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A Generalized Neutrosophic Solid Transportation Model with Insufficient Supply

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Abstract. The classical transportation problem and the solid transportation problem are special types of linear programming problems which are very important in Operations Research. In this paper, a solid transportation model is described, where the total supply of goods is insufficient to fulfil the total demand of goods, due to which the supplier company tries to obtain the required remaining goods from another source. An expression is derived to determine the import plan. The parameters of the model are considered to be uncertain and imprecise and are taken as trapezoidal neutrosophic numbers. The paper gives a general formulation of such a model and an algorithm is proposed to solve the model. The main objective function of the model present in the manuscript is to minimize the total cost. A formula is provided to check the degree of sufficiency of such a solution. The model is elucidated with a numerical example and its solution shows its efficiency and optimality in practical aspect. Finally, the paper provides a brief discussion about the computational time and some relative points of research.

Keywords: Solid Transportation Model, Insufficient supply, Trapezoidal Neutrosophic Number, Ranking function.

1 Introduction

Transportation is the movement of humans, animals, commodities, etc. from one location to another. Modes of transport include air, land (rail and road), water cable, pipeline and space. The field can be divided into infrastructure, vehicles and operations. Transportation is important because it enables trade between people, which is essential for the development of civilizations. It is a key component of growth and globalization.

The transportation problem (TP) was first forwarded by Hitchcock [1] in 1941. It is a popular type of problem in Operations Research where the decision maker wants to find the optimal way to transport goods from source warehouses to destination warehouses. So, there are two types of constraints, namely source constraints and demand constraints. But, real systems may contain other type of constraints too such as product type constraints or transportation mode constraints. This gives a third dimension to the transportation problem and converts the classical transportation problem into the solid transportation problem (STP).

The STP was first stated by Schell [2] in 1955 and later, in 1962, it was formally introduced by Haley [3]. In this paper, we consider that different types of conveyances are required for shipping goods and so, the third type of constraints here are the conveyance constraints.

The classical theories of Mathematics cannot solve problems which simulate real life situations. The information is imprecise and uncertain in nature. To deal with vague information, the fuzzy set theory was introduced by Zadeh [4] in 1965. But, fuzzy sets cannot represent imprecise information efficiently as they only consider the truth membership values of the data. Then, Atanassov [5, 6] introduced the concept of intuitionistic fuzzy sets, where the data are represented by their membership and non-membership values. But, they can only handle incomplete information, not indeterminate or inconsistent information.

Smarandache [8] proposed the concept of neutrosophic set theory by adding an independent indeterminacy membership. The neutrosophic set theory generalizes the concepts of classical set theory, fuzzy set theory, intuitionistic fuzzy set theory, and so on, since it considers all three aspects of decision-making, viz. “agree”, “disagree” and “not sure”.Basset et al. [24] used Neutrosophic theory to solve transition difficulties of Internet of Things identifying some challenge affectingthe process by non-traditional methods. In the article [26], an advance type of Neutrosophic set called type-2 Neutrosophic number are defined with TOPSIS method. A green supply chain model is developed incorporated with neutrosophic set and robust ranking technique and its performance is shown in decision making process [25].

Various researchers like Jiménez and Verdegay [7], Yang and Liu [9], Hussain and Kumar [10], Kundu et al. [11], Singh and Yadav [13], Das et al. [14], Giri et al. [16], Das et al. [18], Aggarwal and Gupta [19], etc. have studied the classical and solid transportation models in different fuzzy and intuitionistic fuzzy environments. A supply chain model is formulated based on some importance matrices based on economic, environment, social aspect as well as information gathering [23]. A hybrid pliogenic decision making approach is developed in this regard. Basset et al. [22] developed an evaluation model to show the performance and efficiency of medical care system with pliogenic set.

In this paper, a mathematical model is developed for the solid transportation model. The model is considered in neutrosophic environment so that we can address the fact of truth, indeterminacy and falsity arises in the data due to factors like unawareness of the scale of the problem, imperfection in data, poor forecasting, etc. As the concept of neutrosophic set theory is relatively new, a few of article is available dealing the transportation or solid transportation models with neutrosophic parameters in literature. A few of them in this context are by Thamaraiselvi and Santhi [15], and Rizk-Allah et al. [21].

The mathematical model present in this paperdescribes a transportation model shipping a homogeneous product from some source warehouses to some destination warehouses by means of heterogeneous conveyances. It is assumed that the conveyances have the necessary overall capacity to transport the whole demanded quantity of the commodity. In this research work, it is considered that the source warehouses do not have the sufficient quantity of goods to supply at a time and they fall short of some amount. At that time, the supplier decides to import the goods from another source. Again, if this new source does not have the requisite amount of goods, it imports the remaining amount from another source, and so on. This process is continued until the fulfilment of the total demand. It terminates after a certain number of sources, since the total original demand of goods is a fixed quantity. The paper addresses the general notion of the situation and also the presence of uncertainties in the data.

The main contribution of the paper is to develop the mathematical model for solid transportation plan to satisfy the demand of customer with insufficient supply of source point.The main objective function of the model is to minimize the total cost. In this research work, parameters of the model are considered in neutrosophic environment. Consideration of neutrosophic number gives an ideal approach of a decision making process dealing the uncertainty with truth, false and in determinant state of information. In this regard, trapezoidal neutrosophic number is used in this STP model. A proposition is provided to establish the relation between the import goods and the cost which define a degree of insufficiency. Hereby, a solution algorithm is given in his manuscript. A numerical example is also shown to discuss the performance of the model.

In this paper, Section 2 contains some preliminary definitions and concepts regarding the model. Section 3 describes the model and gives a general formulation of the model. Section 4 is all about the solution approach to the problem, concerned with the model and Section 5 helps in understanding the model with the help of a numerical example and its solution by the given procedure. Finally, Section 6 briefly discusses the model along with the computational time of the solution process, exemplified by the numerical example. It also suggests some relative points of research and is followed by the conclusion.

2 Preliminaries

In this section, we recall some important definitions and concepts.

2.1 Single-valued neutrosophic set [20]

Let X be a non-empty set. Then a single-valued neutrosophic (SVN) set $\tilde{A}$ of X is defined as

$$\tilde{A}=\{\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\rangle \mid x \in X\},$$

where $T_{\tilde{A}}(x)$, $I_{\tilde{A}}(x)$, $F_{\tilde{A}}(x) \in [0, 1]$ and $0 \le T_{\tilde{A}}(x) + I_{\tilde{A}}(x) + F_{\tilde{A}}(x) \le 3$, $\forall\, x \in X$. $T_{\tilde{A}}(x)$, $I_{\tilde{A}}(x)$ and $F_{\tilde{A}}(x)$ respectively represent truth membership, indeterminacy membership and falsity membership degrees of x in $\tilde{A}$.

2.2 Trapezoidal neutrosophic number [20]

A trapezoidal neutrosophic number (TNN) $\tilde{A}$ is a neutrosophic set in $\boldsymbol{R}$ with the following truth, indeterminacy and falsity membership functions:

$$T_{\tilde{A}}(x) = \begin{cases} \alpha_{\tilde{A}}\left(\dfrac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2, \\ \alpha_{\tilde{A}} & for\ a_2 \le x \le a_3, \\ \alpha_{\tilde{A}}\left(\dfrac{a_4-x}{a_4-a_3}\right) & for\ a_3 \le x \le a_4, \\ 0 & otherwise, \end{cases}$$

$$I_{\tilde{A}}(x) = \begin{cases} \dfrac{a_2 - x + \theta_{\tilde{A}}(x-a_1')}{a_2-a_1'} & for\ a_1' \le x \le a_2, \\ \theta_{\tilde{A}} & for\ a_2 \le x \le a_3, \\ \dfrac{x - a_3 + \theta_{\tilde{A}}(a_4'-x)}{a_4'-a_3} & for\ a_3 \le x \le a_4', \\ 1 & otherwise, \end{cases}$$

$$F_{\tilde{A}}(x) = \begin{cases} \dfrac{a_2 - x + \beta_{\tilde{A}}(x-a_1'')}{a_2-a_1''} & for\ a_1'' \le x \le a_2, \\ \beta_{\tilde{A}} & for\ a_2 \le x \le a_3, \\ \dfrac{x - a_3 + \beta_{\tilde{A}}(a_4''-x)}{a_4''-a_3} & for\ a_3 \le x \le a_4'', \\ 1 & otherwise, \end{cases}$$

where $\alpha_{\tilde{A}}$, $\theta_{\tilde{A}}$ and $\beta_{\tilde{A}}$ represent the maximum degree of truthiness, minimum degree of indeterminacy and minimum degree of falsity respectively, $\alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} \in [0, 1]$. Also, $a_1'' \le a_1 \le a_1' \le a_2 \le a_3 \le a_4' \le a_4 \le a_4''$.

The membership functions of trapezoidal neutrosophic number are shown in Fig. 1.

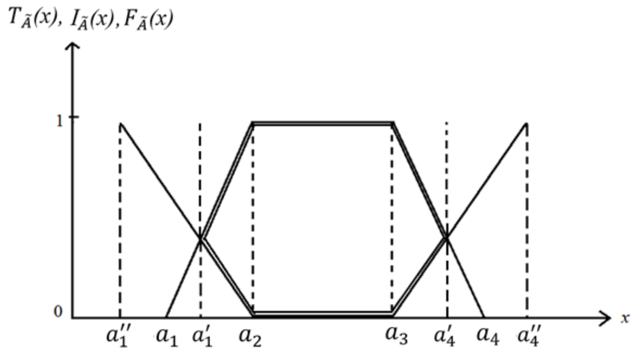

Figure 1: Truth, indeterminacy and falsity membership functions of trapezoidal neutrosophic number.

2.3 Ranking function [20]

A ranking function of neutrosophic numbers is a function $\Re : N(R) \to \boldsymbol{R}$, where $N(R)$ is a set of neutrosophic numbers defined on the set of real numbers, which convert each neutrosophic number into the real line.

Let $\tilde{A} = \langle\!\langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{A}}, \theta_{\tilde{A}}, \beta_{\tilde{A}} \rangle\!\rangle$ and $\tilde{B} = \langle\!\langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{B}}, \theta_{\tilde{B}}, \beta_{\tilde{B}} \rangle\!\rangle$ betwo trapezoidal neutrosophic numbers.

- If $\Re(\tilde{A}) > \Re(\tilde{B})$, then $\tilde{A} \gtrsim \tilde{B}$,
- If $\Re(\tilde{A}) < \Re(\tilde{B})$, then $\tilde{A} \lesssim \tilde{B}$,
- If $\Re(\tilde{A}) = \Re(\tilde{B})$, then $\tilde{A} \approx \tilde{B}$.

3 Description and formulation of model

Real life situations regarding transportation of commodities are complex which give rise to various transportation models. This paper discusses one such situation where the primary "supplier" company (say, Y_1) has shortage of goods to meet the adequate demand of the primary "purchaser" company (say, Y_0).

It may happen that the total required amount of goods cannot be produced due to shortage of time or lack of raw materials or some other factors to fulfill the total demand. So, Company Y_1 decides to import the remaining amount of goods from another company (say, Y_2) and then transport the aggregate amount to Company Y_0. Again, it may happen that Company Y_2 faces the same problem, where it is unable to fulfill the total demand of Company Y_1. So, Company Y_2 imports the remaining amount from another company (say, Y_3). The chain continues until Company Y_N (say) fulfills the total demand of Company Y_{N-1} (say). The process surely terminates, since the total original demand of Company Y_0 is a finite quantity. Here, N is at least 2.

While stating its demand, Company Y_0 may not be sure about the exact quantity of goods it needs. This may be due to the nature of the commodities, uncertain market trend and business scope, etc. Similarly, due to possible production and technical issues, the supply quantity of goods may be uncertain. Also, uncertainty may arise in determining the costs of transportation and the exact capacities of the conveyances due to road issues, weather issues, etc. So, here, all of these parameters in all the *N* steps are considered as trapezoidal neutrosophic numbers.

3.1 Assumptions

- The total supply (in stock) of Company Y_P from its origin warehouses is insufficient to fulfill the total demand of the destination warehouses of Company Y_{P-1} (p = 1, 2, …, N – 1).
- Company Y_{P-1} is indifferent to the arrangement of goods by Company Y_P and Company Y_{P+1} is indifferent to the use of the goods imported by Company Y_P (p = 1, 2, …, N – 1).
- Company Y_P does not have any extra warehouse to import goods. It imports the remaining amount of goods to its existing warehouses (p = 1, 2, …, N – 1).
- The warehouses of company Y_P have the capacity to hold the remaining amount, but the whole amount cannot be stored in a single warehouse and is transported to each of the warehouses in parts (p = 1, 2, …, N – 1).
- The total conveyance capacity of Company Y_P is greater than or equal to the total demand of Company Y_{P-1} (p = 1, 2, …, N).
- Company Y_N can supply the remaining quantity of goods, demanded (required) by Company Y_{N-1}, from its warehouses sufficiently. So, the model saturates in the N^{th} step and thus it is an N-step model.

3.2 Notations

- $c_{ijk}^{(p)}$: Per unit cost of transportation from the i^{th} origin warehouse to the j^{th} destination warehouse by the k^{th} conveyance in the p^{th} step.
- $x_{ijk}^{(p)}$: Amount of goods to be transported from the i^{th} origin warehouse to the j^{th} destination warehouse by the k^{th} conveyance in the p^{th} step.

- : Amount by which the supply falls short in the p^{th} step.
- $A_i^{(p)}$: Original amount of supply of the i^{th} origin in the p^{th} step.
- $a_i^{(p)}$: Total amount of supply of the i^{th} origin in the p^{th} step.
- $b_j^{(p)}$: Amount of demand of the j^{th} destination in the p^{th} step.
- $e_k^{(p)}$: Capacity of the k^{th} conveyance in the p^{th} step.
- m_p : Number of origin warehouses in the p^{th} step.
- m_{p-1} : Number of destination warehouses in the p^{th} step.
- K_p : Number of conveyances in the p^{th} step.
- $AC_i^{(p)}$: Average per unit cost of transportation from the i^{th} origin in the p^{th} step.
- $H^{(p)}$: Harmonic mean of $AC_i^{(p)}$'s ($i = 1, 2, \ldots, m_p$) in the p^{th} step.

3.3 Formulation

The model is formulated mathematically as follows:

$$Min\ z^{(p)} = \sum_{i=1}^{m_p} \sum_{j=1}^{m_{p-1}} \sum_{k=1}^{k_p} c_{ijk}^{(p)} x_{ijk}^{(p)} \quad ; \ p = 1,2,\ldots\ldots,N \qquad (1)$$

Subject to

$$\sum_{j=1}^{m_{p-1}} \sum_{k=1}^{k_p} x_{ijk}^{(p)} \le a_i^{(p)}; \quad p = 1,2,\ldots\ldots,N; \qquad i = 1,2,\ldots\ldots\ldots,m_p \qquad (2)$$

$$\sum_{i=1}^{m_p} \sum_{k=1}^{k_p} x_{ijk}^{(p)} \ge b_j^{(p)}; \quad p = 1,2,\ldots\ldots,N; \qquad j = 1,2,\ldots\ldots\ldots,m_{p-1} \qquad (3)$$

$$\sum_{i=1}^{m_p} \sum_{j=1}^{m_{p-1}} x_{ijk}^{(p)} \le e_k^{(p)}; \quad p = 1,2,\ldots\ldots,N; \qquad k = 1,2,\ldots\ldots\ldots,k_p \qquad (4)$$

$$\text{and} \quad x_{ijk}^{(p)} \ge 0 \quad \forall\ p, i, j, k \qquad (5)$$

where

$$a_i^{(p)} = A_i^{(p)} + b_i^{(p+1)} \quad p = 1,2,\ldots\ldots\ldots,n-1; \quad i = 1,2,\ldots\ldots\ldots\ldots,m_p \qquad (6)$$

$$a_i^{(N)} = A_i^{(N)} i = 1,2,\ldots\ldots\ldots\ldots,m_N \qquad (7)$$

$$b_j^{(p)} = \left\lfloor \frac{x_s^{(p-1)} H^{(p-1)}}{AC_j^{(p-1)} m_{p-1}} \right\rfloor; \quad p = 2,3,\ldots\ldots\ldots\ldots,N; \qquad j = 1,2,\ldots\ldots m_{p-1} \qquad (8)$$

$$x_s^{(p)} = \sum_{j=1}^{m_{p-1}} b_j^{(p)} - \sum_{i=1}^{m_p} A_i^{(p)} > 0; \qquad p = 1,2,\ldots\ldots.N-1 \qquad (9)$$

$$AC_i^{(p)} = \sum_{j=1}^{m_{p-1}} \sum_{k=1}^{k_p} c_{ijk}^{(p)}; \ p = 1,2,\ldots\ldots,N-1; \qquad i = 1,2,\ldots\ldots\ldots,m_p \ (10)$$

$$H^{(p)} = \frac{m_p}{\sum_{i=1}^{m_p} \frac{1}{AC_i^{(p)}}}; \qquad p = 1,2,\ldots\ldots.N-1 \qquad (11)$$

As it can be seen, there are N objective functions in (1) for N steps ($p = 1, 2, \ldots, N$) of the model. Here, the value of N is always a finite natural number greater than or equal to 2. (2), (3) and (4) are the supply, demand and conveyance constraints respectively. The non-negativity constraints (5) are must, since the quantity of goods is always non-negative.

Here, all the parameters and the decision variables$x_{ijk}^{(p)}$ are taken as trapezoidal neutrosophic numbers. But, $x_{ijk}^{(p)}$denote quantities of goods to be transported and in reality, any manager or decision maker would want to obtain the crisp optimal solution of the problem through considering vague, imprecise and inconsistent information while defining the problem.

Equation (8) is used to calculate $b_j^{(p)}$'s (crisp values) after all the given parameters are converted into their corresponding crisp values by a suitable ranking function. So, (8) becomes

$$b_j^{(p)} = \left\lfloor \frac{x_s^{(p-1)} H^{(p-1)}}{AC_j^{(p-1)} m_{p-1}} \right\rfloor; \qquad p = 2, 3, \dots, N, \qquad j = 1, 2, \dots, m_{p-1}. \tag{12}$$

Proposition 3.3.1

If the import plan due to insufficient supply for each supplier Company Y_p (p = 1, 2, …, N – 1) is – "import the highest quantity of goods from Y_{p+1} to that warehouse *j*from which the average per unit cost of transportation of goods to Y_{p-1} is minimum", then the import plan (quantity of goods to be imported to each warehouse *j*) is mathematically given by:

$$b_j^{(p)} = \left\lfloor \frac{x_s^{(p-1)} H^{(p-1)}}{AC_j^{(p-1)} m_{p-1}} \right\rfloor; \qquad p = 2, 3, \dots, N, \qquad j = 1, 2, \dots, m_{p-1}.$$

Proof:

Here, $b_j^{(p)}$'s denote the demands of the destination warehouses in the p^{th} step, which are also the origin warehouses in the $(p-1)^{\text{th}}$ step. In the p^{th} step, we want to import the highest quantity of goods to that warehouse *j*from which the average per unit cost of transportation of goods A $AC_j^{(p-1)}$minimum. So, $b_j^{(p)}$inversely proportional to $AC_j^{(p-1)}$i.e.,

$$b_j^{(p)} \propto \frac{1}{AC_j^{(p-1)}}$$

i.e., $$b_j^{(p)} = \kappa \frac{1}{AC_j^{(p-1)}},$$

where κ is the proportionality constant.

Now, $$\text{total demand } = x_s^{(p-1)}$$

i.e.,$\sum_{j=1}^{m_{p-1}} b_j^{(p)} = x_s^{(p-1)}$

i.e.,$\sum_{j=1}^{m_{p-1}} K \frac{1}{AC_j^{(p-1)}} = x_s^{(p-1)}$

i.e., $K = \frac{x_s^{(p-1)}}{\sum_{j=1}^{m_{p-1}} \frac{1}{AC_j^{(p-1)}}}$

But,$H^{(p-1)} = \frac{m_{p-1}}{\sum_{i=1}^{m_{p-1}} \frac{1}{AC_i^{(p-1)}}} = \frac{m_{p-1}}{\sum_{j=1}^{m_{p-1}} \frac{1}{AC_j^{(p-1)}}}$

Therefore, $$K = \frac{x_s^{(p-1)} H^{(p-1)}}{m_{p-1}}$$

And so, $$b_j^{(p)} = \frac{x_s^{(p-1)} H^{(p-1)}}{AC_j^{(p-1)} m_{p-1}} \tag{13}$$

Now, while calculating $b_j^{(p)}$'s using (13), rounding off their decimals may result in loss of significant quantity of $x_s^{(p-1)}$'s. So, we use the ceiling function and hence, obtain

$$b_j^{(p)} = \left\lceil \frac{x_s^{(p-1)} H^{(p-1)}}{AC_j^{(p-1)} m_{p-1}} \right\rceil; \qquad p = 2, 3, \dots, N, \qquad j = 1, 2, \dots, m_{p-1}.$$

4 Solution approach

An approach is suggested to find the optimal solution of such problems. The step by step procedure is as follows:

Step I. Collect the information for a given problem as trapezoidal neutrosophic numbers from the decision makers with the information that we always want to maximize the truth degree and minimize the indeterminacy and falsity degrees of the data.

Step II. Construct the neutrosophic solid transportation table of the given problem for $p = 1$.

Step III. Convert all the trapezoidal neutrosophic numbers into their equivalent crisp values by the use of the ranking function, proposed by M. Abdel-Basset et al. [20], which is given by:

$$\Re(\tilde{a}) = \frac{a^l + a^u - 3(a^{m1} + a^{m2})}{2} + \textit{confirmation degree},$$

or mathematically,

$$\Re(\tilde{a}) = \frac{a^l + a^u - 3(a^{m1} + a^{m2})}{2} + (T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}), \tag{14}$$

where $T_{\tilde{a}}$, $I_{\tilde{a}}$ and $F_{\tilde{a}}$are respectively the truth, indeterminacy and falsity degrees of the trapezoidal neutrosophic number $\tilde{a} = (a^l, a^u, a^{m1}, a^{m2})$. Here, a^l, a^u, $a^{m1}$and $a^{m2}$are the lower bound, upper bound, first median and second median values of $\tilde{a}$ respectively, which can be obtained from the form $\tilde{a} = (a_1, a_2, a_3, a_4)$ by the transformations: $a^l = a_2$, $a^u = a_3$, $a^{m1} = a_2 - a_1$ and $a^{m2} = a_4 - a_3$.

Step IV. Compute $x_s^{(p)}$ for $p = 1$ using the crisp form of (9).

Step V. Calculate $b_j^{(p)}$'s for $p = 2$ using (12).

Step VI. Construct the crisp solid transportation table for $p = 2$ using the ranking function (14) for the values of supply and conveyance capacity.

Step VII. Repeat Steps (IV – VI) until some $x_s^{(p-1)}$ (say, for $p = N$) is found, which can be totally satisfied by the next supplier company (Y_N).

Step VIII. Calculate $b_j^{(p)}$'s for $p = N$ using (12).

Step IX. Construct the crisp solid transportation table for $p = N$ using the ranking function (14) for the values of supply and conveyance capacity.

Step X. Solve the crisp solid transportation table for $p = N$ using a standard method as used for solving a general crisp STP and obtain the optimal solution for this step.

Step XI. Compute the new crisp values of supply for $p = N - 1$ using the crisp form of (6) and similarly, solve the table for $p = N - 1$ as solved for $p = N$.

Step XII. Repeat Step XI and similarly, solve the tables for $p = N - 2, N - 3, \dots, 1$.

Step XIII. Conclude the solution with the degree of sufficiency η, which is defined as:

$$\eta = \frac{1}{N-1} \sum_{p=1}^{N-1} \eta^{(p)}, \tag{15}$$

where

$$\eta^{(p)} = \frac{\sum_{i=1}^{m_p} a_i^{(p)} - \sum_{i=1}^{m_p} A_i^{(p)}}{x_s^{(p)}} - 1 \tag{16}$$

The proposed solution approach in the paper is a first of its kind. The solution approach is depicted as follows:

Transportation Possible if $X \geq Y$

Transportation Not Possible if $X < Y$

The model proposed in the paper makes the transportation possible in the second case (shown above).

5Numerical example

Suppose, Company Y_1 has to transport a commodity (e.g., wheat) to Company Y_0. But, it falls short of some amount and wants to import the required amount from Company Y_2. Similarly, Company Y_2 does not have the sufficient amount of wheat to fulfill the total demand of Company Y_1, so Company Y_2 imports the required amount from Company Y_3. It is assumed that Company Y_3 has the right amount of wheat to fulfill the total demand of Company Y_2.

The neutrosophic data for the transportations $Y_1 \rightarrow Y_0$, $Y_2 \rightarrow Y_1$ and $Y_3 \rightarrow Y_2$ are given in Table 1, Table 2 and Table 3 respectively. For the sake of simplicity, $(T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}})$ is taken as (0.9, 0.1, 0.1) for all the trapezoidal neutrosophic numbers. The costs are considered in INR and the commodity is measured in kilograms.

$Y_1 \rightarrow Y_0$							**Conveyance Capacity**
	$E_1^{(1)}$			$E_1^{(1)}$			(3150,4500,170,185)
		$E_2^{(1)}$			$E_2^{(1)}$		(4100,5200,200,180)
			$E_3^{(1)}$			$E_3^{(1)}$	(2900,3850,175,190)
		$D_1^{(1)}$			$D_2^{(1)}$		**Supply**
$O_1^{(1)}$	(50,65,7,6)	(40,60,7,7)	(90,110,8,9)	(80,100,6,8)	(40,55,5,7)	(55,70,4,3)	(1300,1600,140,170)
$O_2^{(1)}$	(70,80,6,9)	(80,95,6,4)	(65,75,4,6)	(30,45,7,5)	(50,70,3,5)	(65,85,6,7)	(1650,2000,165,150)
$O_3^{(1)}$	(60,80,6,5)	(45,60,4,5)	(70,85,9,6)	(60,85,6,7)	(75,95,8,5)	(95,115,7,4)	(1050,1400,120,135)
Demand	(4500, 5700,250,230)			(5000,6500,245,260)			

Table 1: Neutrosophic data table for $Y_1 \rightarrow Y_0$.

$Y_2 \rightarrow Y_1$							**Conveyance Capacity**
	$E_1^{(2)}$		$E_1^{(2)}$		$E_1^{(2)}$		(3400,4150,150,130)
		$E_2^{(2)}$		$E_2^{(2)}$		$E_2^{(2)}$	(3250,3900,155,140)
	$D_1^{(2)}$		$D_2^{(2)}$		$D_3^{(2)}$		**Supply**
$O_1^{(2)}$	(90,110,10,8)	(50,70,4,6)	(40,55,5,7)	(70,85,9,6)	(35,55,4,5)	(75,85,8,6)	(1000,1300,120,135)

$O_2^{(2)}$	(75,90,7,8)	(65,80,6,5)	(85,95,6,6)	(50,60,4,3)	(60,80,4,6)	(60,85,9,7)	(1250,1500,145,160)
$O_3^{(2)}$	(80,90,6,7)	(90,105,9,8)	(55,75,8,7)	(75,90,5,5)	(100,120,5,7)	(40,55,6,5)	(1000,1350,105,125)
$O_4^{(2)}$	(30,50,4,4)	(70,85,5,6)	(80,95,5,7)	(60,80,4,6)	(65,75,8,7)	(50,65,4,3)	(1200,1450,110,100)
Demand	( )		( )		( )		

Table 2: Neutrosophic data table for Y$_2$ → Y$_1$.

Y$_3$ → Y$_2$									**Conveyance Capacity**
	$E_1^{(3)}$		$E_1^{(3)}$		$E_1^{(3)}$		$E_1^{(3)}$		(1650,1950,135,140)
		$E_2^{(3)}$		$E_2^{(3)}$		$E_2^{(3)}$		$E_2^{(3)}$	(1800,2200,140,150)
	$D_1^{(3)}$		$D_2^{(3)}$		$D_3^{(3)}$		$D_4^{(3)}$		**Supply**
$O_1^{(3)}$	(85,95,7,8)	(35,50,4,5)	(60,75,8,6)	(70,80,9,7)	(50,60,3,6)	(50,70,4,6)	(80,95,7,5)	(30,50,4,6)	(1150,1350,105,100)
$O_2^{(3)}$	(70,85,6,7)	(45,60,6,5)	(50,70,4,6)	(40,60,4,5)	(80,95,8,6)	(60,70,4,6)	(50,65,4,3)	(85,95,8,7)	(800,1100,90,95)
$O_3^{(3)}$	(50,70,5,4)	(65,75,7,4)	(85,95,7,7)	(75,90,9,6)	(45,60,5,4)	(55,70,5,5)	(35,55,4,5)	(50,65,5,6)	(1400,1700,115,135)
Demand	( )		( )		( )		( )		

Table 3: Neutrosophic data table for Y$_3$ → Y$_2$.

The crisp tables are solved with LINGO 17.0 software. Table 4, Table 5 and Table 6 show the optimal crisp solutions for Y$_3$ → Y$_2$, Y$_2$→ Y$_1$ and Y$_1$→ Y$_0$ respectively.

The minimum values of the objective functions are given below:

$z^{(1)}$: ₹308719.43
$z^{(2)}$: ₹216642.65
$z^{(3)}$: ₹90474.05

The degree of sufficiency η is found out to be 0.00062.

Y$_3$ → Y$_2$									**Conveyance Capacity**
	$E_1^{(3)}$		$E_1^{(3)}$		$E_1^{(3)}$		$E_1^{(3)}$		1388.2
		$E_2^{(3)}$		$E_2^{(3)}$		$E_2^{(3)}$		$E_2^{(3)}$	1565.7
	$D_1^{(3)}$		$D_2^{(3)}$		$D_3^{(3)}$		$D_4^{(3)}$		**Supply**
$O_1^{(3)}$	0 (68.20)	737 (29.70)	0 (47.20)	0 (51.70)	0 (42.20)	0 (45.70)	0 (70.20)	184.7 (25.70)	943.2
$O_2^{(3)}$	0 (58.70)	0 (36.70)	0 (45.70)	644 (37.20)	0 (67.20)	0 (50.70)	0 (47.70)	0 (68.20)	673.2
$O_3^{(3)}$	0 (47.20)	0 (54.20)	0 (69.70)	0 (60.70)	585 (39.70)	0 (48.20)	517.3 (32.20)	0 (41.70)	1175.7
Demand	737		644		585		702		

Table 4: Optimal solution table for Y$_3$ → Y$_2$.

Y$_2$ → Y$_1$							**Conveyance Capacity**
	$E_1^{(2)}$		$E_1^{(2)}$		$E_1^{(2)}$		3355.7
		$E_2^{(2)}$		$E_2^{(2)}$		$E_2^{(2)}$	3133.2
	$D_1^{(2)}$		$D_2^{(2)}$		$D_3^{(2)}$		**Supply (new)**
$O_1^{(2)}$	0 (73.70)	0 (45.70)	1087.9 (30.20)	0 (55.70)	417.3 (32.20)	0 (59.70)	1505.2
$O_2^{(2)}$	0	507.3	0	1052.1	0	0	1562.2

	(60.70)	(56.70)	(72.70)	(45.20)	(55.70)	(49.20)	
$O_3^{(2)}$	0 (66.20)	0 (72.70)	0 (43.20)	0 (68.20)	0 (92.70)	1415.7 (31.70)	1415.7
$O_4^{(2)}$	1712.7 (28.70)	0 (61.70)	0 (70.20)	0 (55.70)	0 (48.20)	0 (47.70)	1712.7
Demand	2220		2140		1833		

Table 5: Optimal solution table for $Y_2 \rightarrow Y_1$.

$Y_1 \rightarrow Y_0$							**Conveyance Capacity**
	$E_1^{(1)}$			$E_1^{(1)}$			3293.2
		$E_2^{(1)}$			$E_2^{(1)}$		4080.7
			$E_3^{(1)}$			$E_3^{(1)}$	2828.2
	$D_1^{(1)}$			$D_2^{(1)}$			**Supply (new)**
$O_1^{(1)}$	0 (38.70)	1705.7 (29.70)	0 (75.20)	0 (69.70)	1500 (30.20)	0 (52.70)	3205.7
$O_2^{(1)}$	0 (53.20)	0 (73.20)	0 (55.70)	3293.2 (20.20)	0 (48.70)	200 (56.20)	3493.2
$O_3^{(1)}$	0 (54.20)	875 (39.70)	1800 (55.70)	0 (53.70)	0 (66.20)	0 (89.20)	2676.2
Demand	4380.7			4993.2			

Table 6: Optimal solution table for $Y_1 \rightarrow Y_0$.

6Discussion

The model, discussed in this paper, is a very interesting solid transportation model, which can be useful in the business sector. It can be safely concluded that the problem of insufficient supply that arises in the model, is dealt with effectively, as the degree of sufficiency η is positive for the given example and is very close to 0 (zero). η should always be non-negative and the closer η is to zero, the more sufficient the solution is for the model.

The numerical example given above is a 3-step model with fewer amounts of data. The computational time for this problem is not too high, but in real systems, the data is greater in amount and so, higher will be the computational time. The computational time *T* for an *N*-step model may roughly be given by the expression:

$$T = Ca(n) + Co(n) + Sol(n),$$

where

Ca(n) is the total time component for calculatio: $b_j^{(p)}$ s ’s,

Co(n) is the total time component for conversion of the trapezoidal neutrosophic numbers into crisp values,

Sol(n) is the total time component for solving the crisp data,

and all these components depend on the amount of data *n*, each of them varying directly with *n*.

Clearly, the amount of data *n* consists of *N* heterogeneous components. So, *Ca(n)*has *N* – 1 sub-components, *Co(n)* has *N* sub-components and *Sol(n)* also has *N* sub-components. For the given numerical example, *Ca(n)* has 2 components, while *Co(n)* and *Sol(n)* have 3 components each.

Equation (12) is a key part of the solution method and a point of research for constructing a more efficient model. The degree of sufficiency η is evidently dependent on (12). The ranking function (14) converts the trapezoidal neutrosophic numbers into their equivalent crisp values effectively by considering the degrees of all three aspects of decision, but efforts can be made to construct a better ranking function to get more accurate crisp models and better results. The model may also be considered with a time constraint (along with some time penalty) for each supplier. Also, as we know that the notion of neutrosophic set theory is relatively new and it broadly covers all the aspects of decision making, so there is a good potential for its extensive research and applications in complex logistic systems.

7 Conclusion and future scope

The solid transportation problem is a significant problem in Operations Research, where the primary goal is to transport commodities from some source warehouses to some destination warehouses via different modes of conveyance. This paper formulates a model, where the source cannot fulfill the total demand and brings in the required amount from another source, which in turn, if unable to supply the necessary amount, brings in the remaining amount from another source, and so on. An expression is derived to provide the distribution of demand of the deficient quantity of goods among the importing warehouses. The paper also considers the impreciseness and uncertainty that may exist in the data and takes the input as trapezoidal neutrosophic numbers. An approach is presented to solve the model and the quality of the solution is checked with the degree of sufficiency. Also, the computational time is shown for the model and it is believed that the model is useful and has an interesting scope.

In this manuscript, the mathematical model has considered the minimization of cost as an objective function. But it is very important to complete the fulfilment of demand of customers as early as possible. Therefore, for future research, one can consider the minimization of time as an objective function. In the business purpose, profit is essential to grow. In this regard, maximization of profit can be treated as an objective function for further study. In this manuscript, uncertainty is used in terms of trapezoidal neutrosophic number. But in the direction of future research, one can use different parameters e.g. uncertain number, fuzzy number, type-2 fuzzy number etc. To solve the problem, genetic algorithm can be developed in future research.

References

[1] F.L. Hitchcock, The distribution of a product from several sources to numerous localities, Journal of Mathematics and Physics, vol. 20, 1941, pp. 224 – 230.

[2] E. Shell, Distribution of a product by several properties, Directorate of Management Analysis, Proceedings of the Second Symposium on Linear Programming, DCS/Comptroller H.Q. U.S.A.F., Washington, DC, vol. 2, 1955, pp. 615 – 642.

[3] K.B. Haley, The solid transportation problem, Operations Research, vol. 10, 1962, pp. 448 – 463.

[4] L.A. Zadeh, Fuzzy sets, Information and Control, vol. 8, 1965, pp. 338 – 353.

[5] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20, 1986, pp. 87 – 96.

[6] K. Atanassov, More on intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 33, 1989, pp. 37 – 46.

[7] F. Jiménez and J.L. Verdegay, Uncertain solid transportation problems, Fuzzy Sets and Systems, vol. 100, 1998, pp. 45 – 57.

[8] F. Smarandache, Neutrosophic set – a generalization of the intuitionistic fuzzy set, International Journal of Pure and Applied Mathematics, vol. 24(3), 2005, pp. 287 – 297.

[9] L. Yang and L. Liu, Fuzzy fixed charge solid transportation problem and algorithm, Applied Soft Computing, vol. 7, 2007, pp. 879 – 889.

[10]R. J. Hussain and P. S. Kumar, Algorithmic approach for solving intuitionistic fuzzy transportation problem, Applied Mathematical Sciences, vol. 6(80), 2012, pp. 3981 – 3989.

[11]P. Kundu, S. Kar and M. Maiti, Some solid transportation models with crisp and rough costs, International Journal of Mathematical and Computational Sciences, vol. 7, 2013, pp. 14 – 21.

[12]P. Liu, Y. Chu, Y. Li and Y. Chen, Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making, International Journal of Fuzzy Systems, vol. 16(2), 2014, pp. 242 – 255.

[13]S. K. Singh and S. P. Yadav, Efficient approach for solving type-1 intuitionistic fuzzy transportation problem, International Journal of System Assurance Engineering and Management, vol. 6(3), 2015, pp. 259 – 267.

[14]A. Das, U.K. Bera and M. Maiti, Defuzzification of trapezoidal type-2 fuzzy variables and its application to solid transportation problem, Journal of Intelligent and Fuzzy Systems, vol. 30(4), 2016, pp. 2431 – 2445.

[15]A. Thamaraiselvi and R. Santhi, A new approach for optimization of real life transportation problem in neutrosophic environment, Mathematical Problems in Engineering, vol. 2016, 2016.

[16]P.K. Giri, M.K. Maiti and M. Maiti, Profit maximization solid transportation problem under budget constraint using fuzzy measures, Iranian Journal of Fuzzy Systems, vol. 13, 2016, pp. 35 – 63.

[17]I. Deli and Y. Şubaş, A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems, Int. J. Mach. Learn. & Cyber., vol. 8, 2017, pp. 1309 – 1322.

[18]A. Das, U.K. Bera and M. Maity, A profit maximizing solid transportation model under a rough interval approach, IEEE Transaction of Fuzzy System, vol. 25(3), 2017, pp. 485 – 498.

[19]S. Aggarwal and C. Gupta, Sensitivity analysis of intuitionistic fuzzy solid transportation problem, International Journal of Fuzzy Systems, vol. 19(6), 2017, pp. 1904 – 1915.

[20]M. Abdel-Basset, M. Gunasekaran, M. Mohamed and F. Smarandache, A novel method for solving the fully

neutrosophic linear programming problems, Neural Computing and Applications, Springer, 2018, pp. 1 – 11.
[21]R.M. Rizk-Allah, A.E. Hassanien and M. Elhoseny, A multi-objective transportation model under neutrosophic environment, Computers and Electrical Engineering, 2018, pp. 1 – 15.
[22]Abdel-Basset, M., El-hoseny, M., Gamal, A., & Smarandache, F. (2019). A novel model for evaluation Hospital medical care systems based on plithogenic sets. Artificial intelligence in medicine, 100, 101710.
[23]Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F. (2019). A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. Symmetry, 11(7), 903.
[24]Abdel-Basset, M., Nabeeh, N. A., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. Enterprise Information Systems, 1-21.
[25]Abdel-Baset, M., Chang, V., & Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel neutrosophic approach. Computers in Industry, 108, 210-220.
[26]Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, 77, 438-452.

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