decision making situation, a rational actor will engage in a search for information before making a decision. However, According to Ball [10], parents seem to use a mixture of rationalities involving an element of the fortuitous and haphazard.

For school choice, parents generally depend on their personal values and judgment as well as others within their social and professional networks in order to collect required information. Parents prefer to choice private schools because they think their children will have better opportunities in private schools. To perform this optimally, parents need to have a clear understanding of school administration and the rules of the school admission process and engage in strategic school choice. In this challenging and demanding process, parents may make technical errors about the rules as well as in judgment in selecting and ordering the schools. Abdulkadiroglu and Sonmez [11] mentioned that the open enrollment school choice programs in Boston, Minneapolis, and Seattle ask parents to make complex school choice decisions, which can result in an inefficient allocation of school seats. In order to deal school choice problem, new model is urgently needed.

Most of the study on school choice is based on assumptions at the theoretical level with little practical situation. Most of the research on school choice is done in crisp environment. Radhakrishnan and Kalaichelvi [12] studied school choice problem based on fuzzy analytic hierarchy process. However, in fuzzy environment degree of indeterminacy is not included. So neutrosophic set theoretic based approach may be helpful to deal this type of problems. Literature review indicates that no research on school choce is done in neutrosophic environment.

Decision making is oriented in every sphere of human activities. However, human being realizes problems in making decision on many normal activities such as education for children, quality of food, transportation, purchasing, selection of partner, healthcare, selection of shelter, etc. A small number of studies are done on edcational problems based on the concept of fuzzy set, neutrosophic set and grey system theory. Pramanik and Mukhopadhyaya [13] presented grey relational analysis based on intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. Mondal and Pramanik [14] presented multi-criteria group decision making approach for teacher recruitment in higher education under simplified neutrosophic environment. In this paper we present a methodological approach to choose the best elementary school for children among particular alternatives to their designated neighbourhood using neutrosophic multi-attribute decision-making with interval weight information based on grey relational analysis. .

A numerical example is developed based on expert opinions from english medium schools of Nadia districts, West Bengal, India . The problem is solved to demonstrate the effectiveness of the proposed neutrosophic decision making model.

Rest of the paper is organized as follows: Section 2 presents preliminaries of neutrosophic sets. Section 3 describes single valued neutrosophic multiple attribute decision making problem based on GRA with interval weight information. Section 4 is devoted to propose neutrosophic decision making model of school choice. Finally, Section 5 presents concluding remarks.

2 Neutrosophic preliminaries

2.1 Definition on neutrosophic sets

The concept of neutrosophic set is originated from neutrosophy, a new branch of philosophy. According to Smarandache [15] ‘‘Neutrosophy studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra``.

Definition1: Let ξ be a space of points (objects) with generic element in ξ denoted by x. Then a neutrosophic set α in ξ is characterized by a truth membership function T_α an indeterminacy membership function I_α and a falsity membership function F_α. The functions T_α and F_α are real standard or non-standard subsets of $]0^-,1^+[$ that is T_α: $\xi \to]0^-,1^+[$; I_α: $\xi \to]0^-,1^+[$; F_α: $\xi \to]0^-,1^+[$.

It should be noted that there is no restriction on the sum of $T_\alpha(x)$, $I_\alpha(x)$, $F_\alpha(x)$ i.e.

$$0^- \le T_\alpha(x) + I_\alpha(x) + F_\alpha(x) \le 3^+$$

Definition2: The complement of a neutrosophic set α is denoted by α^c and is defined by

$$T_{\alpha^c}(x) = \{1^+\} - T_\alpha(x);\ I_{\alpha^c}(x) = \{1^+\} - I_\alpha(x)$$
$$F_{\alpha^c}(x) = \{1^+\} - F_\alpha(x)$$

Definition3: (Containment) A neutrosophic set α is contained in the other neutrosophic set β, $\alpha \subseteq \beta$ if and only if the following result holds.

$$\inf T_\alpha(x) \le \inf T_\beta(x),\ \sup T_\alpha(x) \le \sup T_\beta(x)$$
$$\inf I_\alpha(x) \ge \inf I_\beta(x),\ \sup I_\alpha(x) \ge \sup I_\beta(x)$$
$$\inf F_\alpha(x) \ge \inf F_\beta(x),\ \sup F_\alpha(x) \ge \sup F_\beta(x)$$

for all x in ξ.

Definition4: (Single-valued neutrosophic set). Let ξ be a universal space of points (objects) with a generic element of ξ denoted by x.

A single-valued neutrosophic set S is characterized by a truth membership function $T_s(x)$, an indeterminacy membership function $I_s(x)$, and a falsity membership function $F_s(x)$ with $T_s(x)$, $I_s(x)$, $F_s(x) \in [0, 1]$ for all x in ξ . When ξ is continuous, a SNVS can be written as follows:

$$S = \int_x \langle T_s(x), F_s(x), I_s(x) \rangle / x, \forall x \in \xi$$

and when ξ is discrete, a SVNSs S can be written as follows:

$$S = \sum \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in \xi$$

It should be noted that for a SVNS S,

$$0 \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3, \forall x \in \xi$$

and for a neutrosophic set, the following relation holds:

$$0^- \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3^+, \forall x \in \xi$$

Definition5: The complement of a neutrosophic set S is denoted by S^c and is defined by

$$T_S{}^c(x) = F_S(x);\ I_S{}^c(x) = 1 - I_S(x);\ F_S{}^c(x) = T_S(x)$$

Definition6: A SVNS S_α is contained in the other SVNS S_β , denoted as $S_\alpha \subseteq S_\beta$ iff, $T_{S_\alpha}(x) \le T_{S_\beta}(x)$; $I_{S_\alpha}(x) \ge I_{S_\beta}(x)$; $F_{S_\alpha}(x) \ge F_{S_\beta}(x)$, $\forall x \in \xi$.

Definition7: Two single valued neutrosophic sets S_α and S_β are equal, i.e. $S_\alpha = S_\beta$, if and only if $S_\alpha \subseteq S_\beta$ and $S_\alpha \supseteq S_\beta$

Definition8: (Union) The union of two SVNSs S_α and S_β is a SVNS S_γ , written as $S_\gamma = S_\alpha \cup S_\beta$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to those of S_α and S_β as follows:

$$T_{S_\gamma}(x) = \max(T_{S_\alpha}(x), T_{S_\beta}(x));$$
$$I_{S_\gamma}(x) = \min(I_{S_\alpha}(x), I_{S_\beta}(x));$$

$F_{S_\gamma}(x) = \min\left(F_{S_\alpha}(x), F_{S_\beta}(x)\right)$ for all x in ξ

Definition 9: (intersection) The intersection of two SVNSs, S_α and S_β is a SVNS S_δ, written as $S_\delta = S_\alpha \cap S_\beta$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of S_α an S_β as follows:

$T_{S_\delta}(x) = \min\left(T_{S_\alpha}(x), T_{S_\beta}(x)\right)$;

$I_{S_\delta}(x) = \max\left(I_{S_\alpha}(x), I_{S_\beta}(x)\right)$;

$F_{S_\delta}(x) = \max\left(F_{S_\alpha}(x), F_{S_\beta}(x)\right), \forall x \in \xi$

3. Distance between two neutrosophic sets

The general SVNS can be written by the following form:

$S = \{(x/(T_S(x), I_S(x), F_S(x))) : x \in \xi\}$

Finite SVNSs can be represented by the ordered tetrads:

$$S = \begin{Bmatrix} (x_1/(T_S(x_1), I_S(x_1), F_S(x_1))), \cdots, \\ (x_m/(T_S(x_m), I_S(x_m), F_S(x_m))) \end{Bmatrix}, \forall x \in \xi \quad (1)$$

Definition 10: Let

$$S_\alpha = \begin{Bmatrix} (x_1/(T_{S_\alpha}(x_1), I_{S_\alpha}(x_1), F_{\alpha S}(x_1))), \cdots, \\ (x_n/(T_{S_\alpha}(x_n), I_{S_\alpha}(x_n), F_{S_\alpha}(x_n))) \end{Bmatrix} \quad (2)$$

$$S_\beta = \begin{Bmatrix} (x_1/(T_{S_\beta}(x_1), I_{S_\beta}(x_1), F_{S_\beta}(x_1))), \cdots, \\ (x_n/(T_{S_\beta}(x_n), I_{S_\beta}(x_n), F_{S_\beta}(x_n))) \end{Bmatrix} \quad (3)$$

be two single-valued neutrosophic sets (SVNSs) in $x = \{x_1, x_2, x_3, \ldots, x_n\}$

Then the Hamming distance between two SVNSs as S_α and S_β is defined as follows:

$$d_S(S_\alpha, S_\beta) = \sum_{i=1}^{n} \left\langle \begin{matrix} \left|T_{S_\alpha}(x) - T_{S_\beta}(x)\right| + \\ \left|I_{S_\alpha}(x) - I_{S_\beta}(x)\right| + \\ \left|F_{S_\alpha}(x) - F_{S_\beta}(x)\right| \end{matrix} \right\rangle \quad (4)$$

and normalized Hamming distance between two SNVS S_α and S_β is defined as follows:

$$^N d_S(S_\alpha, S_\beta) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{matrix} \left|T_{S_\alpha}(x) - T_{S_\beta}(x)\right| + \\ \left|I_{S_\alpha}(x) - I_{S_\beta}(x)\right| + \\ \left|F_{S_\alpha}(x) - F_{S_\beta}(x)\right| \end{matrix} \right\rangle \quad (5)$$

with the following two properties

1. $0 \le d_S(S_\alpha, S_\beta) \le 3n$ (6)
2. $0 \le {}^N d_S(S_\alpha, S_\beta) \le 1$ (7)

Definition11: From the neutrosophic cube [16], it can be stated that the membership grade represents the estimates reliability. Ideal neutrosophic reliability solution INERS[17]

$Q_S^+ = \left\langle q_{S_1}^+, q_{S_2}^+, \cdots, q_{S_n}^+ \right\rangle$ is a solution in which every component is presented by $q_{S_j}^+ = \left\langle T_j^+, I_j^+, F_j^+ \right\rangle$ where $T_j^+ = \max_i \{T_{ij}\}$, $I_j^+ = \min_i \{I_{ij}\}$ and $F_j^+ = \min_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$ for i = 1, 2, …, m, j = 1, 2, …, n

Definition 12: In the neutrosophic cube [16] maximum un-reliability occurs when the indeterminacy membership grade and the degree of falsity membership reaches maximum simultaneously. Therefore, the ideal neutrosophic estimates un-reliability solution (INEURS) [17]

$Q_S^- = \left\langle q_{S_1}^-, q_{S_2}^-, \cdots, q_{S_n}^- \right\rangle$ is a solution in which every component is represented by $q_{S_j}^- = \left\langle T_j^-, I_j^-, F_j^- \right\rangle$ where $T_j^- = \min_i \{T_{ij}\}$, $I_j^- = \max_i \{I_{ij}\}$ and $F_j^- = \max_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$ for i = 1, 2, …, m, j = 1, 2, …, n

3. Single valued neutrosophic multiple attribute decision-making problems based on GRA with interval weight information [17]

A multi-criteria decision making problem with m alternatives and n attributes is here considered. Let $A_1, A_2, \ldots, A_m$ be a discrete set of alternatives, and $C_1, C_2, \ldots, C_n$ be the set of criteria. The decision makers provide the ranking of alternatives. The ranking presents the performances of alternatives A_i against the criteria C_j. The values associated with the alternatives for MADM problem can be presented in the following decision matrix (see Table 1).

Table 1: Decision matrix

$$D = \langle \delta_{ij} \rangle_{m \times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \delta_{11} & \delta_{12} & \ldots & \delta_{1n} \\ A_2 & \delta_{21} & \delta_{22} & \ldots & \delta_{2n} \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ A_m & \delta_{m1} & \delta_{m2} & \ldots & \delta_{mn} \end{array} \quad (8)$$

The weight $\omega_j \in [0, 1]$ (j = 1, 2, ..., n) represents the relative importance of criteria C_j (j = 1, 2, ..., m) to the decision-making process such that $\sum_{j=1}^{n} \omega_j = 1$. S is the set of partially known weight information that can be represented by the following forms due to Kim and Ahn[18] and Park [19].

Form1. A weak ranking: $\omega_i \ge \omega_j$ for $i \ne j$;

Form2. A strict ranking: $\omega_i - \omega_j \ge \psi_i$, $\psi_i > 0$, for $i \ne j$;

Form3. A ranking of differences: $\omega_i - \omega_j \ge \omega_k - \omega_1$, for $j \ne k \ne 1$;

Form4. A ranking with multiples: $\omega_i \ge \sigma_j \omega_j$, $\sigma_j \in [0, 1]$, for $i \ne j$;

Form5. An interval form $\delta_i \le \omega_i \le \delta_i + \varepsilon_i$, $0 \le \delta_i < \delta_i + \varepsilon_i \le 1$

The steps of single valued neutrosophic multiple attribute decision-making based on GRA under SVNS due to Biswal et al.[20] can be presented as follows.

Step1. Construction of the decision matrix with SVNSs

Consider the above mention multi attribte decision making problem(8). The general form of decision matrix as shown in Table1 can be presented after data pre-processing. Here, the ratings of alternatives A_i (i = 1, 2, … m) with respect to attributes C_j (j = 1, 2,…n) are considered as SVNSs. The neutrosophic values associated with the alternatives for MADM problems can be represented in the following decision matrix (see Table 2):

Table2:Decision matrix with SVNS

$\delta_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n}$ =

	C_1	C_2	...	C_n
A_1	$\langle T_{11}, I_{11}, F_{11}\rangle$	$\langle T_{12}, I_{12}, F_{12}\rangle$	...	$\langle T_{1n}, I_{1n}, F_{1n}\rangle$
A_2	$\langle T_{21}, I_{21}, F_{21}\rangle$	$\langle T_{22}, I_{22}, F_{22}\rangle$	...	$\langle T_{2n}, I_{2n}, F_{2n}\rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle T_{m1}, I_{m1}, F_{m1}\rangle$	$\langle T_{m2}, I_{m2}, F_{m2}\rangle$	...	$\langle T_{mn}, I_{mn}, F_{mn}\rangle$

(9)

In the matrix $\mathrm{d_S} = \langle \mathrm{T_{ij}, I_{ij}, F_{ij}} \rangle_{\mathrm{m\times n}}$ T_{ij} I_{ij} and F_{ij} denote the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i with respect to attribute C_j. These three components for SVNS satisfy the following properties:

1. $0 \le T_{ij} \le 1, \ 0 \le I_{ij} \le 1, \ 0 \le F_{ij} \le 1$ (10)
2. $0 \le T_{ij} + I_{ij} + F_{ij} \le 3$ (11)

Step2. Determination of the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS).

The ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS)for single valued neutrosophic decision matrix can be determined from the defintion 11 and 12.

Step3. Calculation of the neutrosophic grey relational coefficient.

Grey relational coefficient of each alternative from INERS can be defined as follows:

$$G_{ij}^{+} = \frac{\min_i \min_j \Delta_{ij}^{+} + \rho \max_i \max_j \Delta_{ij}^{+}}{\Delta_{ij}^{+} + \rho \max_i \max_j \Delta_{ij}^{+}} \quad (12)$$

where $\Delta_{ij}^{+} = d\left(q_{S_j}^{+}, q_{S_{ij}}\right)$, i = 1, 2,…,m. and j = 1, 2,…,n.

Grey relational coefficient of each alternative from INEURS can be defined as follows:

$$G_{ij}^{-} = \frac{\min_i \min_j \Delta_{ij}^{-} + \rho \max_i \max_j \Delta_{ij}^{-}}{\Delta_{ij}^{-} + \rho \max_i \max_j \Delta_{ij}^{-}} \quad (13)$$

where $\Delta_{ij}^{-} = d\left(q_{S_{ij}}, q_{S_j}^{-}\right)$, i = 1, 2,…,m. and j =1, 2,…, n.

$\rho \in [0,1]$ is the distinguishing coefficient or the identification coefficient,. Smaller value of distinguishing coefficient reflects the large range of grey relational coefficient. Generally, ρ = 0.5 is set for decision-making situation.

Step4. Determination of the weights of the criteria

The grey relational coefficient between INERS and itself is (1, 1, …, 1). Similarly, the grey relational coefficient between INEURS and itself is also (1, 1, …, 1). The corresponding deviations are presented as follows:

$$d_i^{+}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_j \quad (14)$$

$$d_i^{-}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}^{-}\right)w_j \quad (15)$$

A satisfactory weight vector W= (w_1, w_2,…, w_n) is determined by making smaller all the distances $d_i^{+}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_j$ and $d_i^{-}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}\right)w_j$

Using the max-min operator [21] to integrate all the distances $d_i^{+}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_j$ for i = 1, 2, …, m and $d_i^{-}(w) = \sum_{j=1}^{n}\left(1 - G_{ij}\right)w_j$ for i = 1, 2, …, m, Biswas et al. [20] formulated the following programming model:

$$Model: 1a: \begin{cases} \min z^{+} \\ subject\ to: \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_{ij} \le z^{+} \end{cases} \quad (16)$$

For i=1, 2, …, m

$$Model: 1b: \begin{cases} \min z^{-} \\ subject\ to: \sum_{j=1}^{n}\left(1 - G_{ij}^{-}\right)w_{ij} \le z^{-} \end{cases} \quad (17)$$

$W \in S$

Here $z^{+} = \max_i \left\langle \sum_{j=1}^{n}\left(1 - G_{ij}^{+}\right)w_j \right\rangle$ and

$z^{-} = \max_i \left\langle \sum_{j=1}^{n}\left(1 - G_{ij}\right)w_j \right\rangle$ for i =1, 2, …, m

Solving these two models (*Model-1a*) and (*Model-1b*), the optimal solutions W^{+}= (w_1^{+}, w_2^{+}, …, w_n^{+}) and W^{-} = (w_1^{-}, w_2^{-}, …, w_n^{-}) can be obtained. Combination of these two optimal solutions provides the weight vector of the criterion i.e.

W= tW^{+}+ (1-t)W^{-} for t ∈]0, 1[. (18)

Step5. Calculation of the neutrosophic grey relational coefficient (NGRC)

The degree of neutrosophic grey relational coefficient of each alternative from Indeterminacy Truthfullness Falsity Positive Ideal Solution (ITFPIS) and Indeterminacy Truthfullness Falsity Negative Ideal Solution (ITFNIS) are obtaoinrd using the following relationss:

$$G_i^{+} = \sum_{j=1}^{n} w_j G_{ij}^{+} \quad (19)$$

$$G_i^{-} = \sum_{j=1}^{n} w_j G_{ij}^{-} \quad (20)$$

Step6. Calculation of the neutrosophic relative relational degree (NRD)

Neutrosophic relative relational degree of each alternative from ITFPIS can be obtained by employing the following equation:

$$R_i = \frac{G_i^+}{G_i^+ + G_i^-} \quad (21)$$

Step7. **Ranking of the alternatives**

The highest value of neutrosophic relative relational degree R_i reflects the most desired alternative.

4. Single valued neutrosophic decision making model of school choice

Based on the field study, five major criteria for are identified by domain experts for developing a model for the selection of the best school by the parents for their children. The details are presented as follows.

1) Facility of transportation (C_1): It includes the cost of transportation facility availed by the child provided by school administration from child's house to the school.

2) Cost (C_2): It includes reasonable admission fees and other fees stipulated by the school administration.

3) Staff and curriculums (C_3): The degree of capability of the school administration in providing good competent staff, teaching and coaching, and extra curricular activities.

4) Healthy environmnet and medical facility(C_4): The degree of providing modern infrastructure, campus discipline, security, and medical facilities to the students by the school administration..

5) Administration(C_5): The degree of capability of administration in dealing with academic performance, staff and student welfare, reporting to parents.

After the initial screening, three schools listed below were considered as alternatives and an attempt has been made to develop a model to select the best one based on the above mentioned criteria.

A₁: Ananda Niketan Nursery & KG School, Santipur

A₂:Krishnagar Academy English Medium Public School, Krishnagar

A₃: Sent Mary's English School, Ranaghat

We obtain the following single-valued neutrosophic decision matrix (see Table 3) based on the experts' assessment:

Table3: Decision matrix with SVNS

$\delta_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{3\times5} =$

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle .8,.1,.2 \rangle$	$\langle .7,.2,.3 \rangle$	$\langle .8,.3,.3 \rangle$	$\langle .7,.2,.4 \rangle$	$\langle .7,.4,.3 \rangle$
A_2	$\langle .7,.2,.2 \rangle$	$\langle .8,.2,.3 \rangle$	$\langle .8,.2,.3 \rangle$	$\langle .7,.3,.4 \rangle$	$\langle .8,.3,.3 \rangle$
A_3	$\langle .8,.2,.3 \rangle$	$\langle .7,.2,.2 \rangle$	$\langle .7,.1,.3 \rangle$	$\langle .8,.3,.3 \rangle$	$\langle .9,.1,.2 \rangle$

(22)

Information of the attribute weights is partially known. The known weight information is given as follows:

$0.16 \le w_1 \le .22,\quad 0.15 \le w_2 \le 0.25,\quad 0.19 \le w_3 \le 0.3,$

$0.13 \le w_4 \le 0.21,\quad 0.17 \le w_5 \le 0.2,$

$\sum_{j=1}^{5} w_j = 1$

and $w_j \ge 0$ for $j = 1, 2, 3, 4, 5$

The problem is solved by the following steps:

Step1: Determination of the ideal neutrosophic estimates reliability solution

The ideal neutrosophic estimates reliability solution (INERS) from the given decision matrix (see Table 3) can be obtained as follows:

$$Q_S^+ = \left[q_{S_1}^+, q_{S_2}^+, q_{S_3}^+, q_{S_4}^+, q_{S_5}^+\right] = \begin{bmatrix} \left\langle \max_i \{T_{i1}\}, \min_i \{I_{i1}\}, \min_i \{F_{i1}\} \right\rangle, \\ \left\langle \max_i \{T_{i2}\}, \min_i \{I_{i2}\}, \min_i \{F_{i2}\} \right\rangle, \\ \left\langle \max_i \{T_{i3}\}, \min_i \{I_{i3}\}, \min_i \{F_{I3}\} \right\rangle, \\ \left\langle \max_i \{T_{i4}\}, \min_i \{I_{i4}\}, \min_i \{F_{i4}\} \right\rangle, \\ \left\langle \max_i \{T_{i5}\}, \min_i \{I_{i5}\}, \min_i \{F_{i5}\} \right\rangle \end{bmatrix} \quad (23)$$

$$= \begin{bmatrix} \langle .8,.1,.2 \rangle, \langle .8,.2,.2 \rangle, \langle .8,.1,.3 \rangle, \\ \langle .8,.2,.3 \rangle, \langle .9,.1,.2 \rangle \end{bmatrix}$$

Step2. **Determination of the ideal neutrosophic estimates un-reliability solution**

The ideal neutrosophic estimates un-reliability solution can be obtained as follows:

$$Q_S^- = \left[q_{S_1}^-, q_{S_2}^-, q_{S_3}^-, q_{S_4}^-, q_{S_5}^-\right] = \begin{bmatrix} \left\langle \min_i \{T_{i1}\}, \max_i \{I_{i1}\}, \max_i \{F_{i1}\} \right\rangle, \\ \left\langle \min_i \{T_{i2}\}, \max_i \{I_{i2}\}, \max_i \{F_{i2}\} \right\rangle, \\ \left\langle \min_i \{T_{i3}\}, \max_i \{I_{i3}\}, \max_i \{F_{i3}\} \right\rangle, \\ \left\langle \min_i \{T_{i4}\}, \max_i \{I_{i4}\}, \max_i \{F_{i4}\} \right\rangle, \\ \left\langle \min_i \{T_{i5}\}, \max_i \{I_{i5}\}, \max_i \{F_{i5}\} \right\rangle \end{bmatrix} \quad (24)$$

$$= \begin{bmatrix} \langle .7,.2,.3 \rangle, \langle .7,.2,.3 \rangle, \langle .7,.3,.3 \rangle, \\ \langle .7,.3,.4 \rangle, \langle .7,.4,.3 \rangle \end{bmatrix}$$

Step3. Calculation of the neutrosophic grey relational coefficient of each alternative from INERS and INEURS

Using equation (12), the neutrosophic grey relational coefficient of each alternative from INERS can be obtained as follows:

$\left\langle G_{ij}^{+} \right\rangle_{3\times5}=$

$$\begin{bmatrix} 1.0000 & 0.5692 & 0.4805 & 0.5692 & 0.3333 \\ 0.5692 & 0.6491 & 0.6491 & 0.5211 & 0.4253 \\ 0.5692 & 0.6491 & 0.6491 & 0.6491 & 1.0000 \end{bmatrix} \quad (25)$$

and from equation (13), the neutrosophic grey relational coefficient of each alternative from INEUS is obtained as follows:

$\left\langle G_{ij}^{-} \right\rangle_{3\times5}=$

$$\begin{bmatrix} 0.5211 & 1.0000 & 0.6491 & 0.6411 & 1.0000 \\ 0.6411 & 0.6411 & 0.5692 & 1.0000 & 0.5692 \\ 0.6411 & 0.6411 & 0.4805 & 0.5692 & 0.3333 \end{bmatrix} \quad (26)$$

Step4. Determination of the weights of attribute

Case1. Using the model (*Model-1a*) and (*Model-2b*), the single objective LPP models is formulated as follows:

Case1a:

Min z^{+}

Subject to,

$0.4308w_2+0.5195w_3+0.4308w_4+0.6667w_5 \le z^{+}$;

$0.4308w_1+0.3509w_2+0.3509w_3+0.4789w_4+0.5747w_5 \le z^{+}$;

$0.4308w_1+0.3509w_2+0.3509w_3+0.3509w_4 \le z^{+}$;

$0.16 \le w_1 \le 0.22$;

$0.15 \le w_2 \le 0.25$;

$0.19 \le w_3 \le 0.30$;

$0.13 \le w_4 \le 0.21$;

$0.17 \le w_5 \le 0.20$;

$w_1+ w_2+ w_3+ w_4+ w_5=1$;

$w_j \ge 0$, j= 1, 2, 3, 4, 5

Case1b:

Min z^{-}

Subject to,

$0.4789w_1+0.3509w_3+0.3589w_4 \le z^{-}$;

$0.3509w_1+0.3509w_2+0.4308w_3+0.4308w_5 \le z^{-}$;

$0.3589w_1+0.3509w_2+0.5195w_3+0.4308w_4 + 0.6667w_5 \le z^{-}$;

$0.16 \le w_1 \le .22$;

$0.15 \le w_2 \le 0.25$;

$0.19 \le w_3 \le 0.3$;

$0.13 \le w_4 \le 0.21$;

$0.17 \le w_5 \le .2$;

$w_1+ w_2+ w_3+ w_4+ w_5=1$;

$w_j \ge 0$, j= 1, 2, 3, 4, 5

After solving Case1a and Case1b separately, we obtain the solution set W^{+} = (0.1602, 0.15, 0.30, 0.21, 0.1798), W^{-} = (0.16, 0.15, 0.30, 0.21, 0.18) Therefore, the obtained weight vector of criteria is W = (0.1601, 0.15, 0.30, 0.21, 0.1799).

Step5. Determination of the degree of neutrosophic grey relational co-efficient (NGRC) of each alternative from INERS and INEUS.

The required neutrosophic grey relational co-efficient of each alternative from INERS is determined using equation (19). Tthe corresponding obtained weight vector W for Case-1 and Case-2 is presented in the Table 4. Similarly, the neutrosophic grey relational co-efficient of each alternative from INEURS is obtained with the help of equation (20) (see the Table 4).

Step6. Calculation of the neutrosophic relative relational degree (NRD)

Neutrosophic relative degree (NRD) of each alternative from INERS is obtained with the help of equation (21) (see the Table 4).

Table4: Ranking of the alternatives

Weight vector	(0.1601, 0.15, 0.30, 0.21, 0.1799)
NGRC from INERS	(0.5691, 0.5692, 0.6994)
NGRC from INEURS	(0.7427, 0.6820, 0.5224)
NRD from INERS	(0.4338, 0.4549, 0.5724)
Ranking Result	$R_3 > R_2 > R_1$
Selection	$\boldsymbol{R_3}$

Step7. Ranking of the alternatives

From Table4, we observe that $R_3 > R_2 > R_1$ i.e. Sent Mary's English School, Ranaghat ($\boldsymbol{A_3}$) is the best school for admission of children.

Conclusion

In this paper, we showed the application of single valued neutrosophic decision making model on school choice based on hybridization of grey system theory and single valed neutrosophic set. Five criteria are used to modeling the school choice problem in neutrosophic environment which are realistic in nature. New criterion can be easily incorporated in the model for decision making if it is needed. Application of the single-valued neutrosophic multiple attribte decision-making in real life problems helps the people to take a correct decision from the available alternatives in grey and neutrosophic hybrid environment. The concept presented in this paper can also be easily extended when the weight information are incomplete.

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