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Covering-Based Rough Single Valued Neutrosophic Sets

Yingcang Ma, Wanying Zhou, Qing Wan

School of Science, Xi'an Polytechnic University, Xi'an Shaanxi 710048, China. E-mail: mayingcang@126.com, 892443522@qq.com, wqysbe@163.com

Abstract: Rough sets theory is a powerful tool to deal with uncertainty and incompleteness of knowledge in information systems. Wang et al. proposed single valued neutrosophic sets as an extension of intuitionistic fuzzy sets to deal with real-world problems. In this paper, we propose the covering-based rough single valued neutrosophic sets by combining covering-based rough sets and single valued neutrosophic sets. Firstly, three types of covering-based rough single valued neutrosophic sets models are built and the properties of lower/upper approximation operators are explored. Secondly, the lower/upper approximations in two different covering approximation spaces are studied. The sufficient and necessary condition for generating the same lower/upper approximations from two different covering approximation spaces is discussed. Moreover, the relations of the three models are discussed and the equivalence conditions for three models are given.

Keywords: covering-based rough sets, single valued neutrosophic sets, neutrosophic sets, covering-based rough single valued neutrosophic sets.

1 Introduction

Rough set theory (RST), proposed by Pawlak[1] in 1982, is one of the effective mathematical tools for processing fuzzy and uncertainty knowledge. The classical rough set theory is based on the equivalence relation on the domain. In many practical problems, the relation between objects is essentially no equivalence relation, so this equivalence relation as the basis of the classic rough set model cannot fully meet the actual needs. For this a lot of extension models of Pawlak rough set are given. One approach is to extend the equivalence realtion to similarity relations[2], tolerance relations[3], ordinary binary relations[4], reflexive and transitive relations[5] and others. The other approach is combining the other theory to get more flexible and expressive framework for modeling and processing incomplete information in information systems. Mi et al.[6] introduced the definitions for generalized fuzzy lower and upper approximation operators determined by a residual implication. Pei [7] studied generalized fuzzy rough sets. Zhang et al.[8] gave a general framework of intuitionistic fuzzy rough set theory. Yang et al. [9]proposed hesitant fuzzy rough sets and studied the models axiomatic characterizations by combining hesitant fuzzy sets and rough sets.Zhang et al.[10] further gave the construction and axiomatic characterizations of interval-valued hesitant fuzzy rough sets, and illustrated the application of the model.

Covering rough sets theory is an important rough sets theory. Covering rough set model, first proposed by Zakowski[11] in 1983, Bonikowski et al. later studied the structures of covering[12]. Chen et al. [13]discussed the covering rough sets within the framework of a completely distributive lattice. Zhu and Wang [14]proposed the reduction of covering rough sets to reduce the "redundant" members in a covering in order to find the "smallest" covering. Deng et al. [15] established fuzzy rough set models based on a covering. Li et al. [16] proposed a generalized fuzzy rough approximation operators based on fuzzy coverings. Wei et al. [17]and Xu et al. [18] established the first and second types of rough fuzzy set models based on a covering. Hu et al.[19] proposed the third type of rough fuzzy set models based on a covering. Tang et al. [20] gave the fourth type of rough fuzzy set models based on a covering.

Smarandache [21] proposed neutrosophic sets to deal with real-world problems. A neutrosophic set has three membership functions: truth membership function, indeterminacy membership function and falsity membership function, in which each membership degree is a real standard or non-standard subset of the nonstandard unit interval $]0-,1+[$. Wang et al. [22] introduced single valued neutrosophic sets (SVNSs) that is a generalization of intuitionistic fuzzy sets, in which three membership functions are independent and their values belong to the unit interval $[0,1]$. Further studies have done in recent years. Such as, Majumdar and Samanta [23] studied similarity and entropy of SVNSs. Ye [24] proposed correlation coefficients of SVNSs, and applied it to single valued neutrosophic decision-making problems, etc.

SVNSs and covering rough sets are two different tools of dealing with uncertainty information. In order to use the advantages of SVNSs and covering rough sets, we establish a hybrid model of SVNSs and covering rough sets. Broumi and Smarandache proposed single valued neutrosophic information systems based on rough set theory [25]. Yang et al. proposed single valued neutrosophic rough set model and single valued neutrosophic refined rough set model[26,27]. In the present paper, we shall propose covering-based rough single valued neutrosophic sets by fusing SVNSs and covering rough sets, and explore a general framework of the study of covering-based rough single valued neutrosophic sets.

The paper is organized as follows. After this introduction, In section 2, we provide the basic notions and operations of Pawlak rough sets, covering rough sets and SVNSs. Based on a SVNR, Sect. 3 proposes three types of covering-based rough single valued neutrosophic sets. Properties of lower/upper approximation operators are studied. In Sect. 4, we investigate the relations of the three types models. The last section summarizes the conclusions and gives an outlook for future research.

2 Preliminaries

In this section, we give basic notions and operations on Pawlak tough sets, covering-based rough sets and SVNSs.

Definition 2.1 *Let U be a non-empty finite university and R be an equivalence relations on U. (U, R) is called a Pawlak approximation space. $\forall X \subseteq U$, the lower and upper approximations of X, denoted by $\underline{R}(X)$ and $\overline{R}(X)$, are defined as follows, respectively:*

$\underline{R}(X = \{x \in U | [x]_R \subseteq X\}$,

$\overline{R}(X = \{x \in U | [x]_R \cap X \neq \emptyset\}$,

where $[x]_R = \{y \in U | (x, y) \in R\}$. $\underline{R}(X)$ and $\overline{R}(X)$ are called as lower and upper approximations operators, respectively. The pair $(\underline{R}(X), \overline{R}(X))$ is called a Pawlak rough set.

Definition 2.2 *Let U be a non-empty finite university, C is a family of subsets of U. If none subsets in C is empty and $\cup C = U$, then C is a covering of U.*

Definition 2.3 *Let C be a covering of U, $x \in U$. $Md_C(x) = \{K \in C \wedge (\forall S \in C \wedge x \in S \wedge S \subseteq K \Rightarrow K = S)\}$ is called the minimal description of x, When the covering is clear, we omit the lowercase C in the minimal description.*

Definition 2.4 *Let U be a space of points (objects), with a generic element in U denoted by u. A SVNS A in U is characterized by three membership functions, a truth membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A, where $\forall u \in U, T_A(u), I_A(u), F_A(u) \in [0, 1]$. That is $T_A : U \to [0, 1], I_A : U \to [0, 1]$ and $F_A : U \to [0, 1]$. There is no restriction on the sum of $T_A(u), I_A(u)$ and $F_A(u)$, thus $0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3$.*

Here A can be denoted by $A = \{\langle u, T_A(u), I_A(u), F_A(u)\rangle | u \in U\}$, $\forall u \in U, (T_A(u), I_A(u), F_A(u))$ is called a single valued neutrosophic number(SVNN).

Definition 2.5 *Let A and B be two SVNSs on U. If for any $u \in U$, $T_A(u) \leq T_B(u), I_A(u) \geq I_B(u), F_A(u) \geq F_B(u)$, then we called A is contained in B, denoted by $A \Subset B$.*

If $A \Subset B$ and $B \Subset A$, then we called A is equal to B, denoted by $A = B$.

Definition 2.6 *Let A be a SVNS on U. The complement of A is denoted by A^c, where $\forall u \in U$, $T_{A^c}(u) = F_A(u), I_{A^c}(u) = 1 - I_A(u), F_{A^c}(u) = T_A(u)$.*

Definition 2.7 *Let A and B be two SVNS on U. The union of A and B is a SVNS C, denoted by $C = A \Cup B$, where $\forall u \in U$, $T_C(u) = \max\{T_A(u), T_B(u)\}$, $I_C(u) = \min\{I_A(u), I_B(u)\}$, $F_C(u) = \min\{F_A(u), F_B(u)\}$.*

The intersection of A and B is a SVNS D, denoted by $D = A \Cap B$, where $\forall u \in U$, $T_D(u) = \min\{T_A(u), T_B(u)\}$, $I_C(u) = \max\{I_A(u), I_B(u)\}$, $F_C(u) = \max\{F_A(u), F_B(u)\}$.

Proposition 2.8 *[26] Let A and B be two SVNS on U. The following results hold:*

(1) $A \Subset A \Cup B$ and $B \Subset A \Cup B$;

(2) $A \Cap B \Subset A$ and $A \Cap B \Subset B$;

(3) $(A^c)^c = A$;

(4) $(A \Cup B)^c = A^c \Cap B^c$;

(5) $(A \Cap B)^c = A^c \Cup B^c$.

3 Covering-based rough neutrosophic sets

Definition 3.1 *Let U be a non-empty finite university, C is a covering of U, (U, C) be a covering approximation space. A is a SVNS of U. The first type of lower and upper approximations of A with respect to (U, C), denoted by $FL(A)$ and $FU(A)$, are two SVNSs whose membership functions are defined as $\forall u \in U$,*

$T_{FL(A)}(u) = \inf\{T_A(v) | v \in \cup Md(u)\}$,

$I_{FL(A)}(u) = \sup\{I_A(v) | v \in \cup Md(u)\}$,

$F_{FL(A)}(u) = \sup\{F_A(v) | v \in \cup Md(u)\}$,

$T_{FU(A)}(u) = \sup\{T_A(v) | v \in \cup Md(u)\}$,

$I_{FU(A)}(u) = \inf\{I_A(v) | v \in \cup Md(u)\}$,

$F_{FU(A)}(u) = \inf\{F_A(v) | v \in \cup Md(u)\}$.

The pair $(FL(A), FU(A))$ is called the first type of rough single valued neutrosophic set based on covering C. $FL(A)$ and $FU(A)$ are called as the first lower and upper approximations operators, respectively.

Definition 3.2 *Let U be a non-empty finite university, C is a covering of U, (U, C) be a covering approximation space. A is a SVNS of U. The second type of lower and upper approximations of A with respect to (U, C), denoted by $SL(A)$ and $SU(A)$, are two SVNSs whose membership functions are defined as $\forall u \in U$,*

$T_{SL(A)}(u) = \inf\{T_A(v) | v \in \cap Md(u)\}$,

$I_{SL(A)}(u) = \sup\{I_A(v) | v \in \cap Md(u)\}$,

$F_{SL(A)}(u) = \sup\{F_A(v) | v \in \cap Md(u)\}$,

$T_{SU(A)}(u) = \sup\{T_A(v) | v \in \cap Md(u)\}$,

$I_{SU(A)}(u) = \inf\{I_A(v) | v \in \cap Md(u)\}$,

$F_{SU(A)}(u) = \inf\{F_A(v) | v \in \cap Md(u)\}$.

The pair $(SL(A), SU(A))$ is called the second type of rough single valued neutrosophic set based on covering C. $SL(A)$ and $SU(A)$ are called as the second lower and upper approximations operators, respectively.

Definition 3.3 *Let U be a non-empty finite university, C is a covering of U, (U, C) be a covering approximation space. A is a SVNS of U. The third type of lower and upper approximations* *of A with respect to (U, C), denoted by $TL(A)$ and $TU(A)$, are two SVNSs whose membership functions are defined as $\forall u \in U$,*

$T_{TL(A)}(u) = \sup_{K \in Md(u)}\{\inf_{v \in K}\{T_A(v)\}\}$,
$I_{TL(A)}(u) = \inf_{K \in Md(u)}\{\sup_{v \in K}\{I_A(v)\}\}$,
$F_{TL(A)}(u) = \inf_{K \in Md(u)}\{\sup_{v \in K}\{F_A(v)\}\}$.
$T_{TU(A)}(u) = \inf_{K \in Md(u)}\{\sup_{v \in K}\{T_A(v)\}\}$,
$I_{TU(A)}(u) = \sup_{K \in Md(u)}\{\inf_{v \in K}\{I_A(v)\}\}$,
$F_{TU(A)}(u) = \sup_{K \in Md(u)}\{\inf_{v \in K}\{F_A(v)\}\}$,

The pair $(TL(A), TU(A))$ is called the third type of rough single valued neutrosophic set based on covering C. $TL(A)$ and $TU(A)$ are called as the third lower and upper approximations operators, respectively.

Example 3.4 *Let $U = \{a, b, c, d\}$, $K_1 = \{a, b\}, K_2 = \{b, c\}, K_3 = \{c, d\}$, $C = \{K_1, K_2, K_3\}$. A single valued neutrosophic set $A = \{\langle a, (0.2, 0.8, 0.1)\rangle, \langle b, (1, 0.3, 1)\rangle, \langle c, (0.5, 0.3, 0)\rangle, \langle d, (0.6, 0.7, 0.5)\rangle\}$, then $Md(a) = \{\{a, b\}\}$, $Md(b) = \{\{a, b\}, \{b, c\}\}$, $Md(c) = \{\{b, c\}, \{c, d\}\}$, $Md(d) = \{\{c, d\}\}$. Thus,*

$T_{FL(A)}(a) = \inf\{T_A(v)|v \in \cup Md(a)\} = \inf\{T_A(a), T_A(b)\} = \inf\{0.2, 1\} = 0.2$.
$T_{FL(A)}(b) = \inf\{T_A(v)|v \in \cup Md(b)\} = \inf\{T_A(a), T_A(b), T_A(c)\} = \inf\{0.2, 1, 0.5\} = 0.2$.
$T_{FL(A)}(c) = \inf\{T_A(v)|v \in \cup Md(c)\} = \inf\{T_A(b), T_A(c), T_A(d)\} = \inf\{1, 0.5, 0.6\} = 0.5$.
$T_{FL(A)}(d) = \inf\{T_A(v)|v \in \cup Md(d)\} = \inf\{T_A(c), T_A(d))\} = \inf\{0.5, 0.6\} = 0.5$.
$T_{FU(A)}(a) = \sup\{T_A(v)|v \in \cup Md(a)\} = \sup\{T_A(a), T_A(b)\} = \sup\{0.2, 1\} = 1$.
$T_{FU(A)}(b) = \sup\{T_A(v)|v \in \cup Md(b)\} = \sup\{T_A(a), T_A(b), T_A(c)\} = \sup\{0.2, 1, 0.5\} = 1$.
$T_{FU(A)}(c) = \sup\{T_A(v)|v \in \cup Md(c)\} = \sup\{T_A(b), T_A(c), T_A(d)\} = \sup\{1, 0.5, 0.6\} = 1$.
$T_{FU(A)}(d) = \sup\{T_A(v)|v \in \cup Md(d)\} = \sup\{T_A(c), T_A(d))\} = \sup\{0.5, 0.6\} = 0.6$.
$I_{FL(A)}(a) = \sup\{I_A(v)|v \in \cup Md(a)\} = \sup\{I_A(a), I_A(b)\} = \sup\{0.8, 0.3\} = 0.8$.
$I_{FL(A)}(b) = \sup\{I_A(v)|v \in \cup Md(b)\} = \sup\{I_A(a), I_A(b), T_A(c)\} = \sup\{0.8, 0.3, 0.3\} = 0.8$.
$I_{FL(A)}(c) = \sup\{I_A(v)|v \in \cup Md(c)\} = \sup\{I_A(b), I_A(c), I_A(d)\} = \sup\{0.3, 0.3, 0.7\} = 0.7$.
$I_{FL(A)}(d) = \sup\{I_A(v)|v \in \cup Md(d)\} = \sup\{I_A(c), I_A(d))\} = \sup\{0.3, 0.7\} = 0.7$.
$I_{FU(A)}(a) = \inf\{I_A(v)|v \in \cup Md(a)\} = \inf\{I_A(a), I_A(b)\} = \inf\{0.8, 0.3\} = 0.3$.
$I_{FU(A)}(b) = \inf\{I_A(v)|v \in \cup Md(b)\} = \inf\{I_A(a), I_A(b), I_A(c)\} = \inf\{0.8, 0.3, 0.3\} = 0.3$.
$I_{FU(A)}(c) = \inf\{I_A(v)|v \in \cup Md(c)\} = \inf\{I_A(b), I_A(c), I_A(d)\} = \inf\{0.3, 0.3, 0.7\} = 0.3$.
$I_{FU(A)}(d) = \inf\{I_A(v)|v \in \cup Md(d)\} = \inf\{I_A(c), I_A(d))\} = \inf\{0.3, 0.7\} = 0.3$.
$F_{FL(A)}(a) = \sup\{F_A(v)|v \in \cup Md(a)\} = \sup\{F_A(a), F_A(b)\} = \sup\{0.1, 1\} = 1$.
$F_{FL(A)}(b) = \sup\{F_A(v)|v \in \cup Md(b)\} = \sup\{F_A(a), F_A(b), T_A(c)\} = \sup\{0.1, 1, 0\} = 1$.
$F_{FL(A)}(c) = \sup\{F_A(v)|v \in \cup Md(c)\} = \sup\{F_A(b), F_A(c), F_A(d)\} = \sup\{1, 0, 0.5\} = 1$.
$F_{FL(A)}(d) = \sup\{F_A(v)|v \in \cup Md(d)\} = \sup\{F_A(c), F_A(d))\} = \sup\{0, 0.5\} = 0.5$.
$F_{FU(A)}(a) = \inf\{F_A(v)|v \in \cup Md(a)\} = \inf\{F_A(a), F_A(b)\} = \inf\{0.1, 1\} = 0.1$.
$F_{FU(A)}(b) = \inf\{F_A(v)|v \in \cup Md(b)\} = \inf\{F_A(a), F_A(b), F_A(c)\} = \inf\{0.1, 1, 0\} = 0$.
$F_{FU(A)}(c) = \inf\{F_A(v)|v \in \cup Md(c)\} = \inf\{F_A(b), F_A(c), F_A(d)\} = \inf\{1, 0, 0.5\} = 0$.
$F_{FU(A)}(d) = \inf\{F_A(v)|v \in \cup Md(d)\} = \inf\{F_A(c), F_A(d))\} = \inf\{0, 0.5\} = 0$.

Thus,

$FL(A) = \{\langle a, (0.2, 0.8, 1)\rangle, \langle b, (0.2, 0.8, 1)\rangle, \langle c, (0.5, 0.7, 1)\rangle, \langle d, (0.5, 0.7, 0.5)\rangle\}$,
$FU(A) = \{\langle a, (1, 0.3, 0.1)\rangle, \langle b, (1, 0.3, 0)\rangle, \langle c, (1, 0.3, 0)\rangle, \langle d, (0.6, 0.3, 0)\rangle\}$.

Similarly,

$SL(A) = \{\langle a, (0.2, 0.8, 1)\rangle, \langle b, (1, 0.3, 1)\rangle, \langle c, (0.5, 0.3, 0)\rangle, \langle d, (0.5, 0.7, 0.5)\rangle\}$,
$SU(A) = \{\langle a, (1, 0.3, 0.1)\rangle, \langle b, (1, 0.3, 1)\rangle, \langle c, (0.5, 0.3, 0)\rangle, \langle d, (0.6, 0.3, 0)\rangle\}$.
$TL(A) = \{\langle a, (0.2, 0.8, 1)\rangle, \langle b, (0.5, 0.3, 1)\rangle, \langle c, (0.5, 0.3, 0.5)\rangle, \langle d, (0.5, 0.7, 0.5)\rangle\}$,
$TU(A) = \{\langle a, (1, 0.3, 0.1)\rangle, \langle b, (1, 0.3, 0.1)\rangle, \langle c, (0.6, 0.3, 0)\rangle, \langle d, (0.6, 0.3, 0)\rangle\}$.

Proposition 3.5 *The first type of rough single valued neutrosophic lower and upper approximation operators defined in Definition 3.1 has the following properties: $\forall A, B \in SVNS(U)$,*

(1) $FL(U) = U, FU(U) = U$;
(2) $FL(\emptyset) = \emptyset, FU(\emptyset) = \emptyset$;
(3) $FL(A) \Subset A \Subset FU(A)$;
(4) $FL(A \Cap B) = FL(A) \Cap FL(B)$, $FU(A \Cup B) = FU(A) \Cup FL(B)$;
(5) $A \Subset B \Rightarrow FL(A) \Subset FL(B)$, $A \Subset B \Rightarrow FU(A) \Subset FU(B)$;
(6) $FU(A \Cap B) \Subset FU(A) \Cap FU(B)$, $FL(A \Cup B) \Supset FL(A) \Cup FL(B)$;
(7) $FL(A^c) = (FU(A))^c$, $FU(A^c) = (FL(A))^c$.

Proof: (1) $T_{FL(U)}(u) = \inf\{T_U(v)|v \in \cup Md(u)\} = 1$, $T_{FU(U)}(u) = \sup\{T_U(v)|v \in \cup Md(u)\} = 1$, $I_{FL(U)}(u) = \sup\{I_U(v)|v \in \cup Md(u)\} = 0$, $I_{FU(U)}(u) = \inf\{I_U(v)|v \in \cup Md(u)\} = 0$, $F_{FL(U)}(u) = \sup\{F_U(v)|v \in \cup Md(u)\} = 0$, $F_{FU(U)}(u) = \inf\{F_U(v)|v \in \cup Md(u)\} = 0$, thus $FL(U) = U, FU(U) = U$.

(2) $T_{FL(\emptyset)}(u) = \inf\{T_\emptyset(v)|v \in \cup Md(u)\} = 0$, $T_{FU(\emptyset)}(u) = \sup\{T_\emptyset(v)|v \in \cup Md(u)\} = 0$, $I_{FL(\emptyset)}(u) = \sup\{I_\emptyset(v)|v \in \cup Md(u)\} = 1$, $I_{FU(\emptyset)}(u) = \inf\{I_\emptyset(v)|v \in \cup Md(u)\} = 1$, $F_{FL(\emptyset)}(u) = \sup\{F_\emptyset(v)|v \in \cup Md(u)\} = 1$, $F_{FU(\emptyset)}(u) = \inf\{F_\emptyset(v)|v \in \cup Md(u)\} = 1$, thus $FL(\emptyset) = \emptyset, FU(\emptyset) = \emptyset$.

(3) Being $u \in \cup Md(u)$, so $T_{FL(A)}(u) = \inf\{T_A(v)|v \in \cup Md(u)\} \leq T_A(u) \leq T_{FU(A)}(u) = \sup\{T_A(v)|v \in \cup Md(u)\} =$, $I_{FL(A)}(u) = \sup\{I_A(v)|v \in \cup Md(u)\} \geq$

$I_A(u) \geq I_{FU(A)}(u) = \inf\{I_A(v)|v \in \cup Md(u)\} =$, $F_{FL(A)}(u) = \sup\{F_A(v)|v \in \cup Md(u)\} \geq F_A(u) \geq F_{FU(A)}(u) = \inf\{F_A(v)|v \in \cup Md(u)\} =$, thus, $FL(A) \Subset A \Subset FU(A)$.

(4) $T_{FL}(A \Cap B)(u) = \inf\{T_{A\Cap B}(v)|v \in \cup Md(u)\} = \inf\{\min\{T_A(v), T_B(v)\}|v \in \cup Md(u)\} = \min\{\inf\{T_A(v)|v \in \cup Md(u)\}, \inf\{T_B(v)\}|v \in \cup Md(u)\} = \min\{T_{FL(A)}(u), T_{FL(B)}(u)\}$.

$I_{FL}(A \Cap B)(u) = \sup\{I_{A\Cap B}(v)|v \in \cup Md(u)\} = \sup\{\max\{I_A(v), I_B(v)\}|v \in \cup Md(u)\} = \max\{\sup\{I_A(v)|v \in \cup Md(u)\}, \sup\{I_B(v)\}|v \in \cup Md(u)\} = \max\{I_{FL(A)}(u), I_{FL(B)}(u)\}$.

$F_{FL}(A \Cap B)(u) = \sup\{F_{A\Cap B}(v)|v \in \cup Md(u)\} = \sup\{\max\{F_A(v), F_B(v)\}|v \in \cup Md(u)\} = \max\{\sup\{F_A(v)|v \in \cup Md(u)\}, \sup\{F_B(v)\}|v \in \cup Md(u)\} = \max\{F_{FL(A)}(u), F_{FL(B)}(u)\}$. Thus, $FL(A \Cap B) = FL(A) \Cap FL(B)$.

$T_{FU}(A \Cup B)(u) = \sup\{T_{A\Cup B}(v)|v \in \cup Md(u)\} = \sup\{\max\{T_A(v), T_B(v)\}|v \in \cup Md(u)\} = \max\{\sup\{T_A(v)|v \in \cup Md(u)\}, \sup\{T_B(v)\}|v \in \cup Md(u)\} = \max\{T_{FU(A)}(u), T_{FU(B)}(u)\}$.

$I_{FU}(A \Cup B)(u) = \inf\{I_{A\Cup B}(v)|v \in \cup Md(u)\} = \inf\{\min\{I_A(v), I_B(v)\}|v \in \cup Md(u)\} = \min\{\inf\{I_A(v)|v \in \cup Md(u)\}, \inf\{I_B(v)\}|v \in \cup Md(u)\} = \min\{I_{FU(A)}(u), I_{FU(B)}(u)\}$.

$F_{FU}(A \Cup B)(u) = \inf\{F_{A\Cup B}(v)|v \in \cup Md(u)\} = \inf\{\min\{F_A(v), F_B(v)\}|v \in \cup Md(u)\} = \min\{\inf\{F_A(v)|v \in \cup Md(u)\}, \inf\{F_B(v)\}|v \in \cup Md(u)\} = \min\{F_{FL(A)}(u), F_{FL(B)}(u)\}$. Thus, $FL(A \Cup B) = FL(A) \Cup FL(B)$.

So (4) holds.

(5) If $A \Subset B$, then $T_{FL(A)}(u) = \inf\{T_A(v)|v \in \cup Md(u)\} \leq \inf\{T_B(v)|v \in \cup Md(u)\} = T_{FL(B)}(u)$, $I_{FL(A)}(u) = \sup\{I_A(v)|v \in \cup Md(u)\} \geq \sup\{I_B(v)|v \in \cup Md(u)\} = I_{FL(B)}(u)$, $F_{FL(A)}(u) = \sup\{F_A(v)|v \in \cup Md(u)\} \geq \sup\{F_B(v)|v \in \cup Md(u)\} = F_{FL(B)}(u)$. So, $FL(A) \Subset FL(B)$.

The similar method we can get $A \Subset B \Rightarrow FU(A) \Subset FU(B)$. So (5) holds.

(6) Being $A \Cap B \Subset A \Subset A \Cup B$, $A \Cap B \Subset B \Subset A \Cup B$, from (5), (6) holds.

(7) $T_{FL(A^c)}(u) = \inf\{T_{A^c}(v)|v \in \cup Md(u)\} = \inf\{F_A(v)|v \in \cup Md(u)\} = F_{FU(A)}(u) = T_{(FU(A))^c})(u)$.

$I_{FL(A^c)}(u) = \sup\{I_{A^c}(v)|v \in \cup Md(u)\} = \sup\{1 - I_A(v)|v \in \cup Md(u)\} = 1 - \inf\{I_A(v)|v \in \cup Md(u)\} = 1 - I_{FU(A)})(u) = I_{(FU(A))^c}(u)$.

$F_{FL(A^c)}(u) = \sup\{F_{A^c}(v)|v \in \cup Md(u)\} = \sup\{T_A(v)|v \in \cup Md(u)\} = T_{FU(A)}(u) = F_{(FU(A))^c})(u)$.

So, $FL(A^c) = (FU(A))^c$. The similar method we can get $FU(A^c) = (FL(A))^c$, thus (7) holds.

Remark: $FL(FL(A)) = FL(A)$ and $FU(FU(A)) = FU(A)$ do not hold generally.

Similarly, we can get the following proposition.

Proposition 3.6 *The second type of rough single valued neutrosophic lower and upper approximation operators defined in Definition 3.2 has the following properties:* $\forall A, B \in SVNS(U)$,

(1) $SL(U) = U, SU(U) = U$;

(2) $SL(\emptyset) = \emptyset, SU(\emptyset) = \emptyset$;

(3) $SL(A) \Subset A \Subset SU(A)$;

(4) $SL(A \Cap B) = SL(A) \Cap SL(B)$, $SU(A \Cup B) = SU(A) \Cup SL(B)$;

(5) $A \Subset B \Rightarrow SL(A) \Subset SL(B)$, $A \Subset B \Rightarrow SU(A) \Subset SU(B)$;

(6) $SU(A \Cap B) \Subset SU(A) \Cap SU(B)$, $SL(A \Cup B) \Supset SL(A) \Cup SL(B)$;

(7) $SL(A^c) = (SU(A))^c$, $SU(A^c) = (SL(A))^c$.

Proposition 3.7 *The third type of rough single valued neutrosophic lower and upper approximation operators defined in Definition 3.3 has the following properties:* $\forall A, B \in SVNS(U)$,

(1) $TL(U) = U, TU(U) = U$;

(2) $TL(\emptyset) = \emptyset, TU(\emptyset) = \emptyset$;

(3) $TL(A) \Subset A \Subset TU(A)$;

(4) $A \Subset B \Rightarrow TL(A) \Subset TL(B)$, $A \Subset B \Rightarrow TU(A) \Subset TU(B)$;

(5) $TU(A \Cap B) \Subset TU(A) \Cap FU(B)$, $TL(A \Cup B) \Supset TL(A) \Cup TL(B)$;

(6) $TL(A^c) = (TU(A))^c$, $TU(A^c) = (TL(A))^c$.

(7) $TL(TL(A)) = TL(A)$, $TU(TU(A)) = TU(A)$.

Proof: The proofs of (1)-(6) are similar to the Proposition 3.5, we only show (7).

Let $u \in U, Md(u) = \{K_1, K_2, \cdots, K_m\}$.

$T_{TL(A)}(u) = \sup_{K\in Md(u)}\{\inf_{v\in K}(T_{(A)}(v))\} = \sup\{\inf_{v_1\in K_1}\{T_A(v_1)\}, \inf_{v_2\in K_2}\{T_A(v_2)\}, \cdots, \inf_{v_m\in K_m}\{T_A(v_m)\},\}$. Without loss of generality, let $K_i \in Md(u)$, $T_{TL(A)}(u) = \inf_{v_i\in K_i}\{T_A(v_i)\}$, then for $j \neq i, \inf_{v_i\in K_i}\{T_A(v_i)\} \geq \inf_{v_j\in K_j}\{T_A(v_j)\}$. Let $v_i \in K_i$, from Definition 3.3, we have $T_{TL(A)}(v_i) = \sup_{K\in Md(v_i)}\{\inf_{v\in K}(T_(A)(v))\} \geq \inf_{v_i\in K_i}(T_(A)(v_i)) = T_{TL(A)}(u)$. Being $\forall_{v_i\in K_i}(T_{TL(A)}(v_i) \geq T_{TL(A)(u)})$, so $\inf_{v_i\in K_i}\{T_{TL(A)(v_i)}\} = T_{TL(A)}(u)$. Let $v_j \in K_j, j \neq i$, so $\inf_{y_j\in K_j}\{T_{TL(A)}(v_j)\} \leq T_{TL(A)})(u)$ holds. Thus, $T_{TL(TL(A))}(u) = \sup_{K\in Md(u)}\{\inf_{v\in K}\{T_{TL(A)}(v)\}\} = \sup\{\inf_{v_1\in K_1}\{T_{TL(A)(v_1)}\}, \inf_{v_2\in K_2}\{T_{TL(A)(v_2)}\}, \cdots, \inf_{v_m\in K_m}\{T_{TL(A)(v_m)}\}\} = T_{TL(A)}(u)$.

$I_{TL(A)}(u) = \inf_{K\in Md(u)}\{\sup_{v\in K}(I_{(A)}(v))\} = \inf\{\sup_{v_1\in K_1}\{I_A(v_1)\}, \sup_{v_2\in K_2}\{I_A(v_2)\}, \cdots, \sup_{v_m\in K_m}\{I_A(v_m)\},\}$. Without loss of generality, let $K_i \in Md(u)$, $I_{TL(A)}(u) = \sup_{v_i\in K_i}\{I_A(v_i)\}$, then for $j \neq i, \sup_{v_i\in K_i}\{I_A(v_i)\} \leq \sup_{v_j\in K_j}\{I_A(v_j)\}$. Let $v_i \in K_i$, from Definition 3.3, we have $I_{TL(A)}(v_i) = \inf_{K\in Md(v_i)}\{\sup_{v\in K}(I_(A)(v))\} \leq \sup_{v_i\in K_i}(I_(A)(v_i)) = I_{TL(A)}(u)$. Being $\forall_{v_i\in K_i}(I_{TL(A)}(v_i) \leq I_{TL(A)(u)})$, so $\sup_{v_i\in K_i}\{I_{TL(A)(v_i)}\} = I_{TL(A)}(u)$. Let $v_j \in K_j, j \neq i$, so $\sup_{y_j\in K_j}\{I_{TL(A)}(v_j)\} \geq I_{TL(A)})(u)$ holds. Thus, $I_{TL(TL(A))}(u) = \inf_{K\in Md(u)}\{\sup_{v\in K}\{I_{TL(A)}(v)\}\} = \inf\{\sup_{v_1\in K_1}\{I_{TL(A)(v_1)}\}, \sup_{v_2\in K_2}\{I_{TL(A)(v_2)}\}, \cdots, \sup_{v_m\in K_m}\{I_{TL(A)(v_m)}\}\} = I_{TL(A)}(u)$.

$F_{TL(A)}(u) = \inf_{K\in Md(u)}\{\sup_{v\in K}(F_{(A)}(v))\} = \inf\{\sup_{v_1\in K_1}\{F_A(v_1)\}, \sup_{v_2\in K_2}\{F_A(v_2)\}, \cdots, \sup_{v_m\in K_m}\{F_A(v_m)\},\}$. Without loss of generality, let $K_i \in Md(u)$, $F_{TL(A)}(u) = \sup_{v_i\in K_i}\{F_A(v_i)\}$, then for $j \neq i, \sup_{v_i\in K_i}\{F_A(v_i)\} \leq \sup_{v_j\in K_j}\{F_A(v_j)\}$. Let $v_i \in K_i$, from Definition 3.3, we have $F_{TL(A)}(v_i) = \inf_{K\in Md(v_i)}\{\sup_{v\in K}(F(A)(v))\} \leq \sup_{v_i\in K_i}(F(A)(v_i)) = F_{TL(A)}(u)$. Being $\forall_{v_i\in K_i}(F_{TL(A)}(v_i) \leq F_{TL(A)(u)})$, so $\sup_{v_i\in K_i}\{F_{TL(A)(v_i)}\} = F_{TL(A)}(u)$. Let $v_j \in K_j, j \neq i$, so $\sup_{y_j\in K_j}\{F_{TL(A)}(v_j)\} \geq F_{TL(A)})(u)$ holds. Thus, $F_{TL(TL(A))}(u) = \inf_{K\in Md(u)}\{\sup_{v\in K}\{F_{TL(A)}(v)\}\} = \inf\{\sup_{v_1\in K_1}\{F_{TL(A)(v_1)}\}, \sup_{v_2\in K_2}\{F_{TL(A)(v_2)}\}\cdots, \sup_{v_m\in K_m}\{F_{TL(A)(v_m)}\}\} = F_{TL(A)}(u)$.

That is, $TL(TL(A)) = TL(A)$, the similar way we can get $TU(TU(A)) = TU(A)$. So (7) holds.

Remark: $TL(A \Cap B) = TL(A) \Cap TL(B)$ and $TU(A \Cup B) = TU(A) \Cup TL(B)$ do not hold generally.

4 The relations among the three types of covering-based rough single valued neutrosophic sets models

Definition 4.1 *Let C_1, C_2 are two coverings on a non-empty finite university U, $u \in U$, $\forall K \in Md_{C_1}(u)$, there exists $K' \in Md_{C_2}(u)$, such that $K' \subseteq K$, which is called C_2 is thinner than C_1, denoted by $C_2 \preceq C_1$. If $C_2 \preceq C_1$ and $C_1 \preceq C_2$, which is called C_1 equals C_2, denoted by $C_1 = C_2$. otherwise, which is called C_1 does not equal C_2, denoted by $C_1 \neq C_2$. If $C_2 \leq C_1$ and $C_1 \neq C_2$, it is called C_2 is strict thinner than C_1, denoted by $C_2 < C_1$. If $\forall K \in U, K \in C_1 \Leftrightarrow K \in C_2$, it is called C_1 identity to C_2, denoted by $C_1 \equiv C_2$.*

Proposition 4.2 *Let C_1, C_2 are two coverings on a non-empty finite university U, $C_1 \preceq C_2$, A is a single valued neutrosophic set on U. We have:*

(1) $FL_{C_2}(A) \Subset FL_{C_1}(A) \Subset A \Subset FU_{C_1}(A) \Subset FU_{C_2}(A)$;

(2) $SL_{C_2}(A) \Subset SL_{C_1}(A) \Subset A \Subset SU_{C_1}(A) \Subset SU_{C_2}(A)$;

(3) $TL_{C_2}(A) \Subset TL_{C_1}(A) \Subset A \Subset TU_{C_1}(A) \Subset TU_{C_2}(A)$.

Proof: We only show (3).

Let $u \in U$, $T_{TL_{C_1}(A)}(u) = \sup_{K\in Md(u)}\{\inf\{T_A(v)|v \in K\}\}$, $T_{TL_{C_2}(A)}(u) = \sup_{K'\in Md(u)}\{\inf\{T_A(v)|v \in K'\}\}$, being $C_1 \preceq C_2$, then $\forall K' \in Md_{C_2}(u), \exists K \in Md_{C_1}(u)$, such that $K \subseteq K'$, so $\inf_{v\in K}\{T_A(v)\} \geq \inf_{v\in K'}\{T_A(v)\}$. So $\sup_{K\in Md_{C_1}(u)}\{\inf_{v\in K}\{T_A(v)\}\} \geq \sup_{K'\in Md_{C_2}(u)}\{\inf_{v\in K'}\{T_A(v)\}\}$, that is $T_{TL_{C_1}(A)} \geq T_{TL_{C_2}(A)}$.

$I_{TL_{C_1}(A)}(u) = \inf_{K\in Md(u)}\{\sup\{I_A(v)|v \in K\}\}$, $I_{TL_{C_2}(A)}(u) = \inf_{K'\in Md(u)}\{\sup\{I_A(v)| v \in K'\}\}$, being $C_1 \preceq C_2$, then $\forall K' \in Md_{C_2}(u), \exists K \in Md_{C_1}(u)$, such that $K \subseteq K'$, so $\sup_{v\in K}\{I_A(v)\} \leq \sup_{v\in K'}\{T_A(v)\}$. So $\inf_{K\in Md_{C_1}(u)}\{\sup_{v\in K}\{I_A(v)\}\} \leq \inf_{K'\in Md_{C_2}(u)} \{\sup_{v\in K'}\{I_A(v)\}\}$, that is $I_{TL_{C_1}(A)} \leq I_{TL_{C_2}(A)}$.

$F_{TL_{C_1}(A)}(u) = \inf_{K\in Md(u)}\{\sup\{F_A(v)|v \in K\}\}$, $F_{TL_{C_2}(A)}(u) = \inf_{K'\in Md(u)}\{\sup\{F_A(v)| v \in K'\}\}$, being $C_1 \preceq C_2$, then $\forall K' \in Md_{C_2}(u), \exists K \in Md_{C_1}(u)$, such that $K \subseteq K'$, so $\sup_{v\in K}\{F_A(v)\} \leq \sup_{v\in K'}\{T_A(v)\}$. So $\inf_{K\in Md_{C_1}(u)}\{\sup_{v\in K}\{I_A(v)\}\} \leq \inf_{K'\in Md_{C_2}(u)} \{\sup_{v\in K'}\{F_A(v)\}\}$, that is $F_{TL_{C_1}(A)} \leq F_{TL_{C_2}(A)}$.

Thus we can get $TL_{C_2}(A) \Subset TL_{C_1}(A)$, the similar way we can get $TU_{C_1}(A) \Subset TU_{C_2}(A)$. According Proposition 3.7, we can get $TL_{C_2}(A) \Subset TL_{C_1}(A) \Subset A \Subset TU_{C_1}(A) \Subset TU_{C_2}(A)$ holds.

Definition 4.3 *Let C be a covering of a domain U and $K \in C$. If K is a union of some sets in $C - K$, we say K is reducible in C, otherwise K is irreducible. Let C be a covering of U. If every element in C is irreducible, we say C is irreducible; otherwise C is reducible. $\forall K \in C$, if K is reducible in C, then we can omit K from C, until C is irreducible, which is called a reduction of C, denoted by $reduct(C)$.*

Let (U, C) be a covering approximation space, $reduct(C)$ is the reduction of C, being $\forall u \in U$, $Md(u)$ is same in C and $reduct(C)$, so $C = reduct(C)$, so we can get the following result.

Proposition 4.4 *Let (U, C) be a covering approximation space, $reduct(C)$ is the reduction of C, then $\forall A \in SVNS(U)$, C and $reduct(C)$ generate the same covering-based lower/upper approximations for each type of covering-base rough single valued neutrosophic set.*

Proposition 4.5 *Let C_1, C_2 are two coverings on a non-empty finite university U, then $\forall A$, the lower/upper approximations for each type of covering-base rough single valued neutrosophic set are same in (U, C_1) and (U, C_2) iff $reduct(C_1) = reduct(C_2)$.*

Proof: $\Leftarrow$ Being $reduct(C_1) = reduct(C_2), \forall A$, A is a single valued neutrosophic set on U, from Proposition 4.2 we can get the results hold.

$\Rightarrow$ We just prove the third types of rough single valued neutrosophic set model, the others are similarly.

Proof by contradiction. Assume $reduct(C_1) \neq reduct(C_2)$, let $K \in reduct(C_1), K \notin reduct(C_2)$. We have $FL_{reduct(C_1)}(K) = K$ (here K be a single valued neutrosophic set, $T_K(u) = 1$, if $u \in K$, otherwise $T_K(u) = 0$. $I_K(u) = 0$, if $u \in K$, otherwise $I_K(u) = 1$. $F_K(u) = 0$, if $u \in K$, otherwise $F_K(u) = 1$). From Proposition 4.4, if K has the same covering-based rough single valued neutrosophic set in (U, C_1) and (U, C_2), then K has the same covering-based rough single valued neutrosophic set in $(U, reduct(C_1))$ and $(U, reduct(C_2))$, so $FL_{reduct(C_2)}(K) = K$. Being $K \notin reduct(C_2)$, then there exist $k_1, k_2, \cdots, k_n \in reduct(C_2)$, such that $K = \cup_{1\leq i\leq n} k_i$. For each $k_i \in reduct(C_2)$, there exist $k_{i1}, k_{i2}, \cdots, k_{im_i} \in reduct(C_1)$, such that $k_i = \cup_{1\leq j\leq m_i} k_{ij}$, so $K = \cup_{1\leq i\leq n} \cup_{1\leq j\leq m_i} k_{ij}$, that is K is reducible in

$reduct(C_1)$, which is contradiction that $reduct(C)$ is a reduction of C. So the result holds.

$\forall u \in U, \forall K \in Md(u)$, it is obviously that $\cap Md(u) \subseteq K \subseteq \cup Md(u)$, so we can get the following proposition.

Proposition 4.6 *Let (U, C) be a covering approximation space, A is a single valued neutrosophic set, then $FL(A) \Subset TL(A) \Subset SL(A) \Subset A \Subset SU(A) \Subset TU(A) \Subset FU(A)$.*

Proposition 4.7 *Let (U, C) be a covering approximation space, A is a single valued neutrosophic set, then the three types covering-based rough single valued neutrosophic sets are equivalence iff $\forall u \in U$, $\inf\{A(v)|v \in \cup Md(u)\} = \inf\{A(v)|v \in \cap Md(u)\}$ and $\forall u \in U$, $\sup\{A(v)|v \in \cup Md(u)\} = \sup\{A(v)|v \in \cap Md(u)\}$*

Proof: $\Leftarrow$ From Proposition 4.6 we can get $TL_{C_2}(A) \Subset TL_{C_1}(A) \Subset A \Subset TU_{C_1}(A) \Subset TU_{C_2}(A)$, being $\forall u \in U$, $\inf\{A(v)|v \in \cup Md(u)\} = \inf\{A(v)|v \in \cap Md(u)\}$, from Definition 3.1, 3.2, 3.3, we can get $FL(A) = SL(A) = TL(A)$ and $FU(A) = SU(A) = TU(A)$.

$\Rightarrow$ If the three types covering-based rough single valued neutrosophic sets are same, from Definition 3.1, 3.2, 3.3, we can easily get $\forall u \in U$, $\inf\{A(v)|v \in \cup Md(u)\} = \inf\{A(v)|v \in \cap Md(u)\}$ and $\sup\{A(v)|v \in \cup Md(u)\} = \sup\{A(v)|v \in \cap Md(u)\}$.

5 Conclusion

In this paper, we proposed the hybrid models of single valued neutrosophic refined sets, covering-based rough sets and covering-based rough single valued neutrosophic sets. Specifically, we explored the hybrid models through three different definitions and give the basic properties. Moreover, we discussed the relations of the three models. For the future prospects, we plan to explore the application of the proposed model to data mining and attribute reduction.

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