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Neutrosophic ideal of Subtraction Algebras

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Abstract: The notion of neutrosophic ideal in subtraction algebras is introduced, and several properties are investigated. Also we give conditions for a neutrosophic set to be a neutrosophic ideal. Characterization of neutorsophic ideal are discussed.

Keywords: Subtraction algebra,Neutrosophic set,Neutrosophic ideal

1 Introduction

The concept of Neutrosophic set, first introduced by Smarandache [17],is a powerful general formal framework that generalizes the concept of fuzzy set and intuitionistic fuzzy set. Recently, many researchers have been involved in extending the concepts and results of abstract algebra to the broader framework of the neutrosophic set theory[2, 3, 4, 5, 19]. Smarandache[17] and Wang et al.[18] introduced the concept of a single valued neutrosophic set as a subclass of the neutrosophic set and specified the definition of a neutrosophic set to make more applicable the theory to real life problems. In 1992, B. M. Schein have considered systems of the form $(\Phi; \circ, \backslash)$ [16], where Φ is a set of functions closed under the composition "$\circ$" of functions (and hence $(\Phi; \circ)$ is a function semigroup) and the set theoretic subtraction "$\backslash$" (and hence $(\Phi; \backslash)$ is a subtraction algebra in the sense of [1]). Jun et al. introduced the concept of ideal in subtraction algebras and continued studying on ideals in subtraction algebras[6, 8, 9, 14]. K. J. Lee and C. H. Park [11] introduced the concept of a fuzzy ideal in subtraction algebras and investigated some conditions for a fuzzy set to be a fuzzy ideal in subtraction algebras. Since then many reseachers worked in this area[7, 10, 12, 13].

In this paper, we apply the notion of neutrosophic sets in subtraction algebras. Also, we introduce the notion of neutrosophic ideal and give some conditions for a neutrosophic set to be a neutrosophic ideal in substraction algebras. Finally,we showed that neutrosophic image and neutrosophic inverse image of neutrosophic ideal are both neutrosophic ideal under certain conditions

2 Preliminaries

We review some definitions and properties that are necessary for this paper.

Definition 2.1. [1] An algebra $(X, -)$ is called a subtraction algebra if a single binary operation $-$ satisfies the following identities: for any $x, y, z \in X$,

(SA1) $x - (y - x) = x$,

(SA2) $x - (x - y) = y - (y - x)$,

(SA3) $(x - y) - z = (x - z) - y$,

We introduced an order relation X on a subtraction algebras: $a \leq b \Leftrightarrow a - b = 0$, where $0 = a - a$ is an element that does not depend on the choice of $a \in X$.

Proposition 2.2. [9] Let $(X, -)$ be a subtraction algebra. Then we have the following axioms:

(SP1) $(x - y) - y = x - y$,

(SP2) $x - 0 = x$ and $0 - x = 0$,

(SP3) $(x - y) - x = 0$,

(SP4) $x - (x - y) \leq y$,

(SP5) $(x - y) - (y - x) = x - y$,

(SP6) $x - (x - (x - y)) = x - y$,

(SP7) $(x - y) - (z - y) \leq x - z$,

(SP8) $x \leq y$ if and only if $x = y - w$ for some $w \in X$,

(SP9) $x \leq y$ implies $x - z \leq y - z$ and $z - y \leq z - x$ for all $z \in X$,

(SP10) $x, y \leq z$ implies $x - y = x \wedge (z - y)$,

(SP11) $(x \wedge y) - (x \wedge z) \leq x \wedge (y - z)$, for all $x, y, z \in X$.

Definition 2.3. [9] A nonempty subset A of a subtraction algebra X is called an ideal of X, denoted by $A \lhd X$, if it satisfies:

(SI1) $a - x \in A$ for all $a \in A$ and $x \in X$,

(SI2) for all $a, b \in A$, whenever $a \vee b$ exists in X then $a \vee b \in A$.

Proposition 2.4. [9] Let X be a subtraction algebra and let $x, y \in X$. If $w \in X$ is an upper bound for x and y, then the element

$$x \vee y := w - ((w - y) - x)$$

is a least upper bound for x and y.

Definition 2.5. [11] A fuzzy set μ in X is called a fuzzy ideal of X if it satisfies:

(SFI1) $\mu(x - y) \geq \mu(x)$,

(SFI2) $\exists\, x \vee y \Rightarrow \mu(x \vee y) \geq \min\{\mu(x), \mu(y)\})$ for all $x, y \in X$.

We give some preliminaries about single valued neutrosophic sets and set operations, which will be called neutrosophic sets, for simplicity.

Definition 2.6. [18] Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set A on X is characterized by truth-membership function t_A, indeterminacy-membership function i_A and falsity-membership function f_A. For each point x in $X, t_A(x), i_A(x), f_A(x) \in [0,1]$. A neutrosophic set A can be written as denoted by a mapping defined as $A : X \to [0,1] \times [0,1] \times [0,1]$ and

$$A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$$

for simplicity.

Definition 2.7. [15, 18] Let A and B be two neutrosophic sets on X. Then

(1) A is contained in B, denoted as $A \subseteq B$, if and only if $\mathscr{N}_A(x) \leq \mathscr{N}_B(x)$. i.e., $t_A(x) \leq t_B(x), i_A(x) \leq i_B(x)$ and $f_A(x) \geq f_B(x)$. Two sets A and B is called equal, i.e., $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(2) the union of A and B is denoted by $C = A \cup B$ and defined as $\mathscr{N}_C(x) = \mathscr{N}_A(x) \vee \mathscr{N}_B(x)$ where $\mathscr{N}_A(x) \vee \mathscr{N}_B(x) = (t_A(x) \vee t_B(x), i_A(x) \vee i_B(x), f_A(x) \wedge f_B(x))$, for each $x \in X$. i.e., $t_C(x) = \max\{t_A(x), t_B(x)\}, i_C(x) = \max\{i_A(x), i_B(x)\}$ and $f_C(x) = \min\{f_A(x), f_B(x)\}$.

(3) the intersection of A and B is denoted by $C = A \cap B$ and defined as $\mathscr{N}_C(x) = \mathscr{N}_A(x) \wedge \mathscr{N}_B(x)$ where $\mathscr{N}_A(x) \wedge \mathscr{N}_B(x) = (t_A(x) \wedge t_B(x), i_A(x) \wedge i_B(x), f_A(x) \vee f_B(x))$, for each $x \in X$.i.e., $t_C(x) = \min\{t_A(x), t_B(x)\}, i_C(x) = \min\{i_A(x), i_B(x)\}$ and $f_C(x) = \max\{f_A(x), f_B(x)\}$.

(4) the complement of A is denoted by A^c and defined as $\mathscr{N}_A^c(x) = (f_A(x), 1 - i_A(x), t_A(x))$, for each $x \in X$.

Definition 2.8. [4] Let $g : X_1 \to X_2$ be a function and A, B be the neutrosophic sets of X_1 and X_2, respectively. Then the image of a neutrosophic set A is a neutrosophic set of X_2 and it is defined as follows:$\forall y \in X_2$

$$\begin{aligned} g(A)(y) &= (t_{g(A)}(y), i_{g(A)}(y), f_{g(A)}(y)) \\ &= (g(t_A)(y), g(i_A)(y), g(f_A)(y)), \end{aligned}$$

where

$$g(t_A)(y) = \begin{cases} \bigvee t_A(x) & \text{if } x \in g^{-1}(y), \\ 0 & \text{otherwise,} \end{cases}$$

$$g(i_A)(y) = \begin{cases} \bigvee i_A(x) & \text{if } x \in g^{-1}(y), \\ 0 & \text{otherwise,} \end{cases}$$

$$g(f_A)(y) = \begin{cases} \bigwedge t_A(x) & \text{if } x \in g^{-1}(y), \\ 0 & \text{otherwise,} \end{cases}$$

And the preimage of a neutrosophic set B is a neutrosophic set of X_1 and it is defined as follows:

$$\begin{aligned} g^{-1}(B)(x) &= (t_{g^{-1}(B)}(x), i_{g^{-1}(B)}(x), f_{g^{-1}(B)}(x)) \\ &= (t_B(g(x)), i_B(g(x)), f_B(g(x))) \\ &= B(g(x)), \forall x \in X_1. \end{aligned}$$

Definition 2.9. [4] Let $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ be a neutrosophic set on X and $\alpha \in [0,1]$. Define the α-level sets of A as follows: $(t_A)_\alpha = \{x \in X \mid t_A(x) \geq \alpha\}, (i_A)_\alpha = \{x \in X \mid i_A(x) \geq \alpha\}$, and $(f_A)^\alpha = \{x \in X \mid f_A(x) \leq \alpha\}$.

3 Neutrosophic ideals

In what follows, let X be a subtraction algebra unless otherwise specified.

Definition 3.1. A neutrosophic set A of X is called a neutrosophic ideal of X if the following conditions are true: $\forall x, y \in X$,

(SNI1) $\mathscr{N}_A(x-y) \geq \mathscr{N}_A(x)$ i.e., $t_A(x-y) \geq t_A(x), i_A(x-y) \geq i_A(x)$ and $f_A(x-y) \leq f_A(x)$;

(SNI2) $\exists x \vee y \Rightarrow \mathscr{N}_A(x \vee y) \geq \mathscr{N}_A(x) \wedge \mathscr{N}_A(y)$,i.e., $t_A(x \vee y) \geq t_A(x) \wedge t_A(y), i_A(x \vee y) \geq i_A(x) \wedge i_A(y)$ and $f_A(x \vee y) \leq f_A(x) \vee f_A(y)$ whenever there exists $x \vee y$.

Proposition 3.2. *If a neutrosophic set A of X satisfies*

$$(\forall x, a, b \in X)\left(\mathscr{N}_A(x-((x-a)-b)) \geq \mathscr{N}_A(a) \wedge \mathscr{N}_A(b) \right) \tag{3.1}$$

then A is a neutrosophic ideal of X.

Proof. Let $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ be a neutrosophic set of X that satisfies (3.1). By (SP2) and (SP3) we have $(x-y)-(((x-y)-x)-x) = (x-y)-(0-x) = (x-y)-0 = x-y$. From this we get

$$\begin{aligned} t_A(x-y) &= t_A((x-y)-(((x-y)-x)-x)) \geq t_A(x) \wedge t_A(x) = t_A(x), \\ i_A(x-y) &= i_A((x-y)-(((x-y)-x)-x)) \geq i_A(x) \wedge i_A(x) = i_A(x), \\ f_A(x-y) &= f_A((x-y)-(((x-y)-x)-x)) \leq f_A(x) \vee f_A(x) = f_A(x). \end{aligned}$$

Now suppose $x \vee y$ exists for $x, y \in X$. If we take $w = x \vee y$, we have $x \vee y = w-((w-x)-y)$ by Proposition 2.4. It follows from (3.1) that

$$\begin{aligned} t_A(x \vee y) &= t_A(w-((w-x)-y)) \geq t_A(x) \wedge t_A(y), \\ i_A(x \vee y) &= i_A(w-((w-x)-y)) \geq i_A(x) \wedge i_A(y), \\ f_A(x \vee y) &= f_A(w-((w-x)-y)) \leq f_A(x) \vee f_A(y). \end{aligned}$$

Hence A is a neutrosophic ideal of X. □

Proposition 3.3. *For every neutrosophic ideal A of X, we have the following inequality:*

$$(\forall x \in X)\,(\mathscr{N}_A(0) \geq \mathscr{N}_A(x)). \tag{3.2}$$

Proof. Let $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ be a neutrosophic ideal of X. Putting $y = x$ in (SNI1), then

$$t_A(0) = t_A(x-x) \geq t_A(x), i_A(0) = i_A(x-x) \geq i_A(x), f_A(0) = f_A(x-x) \leq f_A(x).$$

Hence (3.2) is valid. □

Proposition 3.4. *Let A be a neutrosophic set of X such that*

(SNI3) $(\forall x \in X) \quad (\mathscr{N}_A(0) \geq \mathscr{N}_A(x))$, (SNI4) $(\forall x, y, z \in X) \quad (\mathscr{N}_A(x - z) \geq \mathscr{N}_A((x - y) - z) \wedge \mathscr{N}_A(y).)$

Then we have the following implication:

$$(\forall a, x \in X)(x \leq a \Rightarrow \mathscr{N}_A(x) \geq \mathscr{N}_A(a)). \tag{3.3}$$

Proof. Let $a, x \in X$ be such that $x \leq a$. Then

$$\begin{aligned} t_A(x) &= t_A(x - 0) \geq t_A((x - a) - 0) \wedge t_A(a) = t_A(0) \wedge t_A(a) = t_A(a), \\ i_A(x) &= i_A(x - 0) \geq i_A((x - a) - 0) \wedge i_A(a) = i_A(0) \wedge i_A(a) = i_A(a), \\ f_A(x) &= f_A(x - 0) \leq f_A((x - a) - 0) \vee f_A(a) = f_A(0) \vee f_A(a) = f_A(a). \end{aligned}$$

Hence $\mathscr{N}_A(x) \geq \mathscr{N}_A(a)$. □

Theorem 3.5. *If a neutrosophic set A in X satisfies (SNI3) and (SNI4), then A is a neutrosophic ideal of X.*

Proof. Let A be aneutrosophic in X satisfying (SNI3) and (SNI4), and let $x, y \in X$. Then $x - y \leq x$ by (SP3). It follows from Proposition 3.4 that

$$\mathscr{N}_A(x - y) \geq \mathscr{N}_A(x),$$

i.e., (SNI1) is valid. Also, we have

$$\mathscr{N}_A(x \vee y) \geq \mathscr{N}_A(x)$$

whenever $x \vee y$ exists in X by using Proposition 3.4 and so

$$\mathscr{N}_A(x \vee y) \geq \mathscr{N}_A(x) \wedge \mathscr{N}_A(y).$$

Thus (SNI2) is valid. Therefore $\mathscr{N}_A$ is a neutrosophic ideal of X. □

Proposition 3.6. *A necessary and sufficient condition for a neutrosophic set A of X to be a neutrosophic ideal of X is that t_A ,i_A and $1 - f_A$ are fuzzy ideals of X.*

Proof. Assume that $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ is a neutrosophic ideal of X. For any $x, y \in X$, we have $t_A(x - y) \geq t_A(x), i_A(x - y) \geq i_A(x)$ and $f_A(x - y) \leq f_A(x)$. Thus

$$(1 - f_A)(x - y) \geq (1 - f_A(x)).$$

Now suppose $x \vee y$ exists for $x, y \in X$. We have $t_A(x \vee y) \geq t_A(x) \wedge, t_A(y), i_A(x \vee y) \geq i_A(x) \wedge i_A(y)$ and $f_A(x \vee y) \leq f_A(x) \vee f_A(y)$. Thus

$$(1 - f_A)(x \vee y) \geq (1 - f_A(x)) \wedge (1 - f_A(y)).$$

Hence t_A, i_A and $1 - f_A$ are fuzzy ideal of X.

Conversely,assume that t_A, i_A and $1 - f_A$ are fuzzy ideal of X and $x, y \in R$. Then $t_A(x - y) \geq t_A(x), i_A(x - y) \geq i_A(x)$ and $1 - f_A(x - y) \geq (1 - f_A(x))$. Thus

$$f_A(x - y) = 1 - (1 - f_A(x - y)) \leq 1 - (1 - f_A(x)) = f_A(x).$$

It follows that $\mathscr{N}_A(x-y) \geq \mathscr{N}_A(x) \wedge \mathscr{N}_A(y)$. Suppose $x \vee y$ exists for $x, y \in X$, we have $t_A(x \vee y) \geq t_A(x) \wedge t_A(y), i_A(x \vee y) \geq i_A(x) \wedge i_A(y)$ and $(1-f_A)(x \vee y) \geq 1 - f_A(x) \wedge 1 - f_A(y)$. Thus

$$f_A(x \vee y) \leq f_A(x) \vee f_A(y).$$

It follows that

$$\mathscr{N}_A(x \vee y) \geq \mathscr{N}_A(x) \wedge \mathscr{N}_A(y).$$

Hence A is a neutrosophic ideal of X. □

Theorem 3.7. *A is a neutrosophic ideal of X if and only if for all $\alpha \in [0,1]$, the α-level sets of A, $(t_A)_\alpha$,$(i_A)_\alpha$ and $(f_A)^\alpha$ are ideals of X.*

Proof. Assume that $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ is a neutrosophic ideal of X. Let $x \in X$, $a \in (t_A)_\alpha, a \in (i_A)_\alpha$ and $a \in (f_A)^\alpha$. Then $t_A(a) \geq \alpha, i_A(a) \geq \alpha$, and $f_A(a) \leq \alpha$. By Definition 3.1(SNI1), we have

$$t_A(a-x) \geq t_A(a) \geq \alpha, i_A(a-x) \geq i_A(a) \geq \alpha, f_A(a-x) \leq f_A(a) \leq \alpha.$$

Hence $a - x \in (t_A)_\alpha$, $a - x \in (t_A)_\alpha$ and $a - x \in (t_A)_\alpha$. Let $a, b \in (t_A)_\alpha$ $a, b \in (i_A)_\alpha$ and $a, b \in (f_A)^\alpha$ and assume that there exists $a \vee b$. Then $t_A(a) \geq \alpha$ and $t_A(b) \geq \alpha$, which imply from Definition 3.1(SNI2) that

$$t_A(a \vee b) \geq t_A(a) \wedge t_A(b) \geq \alpha, i_A(a \vee b) \geq i_A(a) \wedge i_A(b) \geq \alpha, f_A(a \vee b) \leq f_A(a) \vee f_A(b) \leq \alpha.$$

and so that $a \vee b \in (t_A)_\alpha$,$a \vee b \in (i_A)_\alpha$ and $a \vee b \in (f_A)^\alpha$. Therefore $(t_A)_\alpha$,$(i_A)_\alpha$ and $(f_A)^\alpha$ are ideals of X. Conversely, assume that $t_A(x-y) < t_A(x)$ for some $x, y \in X$. Then

$$t_A(x-y) < \alpha < t_A(x)$$

for some $\alpha \in (0,1]$. This implies that $x \in (t_A)_\alpha$ but $x-y \notin (t_A)_\alpha$. This is contradiction. Therefore $t_A(x-y) \geq t_A(x)$ for all $x, y \in X$. Similary $i_A(x-y) \geq i_A(x)$. If $f_A(x-y) > f_A(x)$ for all $x, y \in X$. Then

$$t_A(x-y) > \alpha > f_A(x)$$

for some $\alpha \in (0,1]$. This implies that $x \in (f_A)^\alpha$ but $x - y \notin (f_A)^\alpha$. This is contradiction. Therefore $f_A(x-y) \leq f_A(x)$ for all $x, y \in X$. Suppose that $x \vee y$ exists such that $t_A(x \vee y) < t_A(x) \wedge t_A(y)$ for some $x, y \in X$, Then

$$t_A(x \vee y) < \alpha < t_A(x) \wedge t_A(y)$$

for some $\alpha \in (0,1]$. It follows that $x, y \in (t_A)_\alpha$ and $x \vee y \notin (t_A)_\alpha$. This is contradiction. Therefore $t_A(x \vee y) \geq t_A(x) \wedge t_A(y)$ for all $x, y \in X$. Similary $i_A(x \vee y) \geq i_A(x) \wedge i_A(y)$. If $x \vee y$ exists such that $f_A(x \vee y) > f_A(x) \wedge t_A(y)$ for some $x, y \in X$, Then

$$f_A(x \vee y) > \alpha > f_A(x) \vee f_A(y)$$

for some $\alpha \in (0,1]$. It follows that $x, y \in (f_A)^\alpha$ and $x \vee y \notin (f_A)^\alpha$. This is contradiction. Therefore $f_A(x \vee y) \leq f_A(x) \vee f_A(y)$ for all $x, y \in X$. Hence A is a neutrosophic ideal of X. □

Theorem 3.8. *Let A and B are neutrosophic ideals of X. Then $A \cap B$ is a neutrosophic ideal of X.*

Proof. Suppse that $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ and $B = \{< x, t_B(x), i_B(x), f_B(x) >, x \in X\}$ are neutrosophic ideals of X and let $x, y \in X$. By Definition 3.1 , we have

$$\begin{aligned} t_{A\cap B}(x-y) &= t_A(x-y) \wedge t_B(x-y) \geq t_A(x) \wedge t_B(x) = t_{A\cap B}(x), \\ i_{A\cap B}(x-y) &= i_A(x-y) \wedge i_B(x-y) \geq i_A(x) \wedge i_B(x) = i_{A\cap B}(x), \\ f_{A\cap B}(x-y) &= f_A(x-y) \vee f_B(x-y) \leq f_A(x) \vee f_B(x) = f_{A\cap B}(x). \end{aligned}$$

Now suppose $x \vee y$ exists for $x, y \in X$. By Definition 3.1, we have

$$\begin{aligned} t_{A\cap B}(x \vee y) &= t_A(x \vee y) \wedge t_B(x \vee y) \\ &\geq (t_A(x) \wedge t_A(y)) \wedge (t_B(x) \wedge t_B(y)) \\ &= (t_A(x) \wedge t_B(x)) \wedge (t_A(y) \wedge t_B(y)) \\ &= t_{A\cap B}(x) \wedge t_{A\cap B}(y). \end{aligned}$$

Similary we get $i_{A\cap B}(x \vee y) \geq i_{A\cap B}(x) \wedge i_{A\cap B}(y)$. Also we obtain

$$\begin{aligned} f_{A\cap B}(x \vee y) &= f_A(x \vee y) \vee f_B(x \vee y) \\ &\leq (f_A(x) \vee f_A(y)) \vee (f_B(x) \vee f_B(y)) \\ &= (f_A(x) \vee f_B(x)) \vee (f_A(y) \vee f_B(y)) \\ &= f_{A\cap B}(x) \vee f_{A\cap B}(y). \end{aligned}$$

Hence A is a neutrosophic ideal of X. □

Theorem 3.9. *Let A be a neutrosophic ideal of X. Then the set*

$$K := \{x \in X \mid \mathscr{N}_A(x) = \mathscr{N}_A(0)\}$$

is an ideal of X.

Proof. Let A be a neutrosophic ideal of X and $a \in K$. Then $\mathscr{N}_A(a) = \mathscr{N}_A(0)$. By (SNI1), we have

$$\mathscr{N}_A(a - x) \geq \mathscr{N}_A(a) = \mathscr{N}_A(0)$$

for $x \in X$. It follows from (3.2) that $\mathscr{N}_A(a - x) = \mathscr{N}_A(0)$ so that $a - x \in K$. Let $a, b \in K$ and assume that there exists $a \vee b$. By means of (SNI2), we know that

$$\mathscr{N}_A(a \vee b) \geq \min\{\mathscr{N}_A(a), \mathscr{N}_A(b)\} = \mathscr{N}_A(0).$$

Thus $\mathscr{N}_A(a \vee b) = \mathscr{N}_A(0)$ by (3.2), and so $a \vee b \in K$. Therefore K is an ideal of X. □

Theorem 3.10. *Let $g : X_1 \to X_2$ be a homomorphism. Then the image $f(A)$ of a neutrosophic ideal A of X_1 is a neutrosophic ideal of X_2.*

Proof. For any $y_1, y_2 \in f(X_1)$, Consider the set

$$S = \{a_1 - a_2 \mid a_1 \in g^{-1}(y_1), a_2 \in g^{-1}(y_2)\}.$$

If $x \in S$ then $x = x_1 - x_2$ for $x_1 \in g^{-1}(y_1)$ and $x_2 \in g^{-1}(y_2)$ and so

$$f(x) = f(x_1 - x_2) = f(x_1) - f(x_2) = y_1 - y_2,$$

that is, $x = x_1 - x_2 \in f^{-1}(y_1 - y_2)$. It follows that

$$\begin{aligned}
g(t_A)(y_1 - y_2) &= \bigvee_{x \in f^{-1}(y_1 - y_2)} t_A(x) \geq t_A(x_1 - x_2) \geq t_A(x_1) \\
g(i_A)(y_1 - y_2) &= \bigvee_{x \in f^{-1}(y_1 - y_2)} i_A(x) \geq i_A(x_1 - x_2) \geq i_A(x_1) \\
g(f_A)(y_1 - y_2) &= \bigwedge_{x \in f^{-1}(y_1 - y_2)} f_A(x) \leq f_A(x_1 - x_2) \leq f_A(x_1).
\end{aligned}$$

Then

$$\begin{aligned}
g(A)(y_1 - y_2) &= (g(t_A)(y_1 - y_2), g(i_A)(y_1 - y_2), g(f_A)(y_1 - y_2))) \\
&= (\bigvee_{x \in f^{-1}(y_1 - y_2)} t_A(x), \bigvee_{x \in f^{-1}(y_1 - y_2)} i_A(x), \bigwedge_{x \in f^{-1}(y_1 - y_2)} f_A(x)) \\
&\geq (t_A(x_1 - x_2), i_A(x_1 - x_2), f_A(x_1 - x_2)) \\
&\geq (t_A(x_1), i_A(x_1), f_A(x_1)).
\end{aligned}$$

Cnsequently,

$$\begin{aligned}
g(A)(y_1 - y_2) &\geq (\bigvee_{x_1 \in f^{-1}(y_1)} t_A(x_1), \bigvee_{x_1 \in f^{-1}(y_1)} i_A(x_1), \bigwedge_{x_1 \in f^{-1}(y_1 - y_2} f_A(x_1)) \\
&= (g(t_A)(y_1), g(i_A)(y_1), g(f_A)(y_1)) \\
&= g(A)(y_1).
\end{aligned}$$

If $y_1 \vee y_2$ exist for any $y_1, y_2 \in f(X_1)$. We first consider the set

$$T = \{a_1 \vee a_2 \mid a_1 \in g^{-1}(y_1), a_2 \in g^{-1}(y_2)\}.$$

If $x \in T$ then $x = x_1 \vee x_2$ for $x_1 \in g^{-1}(y_1)$ and $x_2 \in g^{-1}(y_2)$ and so

$$f(x) = f(x_1 \vee x_2) = f(x_1) \vee f(x_2) = y_1 \vee y_2,$$

that is, $x = x_1 \vee x_2 \in f^{-1}(y_1 \vee y_2)$. It follows that

$$\begin{aligned}
g(t_A)(y_1 \vee y_2) &= \bigvee_{x \in f^{-1}(y_1 \vee y_2)} t_A(x) \geq t_A(x_1 \vee x_2), \\
g(i_A)(y_1 \vee y_2) &= \bigvee_{x \in f^{-1}(y_1 \vee y_2)} i_A(x) \geq i_A(x_1 \vee x_2), \\
g(f_A)(y_1 \vee y_2) &= \bigwedge_{x \in f^{-1}(y_1 \vee y_2)} f_A(x) \leq f_A(x_1 \vee x_2).
\end{aligned}$$

Then

$$\begin{aligned} g(A)(y_1 \vee y_2) &= (g(t_A)(y_1 \vee y_2), g(i_A)(y_1 \vee y_2), g(f_A)(y_1 \vee y_2))) \\ &= (\bigvee_{x \in f^{-1}(y_1 \vee y_2)} t_A(x), \bigvee_{x \in f^{-1}(y_1 \vee y_2)} i_A(x), \bigwedge_{x \in f^{-1}(y_1 \vee y_2)} f_A(x)) \\ &\geq (t_A(x_1 \vee x_2), i_A(x_1 \vee x_2), f_A(x_1 \vee x_2)) \\ &\geq (t_A(x_1) \wedge t_A(x_2), i_A(x_1) \wedge i_A(x_2), f_A(x_1) \vee f_A(x_2)) \\ &= (t_A(x_1), i_A(x_1), f_A(x_1)) \wedge (t_A(x_2), i_A(x_2), f_A(x_2)). \end{aligned}$$

Cnsequently,

$$\begin{aligned} g(A)(y_1 - y_2) &\geq (\bigvee_{x_1 \in f^{-1}(y_1)} t_A(x_1), \bigvee_{x_1 \in f^{-1}(y_1)} i_A(x_1), \bigwedge_{x_1 \in f^{-1}(y_1)} f_A(x_1)) \\ &\wedge (\bigvee_{x_2 \in f^{-1}(y_2)} t_A(x_2), \bigvee_{x_2 \in f^{-1}(y_2)} i_A(x_2) \bigwedge_{x_2 \in f^{-1}(y_2} f_A(x_1)) \\ &= (g(t_A)(y_1), g(i_A)(y_1), g(f_A)(y_1)) \wedge (g(t_A)(y_2), g(i_A)(y_2), g(f_A)(y_2)) \\ &= g(A)(y_1) \wedge g(A)(y_2). \end{aligned}$$

Hence $g(A)$ is a neutrosophic ideal of $f(X_1)$. □

Theorem 3.11. *Let $g : X_1 \to X_2$ be a homomorphism. Then the preimage $f^{-1}(B)$ of a neutrosophic ideal B of X_2 is a neutrosophic ideal of X_1.*

Proof. Let $B = \{< x, t_B(x), i_B(x), f_B(x) >, x \in X_2\}$ be a neutrosophic ideal of X_2 and $x, y \in X_1$. Then

$$\begin{aligned} g^{-1}(B)(x - y) &= (t_B(g(x - y)), i_B(g(x - y)), f_B(g(x - y)) \\ &= (t_B(g(x) - g(y)), i_B(g(x) - g(y)), f_B(g(x) - g(y)) \\ &\geq (t_B(g(x)), i_B(g(x)), f_B(g(x)) \\ &= g^{-1}(B)(x). \end{aligned}$$

Now suppse $x \vee y$ exists for $x, y \in X_1$. Then

$$\begin{aligned} g^{-1}(B)(x \vee y) &= (t_B(g(x \vee y)), i_B(g(x \vee y)), f_B(g(x \vee y)) \\ &= (t_B(g(x) \vee g(y)), i_B(g(x) \vee g(y)), f_B(g(x) \vee g(y)) \\ &\geq t_B(g(x)) \wedge i_B(g(y)), i_B(g(x) \wedge i_B(g(y)), f_B(g(x) \vee f_B(g(y))) \\ &= (t_B(g(x)), i_B(g(x)), f_B(g(x)) \wedge (t_B(g(y)), i_B(g(y)), f_B(g(y)) \\ &= g^{-1}(B)(x) \wedge g^{-1}(B)(y) \end{aligned}$$

Hence $g^{-1}(B)$, is a neutrosophic ideal of X_1. □

4 conclusions

F.Smarandache introduced the concept of neutrosophic sets, which can be seen as a new mathematical tool for dealing with uncertainty. In this paper, we apply the notion of neutrosophic sets in subtraction algebras.

Also, we introduce the notion of neutrosophic ideal and give some conditions for a neutrosophic set to be a neutrosophic ideal in substraction algebras. Finally,we showed that neutrosophic image and neutrosophic inverse image of neutrosophic ideal are both neutrosophic ideal under certain conditions Based on these results,we could apply neutrosophic sets to other types of ideals in subtraction algebra. Also,we believe that such a results applied for other algebraic structure.

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