



Translative and Multiplicative Interpretation of Neutrosophic Cubic Set

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Abstract: In this paper, we introduce the idea of neutrosophic cubic translation (NCT) and neutrosophic cubic multiplication (NCM) and provide entirely new type of conditions for neutrosophic cubic translation and neutrosophic cubic multiplication on BF-algebra. This is the new kind of approach towards translation and multiplication which involves the indeterminacy membership function. We also define neutrosophic cubic magnified translation (NCMT) on BF-algebra which handles the neutrosophic cubic translation and neutrosophic cubic multiplication at the same time on membership function, indeterminacy membership function and non-membership function. We present the examples for better understanding of neutrosophic cubic translation, neutrosophic cubic multiplication, and neutrosophic cubic magnified translation, and investigate significant results of BF-ideal and BF-subalgebra by applying the ideas of NCT, NCM and NCMT. Intersection and union of neutrosophic cubic BF-ideals are also explained through this new type of translation and multiplication.

Keywords: BF-algebra, neutrosophic cubic translation, neutrosophic cubic multiplication, neutrosophic cubic BF ideal, neutrosophic cubic BF subalgebra, neutrosophic cubic magnified translation.

1. Introduction

Zadeh [1] presented the theory of fuzzy set in 1965. Fuzzy idea has been applied to different algebraic structures like groups, rings, modules, vector spaces and topologies. In this way, Iseki and Tanaka [2] introduced the idea of BCK-algebra in 1978. Iseki [3] introduced the idea of BCI-algebra in 1980 and it is obvious that the class of BCK-algebra is a proper sub class of the class of BCI-algebra. Lee et al. [4] studied the fuzzy translation,

(normalized, maximal) fuzzy extension and fuzzy multiplication of fuzzy subalgebra in BCK/BCI-algebra. Link among fuzzy translation, (normalized, maximal) fuzzy extension and fuzzy multiplication are also discussed. Ansari and Chandramouleeswaran [5] introduced the idea of fuzzy translation, fuzzy extension and fuzzy multiplication of fuzzy β ideal of β -algebra and investigated some of their properties. Satyanarayana [6] introduced the concepts of fuzzy ideals, fuzzy implicative ideals and fuzzy p-ideals in BF-algebras and investigate some of its properties. Andrzej [7] defined the BF-algebra. Lekkoksung [8] focused on fuzzy magnified translation in ternary hemirings, which is a extension of BCI / BCK/Q / KU / d-algebra. Senapati et al. [9] have done much work on intuitionistic fuzzy H-ideal in BCK/BCI-algebra. Jana et al. [10] wrote on intuitionistic fuzzy G-algebra. Senapati et al. [11] studied fuzzy translations of fuzzy H-ideals in BCK/BCI-algebra. Atanassov [12] introduced the intuitionistic fuzzy sets. Senapati [13] investigated the relationship among intuitionistic fuzzy translation, intuitionistic fuzzy extension and intuitionistic fuzzy multiplication in B-algebra. Kim and Jeong [14] defined the intuitionistic fuzzy structure of B-algebra. Senapati et al. [15] introduced the cubic subalgebras and cubic closed ideals of B-algebras. Senapati et al. [16] discussed the fuzzy dot subalgebra and fuzzy dot ideal of B-algebras. Priya and Ramachandran [17] worked on fuzzy translation and fuzzy multiplication in PS-algebra. Chandramouleeswaran et al. [18] worked on fuzzy translation and fuzzy multiplication in BF/BG-algebra. Jana et al. [19] studied the cubic G-subalgebra of G-algebra. Smarandache [20,21] extended the intuitionistic fuzzy set, paraconsistent set, and intuitionistic set to the neutrosophic set through Several examples. Jun et al. [22] studied the Cubic set and apply the idea of cubic sets in group and gave the definition of cubic subgroups. Saeid and Rezvani [23] introduced and studied fuzzy BF-algebra, fuzzy BF-subalgebras, level subalgebras, fuzzy topological BF-algebra. Jun et al. [24] defined the neutrosophic cubic set introduced truth-internal and truth-external and discuss the many properties. Jun et al. [25] investigated the commutative falling neutrosophic ideals in BCK-algebra. C. H. Park [26] defined the neutrosophic ideal in subtraction algebra and studied it through several properties, he also discussed conditions for a neutrosophic set to be a neutrosophic ideal along with properties of neutrosophic ideal. Khalid et al. [27] investigated the neutrosophic soft cubic subalgebra through significant characteristic like P-union, R-intersection etc. Khalid et al. [28] interestingly investigated the intuitionistic fuzzy translation and multiplication through subalgebra and ideals. Khalid et al. [29] defined the T-neutrosophic cubic set and studied this set through ideals and subalgebras and investigated many results.

The purpose of this paper is to introduce the idea of neutrosophic cubic translation (**NCT**), neutrosophic cubic multiplication (**NCM**) and neutrosophic cubic magnified translation (**NCMT**) on BF-algebra. In second section we discuss some fundamental definitions which are used to develop the paper. In third's first subsection we discuss the neutrosophic cubic translation (**NCT**) and neutrosophic cubic multiplication (**NCM**) of BF subalgebra. In second subsection we discuss the neutrosophic cubic translation (**NCT**) and neutrosophic cubic multiplication (**NCM**) of BF ideal. In third subsection we discuss the neutrosophic cubic magnified translation (**NCMT**) of BF ideal and BF subalgebra.

2 Preliminaries

First we discuss some definitions which are used to present this paper.

Definition 2.1 [3] An algebra $(Y, *, 0)$ of type $(2,0)$ is called a BCI-algebra if it satisfies the following conditions:

- i) $(t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2)$,
- ii) $t_1 * (t_1 * t_2) \leq t_2$,
- iii) $t_1 \leq t_1$,
- iv) $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$,
- v) $t_1 \leq 0 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, for all $t_1, t_2, t_3 \in Y$.

Definition 2.2 [1] An algebra $(Y, *, 0)$ of type $(2,0)$ is called a BCK-algebra if it satisfies the following conditions:

- i) $(t_1 * t_2) * (t_1 * t_3) \leq (t_3 * t_2)$,
- ii) $t_1 * (t_1 * t_2) \leq t_2$,
- iii) $t_1 \leq t_1$,
- iv) $t_1 \leq t_2$ and $t_2 \leq t_1 \Rightarrow t_1 = t_2$,
- v) $0 \leq t_1 \Rightarrow t_1 = 0$, where $t_1 \leq t_2$ is defined by $t_1 * t_2 = 0$, for all $t_1, t_2, t_3 \in Y$.

Definition 2.3 [7] A nonempty set Y with a constant 0 and a binary operation $*$ is called BF-algebra when it fulfills these axioms.

- i) $t_1 * t_1 = 0$
- ii) $t_1 * 0 = 0$
- iii) $0 * (t_1 * t_2) = t_2 * t_1$ for all $t_1, t_2 \in Y$.

A BF-algebra is denoted by $(Y, *, 0)$.

Definition 2.4 [7] Let S be a nonempty subset of BF-algebra Y , then S is called a BF-subalgebra of Y if $t_1 * t_2 \in S$, for all $t_1, t_2 \in S$.

Definition 2.5 [6] Let Y ba a BF-algebra and I is a subset of Y , then I is called a BF ideal of Y if it satisfies the following conditions:

- i) $0 \in I$,
- ii) $t_2 * t_1 \in I$ and $t_2 \in I \rightarrow t_1 \in I$.

Definition 2.6 [6] Let Y be a BF-algebra. A fuzzy set B of Y is called a fuzzy BF ideal of Y if it satisfies the following conditions:

- i) $\kappa(0) \geq \kappa(t_1)$,
- ii) $\kappa(t_1) \geq \min\{\kappa(t_2 * t_1), \kappa(t_2)\}$, for all $t_1, t_2 \in Y$.

Definition 2.7 [1] Let Y be a group of objects denoted generally by t_1 . Then a fuzzy set B of Y is defined as $B = \{< t_1, \kappa_B(t_1) > | t_1 \in Y\}$, where $\kappa_B(t_1)$ is called the membership value of t_1 in B and $\kappa_B(t_1) \in [0,1]$.

Definition 2.8 [23] A fuzzy set B of BF-algebra Y is called a fuzzy PS subalgebra of Y if $\kappa(t_1 * t_2) \geq \min\{\kappa(t_1), \kappa(t_2)\}$, for all $t_1, t_2 \in Y$.

Definition 2.9 [4,5] Let a fuzzy subset B of Y and $\alpha \in [0,1 - \sup\{\kappa_B(t_1) | t_1 \in Y\}]$. A mapping $(\kappa_B)_\alpha^T | Y \in [0,1]$ is said to be a fuzzy α translation of κ_B if it satisfies $(\kappa_B)_\alpha^T(t_1) = \kappa_B(t_1) + \alpha$, for all $t_1 \in Y$.

Definition 2.9 [4,5] Let a fuzzy subset B of Y and $\alpha \in [0,1]$. A mapping $(\kappa_B)_\alpha^M : Y \rightarrow [0,1]$ is said to be a fuzzy α multiplication of B if it satisfies $(\kappa_B)_\alpha^M(t_1) = \alpha \cdot (\kappa_B)(t_1)$, for all $t_1 \in Y$.

Definition 2.10 [12] An intuitionistic fuzzy set (**IFS**) B over Y is an object having the form $B = \{(t_1, \kappa_B(t_1), v_B(t_1)) | t_1 \in Y\}$, where $\kappa_B(t_1) : Y \rightarrow [0,1]$ and $v_B(t_1) : Y \rightarrow [0,1]$, with the condition $0 \leq \kappa_B(t_1) + v_B(t_1) \leq 1$, for all $t_1 \in Y$. $\kappa_B(t_1)$ and $v_B(t_1)$ represent the degree of membership and the degree of non-membership of the element t_1 in the set B respectively.

Definition 2.11 [12] Let $B = \{(t_1, \kappa_B(t_1), v_B(t_1)) | t_1 \in Y\}$ and $B = \{(t_1, \kappa_B(t_1), v_B(t_1)) | t_1 \in Y\}$ be two **IFSs** on Y . Then intersection and union of A and B are indicated by $A \cap B$ and $A \cup B$ respectively and are given by

$$A \cap B = \{(t_1, \min(\kappa_A(t_1), \kappa_B(t_1)), \max(v_A(t_1), v_B(t_1))) | t_1 \in Y\},$$

$$A \cup B = \{(t_1, \max(\kappa_A(t_1), \kappa_B(t_1)), \min(v_A(t_1), v_B(t_1))) | t_1 \in Y\}.$$

Definition 2.12 [14] An **IFS** $B = \{(t_1, \kappa_B(t_1), v_B(t_1)) | t_1 \in Y\}$ of Y is called an **IFSU** of Y if it satisfies these two conditions:

- (i) $\kappa_B(t_1 * t_2) \geq \min\{\kappa_B(t_1), \kappa_B(t_2)\}$,
- (ii) $v_B(t_1 * t_2) \leq \max\{v_B(t_1), v_B(t_2)\}$, for all $t_1, t_2 \in Y$.

Definition 2.13 An **IFS** $B = \{(t_1, \kappa_B(t_1), v_B(t_1)) | t_1 \in Y\}$ of Y is said to be an **IFID** of Y if it satisfies these three conditions:

- (i) $\kappa_B(0) \geq \kappa_B(t_1), v_B(0) \leq v_B(t_1)$,
- (ii) $\kappa_B(t_1) \geq \min\{\kappa_B(t_1 * t_2), \kappa_B(t_2)\}$,
- (iii) $v_B(t_1) \leq \max\{v_B(t_1 * t_2), v_B(t_2)\}$, for all $t_1, t_2 \in Y$.

Definition 2.14 [8] Let κ be a fuzzy subset of Y , $\alpha \in [0, T]$ and $\beta \in [0, 1]$. A mapping $\kappa_{\beta \alpha}^{MT} | Y \rightarrow [0, 1]$ is said to be a fuzzy magnified $\beta\alpha$ translation of κ if it satisfies: $\kappa_{\beta \alpha}^{MT}(t_1) = \beta \cdot \kappa(t_1) + \alpha$ for all $t_1 \in Y$.

Jun et al. [22,24] introduced neutrosophic cubic set and investigated several properties.

Definition 2.15 [24] Suppose X be a nonempty set. A neutrosophic cubic set in X is pair $C = (\kappa, \sigma)$ where $\kappa = \{(t_1; \kappa_E(t_1), \kappa_I(t_1), \kappa_N(t_1)) | t_1 \in X\}$ is an interval neutrosophic set in X and $\sigma = \{(t_1; \sigma_E(t_1), \sigma_I(t_1), \sigma_N(t_1)) | t_1 \in X\}$ is a neutrosophic set in X .

Definition 2.16 [15] Let $C = \{(t_1, \kappa(t_1), \sigma(t_1))\}$ be a cubic set, where $\kappa(t_1)$ is an interval-valued fuzzy set in X , $\sigma(t_1)$ is a fuzzy set in X . Then C is cubic subalgebra under binary operation " $*$ ", if following axioms are satisfied:

- i) $\kappa(t_1 * t_2) \geq \text{rmin}\{\kappa(t_1), \kappa(t_2)\}$,
- ii) $\sigma(t_1 * t_2) \leq \max\{\sigma(t_1), \sigma(t_2)\} \quad \forall t_1, t_2 \in X$.

Definition 2.17 [28] Let $A = (\kappa_A, v_A)$ be an IFS of G-algebra and let $\alpha \in [0, \mathbb{Y}]$. An object of the form $A_\alpha^T = ((\kappa_A)_\alpha^T, (v_A)_\alpha^T)$ is called an intuitionistic fuzzy α -translation (**IFAT**) of A when $(\kappa_A)_\alpha^T(t_1) = \kappa_A(t_1) + \alpha$ and $(v_A)_\alpha^T(t_1) = v_A(t_1) - \alpha$ for all $t_1 \in Y$.

3 Translative and Multiplicative Interpretation of Neutrosophic Cubic Set

For our simplicity, we use the notation $B = (\kappa_{T,I,F}, v_{T,I,F})$ for the NCS $B = \{(t_1, \kappa_{T,I,F}(t_1), v_{T,I,F}(t_1)) | t_1 \in Y\}$. In this paper, we used $\gamma = [1,1] - \text{rsup}\{\kappa_{\{T,I\}}(t_1) | t_1 \in Y\}$, $\mathbb{Y} = \text{rinf}\{\kappa_F(t_1) | t_1 \in Y\}$, $\Gamma = 1 - \text{sup}\{v_{\{T,I\}}(t_1) | t_1 \in Y\}$, $\mathbb{E} = \text{inf}\{v_F(t_1) | t_1 \in Y\}$ for any NCS $B = (\kappa_{T,I,F}, v_{T,I,F})$ of Y .

3.1 Translative and Multiplicative Interpretation of Neutrosophic Cubic Subalgebra

Definition 3.1.1 Let $B = (\kappa_{T,I,F}, v_{T,I,F})$ be a NCS of Y and for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], \gamma]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathbb{E}]$. An object of the form $B_{\alpha,\beta,\gamma}^T = ((\kappa_{T,I,F})_{\alpha,\beta,\gamma}^T, (v_{T,I,F})_{\alpha,\beta,\gamma}^T)$ is called a **NCT** of B , when $(\kappa_T)_\alpha^T(t_1) = \kappa_B(t_1) + \alpha$, $(\kappa_I)_\beta^T(t_1) = \kappa_B(t_1) + \beta$, $(\kappa_F)_\gamma^T(t_1) = \kappa_F(t_1) - \gamma$ and $(v_T)_\alpha^T(t_1) = v_B(t_1) + \alpha$, $(v_I)_\beta^T(t_1) = v_B(t_1) + \beta$, $(v_F)_\gamma^T(t_1) = v_B(t_1) - \gamma$ for all $t_1 \in Y$.

Example 3.1.1 Let $Y = \{0,1,2\}$ be a BF-algebra with the following Cayley table:

*	0	1	2
0	0	1	2
1	0	0	1
2	0	2	0

Let $B = (\kappa_{T,I,F}, v_{T,I,F})$ be a NCS of Y is defined as

$$\kappa_T(t_1) = \begin{cases} [0.1, 0.3] & \text{if } t_1 = 0 \\ [0.4, 0.7] & \text{if otherwise} \end{cases}$$

$$\kappa_I(t_1) = \begin{cases} [0.2, 0.4] & \text{if } t_1 = 0 \\ [0.5, 0.7] & \text{if otherwise} \end{cases}$$

$$\kappa_F(t_1) = \begin{cases} [0.4, 0.6] & \text{if } t_1 = 0 \\ [0.3, 0.8] & \text{if otherwise} \end{cases}$$

and

$$v_T(t_1) = \begin{cases} 0.1 & \text{if } t_1 = 0 \\ 0.4 & \text{if otherwise} \end{cases}$$

$$v_I(t_1) = \begin{cases} 0.2 & \text{if } t_1 = 0 \\ 0.3 & \text{if otherwise} \end{cases}$$

$$v_F(t_1) = \begin{cases} 0.5 & \text{if } t_1 = 0 \\ 0.7 & \text{if otherwise.} \end{cases}$$

Then B is a neutrosophic cubic subalgebra. Here we choose for $v_{T,I,F}$, $\alpha = 0.01, \beta = 0.02, \gamma = 0.03$, and for $\kappa_{T,I,F}$, $\alpha = [0.1, 0.2]$, $\beta = [0.2, 0.25]$, $\gamma = [0.2, 0.3]$ then the mapping $B^T: Y \rightarrow [0,1]$ is given by

$$(\kappa_T)_{[0.1,0.2]}^T(t_1) = \begin{cases} [0.2, 0.5] & \text{if } t_1 = 0 \\ [0.5, 0.9] & \text{if otherwise} \end{cases}$$

$$(\kappa_I)_{[0.2,0.25]}^T(t_1) = \begin{cases} [0.4, 0.7] & \text{if } t_1 = 0 \\ [0.7, 0.95] & \text{if otherwise} \end{cases}$$

$$(\kappa_F)_{[0.2,0.3]}^T(t_1) = \begin{cases} [0.2, 0.3] & \text{if } t_1 = 0 \\ [0.1, 0.5] & \text{if otherwise} \end{cases}$$

and

$$(v_T)_{0.01}^T(t_1) = \begin{cases} 0.11 & \text{if } t_1 = 0 \\ 0.41 & \text{if otherwise.} \end{cases}$$

$$(v_I)_{0.02}^T(t_1) = \begin{cases} 0.22 & \text{if } t_1 = 0 \\ 0.32 & \text{if otherwise.} \end{cases}$$

$$(v_F)_{0.03}^T(t_1) = \begin{cases} 0.47 & \text{if } t_1 = 0 \\ 0.67 & \text{if otherwise,} \end{cases}$$

which imply $(\kappa_T)_{[0.1,0.2]}^T(t_1) = \kappa_T(t_1) + [0.1, 0.2]$, $(\kappa_I)_{[0.2,0.25]}^T(t_1) = \kappa_I(t_1) + [0.2, 0.25]$, $(\kappa_F)_{[0.2,0.3]}^T(t_1) = \kappa_F(t_1) - [0.2, 0.3]$ and $(v_T)_{0.01}^T(t_1) = v_T(t_1) + 0.01$, $(v_I)_{0.02}^T(t_1) = v_I(t_1) + 0.02$, $(v_F)_{0.03}^T(t_1) = v_F(t_1) - 0.03$ for all $t_1 \in Y$. Hence B^T is a neutrosophic cubic translation.

Theorem 3.1.1 Let B be a NCSU of Y and for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], \gamma]$ and $\gamma \in [[0,0], \psi]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \xi]$. Then NCT $B_{\alpha, \beta, \gamma}^T$ of B is a NCSU of Y .

Proof. Assume $t_1, t_2 \in Y$. Then

$$\begin{aligned} (\kappa_T)_\alpha^T(t_1 * t_2) &= \kappa_T(t_1 * t_2) + \alpha \\ &\geq rmin\{\kappa_T(t_1), \kappa_T(t_2)\} + \alpha \\ &= rmin\{\kappa_T(t_1) + \alpha, \kappa_T(t_2) + \alpha\} \\ (\kappa_T)_\alpha^T(t_1 * t_2) &= rmin\{(\kappa_T)_\alpha^T(t_1), (\kappa_T)_\alpha^T(t_2)\}, \\ (\kappa_I)_\beta^T(t_1 * t_2) &= \kappa_I(t_1 * t_2) + \beta \\ &\geq rmin\{\kappa_I(t_1), \kappa_I(t_2)\} + \beta \\ &= rmin\{\kappa_I(t_1) + \beta, \kappa_I(t_2) + \beta\} \end{aligned}$$

$$(\kappa_I)_{\beta}^T(t_1 * t_2) = \text{rmin}\{(\kappa_I)_{\beta}^T(t_1), (\kappa_I)_{\beta}^T(t_2)\},$$

$$(\kappa_F)_{\gamma}^T(t_1 * t_2) = \kappa_F(t_1 * t_2) - \gamma$$

$$\geq \text{rmin}\{\kappa_F(t_1), \kappa_F(t_2)\} - \gamma$$

$$= \text{rmin}\{\kappa_F(t_1) - \gamma, \kappa_F(t_2) - \gamma\}$$

$$(\kappa_F)_{\gamma}^T(t_1 * t_2) = \text{rmin}\{(\kappa_F)_{\gamma}^T(t_1), (\kappa_F)_{\gamma}^T(t_2)\}$$

and

$$(v_T)_{\alpha}^T(t_1 * t_2) = v_T(t_1 * t_2) + \alpha$$

$$\leq \max\{v_T(t_1), v_T(t_2)\} + \alpha$$

$$= \max\{v_T(t_1) + \alpha, v_T(t_2) + \alpha\}$$

$$(v_T)_{\alpha}^T(t_1 * t_2) = \max\{(v_T)_{\alpha}^T(t_1), (v_T)_{\alpha}^T(t_2)\},$$

$$(v_I)_{\beta}^T(t_1 * t_2) = v_I(t_1 * t_2) + \beta$$

$$\leq \max\{v_I(t_1), v_I(t_2)\} + \beta$$

$$= \max\{v_I(t_1) + \beta, v_I(t_2) + \beta\}$$

$$(v_I)_{\beta}^T(t_1 * t_2) = \max\{(v_I)_{\beta}^T(t_1), (v_I)_{\beta}^T(t_2)\},$$

$$(v_F)_{\gamma}^T(t_1 * t_2) = v_F(t_1 * t_2) - \gamma$$

$$\leq \max\{v_F(t_1), v_F(t_2)\} - \gamma$$

$$= \max\{v_F(t_1) - \gamma, v_F(t_2) - \gamma\}$$

$$(v_F)_{\gamma}^T(t_1 * t_2) = \max\{(v_F)_{\gamma}^T(t_1), (v_F)_{\gamma}^T(t_2)\}.$$

Hence **NCT** $B_{\alpha,\beta,\gamma}^T$ of B is a **NCSU** of Y .

Theorem 3.1.2 Let B be a **NCS** of Y such that **NCT** $B_{\alpha,\beta,\gamma}^T$ of B is a **NCSU** of Y for some $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathbb{E}]$. Then B is a **NCSU** of Y .

Proof. Let $B_{\alpha,\beta,\gamma}^T = ((\kappa_{T,I,F})_{\alpha,\beta,\gamma}^T, (v_{T,I,F})_{\alpha,\beta,\gamma}^T)$ be a **NCSU** of Y for some $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathbb{E}]$ and $t_1, t_2 \in Y$. Then

$$\kappa_T(t_1 * t_2) + \alpha = (\kappa_T)_{\alpha}^T(t_1 * t_2)$$

$$\geq \text{rmin}\{(\kappa_T)_{\alpha}^T(t_1), (\kappa_T)_{\alpha}^T(t_2)\}$$

$$= \text{rmin}\{\kappa_T(t_1) + \alpha, \kappa_T(t_2) + \alpha\}$$

$$\kappa_T(t_1 * t_2) + \alpha = \text{rmin}\{\kappa_T(t_1), \kappa_T(t_2)\} + \alpha,$$

$$\kappa_I(t_1 * t_2) + \beta = (\kappa_I)_{\beta}^T(t_1 * t_2)$$

$$\geq \text{rmin}\{(\kappa_I)_{\beta}^T(t_1), (\kappa_I)_{\beta}^T(t_2)\}$$

$$\begin{aligned}
&= \text{rmin}\{\kappa_l(t_1) + \beta, \kappa_l(t_2) + \beta\} \\
\kappa_l(t_1 * t_2) + \beta &= \text{rmin}\{\kappa_l(t_1), \kappa_l(t_2)\} + \beta,
\end{aligned}$$

$$\begin{aligned}
\kappa_F(t_1 * t_2) - \gamma &= (\kappa_F)_Y^T(t_1 * t_2) \\
&\geq \text{rmin}\{(\kappa_F)_Y^T(t_1), (\kappa_F)_Y^T(t_2)\} \\
&= \text{rmin}\{\kappa_F(t_1) - \gamma, \kappa_F(t_2) - \gamma\} \\
\kappa_F(t_1 * t_2) - \gamma &= \text{rmin}\{\kappa_F(t_1), \kappa_F(t_2)\} - \gamma
\end{aligned}$$

and

$$\begin{aligned}
v_T(t_1 * t_2) + \alpha &= (v_T)_\alpha^T(t_1 * t_2) \\
&\leq \text{max}\{(v_T)_\alpha^T(t_1), (v_T)_\alpha^T(t_2)\} \\
&= \text{max}\{v_T(t_1) + \alpha, v_B(t_2) + \alpha\} \\
v_T(t_1 * t_2) + \alpha &= \text{max}\{v_T(t_1), v_T(t_2)\} + \alpha,
\end{aligned}$$

$$\begin{aligned}
v_I(t_1 * t_2) + \beta &= (v_I)_\beta^T(t_1 * t_2) \\
&\leq \text{max}\{(v_I)_\beta^T(t_1), (v_I)_\beta^T(t_2)\} \\
&= \text{max}\{v_I(t_1) + \beta, v_B(t_2) + \beta\} \\
v_I(t_1 * t_2) + \beta &= \text{max}\{v_I(t_1), v_I(t_2)\} + \beta,
\end{aligned}$$

$$\begin{aligned}
v_F(t_1 * t_2) - \gamma &= (v_F)_Y^T(t_1 * t_2) \\
&\leq \text{max}\{(v_F)_Y^T(t_1), (v_F)_Y^T(t_2)\} \\
&= \text{max}\{v_F(t_1) - \gamma, v_B(t_2) - \gamma\} \\
v_F(t_1 * t_2) - \gamma &= \text{max}\{v_F(t_1), v_F(t_2)\} - \gamma,
\end{aligned}$$

which imply $\kappa_T(t_1 * t_2) \geq \text{rmin}\{\kappa_T(t_1), \kappa_T(t_2)\}$, $\kappa_l(t_1 * t_2) \geq \text{rmin}\{\kappa_l(t_1), \kappa_l(t_2)\}$, $\kappa_F(t_1 * t_2) \geq \text{rmin}\{\kappa_F(t_1), \kappa_F(t_2)\}$, and $v_T(t_1 * t_2) \leq \text{max}\{v_T(t_1), v_T(t_2)\}$, $v_I(t_1 * t_2) \leq \text{max}\{v_I(t_1), v_I(t_2)\}$, $v_F(t_1 * t_2) \leq \text{max}\{v_F(t_1), v_F(t_2)\}$, for all $t_1, t_2 \in Y$. Hence B is a NCSU of Y .

Definition 3.1.2 Let B be a NCS of Y and $\delta \in [0,1]$. An object having the form $B_\delta^M = (((\kappa_T)_\delta^M, (\kappa_l)_\delta^M, (\kappa_F)_\delta^M), ((v_T)_\delta^M, (v_I)_\delta^M, (v_F)_\delta^M))$ is called a NCM of B , when $(\kappa_T)_\delta^M(t_1) = \delta \cdot \kappa_T(t_1)$, $(\kappa_l)_\delta^M(t_1) = \delta \cdot \kappa_l(t_1)$, $(\kappa_F)_\delta^M(t_1) = \delta \cdot \kappa_F(t_1)$ and $(v_T)_\delta^M(t_1) = \delta \cdot v_T(t_1)$, $(v_I)_\delta^M(t_1) = \delta \cdot v_I(t_1)$, $(v_F)_\delta^M(t_1) = \delta \cdot v_F(t_1)$ for all $t_1 \in Y$.

Example 3.1.2 Let $Y = \{0,1,2\}$ be a BF-algebra with the following Cayley table:

*	0	1	2
0	0	1	2
1	0	0	1
2	0	2	0

Let $B = (\kappa_{T,I,F}, v_{T,I,F})$ be a NCS of Y is defined as

$$\kappa_T(t_1) = \begin{cases} [0.1, 0.3] & \text{if } t_1 = 0 \\ [0.4, 0.7] & \text{if otherwise} \end{cases}$$

$$\kappa_I(t_1) = \begin{cases} [0.2, 0.4] & \text{if } t_1 = 0 \\ [0.5, 0.7] & \text{if otherwise} \end{cases}$$

$$\kappa_F(t_1) = \begin{cases} [0.4, 0.6] & \text{if } t_1 = 0 \\ [0.3, 0.8] & \text{if otherwise} \end{cases}$$

and

$$v_T(t_1) = \begin{cases} 0.1 & \text{if } t_1 = 0 \\ 0.4 & \text{if otherwise} \end{cases}$$

$$v_I(t_1) = \begin{cases} 0.2 & \text{if } t_1 = 0 \\ 0.3 & \text{if otherwise} \end{cases}$$

$$v_F(t_1) = \begin{cases} 0.5 & \text{if } t_1 = 0 \\ 0.7 & \text{if otherwise.} \end{cases}$$

Then B is a neutrosophic cubic subalgebra, choose $\delta = 0.01$ for v and $\delta = [0.1, 0.2]$ for κ then the mapping $B_\delta^M | Y \rightarrow [0,1]$ is given by

$$(\kappa_T)_{[0.1, 0.2]}^M(t_1) = \begin{cases} [0.01, 0.06] & \text{if } t_1 = 1 \\ [0.04, 0.14] & \text{if otherwise,} \end{cases}$$

$$(\kappa_I)_{[0.1, 0.2]}^M(t_1) = \begin{cases} [0.02, 0.08] & \text{if } t_1 = 1 \\ [0.05, 0.14] & \text{if otherwise,} \end{cases}$$

$$(\kappa_F)_{[0.1, 0.2]}^M(t_1) = \begin{cases} [0.04, 0.12] & \text{if } t_1 = 1 \\ [0.03, 0.16] & \text{if otherwise} \end{cases}$$

and

$$(v_T)_{0.01}^M(t_1) = \begin{cases} 0.001 & \text{if } t_1 = 0 \\ 0.004 & \text{if otherwise,} \end{cases}$$

$$(v_I)_{0.01}^M(t_1) = \begin{cases} 0.002 & \text{if } t_1 = 0 \\ 0.003 & \text{if otherwise,} \end{cases}$$

$$(v_F)_{0.01}^M(t_1) = \begin{cases} 0.005 & \text{if } t_1 = 0 \\ 0.007 & \text{if otherwise,} \end{cases}$$

which imply $(\kappa_T)_{[0.1,0.2]}^M(t_1) = \kappa_T(t_1) \cdot [0.1,0.2]$, $(\kappa_I)_{[0.1,0.2]}^M(t_1) = \kappa_I(t_1) \cdot [0.1,0.2]$, $(\kappa_F)_{[0.1,0.2]}^M(t_1) = \kappa_F(t_1) \cdot [0.1,0.2]$ and $(v_T)_{0.01}^M(t_1) = v_T(t_1) \cdot (0.01)$, $(v_I)_{0.01}^M(t_1) = v_I(t_1) \cdot (0.01)$, $(v_F)_{0.01}^M(t_1) = v_F(t_1) \cdot (0.01)$ for all $t_1 \in Y$. Hence B_δ^M is a neutrosophic cubic multiplication.

Theorem 3.1.3 Let B be a NCS of Y such that NCM B_δ^M of B is a NCSU of Y for some $\delta \in [0,1]$. Then B is a NCSU of Y .

Proof. Assume B_δ^M of B is a NCSU of Y for some $\delta \in [0,1]$. Now for all $t_1, t_2 \in Y$, we have

$$\begin{aligned}\kappa_T(t_1 * t_2) \cdot \delta &= (\kappa_T)_\delta^M(t_1 * t_2) \\ &\geq \text{rmin}\{(\kappa_T)_\delta^M(t_1), (\kappa_T)_\delta^M(t_2)\} \\ &= \text{rmin}\{\kappa_T(t_1) \cdot \delta, \kappa_T(t_2) \cdot \delta\} \\ \kappa_T(t_1 * t_2) \cdot \delta &= \text{rmin}\{\kappa_T(t_1), \kappa_T(t_2)\} \cdot \delta,\end{aligned}$$

$$\begin{aligned}\kappa_I(t_1 * t_2) \cdot \delta &= (\kappa_I)_\delta^M(t_1 * t_2) \\ &\geq \text{rmin}\{(\kappa_I)_\delta^M(t_1), (\kappa_I)_\delta^M(t_2)\} \\ &= \text{rmin}\{\kappa_I(t_1) \cdot \delta, \kappa_I(t_2) \cdot \delta\} \\ \kappa_I(t_1 * t_2) \cdot \delta &= \text{rmin}\{\kappa_I(t_1), \kappa_I(t_2)\} \cdot \delta,\end{aligned}$$

$$\begin{aligned}\kappa_F(t_1 * t_2) \cdot \delta &= (\kappa_F)_\delta^M(t_1 * t_2) \\ &\geq \text{rmin}\{(\kappa_F)_\delta^M(t_1), (\kappa_F)_\delta^M(t_2)\} \\ &= \text{rmin}\{\kappa_F(t_1) \cdot \delta, \kappa_F(t_2) \cdot \delta\} \\ \kappa_F(t_1 * t_2) \cdot \delta &= \text{rmin}\{\kappa_F(t_1), \kappa_F(t_2)\} \cdot \delta\end{aligned}$$

and

$$\begin{aligned}v_T(t_1 * t_2) \cdot \delta &= (v_T)_\delta^M(t_1 * t_2) \\ &\leq \text{max}\{(v_T)_\delta^M(t_1), (v_T)_\delta^M(t_2)\} \\ &= \text{max}\{v_T(t_1) \cdot \delta, v_T(t_2) \cdot \delta\} \\ v_T(t_1 * t_2) \cdot \delta &= \text{max}\{v_T(t_1), v_T(t_2)\} \cdot \delta,\end{aligned}$$

$$\begin{aligned}v_I(t_1 * t_2) \cdot \delta &= (v_I)_\delta^M(t_1 * t_2) \\ &\leq \text{max}\{(v_I)_\delta^M(t_1), (v_I)_\delta^M(t_2)\} \\ &= \text{max}\{v_I(t_1) \cdot \delta, v_I(t_2) \cdot \delta\} \\ v_I(t_1 * t_2) \cdot \delta &= \text{max}\{v_I(t_1), v_I(t_2)\} \cdot \delta,\end{aligned}$$

$$v_F(t_1 * t_2) \cdot \delta = (v_F)_\delta^M(t_1 * t_2)$$

$$\begin{aligned}
&\leq \max\{(v_F)_\delta^M(t_1), (v_F)_\delta^M(t_2)\} \\
&= \max\{v_F(t_1) \cdot \delta, v_F(t_2) \cdot \delta\} \\
&v_F(t_1 * t_2) \cdot \delta = \max\{v_F(t_1), v_F(t_2)\} \cdot \delta,
\end{aligned}$$

which imply $\kappa_T(t_1 * t_2) \geq rmin\{\kappa_T(t_1), \kappa_T(t_2)\}$, $\kappa_I(t_1 * t_2) \geq rmin\{\kappa_I(t_1), \kappa_I(t_2)\}$, $\kappa_F(t_1 * t_2) \geq rmin\{\kappa_F(t_1), \kappa_F(t_2)\}$ and $v_T(t_1 * t_2) \leq \max\{v_T(t_1), v_T(t_2)\}$, $v_I(t_1 * t_2) \leq \max\{v_I(t_1), v_I(t_2)\}$, $v_F(t_1 * t_2) \leq \max\{v_F(t_1), v_F(t_2)\}$ for all $t_1, t_2 \in Y$. Hence B is a NCSU of Y .

Theorem 3.1.4 Let B be a NCSU of Y for $\delta \in [0,1]$. Then **NCM** B_δ^M of B is a NCSU of Y .

Proof. Assume $t_1, t_2 \in Y$. Then

$$\begin{aligned}
(\kappa_T)_\delta^M(t_1 * t_2) &= \delta \cdot \kappa_T(t_1 * t_2) \\
&\geq \delta \cdot rmin\{(\kappa_T)(t_1), (\kappa_T)(t_2)\} \\
&= rmin\{\delta \cdot \kappa_T(t_1), \delta \cdot \kappa_T(t_2)\} \\
&= rmin\{(\kappa_T)_\delta^M(t_1), (\kappa_T)_\delta^M(t_2)\} \\
(\kappa_T)_\delta^M(t_1 * t_2) &\geq rmin\{(\kappa_T)_\delta^M(t_1), (\kappa_T)_\delta^M(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(\kappa_I)_\delta^M(t_1 * t_2) &= \delta \cdot \kappa_I(t_1 * t_2) \\
&\geq \delta \cdot rmin\{(\kappa_I)(t_1), (\kappa_I)(t_2)\} \\
&= rmin\{\delta \cdot \kappa_I(t_1), \delta \cdot \kappa_I(t_2)\} \\
&= rmin\{(\kappa_I)_\delta^M(t_1), (\kappa_I)_\delta^M(t_2)\} \\
(\kappa_I)_\delta^M(t_1 * t_2) &\geq rmin\{(\kappa_I)_\delta^M(t_1), (\kappa_I)_\delta^M(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(\kappa_F)_\delta^M(t_1 * t_2) &= \delta \cdot \kappa_F(t_1 * t_2) \\
&\geq \delta \cdot rmin\{(\kappa_F)(t_1), (\kappa_F)(t_2)\} \\
&= rmin\{\delta \cdot \kappa_F(t_1), \delta \cdot \kappa_F(t_2)\} \\
&= rmin\{(\kappa_F)_\delta^M(t_1), (\kappa_F)_\delta^M(t_2)\} \\
(\kappa_F)_\delta^M(t_1 * t_2) &\geq rmin\{(\kappa_F)_\delta^M(t_1), (\kappa_F)_\delta^M(t_2)\}
\end{aligned}$$

and

$$\begin{aligned}
(v_T)_\delta^M(t_1 * t_2) &= \delta \cdot v_T(t_1 * t_2) \\
&\leq \delta \cdot \max\{(v_T)(t_1), (v_T)(t_2)\} \\
&= \max\{\delta \cdot v_T(t_1), \delta \cdot v_T(t_2)\} \\
&= \max\{(\kappa_B)_\delta^M(t_1), (\kappa_B)_\delta^M(t_2)\} \\
(v_T)_\delta^M(t_1 * t_2) &\leq \max\{(v_T)_\delta^M(t_1), (v_T)_\delta^M(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(v_I)_\delta^M(t_1 * t_2) &= \delta \cdot v_I(t_1 * t_2) \\
&\leq \delta \cdot \max\{(v_I)(t_1), (v_I)(t_2)\} \\
&= \max\{\delta \cdot v_I(t_1), \delta \cdot v_I(t_2)\} \\
&= \max\{(\kappa_B)_\delta^M(t_1), (\kappa_B)_\delta^M(t_2)\} \\
(v_I)_\delta^M(t_1 * t_2) &\leq \max\{(v_I)_\delta^M(t_1), (v_I)_\delta^M(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(v_F)_\delta^M(t_1 * t_2) &= \delta \cdot v_F(t_1 * t_2) \\
&\leq \delta \cdot \max\{(v_F)(t_1), (v_F)(t_2)\} \\
&= \max\{\delta \cdot v_F(t_1), \delta \cdot v_F(t_2)\} \\
&= \max\{(\kappa_B)_\delta^M(t_1), (\kappa_B)_\delta^M(t_2)\} \\
(v_F)_\delta^M(t_1 * t_2) &\leq \max\{(v_F)_\delta^M(t_1), (v_F)_\delta^M(t_2)\},
\end{aligned}$$

which imply $\kappa_T(t_1 * t_2) \geq r\min\{\kappa_T(t_1), \kappa_T(t_2)\}$, $\kappa_I(t_1 * t_2) \geq r\min\{\kappa_I(t_1), \kappa_I(t_2)\}$, $\kappa_F(t_1 * t_2) \geq r\min\{\kappa_F(t_1), \kappa_F(t_2)\}$ and $v_T(t_1 * t_2) \leq \max\{v_T(t_1), v_T(t_2)\}$, $v_I(t_1 * t_2) \leq \max\{v_I(t_1), v_I(t_2)\}$, $v_F(t_1 * t_2) \leq \max\{v_F(t_1), v_F(t_2)\}$ for all $t_1, t_2 \in Y$. Hence B_δ^M is a NCSU of Y .

3.2 Translative and Multiplicative Interpretation of Neutrosophic Cubic Ideal

In this section, neutrosophic cubic translation of **NCID**, neutrosophic cubic multiplication of **NCID**, union and intersection of neutrosophic cubic translation of **NCID** are investigated through some results.

Theorem 3.2.1 If **NCT** $B_{\alpha,\beta,\gamma}^T$ of B is a neutrosophic cubic BF ideal, then it fulfills the conditions $(\kappa_T)_\alpha^T(t_1 * (t_2 * t_1)) \geq (\kappa_T)_\alpha^T(t_2)$, $(\kappa_I)_\beta^T(t_1 * (t_2 * t_1)) \geq (\kappa_I)_\beta^T(t_2)$, $(\kappa_F)_\gamma^T(t_1 * (t_2 * t_1)) \geq (\kappa_F)_\gamma^T(t_2)$ and $(v_T)_\alpha^T(t_1 * (t_2 * t_1)) \leq (v_T)_\alpha^T(t_2)$, $(v_I)_\beta^T(t_1 * (t_2 * t_1)) \leq (v_I)_\beta^T(t_2)$, $(v_F)_\gamma^T(t_1 * (t_2 * t_1)) \leq (v_F)_\gamma^T(t_2)$.

Proof. Let **NCT** $B_{\alpha,\beta,\gamma}^T$ of B be a neutrosophic cubic BF ideal. Then

$$\begin{aligned}
(\kappa_T)_\alpha^T(t_1 * (t_2 * t_1)) &= \kappa_T(t_1 * (t_2 * t_1)) + \alpha \\
&\geq r\min\{\kappa_T(t_2 * (t_1 * (t_2 * t_1))), \kappa_T(t_2) + \alpha\} \\
&= r\min\{\kappa_T(0) + \alpha, \kappa_T(t_2) + \alpha\} \\
&= r\min\{(\kappa_T)_\alpha^T(0), (\kappa_T)_\alpha^T(t_2)\} \\
(\kappa_T)_\alpha^T(t_1 * (t_2 * t_1)) &= (\kappa_T)_\alpha^T(t_2),
\end{aligned}$$

$$\begin{aligned}
(\kappa_I)_\beta^T(t_1 * (t_2 * t_1)) &= \kappa_I(t_1 * (t_2 * t_1)) + \beta \\
&\geq r\min\{\kappa_I(t_2 * (t_1 * (t_2 * t_1))), \kappa_I(t_2) + \beta\} \\
&= r\min\{\kappa_I(0) + \beta, \kappa_I(t_2) + \beta\}
\end{aligned}$$

$$= \text{rmin}\{(\kappa_I)_\beta^T(0), (\kappa_I)_\beta^T(t_2)\} \\ (\kappa_I)_\beta^T(t_1 * (t_2 * t_1)) = (\kappa_I)_\beta^T(t_2),$$

$$(\kappa_F)_\gamma^T(t_1 * (t_2 * t_1)) = \kappa_F(t_1 * (t_2 * t_1)) - \gamma \\ \geq \text{rmin}\{\kappa_F(t_2 * (t_1 * (t_2 * t_1))) - \gamma, \kappa_F(t_2) - \gamma\} \\ = \text{rmin}\{\kappa_F(0) - \gamma, \kappa_F(t_2) - \gamma\} \\ = \text{rmin}\{(\kappa_F)_\gamma^T(0), (\kappa_F)_\gamma^T(t_2)\} \\ (\kappa_F)_\gamma^T(t_1 * (t_2 * t_1)) = (\kappa_F)_\gamma^T(t_2)$$

and

$$(\nu_T)_\alpha^T(t_1 * (t_2 * t_1)) = \nu_T(t_1 * (t_2 * t_1)) + \alpha \\ \leq \max\{\nu_T(t_2 * (t_1 * (t_2 * t_1))) + \alpha, \nu_T(t_2) + \alpha\} \\ = \max\{\nu_T(0) + \alpha, \nu_T(t_2) + \alpha\} \\ = \max\{(\nu_T)_\alpha^T(0), (\nu_T)_\alpha^T(t_2)\} \\ (\nu_T)_\alpha^T(t_1 * (t_2 * t_1)) = (\nu_T)_\alpha^T(t_2),$$

$$(\nu_I)_\alpha^T(t_1 * (t_2 * t_1)) = \nu_I(t_1 * (t_2 * t_1)) + \beta \\ \leq \max\{\nu_I(t_2 * (t_1 * (t_2 * t_1))) + \beta, \nu_I(t_2) + \beta\} \\ = \max\{\nu_I(0) + \beta, \nu_I(t_2) + \beta\} \\ = \max\{(\nu_I)_\beta^T(0), (\nu_I)_\beta^T(t_2)\} \\ (\nu_I)_\beta^T(t_1 * (t_2 * t_1)) = (\nu_I)_\beta^T(t_2),$$

$$(\nu_F)_\gamma^T(t_1 * (t_2 * t_1)) = \nu_F(t_1 * (t_2 * t_1)) - \gamma \\ \leq \max\{\nu_F(t_2 * (t_1 * (t_2 * t_1))) - \gamma, \nu_F(t_2) - \gamma\} \\ = \max\{\nu_F(0) - \gamma, \nu_F(t_2) - \gamma\} \\ = \max\{(\nu_F)_\gamma^T(0), (\nu_F)_\gamma^T(t_2)\} \\ (\nu_F)_\gamma^T(t_1 * (t_2 * t_1)) = (\nu_F)_\gamma^T(t_2).$$

Hence $(\kappa_T)_\alpha^T(t_1 * (t_2 * t_1)) \geq (\kappa_T)_\alpha^T(t_2)$, $(\kappa_I)_\beta^T(t_1 * (t_2 * t_1)) \geq (\kappa_I)_\beta^T(t_2)$, $(\kappa_F)_\gamma^T(t_1 * (t_2 * t_1)) \geq (\kappa_F)_\gamma^T(t_2)$ and $(\nu_T)_\alpha^T(t_1 * (t_2 * t_1)) \leq (\nu_T)_\alpha^T(t_2)$, $(\nu_I)_\beta^T(t_1 * (t_2 * t_1)) \leq (\nu_I)_\beta^T(t_2)$, $(\nu_F)_\gamma^T(t_1 * (t_2 * t_1)) \leq (\nu_F)_\gamma^T(t_2)$.

Theorem 3.2.2 Let B be a **NCID** of Y and for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], \gamma]$ and $\gamma \in [[0,0], \psi]$, where for $\nu_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \xi]$. Then **NCT** $B_{\alpha,\beta,\gamma}^T$ of B is a **NCID** of Y .

Proof. Let B be a **NCID** of Y and for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], \gamma]$ and $\gamma \in [[0,0], \psi]$, where for $\nu_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \xi]$. Then $(\kappa_T)_\alpha^T(0) = \kappa_T(0) + \alpha \geq \kappa_T(t_1) + \alpha = (\kappa_T)_\alpha^T(t_1)$, $(\kappa_I)_\beta^T(0) = \kappa_I(0) + \beta \geq$

$\kappa_I(t_1) + \beta = (\kappa_I)_\beta^T(t_1)$, $(\kappa_F)_Y^T(0) = \kappa_F(0) - \gamma \geq \kappa_F(t_1) - \gamma = (\kappa_F)_Y^T(t_1)$ and $(v_T)_\alpha^T(0) = v_T(0) + \alpha \leq v_T(t_1) + \alpha = (v_T)_\alpha^T(t_1)$, $(v_I)_\beta^T(0) = v_I(0) + \beta \leq v_I(t_1) + \beta = (v_I)_\beta^T(t_1)$, $(v_F)_Y^T(0) = v_F(0) - \gamma \leq v_F(t_1) - \gamma = (v_F)_Y^T(t_1)$. So

$$\begin{aligned}
 & (\kappa_T)_\alpha^T(t_1) = \kappa_T(t_1) + \alpha \\
 & \geq \text{rmin}\{\kappa_T(t_1 * t_2), \kappa_T(t_2)\} + \alpha \\
 & = \text{rmin}\{\kappa_T(t_1 * t_2) + \alpha, \kappa_T(t_2) + \alpha\} \\
 & (\kappa_T)_\alpha^T(t_1) = \text{rmin}\{(\kappa_T)_\alpha^T(t_1 * t_2), (\kappa_T)_\alpha^T(t_2)\}, \\
 & (\kappa_I)_\beta^T(t_1) = \kappa_I(t_1) + \beta \\
 & \geq \text{rmin}\{\kappa_I(t_1 * t_2), \kappa_I(t_2)\} + \beta \\
 & = \text{rmin}\{\kappa_I(t_1 * t_2) + \beta, \kappa_I(t_2) + \beta\} \\
 & (\kappa_I)_\beta^T(t_1) = \text{rmin}\{(\kappa_I)_\beta^T(t_1 * t_2), (\kappa_I)_\beta^T(t_2)\}, \\
 & (\kappa_F)_\alpha^T(t_1) = \kappa_F(t_1) - \gamma \\
 & \geq \text{rmin}\{\kappa_F(t_1 * t_2), \kappa_F(t_2)\} - \gamma \\
 & = \text{rmin}\{\kappa_F(t_1 * t_2) - \gamma, \kappa_F(t_2) - \gamma\} \\
 & (\kappa_F)_Y^T(t_1) = \text{rmin}\{(\kappa_F)_Y^T(t_1 * t_2), (\kappa_F)_Y^T(t_2)\}
 \end{aligned}$$

and

$$\begin{aligned}
 & (v_T)_\alpha^T(t_1) = v_T(t_1) + \alpha \\
 & \leq \max\{v_T(t_1 * t_2), v_T(t_2)\} + \alpha \\
 & = \max\{v_T(t_1 * t_2) + \alpha, v_T(t_2) + \alpha\} \\
 & (v_T)_\alpha^T(t_1) = \max\{(v_T)_\alpha^T(t_1 * t_2), (v_T)_\alpha^T(t_2)\}, \\
 & (v_I)_\beta^T(t_1) = v_I(t_1) + \beta \\
 & \leq \max\{v_I(t_1 * t_2), v_I(t_2)\} + \beta \\
 & = \max\{v_I(t_1 * t_2) + \beta, v_I(t_2) + \beta\} \\
 & (v_I)_\beta^T(t_1) = \max\{(v_I)_\beta^T(t_1 * t_2), (v_I)_\beta^T(t_2)\}, \\
 & (v_F)_Y^T(t_1) = v_F(t_1) - \gamma \\
 & \leq \max\{v_F(t_1 * t_2), v_F(t_2)\} - \gamma \\
 & = \max\{v_F(t_1 * t_2) - \gamma, v_F(t_2) - \gamma\} \\
 & (v_F)_Y^T(t_1) = \max\{(v_F)_Y^T(t_1 * t_2), (v_F)_Y^T(t_2)\},
 \end{aligned}$$

for all $t_1, t_2 \in Y$ and for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], \gamma]$ and $\gamma \in [[0,0], \gamma]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \Gamma]$. Hence $B_{\alpha,\beta,\gamma}^T$ of B is a **NCID** of Y .

Theorem 3.2.3 Let B be a neutrosophic cubic set of Y such that **NCT** $B_{\alpha,\beta,\gamma}^T$ of B is a **NCID** of Y for all $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], \gamma]$ and $\gamma \in [[0,0], \gamma]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \Gamma]$. Then B is a **NCID** of Y .

Proof. Suppose $B_{\alpha,\beta,\gamma}^T$ is a **NCID** of Y , where for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], \gamma]$ and $\gamma \in [[0,0], \gamma]$, and for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \Gamma]$ and $t_1, t_2 \in Y$. Then

$$\kappa_T(0) + \alpha = (\kappa_T)_\alpha^T(0) \geq (\kappa_T)_\alpha^T(t_1) = \kappa_T(t_1) + \alpha,$$

$$\kappa_I(0) + \beta = (\kappa_I)_\beta^T(0) \geq (\kappa_I)_\beta^T(t_1) = \kappa_I(t_1) + \beta,$$

$$\kappa_F(0) - \gamma = (\kappa_F)_\gamma^T(0) \geq (\kappa_F)_\gamma^T(t_1) = \kappa_F(t_1) - \gamma,$$

and

$$v_T(0) + \alpha = (v_T)_\alpha^T(0) \leq (v_T)_\alpha^T(t_1) = v_T(t_1) + \alpha,$$

$$v_I(0) + \beta = (v_I)_\beta^T(0) \leq (v_I)_\beta^T(t_1) = v_I(t_1) + \beta$$

$$v_F(0) - \gamma = (v_F)_\gamma^T(0) \leq (v_F)_\gamma^T(t_1) = v_F(t_1) - \gamma,$$

which imply $\kappa_T(0) \geq \kappa_T(t_1)$, $\kappa_I(0) \geq \kappa_I(t_1)$, $\kappa_F(0) \geq \kappa_F(t_1)$ and $v_T(0) \leq v_T(t_1)$, $v_I(0) \leq v_I(t_1)$,

$v_F(0) \leq v_F(t_1)$, now

$$\kappa_T(t_1) + \alpha = (\kappa_T)_\alpha^T(t_1) \geq \text{rmin}\{(\kappa_T)_\alpha^T(t_1 * t_2), (\kappa_T)_\alpha^T(t_2)\}$$

$$= \text{rmin}\{\kappa_T(t_1 * t_2) + \alpha, \kappa_T(t_2) + \alpha\}$$

$$\kappa_T(t_1) + \alpha = \text{rmin}\{\kappa_T(t_1 * t_2), \kappa_T(t_2)\} + \alpha,$$

$$\kappa_I(t_1) + \beta = (\kappa_I)_\beta^T(t_1) \geq \text{rmin}\{(\kappa_I)_\beta^T(t_1 * t_2), (\kappa_I)_\beta^T(t_2)\}$$

$$= \text{rmin}\{\kappa_I(t_1 * t_2) + \beta, \kappa_I(t_2) + \beta\}$$

$$\kappa_I(t_1) + \beta = \text{rmin}\{\kappa_I(t_1 * t_2), \kappa_I(t_2)\} + \beta,$$

$$\kappa_F(t_1) - \gamma = (\kappa_F)_\gamma^T(t_1) \geq \text{rmin}\{(\kappa_F)_\gamma^T(t_1 * t_2), (\kappa_F)_\gamma^T(t_2)\}$$

$$= \text{rmin}\{\kappa_F(t_1 * t_2) - \gamma, \kappa_F(t_2) - \gamma\}$$

$$\kappa_F(t_1) - \gamma = \text{rmin}\{\kappa_F(t_1 * t_2), \kappa_F(t_2)\} - \gamma,$$

and

$$v_T(t_1) + \alpha = (v_T)_\alpha^T(t_1) \leq \text{max}\{(v_T)_\alpha^T(t_1 * t_2), (v_T)_\alpha^T(t_2)\}$$

$$= \text{max}\{v_T(t_1 * t_2) + \alpha, v_T(t_2) + \alpha\}$$

$$\begin{aligned}
v_T(t_1) + \alpha &= \max\{v_T(t_1 * t_2), v_T(t_2)\} + \alpha, \\
v_I(t_1) + \beta &= (v_I)_\beta^T(t_1) \leq \max\{(v_I)_\beta^T(t_1 * t_2), (v_I)_\beta^T(t_2)\} \\
&= \max\{v_I(t_1 * t_2) + \beta, v_I(t_2) + \beta\} \\
v_I(t_1) + \beta &= \max\{v_I(t_1 * t_2), v_I(t_2)\} + \beta,
\end{aligned}$$

$$\begin{aligned}
v_F(t_1) - \gamma &= (v_F)_\gamma^T(t_1) \leq \max\{(v_F)_\gamma^T(t_1 * t_2), (v_F)_\gamma^T(t_2)\} \\
&= \max\{v_F(t_1 * t_2) - \gamma, v_F(t_2) - \gamma\} \\
v_F(t_1) - \gamma &= \max\{v_F(t_1 * t_2), v_F(t_2)\} - \gamma,
\end{aligned}$$

which imply $\kappa_T(t_1) \geq r\min\{\kappa_T(t_1 * t_2), \kappa_T(t_2)\}$, $\kappa_I(t_1) \geq r\min\{\kappa_I(t_1 * t_2), \kappa_I(t_2)\}$, $\kappa_F(t_1) \geq r\min\{\kappa_F(t_1 * t_2), \kappa_F(t_2)\}$ and $v_T(t_1) \leq \max\{v_T(t_1 * t_2), v_T(t_2)\}$, $v_I(t_1) \leq \max\{v_I(t_1 * t_2), v_I(t_2)\}$, $v_F(t_1) \leq \max\{v_F(t_1 * t_2), v_F(t_2)\}$ for all $t_1, t_2 \in Y$. Hence B is a **NCID** of Y .

Theorem 3.2.4 Let B be a **NCID** of Y for some $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathcal{E}]$. Then **NCT** $B_{\alpha,\beta,\gamma}^T$ of B is a **NCSU** of Y .

Proof. Assume $t_1, t_2 \in Y$. Then

$$\begin{aligned}
(\kappa_T)_\alpha^T(t_1 * t_2) &= \kappa_T(t_1 * t_2) + \alpha \\
&\geq r\min\{\kappa_T(t_2 * (t_1 * t_2)), \kappa_T(t_2)\} + \alpha \\
&= r\min\{\kappa_T(0), \kappa_T(t_2)\} + \alpha \\
&\geq r\min\{\kappa_T(t_1), \kappa_T(t_2)\} + \alpha \\
&= r\min\{\kappa_T(t_1) + \alpha, \kappa_T(t_2) + \alpha\} \\
(\kappa_T)_\alpha^T(t_1 * t_2) &= r\min\{(\kappa_T)_\alpha^T(t_1), (\kappa_T)_\alpha^T(t_2)\} \\
(\kappa_T)_\alpha^T(t_1 * t_2) &\geq r\min\{(\kappa_T)_\alpha^T(t_1), (\kappa_T)_\alpha^T(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(\kappa_I)_\beta^T(t_1 * t_2) &= \kappa_I(t_1 * t_2) + \beta \\
&\geq r\min\{\kappa_I(t_2 * (t_1 * t_2)), \kappa_I(t_2)\} + \beta \\
&= r\min\{\kappa_I(0), \kappa_I(t_2)\} + \beta \\
&\geq r\min\{\kappa_I(t_1), \kappa_I(t_2)\} + \beta \\
&= r\min\{\kappa_I(t_1) + \beta, \kappa_I(t_2) + \beta\} \\
(\kappa_I)_\beta^T(t_1 * t_2) &= r\min\{(\kappa_I)_\beta^T(t_1), (\kappa_I)_\beta^T(t_2)\} \\
(\kappa_I)_\beta^T(t_1 * t_2) &\geq r\min\{(\kappa_I)_\beta^T(t_1), (\kappa_I)_\beta^T(t_2)\},
\end{aligned}$$

$$(\kappa_F)_\gamma^T(t_1 * t_2) = \kappa_F(t_1 * t_2) - \gamma$$

$$\begin{aligned}
&\geq \text{rmin}\{\kappa_F(t_2 * (t_1 * t_2)), \kappa_F(t_2)\} - \gamma \\
&= \text{rmin}\{\kappa_F(0), \kappa_F(t_2)\} - \gamma \\
&\geq \text{rmin}\{\kappa_F(t_1), \kappa_F(t_2)\} - \gamma \\
&= \text{rmin}\{\kappa_F(t_1) - \gamma, \kappa_F(t_2) - \gamma\} \\
&(\kappa_F)_Y^T(t_1 * t_2) = \text{rmin}\{(\kappa_F)_Y^T(t_1), (\kappa_F)_Y^T(t_2)\} \\
&(\kappa_F)_Y^T(t_1 * t_2) \geq \text{rmin}\{(\kappa_F)_Y^T(t_1), (\kappa_F)_Y^T(t_2)\}
\end{aligned}$$

and

$$\begin{aligned}
(v_T)_\alpha^T(t_1 * t_2) &= v_T(t_1 * t_2) + \alpha \\
&\leq \max\{v_T(t_2 * (t_1 * t_2)), v_T(t_2)\} + \alpha \\
&= \max\{v_T(0), v_T(t_2)\} + \alpha \\
&\leq \max\{v_T(t_1), v_T(t_2)\} + \alpha \\
&= \max\{v_T(t_1) + \alpha, v_T(t_2) + \alpha\} \\
(v_T)_\alpha^T(t_1 * t_2) &= \max\{(v_T)_\alpha^T(t_1), (v_T)_\alpha^T(t_2)\} \\
(v_T)_\alpha^T(t_1 * t_2) &\leq \max\{(v_T)_\alpha^T(t_1), (v_T)_\alpha^T(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(v_I)_\beta^T(t_1 * t_2) &= v_I(t_1 * t_2) + \beta \\
&\leq \max\{v_I(t_2 * (t_1 * t_2)), v_I(t_2)\} + \beta \\
&= \max\{v_I(0), v_I(t_2)\} + \beta \\
&\leq \max\{v_I(t_1), v_I(t_2)\} + \beta \\
&= \max\{v_I(t_1) + \beta, v_I(t_2) + \beta\} \\
(v_I)_\beta^T(t_1 * t_2) &= \max\{(v_I)_\beta^T(t_1), (v_I)_\beta^T(t_2)\} \\
(v_I)_\beta^T(t_1 * t_2) &\leq \max\{(v_I)_\beta^T(t_1), (v_I)_\beta^T(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(v_F)_Y^T(t_1 * t_2) &= v_F(t_1 * t_2) - \gamma \\
&\leq \max\{v_F(t_2 * (t_1 * t_2)), v_F(t_2)\} - \gamma \\
&= \max\{v_F(0), v_F(t_2)\} - \gamma \\
&\leq \max\{v_F(t_1), v_F(t_2)\} - \gamma \\
&= \max\{v_F(t_1) - \gamma, v_F(t_2) - \gamma\} \\
(v_F)_Y^T(t_1 * t_2) &= \max\{(v_F)_Y^T(t_1), (v_F)_Y^T(t_2)\} \\
(v_F)_Y^T(t_1 * t_2) &\leq \max\{(v_F)_Y^T(t_1), (v_F)_Y^T(t_2)\}.
\end{aligned}$$

Hence $B_{\alpha, \beta, \gamma}^T$ is a NCSU of Y.

Theorem 3.2.5 If $\mathbf{NCT}_{\alpha,\beta,\gamma}^T$ of B is a **NCID** of Y for some $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], \gamma]$ and $\gamma \in [[0,0], \psi]$, and for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \varepsilon]$. Then B is a **NCSU** of Y .

Proof. Suppose $\mathbf{B}_{\alpha,\beta,\gamma}^T$ of B is a **NCID** of Y . Then

$$\begin{aligned} (\kappa_T)(t_1 * t_2) + \alpha &= (\kappa_T)_\alpha^T(t_1 * t_2) \\ &\geq rmin\{(\kappa_T)_\alpha^T(t_2 * (t_1 * t_2)), (\kappa_T)_\alpha^T(t_2)\} \\ &= rmin\{(\kappa_T)_\alpha^T(0), (\kappa_T)_\alpha^T(t_2)\} \\ &\geq rmin\{(\kappa_T)_\alpha^T(t_1), (\kappa_T)_\alpha^T(t_2)\} \\ &= rmin\{\kappa_T(t_1) + \alpha, \kappa_T(t_2) + \alpha\} \\ (\kappa_T)(t_1 * t_2) + \alpha &= rmin\{\kappa_T(t_1), \kappa_T(t_2)\} + \alpha, \end{aligned}$$

$$\begin{aligned} (\kappa_I)(t_1 * t_2) + \beta &= (\kappa_I)_\beta^T(t_1 * t_2) \\ &\geq rmin\{(\kappa_I)_\beta^T(t_2 * (t_1 * t_2)), (\kappa_I)_\beta^T(t_2)\} \\ &= rmin\{(\kappa_I)_\beta^T(0), (\kappa_I)_\beta^T(t_2)\} \\ &\geq rmin\{(\kappa_I)_\beta^T(t_1), (\kappa_I)_\beta^T(t_2)\} \\ &= rmin\{\kappa_I(t_1) + \beta, \kappa_I(t_2) + \beta\} \\ (\kappa_I)(t_1 * t_2) + \beta &= rmin\{\kappa_I(t_1), \kappa_I(t_2)\} + \beta, \end{aligned}$$

$$\begin{aligned} (\kappa_F)(t_1 * t_2) - \gamma &= (\kappa_F)_\gamma^T(t_1 * t_2) \\ &\geq rmin\{(\kappa_F)_\gamma^T(t_2 * (t_1 * t_2)), (\kappa_F)_\gamma^T(t_2)\} \\ &= rmin\{(\kappa_F)_\gamma^T(0), (\kappa_F)_\gamma^T(t_2)\} \\ &\geq rmin\{(\kappa_F)_\gamma^T(t_1), (\kappa_F)_\gamma^T(t_2)\} \\ &= rmin\{\kappa_F(t_1) - \gamma, \kappa_F(t_2) - \gamma\} \\ (\kappa_F)(t_1 * t_2) - \gamma &= rmin\{\kappa_F(t_1), \kappa_F(t_2)\} - \gamma \end{aligned}$$

$\Rightarrow \kappa_T(t_1 * t_2) \geq rmin\{\kappa_T(t_1), \kappa_T(t_2)\}$, $\kappa_I(t_1 * t_2) \geq rmin\{\kappa_I(t_1), \kappa_I(t_2)\}$ and $\kappa_F(t_1 * t_2) \geq rmin\{\kappa_F(t_1), \kappa_F(t_2)\}$ and now

$$\begin{aligned} (v_T)(t_1 * t_2) + \alpha &= (v_T)_\alpha^T(t_1 * t_2) \\ &\leq max\{(v_T)_\alpha^T(t_2 * (t_1 * t_2)), (v_T)_\alpha^T(t_2)\} \\ &= max\{(v_T)_\alpha^T(0), (v_T)_\alpha^T(t_2)\} \\ &\leq max\{(v_T)_\alpha^T(t_1), (v_T)_\alpha^T(t_2)\} \\ &= max\{v_T(t_1) + \alpha, v_T(t_2) + \alpha\} \\ (v_T)(t_1 * t_2) + \alpha &= max\{v_T(t_1), v_T(t_2)\} + \alpha, \end{aligned}$$

$$\begin{aligned}
(v_I)(t_1 * t_2) + \beta &= (v_I)_\beta^T(t_1 * t_2) \\
&\leq \max\{(v_I)_\beta^T(t_2 * (t_1 * t_2)), (v_I)_\beta^T(t_2)\} \\
&= \max\{(v_I)_\beta^T(0), (v_I)_\beta^T(t_2)\} \\
&\leq \max\{(v_I)_\beta^T(t_1), (v_I)_\beta^T(t_2)\} \\
&= \max\{v_I(t_1) + \beta, v_I(t_2) + \beta\} \\
(v_I)(t_1 * t_2) + \beta &= \max\{v_I(t_1), v_I(t_2)\} + \beta,
\end{aligned}$$

$$\begin{aligned}
(v_F)(t_1 * t_2) - \gamma &= (v_F)_\gamma^T(t_1 * t_2) \\
&\leq \max\{(v_F)_\gamma^T(t_2 * (t_1 * t_2)), (v_F)_\gamma^T(t_2)\} \\
&= \max\{(v_F)_\gamma^T(0), (v_F)_\gamma^T(t_2)\} \\
&\leq \max\{(v_F)_\gamma^T(t_1), (v_F)_\gamma^T(t_2)\} \\
&= \max\{v_F(t_1) - \gamma, v_F(t_2) - \gamma\} \\
(v_F)(t_1 * t_2) - \gamma &= \max\{v_F(t_1), v_F(t_2)\} - \gamma
\end{aligned}$$

$\Rightarrow v_T(t_1 * t_2) \leq \max\{v_T(t_1), v_T(t_2)\}, v_I(t_1 * t_2) \leq \max\{v_I(t_1), v_I(t_2)\}$ and $v_F(t_1 * t_2) \leq \max\{v_F(t_1), v_F(t_2)\}$. Hence B is a NCSU of Y.

Theorem 3.2.6 Intersection of any two neutrosophic cubic translations of a neutrosophic cubic BF ideals B of Y is a neutrosophic cubic BF ideal of Y.

Proof. Suppose $B_{\alpha,\beta,\gamma}^T$ and $B_{\alpha',\beta',\gamma'}^T$ are two neutrosophic cubic translations of neutrosophic cubic BF ideal B and C of Y respectively, where for $B_{\alpha,\beta,\gamma}^T$, for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$, $\gamma \in [[0,0], \mathbb{Y}]$, for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$, $\gamma \in [0, \mathbb{E}]$ and for $B_{\alpha',\beta',\gamma'}^T$, for $\kappa_{T,I,F}$, $\alpha', \beta' \in [[0,0], 7]$, $\gamma' \in [[0,0], \mathbb{Y}]$, for $v_{T,I,F}$, $\alpha', \beta' \in [0, \Gamma]$, $\gamma' \in [0, \mathbb{E}]$ and $\alpha \leq \alpha'$, $\beta \leq \beta'$, $\gamma \leq \gamma'$ as we know that, $B_{\alpha,\beta,\gamma}^T$ and $B_{\alpha',\beta',\gamma'}^T$ are neutrosophic cubic BF ideals of Y. So

$$\begin{aligned}
((\kappa_T)_\alpha^T \cap (\kappa_T)_{\alpha'}^T)(t_1) &= \text{rmin}\{(\kappa_T)_\alpha^T(t_1), (\kappa_T)_{\alpha'}^T(t_1)\} \\
&= \text{rmin}\{\kappa_T(t_1) + \alpha, \kappa_T(t_1) + \alpha'\} \\
&= \kappa_T(t_1) + \alpha \\
((\kappa_T)_\alpha^T \cap (\kappa_T)_{\alpha'}^T)(t_1) &= (\kappa_T)_\alpha^T(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_I)_\beta^T \cap (\kappa_I)_{\beta'}^T)(t_1) &= \text{rmin}\{(\kappa_I)_\beta^T(t_1), (\kappa_I)_{\beta'}^T(t_1)\} \\
&= \text{rmin}\{\kappa_I(t_1) + \beta, \kappa_I(t_1) + \beta'\} \\
&= \kappa_I(t_1) + \beta \\
((\kappa_I)_\beta^T \cap (\kappa_I)_{\beta'}^T)(t_1) &= (\kappa_I)_\beta^T(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_F)_Y^T \cap (\kappa_F)_{Y'}^T)(t_1) &= \text{rmin}\{(\kappa_F)_Y^T(t_1), (\kappa_F)_{Y'}^T(t_1)\} \\
&= \text{rmin}\{\kappa_F(t_1) - \gamma, \kappa_F(t_1) - \gamma'\} \\
&= \kappa_F(t_1) - \gamma' \\
((\kappa_F)_Y^T \cap (\kappa_F)_{Y'}^T)(t_1) &= (\kappa_F)_{Y'}^T(t_1)
\end{aligned}$$

and

$$\begin{aligned}
((v_T)_\alpha^T \cap (v_T)_{\alpha'}^T)(t_1) &= \max\{(v_T)_\alpha^T(t_1), (v_T)_{\alpha'}^T(t_1)\} \\
&= \max\{v_T(t_1) + \alpha, v_T(t_1) + \alpha'\} \\
&= v_T(t_1) + \alpha' \\
((v_T)_\alpha^T \cap (v_T)_{\alpha'}^T)(t_1) &= (v_T)_{\alpha'}^T(t_1),
\end{aligned}$$

$$\begin{aligned}
((v_I)_\beta^T \cap (v_I)_{\beta'}^T)(t_1) &= \max\{(v_I)_\beta^T(t_1), (v_I)_{\beta'}^T(t_1)\} \\
&= \max\{v_I(t_1) + \beta, v_I(t_1) + \beta'\} \\
&= v_I(t_1) + \beta' \\
((v_I)_\beta^T \cap (v_I)_{\beta'}^T)(t_1) &= (v_I)_{\beta'}^T(t_1),
\end{aligned}$$

$$\begin{aligned}
((v_F)_Y^T \cap (v_F)_{Y'}^T)(t_1) &= \max\{(v_F)_Y^T(t_1), (v_F)_{Y'}^T(t_1)\} \\
&= \max\{v_F(t_1) - \gamma, v_F(t_1) - \gamma'\} \\
&= v_F(t_1) - \gamma' \\
((v_F)_Y^T \cap (v_F)_{Y'}^T)(t_1) &= (v_F)_Y^T(t_1).
\end{aligned}$$

Hence $B_{\alpha,\beta,Y}^T \cap B_{\alpha',\beta',Y'}^T$ is a neutrosophic cubic BF ideal of Y.

Theorem 3.2.7 Union of any two neutrosophic cubic translations of a neutrosophic cubic BF ideals B of Y is a neutrosophic cubic BF ideal of Y.

Proof. Suppose $B_{\alpha,\beta,Y}^T$ and $B_{\alpha',\beta',Y'}^T$ are two neutrosophic cubic translations of neutrosophic cubic BF ideal B of Y respectively, where for $B_{\alpha,\beta,Y}^T$, for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$, $\gamma \in [[0,0], \mathbb{Y}]$, for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$, $\gamma \in [0, \mathcal{E}]$ and for $B_{\alpha',\beta',Y'}^T$, for $\kappa_{T,I,F}$ $\alpha', \beta' \in [[0,0], 7]$, $\gamma' \in [[0,0], \mathbb{Y}]$, for $v_{T,I,F}$, $\alpha', \beta' \in [0, \Gamma]$, $\gamma' \in [0, \mathcal{E}]$ and $\alpha \geq \alpha'$, $\beta \geq \beta'$, $\gamma \geq \gamma'$ as we know that, $B_{\alpha,\beta,Y}^T$ and $B_{\alpha',\beta',Y'}^T$ are neutrosophic cubic BF ideals of Y. Then

$$\begin{aligned}
((\kappa_T)_\alpha^T \cup (\kappa_T)_{\alpha'}^T)(t_1) &= \text{rmax}\{(\kappa_T)_\alpha^T(t_1), (\kappa_T)_{\alpha'}^T(t_1)\} \\
&= \text{rmax}\{\kappa_T(t_1) + \alpha, \kappa_T(t_1) + \alpha'\} \\
&= \kappa_T(t_1) + \alpha \\
((\kappa_T)_\alpha^T \cup (\kappa_T)_{\alpha'}^T)(t_1) &= (\kappa_T)_\alpha^T(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_I)_\beta^T \cup (\kappa_I)_{\beta'}^T)(t_1) &= \text{rmax}\{(\kappa_I)_\beta^T(t_1), (\kappa_I)_{\beta'}^T(t_1)\} \\
&= \text{rmax}\{\kappa_I(t_1) + \beta, \kappa_I(t_1) + \beta'\} \\
&= \kappa_I(t_1) + \beta \\
((\kappa_I)_\beta^T \cup (\kappa_I)_{\beta'}^T)(t_1) &= (\kappa_I)_\beta^T(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_F)_Y^T \cup (\kappa_F)_{Y'}^T)(t_1) &= \text{rmax}\{(\kappa_F)_Y^T(t_1), (\kappa_F)_{Y'}^T(t_1)\} \\
&= \text{rmax}\{\kappa_F(t_1) - \gamma, \kappa_F(t_1) - \gamma'\} \\
&= \kappa_F(t_1) - \gamma' \\
((\kappa_F)_Y^T \cup (\kappa_F)_{Y'}^T)(t_1) &= (\kappa_F)_{Y'}^T(t_1)
\end{aligned}$$

and

$$\begin{aligned}
((v_T)_\alpha^T \cup (v_T)_{\alpha'}^T)(t_1) &= \min\{(v_T)_\alpha^T(t_1), (v_T)_{\alpha'}^T(t_1)\} \\
&= \min\{v_T(t_1) + \alpha, v_T(t_1) + \alpha'\} \\
&= v_T(t_1) + \alpha' \\
((v_T)_\alpha^T \cup (v_T)_{\alpha'}^T)(t_1) &= (v_T)_{\alpha'}^T(t_1),
\end{aligned}$$

$$\begin{aligned}
((v_I)_\beta^T \cup (v_I)_{\beta'}^T)(t_1) &= \min\{(v_I)_\beta^T(t_1), (v_I)_{\beta'}^T(t_1)\} \\
&= \min\{v_I(t_1) + \beta, v_I(t_1) + \beta'\} \\
&= v_I(t_1) + \beta' \\
((v_I)_\beta^T \cup (v_I)_{\beta'}^T)(t_1) &= (v_I)_\beta^T(t_1),
\end{aligned}$$

$$\begin{aligned}
((v_F)_Y^T \cup (v_F)_{Y'}^T)(t_1) &= \min\{(v_F)_Y^T(t_1), (v_F)_{Y'}^T(t_1)\} \\
&= \min\{v_F(t_1) - \gamma, v_F(t_1) - \gamma'\} \\
&= v_F(t_1) - \gamma' \\
((v_F)_Y^T \cup (v_F)_{Y'}^T)(t_1) &= (v_F)_Y^T(t_1)
\end{aligned}$$

Hence $B_{\alpha,\beta,\gamma}^T \cup B_{\alpha',\beta',\gamma'}^T$ is a neutrosophic cubic BF ideal of Y.

Theorem 3.2.8 Let B be a NCS of Y such that NCM B_δ^M of B is a NCID of Y for $\delta \in (0,1]$ then B is a NCID of Y.

Proof. Suppose that B_δ^M is a NCID of Y for $\delta \in (0,1]$ and $t_1, t_2 \in Y$. Then $\delta \cdot \kappa_T(0) = (\kappa_T)_\delta^M(0) \geq (\kappa_T)_\delta^M(t_1) = \delta \cdot \kappa_T(t_1)$, so $\kappa_T(0) \geq \kappa_T(t_1)$, $\delta \cdot \kappa_I(0) = (\kappa_I)_\delta^M(0) \geq (\kappa_I)_\delta^M(t_1) = \delta \cdot \kappa_I(t_1)$, so $\kappa_I(0) \geq \kappa_I(t_1)$, $\delta \cdot \kappa_F(0) = (\kappa_F)_\delta^M(0) \geq (\kappa_F)_\delta^M(t_1) = \delta \cdot \kappa_F(t_1)$, so $\kappa_F(0) \geq \kappa_F(t_1)$ and $\delta \cdot v_T(0) = (v_T)_\delta^M(0) \leq (v_T)_\delta^M(t_1) = \delta \cdot v_T(t_1)$, so $v_T(0) \leq v_T(t_1)$, $\delta \cdot v_I(0) = (v_I)_\delta^M(0) \leq (v_I)_\delta^M(t_1) = \delta \cdot v_I(t_1)$, so $v_I(0) \leq v_I(t_1)$, $\delta \cdot v_F(0) = (v_F)_\delta^M(0) \leq (v_F)_\delta^M(t_1) = \delta \cdot v_F(t_1)$, so $v_F(0) \leq v_F(t_1)$. Now

$$\begin{aligned}
\delta \cdot \kappa_T(t_1) &= (\kappa_T)_\delta^M(t_1) \\
&\geq \text{rmin}\{(\kappa_T)_\delta^M(t_1 * t_2), (\kappa_T)_\delta^M(t_2)\} \\
&= \text{rmin}\{\delta \cdot \kappa_T(t_1 * t_2), \delta \cdot \kappa_T(t_2)\} \\
\delta \cdot \kappa_T(t_1) &= \delta \cdot \text{rmin}\{\kappa_T(t_1 * t_2), \kappa_T(t_2)\},
\end{aligned}$$

$$\delta \cdot \kappa_I(t_1) = (\kappa_I)_\delta^M(t_1)$$

$$\begin{aligned}
&\geq \text{rmin}\{(\kappa_l)_{\delta}^M(t_1 * t_2), (\kappa_l)_{\delta}^M(t_2)\} \\
&= \text{rmin}\{\delta \cdot \kappa_l(t_1 * t_2), \delta \cdot \kappa_l(t_2)\} \\
&\delta \cdot \kappa_l(t_1) = \delta \cdot \text{rmin}\{\kappa_l(t_1 * t_2), \kappa_l(t_2)\},
\end{aligned}$$

$$\begin{aligned}
&\delta \cdot \kappa_F(t_1) = (\kappa_F)_{\delta}^M(t_1) \\
&\geq \text{rmin}\{(\kappa_F)_{\delta}^M(t_1 * t_2), (\kappa_F)_{\delta}^M(t_2)\} \\
&= \text{rmin}\{\delta \cdot \kappa_F(t_1 * t_2), \delta \cdot \kappa_F(t_2)\} \\
&\delta \cdot \kappa_F(t_1) = \delta \cdot \text{rmin}\{\kappa_F(t_1 * t_2), \kappa_F(t_2)\},
\end{aligned}$$

so $\kappa_T(t_1) \geq \text{rmin}\{\kappa_T(t_1 * t_2), \kappa_T(t_2)\}$, $\kappa_l(t_1) \geq \text{rmin}\{\kappa_l(t_1 * t_2), \kappa_l(t_2)\}$ and $\kappa_F(t_1) \geq \text{rmin}\{\kappa_F(t_1 * t_2), \kappa_F(t_2)\}$ and also

$$\begin{aligned}
&\delta \cdot v_T(t_1) = (v_T)_{\delta}^M(t_1) \\
&\leq \text{max}\{(v_T)_{\delta}^M(t_1 * t_2), (v_T)_{\delta}^M(t_2)\} \\
&= \text{max}\{\delta \cdot v_T(t_1 * t_2), \delta \cdot v_T(t_2)\} \\
&\delta \cdot v_T(t_1) = \delta \cdot \text{max}\{v_T(t_1 * t_2), v_T(t_2)\},
\end{aligned}$$

$$\begin{aligned}
&\delta \cdot v_l(t_1) = (v_l)_{\delta}^M(t_1) \\
&\leq \text{max}\{(v_l)_{\delta}^M(t_1 * t_2), (v_l)_{\delta}^M(t_2)\} \\
&= \text{max}\{\delta \cdot v_l(t_1 * t_2), \delta \cdot v_l(t_2)\} \\
&\delta \cdot v_l(t_1) = \delta \cdot \text{max}\{v_l(t_1 * t_2), v_l(t_2)\},
\end{aligned}$$

$$\begin{aligned}
&\delta \cdot v_F(t_1) = (v_F)_{\delta}^M(t_1) \\
&\leq \text{max}\{(v_F)_{\delta}^M(t_1 * t_2), (v_F)_{\delta}^M(t_2)\} \\
&= \text{max}\{\delta \cdot v_F(t_1 * t_2), \delta \cdot v_F(t_2)\} \\
&\delta \cdot v_F(t_1) = \delta \cdot \text{max}\{v_F(t_1 * t_2), v_F(t_2)\},
\end{aligned}$$

so $v_T(t_1) \leq \text{max}\{v_T(t_1 * t_2), v_T(t_2)\}$, $v_l(t_1) \leq \text{max}\{v_l(t_1 * t_2), v_l(t_2)\}$ and $v_F(t_1) \leq \text{max}\{v_F(t_1 * t_2), v_F(t_2)\}$. Hence B is a **NCID** of Y.

Theorem 3.2.9 If B is a **NCID** of Y, then **NCM** B_{δ}^M of B is a **NCID** of Y, for all $\delta \in (0,1]$.

Proof. Let B be a **NCID** of Y and $\delta \in (0,1]$. Then we have $(\kappa_T)_{\delta}^M(0) = \delta \cdot \kappa_T(0) \geq \delta \cdot \kappa_T(t_1) \rightarrow (\kappa_T)_{\delta}^M(0) = (\kappa_T)_{\delta}^M(t_1)$, $(\kappa_l)_{\delta}^M(0) = \delta \cdot \kappa_l(0) \geq \delta \cdot \kappa_l(t_1) \rightarrow (\kappa_l)_{\delta}^M(0) = (\kappa_l)_{\delta}^M(t_1)$, $(\kappa_F)_{\delta}^M(0) = \delta \cdot \kappa_F(0) \geq \delta \cdot \kappa_F(t_1) \rightarrow (\kappa_F)_{\delta}^M(0) = (\kappa_F)_{\delta}^M(t_1)$ and $(v_T)_{\delta}^M(0) = \delta \cdot v_T(0) \leq \delta \cdot v_T(t_1) \rightarrow (v_T)_{\delta}^M(0) = (v_T)_{\delta}^M(t_1)$,

$$(v_I)_{\delta}^M(0) = \delta \cdot v_I(0) \leq \delta \cdot v_I(t_1) \rightarrow (v_I)_{\delta}^M(0) = (v_I)_{\delta}^M(t_1), \quad (v_F)_{\delta}^M(0) = \delta \cdot v_F(0) \leq \delta \cdot v_F(t_1) \rightarrow (v_F)_{\delta}^M(0) = (v_F)_{\delta}^M(t_1).$$

Now

$$\begin{aligned} (\kappa_T)_{\delta}^M(t_1) &= \delta \cdot \kappa_T(t_1) \\ &\geq \delta \cdot \text{rmin}\{\kappa_T(t_1 * t_2), \kappa_T(t_2)\} \\ &= \text{rmin}\{\delta \cdot \kappa_T(t_1 * t_2), \delta \cdot \kappa_T(t_2)\} \\ (\kappa_T)_{\delta}^M(t_1) &= \text{rmin}\{(\kappa_T)_{\delta}^M(t_1 * t_2), (\kappa_T)_{\delta}^M(t_2)\} \\ (\kappa_T)_{\delta}^M(t_1) &\geq \text{rmin}\{(\kappa_T)_{\delta}^M(t_1 * t_2), (\kappa_T)_{\delta}^M(t_2)\}, \end{aligned}$$

$$\begin{aligned} (\kappa_I)_{\delta}^M(t_1) &= \delta \cdot \kappa_I(t_1) \\ &\geq \delta \cdot \text{rmin}\{\kappa_I(t_1 * t_2), \kappa_I(t_2)\} \\ &= \text{rmin}\{\delta \cdot \kappa_I(t_1 * t_2), \delta \cdot \kappa_I(t_2)\} \\ (\kappa_I)_{\delta}^M(t_1) &= \text{rmin}\{(\kappa_I)_{\delta}^M(t_1 * t_2), (\kappa_I)_{\delta}^M(t_2)\} \\ (\kappa_I)_{\delta}^M(t_1) &\geq \text{rmin}\{(\kappa_I)_{\delta}^M(t_1 * t_2), (\kappa_I)_{\delta}^M(t_2)\}, \end{aligned}$$

$$\begin{aligned} (\kappa_F)_{\delta}^M(t_1) &= \delta \cdot \kappa_F(t_1) \\ &\geq \delta \cdot \text{rmin}\{\kappa_F(t_1 * t_2), \kappa_F(t_2)\} \\ &= \text{rmin}\{\delta \cdot \kappa_F(t_1 * t_2), \delta \cdot \kappa_F(t_2)\} \\ (\kappa_F)_{\delta}^M(t_1) &= \text{rmin}\{(\kappa_F)_{\delta}^M(t_1 * t_2), (\kappa_F)_{\delta}^M(t_2)\} \\ (\kappa_F)_{\delta}^M(t_1) &\geq \text{rmin}\{(\kappa_F)_{\delta}^M(t_1 * t_2), (\kappa_F)_{\delta}^M(t_2)\} \end{aligned}$$

and

$$\begin{aligned} (v_T)_{\delta}^M(t_1) &= \delta \cdot v_T(t_1) \\ &\leq \delta \cdot \text{max}\{v_T(t_1 * t_2), v_T(t_2)\} \\ &= \text{max}\{\delta \cdot v_T(t_1 * t_2), \delta \cdot v_T(t_2)\} \\ (v_T)_{\delta}^M(t_1) &= \text{max}\{(v_T)_{\delta}^M(t_1 * t_2), (v_T)_{\delta}^M(t_2)\} \\ (v_T)_{\delta}^M(t_1) &\leq \text{max}\{(v_T)_{\delta}^M(t_1 * t_2), (v_T)_{\delta}^M(t_2)\}, \end{aligned}$$

$$\begin{aligned} (v_I)_{\delta}^M(t_1) &= \delta \cdot v_I(t_1) \\ &\leq \delta \cdot \text{max}\{v_I(t_1 * t_2), v_I(t_2)\} \\ &= \text{max}\{\delta \cdot v_I(t_1 * t_2), \delta \cdot v_I(t_2)\} \\ (v_I)_{\delta}^M(t_1) &= \text{max}\{(v_I)_{\delta}^M(t_1 * t_2), (v_I)_{\delta}^M(t_2)\} \\ (v_I)_{\delta}^M(t_1) &\leq \text{max}\{(v_I)_{\delta}^M(t_1 * t_2), (v_I)_{\delta}^M(t_2)\}, \end{aligned}$$

$$\begin{aligned}
(v_F)_\delta^M(t_1) &= \delta \cdot v_F(t_1) \\
&\leq \delta \cdot \max\{v_F(t_1 * t_2), v_F(t_2)\} \\
&= \max\{\delta \cdot v_F(t_1 * t_2), \delta \cdot v_F(t_2)\} \\
(v_F)_\delta^M(t_1) &= \max\{(v_F)_\delta^M(t_1 * t_2), (v_F)_\delta^M(t_2)\} \\
(v_F)_\delta^M(t_1) &\leq \max\{(v_F)_\delta^M(t_1 * t_2), (v_F)_\delta^M(t_2)\}.
\end{aligned}$$

Hence B_δ^M of B is a **NCID** of Y , for all $\delta \in (0,1]$.

Theorem 3.2.10 Let B be a **NCID** of Y and $\delta \in [0,1]$ then **NCM** B_δ^M of B is a **NCSU** of Y .

Proof. Suppose $t_1, t_2 \in Y$. Then

$$\begin{aligned}
(\kappa_T)_\delta^M(t_1 * t_2) &= \delta \cdot \kappa_T(t_1 * t_2) \\
&\geq \delta \cdot \text{rmin}\{\kappa_T(t_2 * (t_1 * t_2)), \kappa_T(t_2)\} \\
&= \delta \cdot \text{rmin}\{\kappa_T(0), \kappa_T(t_2)\} \\
&\geq \delta \cdot \text{rmin}\{\kappa_T(t_1), \kappa_T(t_2)\} \\
&= \text{rmin}\{\delta \cdot \kappa_T(t_1), \delta \cdot \kappa_T(t_2)\} \\
(\kappa_T)_\delta^M(t_1 * t_2) &= \text{rmin}\{(\kappa_T)_\delta^M(t_1), (\kappa_T)_\delta^M(t_2)\} \\
(\kappa_T)_\delta^M(t_1 * t_2) &\geq \text{rmin}\{(\kappa_T)_\delta^M(t_1), (\kappa_T)_\delta^M(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(\kappa_I)_\delta^M(t_1 * t_2) &= \delta \cdot \kappa_I(t_1 * t_2) \\
&\geq \delta \cdot \text{rmin}\{\kappa_I(t_2 * (t_1 * t_2)), \kappa_I(t_2)\} \\
&= \delta \cdot \text{rmin}\{\kappa_I(0), \kappa_I(t_2)\} \\
&\geq \delta \cdot \text{rmin}\{\kappa_I(t_1), \kappa_I(t_2)\} \\
&= \text{rmin}\{\delta \cdot \kappa_I(t_1), \delta \cdot \kappa_I(t_2)\} \\
(\kappa_I)_\delta^M(t_1 * t_2) &= \text{rmin}\{(\kappa_I)_\delta^M(t_1), (\kappa_I)_\delta^M(t_2)\} \\
(\kappa_I)_\delta^M(t_1 * t_2) &\geq \text{rmin}\{(\kappa_I)_\delta^M(t_1), (\kappa_I)_\delta^M(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(\kappa_F)_\delta^M(t_1 * t_2) &= \delta \cdot \kappa_F(t_1 * t_2) \\
&\geq \delta \cdot \text{rmin}\{\kappa_F(t_2 * (t_1 * t_2)), \kappa_F(t_2)\} \\
&= \delta \cdot \text{rmin}\{\kappa_F(0), \kappa_F(t_2)\} \\
&\geq \delta \cdot \text{rmin}\{\kappa_F(t_1), \kappa_F(t_2)\} \\
&= \text{rmin}\{\delta \cdot \kappa_F(t_1), \delta \cdot \kappa_F(t_2)\} \\
(\kappa_F)_\delta^M(t_1 * t_2) &= \text{rmin}\{(\kappa_F)_\delta^M(t_1), (\kappa_F)_\delta^M(t_2)\}
\end{aligned}$$

$$(\kappa_F)_\delta^M(t_1 * t_2) \geq rmin\{(\kappa_F)_\delta^M(t_1), (\kappa_F)_\delta^M(t_2)\}$$

and

$$\begin{aligned} (v_T)_\delta^M(t_1 * t_2) &= \delta \cdot v_T(t_1 * t_2) \\ &\leq \delta \cdot \max\{v_T(t_2 * (t_1 * t_2)), v_T(t_2)\} \\ &= \delta \cdot \max\{v_T(0), v_T(t_2)\} \\ &\leq \delta \cdot \max\{v_T(t_1), v_T(t_2)\} \\ &= \max\{\delta \cdot v_T(t_1), \delta \cdot v_T(t_2)\} \\ (v_T)_\delta^M(t_1 * t_2) &= \max\{(v_T)_\delta^M(t_1), (v_T)_\delta^M(t_2)\} \\ (v_T)_\delta^M(t_1 * t_2) &\leq \max\{(v_T)_\delta^M(t_1), (v_T)_\delta^M(t_2)\}, \end{aligned}$$

$$\begin{aligned} (v_I)_\delta^M(t_1 * t_2) &= \delta \cdot v_I(t_1 * t_2) \\ &\leq \delta \cdot \max\{v_I(t_2 * (t_1 * t_2)), v_I(t_2)\} \\ &= \delta \cdot \max\{v_I(0), v_I(t_2)\} \\ &\leq \delta \cdot \max\{v_I(t_1), v_I(t_2)\} \\ &= \max\{\delta \cdot v_I(t_1), \delta \cdot v_I(t_2)\} \\ (v_I)_\delta^M(t_1 * t_2) &= \max\{(v_I)_\delta^M(t_1), (v_I)_\delta^M(t_2)\} \\ (v_I)_\delta^M(t_1 * t_2) &\leq \max\{(v_I)_\delta^M(t_1), (v_I)_\delta^M(t_2)\}, \end{aligned}$$

$$\begin{aligned} (v_F)_\delta^M(t_1 * t_2) &= \delta \cdot v_F(t_1 * t_2) \\ &\leq \delta \cdot \max\{v_F(t_2 * (t_1 * t_2)), v_F(t_2)\} \\ &= \delta \cdot \max\{v_F(0), v_F(t_2)\} \\ &\leq \delta \cdot \max\{v_F(t_1), v_F(t_2)\} \\ &= \max\{\delta \cdot v_F(t_1), \delta \cdot v_F(t_2)\} \\ (v_F)_\delta^M(t_1 * t_2) &= \max\{(v_F)_\delta^M(t_1), (v_F)_\delta^M(t_2)\} \\ (v_F)_\delta^M(t_1 * t_2) &\leq \max\{(v_F)_\delta^M(t_1), (v_F)_\delta^M(t_2)\}. \end{aligned}$$

Hence B_δ^M is a **NCSU** of Y .

Theorem 3.2.11 If the **NCM** B_δ^M of B is a **NCID** of Y , for $\delta \in (0,1]$. Then B is a neutrosophic cubic BF-subalgebra of Y .

Proof. Assume B_δ^M of B is a **NCID** of Y . Then

$$\begin{aligned} \delta \cdot (\kappa_T)(t_1 * t_2) &= (\kappa_T)_\delta^M(t_1 * t_2) \\ &\geq rmin\{(\kappa_T)_\delta^M(t_2 * (t_1 * t_2)), (\kappa_T)_\delta^M(t_2)\} \\ &= rmin\{(\kappa_T)_\delta^M(0), (\kappa_T)_\delta^M(t_2)\} \end{aligned}$$

$$\begin{aligned}
&\geq \text{rmin}\{(\kappa_T)_{\delta}^M(t_1), (\kappa_T)_{\delta}^M(t_2)\} \\
&= \text{rmin}\{\delta \cdot \kappa_T(t_1), \delta \cdot \kappa_T(t_2)\} \\
&\delta \cdot (\kappa_T)(t_1 * t_2) = \delta \cdot \text{rmin}\{\kappa_T(t_1), \kappa_T(t_2)\} \\
&\Rightarrow \kappa_T(t_1 * t_2) \geq \text{rmin}\{\kappa_T(t_1), \kappa_T(t_2)\},
\end{aligned}$$

$$\begin{aligned}
&\delta \cdot (\kappa_I)(t_1 * t_2) = (\kappa_I)_{\delta}^M(t_1 * t_2) \\
&\geq \text{rmin}\{(\kappa_I)_{\delta}^M(t_2 * (t_1 * t_2)), (\kappa_I)_{\delta}^M(t_2)\} \\
&= \text{rmin}\{(\kappa_I)_{\delta}^M(0), (\kappa_I)_{\delta}^M(t_2)\} \\
&\geq \text{rmin}\{(\kappa_I)_{\delta}^M(t_1), (\kappa_I)_{\delta}^M(t_2)\} \\
&= \text{rmin}\{\delta \cdot \kappa_I(t_1), \delta \cdot \kappa_I(t_2)\} \\
&\delta \cdot (\kappa_I)(t_1 * t_2) = \delta \cdot \text{rmin}\{\kappa_I(t_1), \kappa_I(t_2)\} \\
&\Rightarrow \kappa_I(t_1 * t_2) \geq \text{rmin}\{\kappa_I(t_1), \kappa_I(t_2)\},
\end{aligned}$$

$$\begin{aligned}
&\delta \cdot (\kappa_F)(t_1 * t_2) = (\kappa_F)_{\delta}^M(t_1 * t_2) \\
&\geq \text{rmin}\{(\kappa_F)_{\delta}^M(t_2 * (t_1 * t_2)), (\kappa_F)_{\delta}^M(t_2)\} \\
&= \text{rmin}\{(\kappa_F)_{\delta}^M(0), (\kappa_F)_{\delta}^M(t_2)\} \\
&\geq \text{rmin}\{(\kappa_F)_{\delta}^M(t_1), (\kappa_F)_{\delta}^M(t_2)\} \\
&= \text{rmin}\{\delta \cdot \kappa_F(t_1), \delta \cdot \kappa_F(t_2)\} \\
&\delta \cdot (\kappa_F)(t_1 * t_2) = \delta \cdot \text{rmin}\{\kappa_F(t_1), \kappa_F(t_2)\} \\
&\Rightarrow \kappa_F(t_1 * t_2) \geq \text{rmin}\{\kappa_F(t_1), \kappa_F(t_2)\}
\end{aligned}$$

and

$$\begin{aligned}
&\delta \cdot (v_T)(t_1 * t_2) = (v_T)_{\delta}^M(t_1 * t_2) \\
&\leq \text{max}\{(v_T)_{\delta}^M(t_2 * (t_1 * t_2)), (v_T)_{\delta}^M(t_2)\} \\
&= \text{max}\{(v_T)_{\delta}^M(0), (v_T)_{\delta}^M(t_2)\} \\
&\leq \text{max}\{(v_T)_{\delta}^M(t_1), (v_T)_{\delta}^M(t_2)\} \\
&= \text{max}\{\delta \cdot v_T(t_1), \delta \cdot v_T(t_2)\} \\
&\delta \cdot (v_T)(t_1 * t_2) = \delta \cdot \text{max}\{v_T(t_1), v_T(t_2)\} \\
&\Rightarrow v_T(t_1 * t_2) \leq \text{max}\{v_T(t_1), v_T(t_2)\},
\end{aligned}$$

$$\begin{aligned}
&\delta \cdot (v_I)(t_1 * t_2) = (v_I)_{\delta}^M(t_1 * t_2) \\
&\leq \text{max}\{(v_I)_{\delta}^M(t_2 * (t_1 * t_2)), (v_I)_{\delta}^M(t_2)\} \\
&= \text{max}\{(v_I)_{\delta}^M(0), (v_I)_{\delta}^M(t_2)\}
\end{aligned}$$

$$\begin{aligned}
&\leq \max\{(v_I)_{\delta}^M(t_1), (v_I)_{\delta}^M(t_2)\} \\
&= \max\{\delta \cdot v_I(t_1), \delta \cdot v_I(t_2)\} \\
&\delta \cdot (v_I)(t_1 * t_2) = \delta \cdot \max\{v_I(t_1), v_I(t_2)\} \\
&\Rightarrow v_I(t_1 * t_2) \leq \max\{v_I(t_1), v_I(t_2)\},
\end{aligned}$$

$$\begin{aligned}
&\delta \cdot (v_F)(t_1 * t_2) = (v_F)_{\delta}^M(t_1 * t_2) \\
&\leq \max\{(v_F)_{\delta}^M(t_2 * (t_1 * t_2)), (v_F)_{\delta}^M(t_2)\} \\
&= \max\{(v_F)_{\delta}^M(0), (v_F)_{\delta}^M(t_2)\} \\
&\leq \max\{(v_F)_{\delta}^M(t_1), (v_F)_{\delta}^M(t_2)\} \\
&= \max\{\delta \cdot v_F(t_1), \delta \cdot v_F(t_2)\} \\
&\delta \cdot (v_F)(t_1 * t_2) = \delta \cdot \max\{v_F(t_1), v_F(t_2)\} \\
&\Rightarrow v_F(t_1 * t_2) \leq \max\{v_F(t_1), v_F(t_2)\}.
\end{aligned}$$

Hence B is a NCSU of Y.

Theorem 3.2.12 Intersection of any two neutrosophic cubic multiplications of a NCID B of Y is a NCID of Y.

Proof. Suppose B_{δ}^M and $B_{\delta'}^M$ are neutrosophic cubic multiplications of NCID B of Y, where $\delta, \delta' \in (0,1]$ and $\delta \leq \delta'$, as we know that B_{δ}^M and $B_{\delta'}^M$ are NCIDs of Y. Then

$$\begin{aligned}
((\kappa_T)_{\delta}^M \cap (\kappa_T)_{\delta'}^M)(t_1) &= rmin\{(\kappa_T)_{\delta}^M(t_1), (\kappa_T)_{\delta'}^M(t_1)\} \\
&= rmin\{\kappa_T(t_1) \cdot \delta, \kappa_T(t_1) \cdot \delta'\} \\
&= \kappa_T(t_1) \cdot \delta \\
((\kappa_T)_{\delta}^M \cap (\kappa_T)_{\delta'}^M)(t_1) &= (\kappa_T)_{\delta}^M(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_I)_{\delta}^M \cap (\kappa_I)_{\delta'}^M)(t_1) &= rmin\{(\kappa_I)_{\delta}^M(t_1), (\kappa_I)_{\delta'}^M(t_1)\} \\
&= rmin\{\kappa_I(t_1) \cdot \delta, \kappa_I(t_1) \cdot \delta'\} \\
&= \kappa_I(t_1) \cdot \delta \\
((\kappa_I)_{\delta}^M \cap (\kappa_I)_{\delta'}^M)(t_1) &= (\kappa_I)_{\delta}^M(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_F)_{\delta}^M \cap (\kappa_F)_{\delta'}^M)(t_1) &= rmin\{(\kappa_F)_{\delta}^M(t_1), (\kappa_F)_{\delta'}^M(t_1)\} \\
&= rmin\{\kappa_F(t_1) \cdot \delta, \kappa_F(t_1) \cdot \delta'\} \\
&= \kappa_F(t_1) \cdot \delta \\
((\kappa_F)_{\delta}^M \cap (\kappa_F)_{\delta'}^M)(t_1) &= (\kappa_F)_{\delta}^M(t_1)
\end{aligned}$$

and

$$\begin{aligned}
((v_T)_{\delta}^M \cap (v_T)_{\delta'}^M)(t_1) &= \max\{(v_T)_{\delta}^M(t_1), (v_T)_{\delta'}^M(t_1)\} \\
&= \max\{v_T(t_1). \delta, v_T(t_1). \delta'\} \\
&= v_T(t_1). \delta' \\
((v_T)_{\delta}^M \cap (v_T)_{\delta'}^M)(t_1) &= (v_T)_{\delta'}^M(t_1),
\end{aligned}$$

$$\begin{aligned}
((v_I)_{\delta}^M \cap (v_I)_{\delta'}^M)(t_1) &= \max\{(v_I)_{\delta}^M(t_1), (v_I)_{\delta'}^M(t_1)\} \\
&= \max\{v_I(t_1). \delta, v_I(t_1). \delta'\} \\
&= v_I(t_1). \delta' \\
((v_I)_{\delta}^M \cap (v_I)_{\delta'}^M)(t_1) &= (v_I)_{\delta'}^M(t_1),
\end{aligned}$$

$$\begin{aligned}
((v_F)_{\delta}^M \cap (v_F)_{\delta'}^M)(t_1) &= \max\{(v_F)_{\delta}^M(t_1), (v_F)_{\delta'}^M(t_1)\} \\
&= \max\{v_F(t_1). \delta, v_F(t_1). \delta'\} \\
&= v_F(t_1). \delta' \\
((v_F)_{\delta}^M \cap (v_F)_{\delta'}^M)(t_1) &= (v_F)_{\delta'}^M(t_1).
\end{aligned}$$

Hence $B_{\delta}^M \cap B_{\delta'}^M$ is **NCID** of Y.

Theorem 3.2.13 Union of any two neutrosophic cubic multiplications of a **NCID** B of Y is a **NCID** of Y.

Proof. Suppose B_{δ}^M and $B_{\delta'}^M$ are neutrosophic cubic multiplications of **NCID** B of Y, where $\delta, \delta' \in (0,1]$ and $\delta \leq \delta'$, as we know that B_{δ}^M and $B_{\delta'}^M$ are **NCIDs** of Y. Then

$$\begin{aligned}
((\kappa_T)_{\delta}^M \cup (\kappa_T)_{\delta'}^M)(t_1) &= rmax\{(\kappa_T)_{\delta}^M(t_1), (\kappa_T)_{\delta'}^M(t_1)\} \\
&= rmax\{\kappa_T(t_1). \delta, \kappa_T(t_1). \delta'\} \\
&= \kappa_T(t_1). \delta' \\
((\kappa_T)_{\delta}^M \cup (\kappa_T)_{\delta'}^M)(t_1) &= (\kappa_T)_{\delta'}^M(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_I)_{\delta}^M \cup (\kappa_I)_{\delta'}^M)(t_1) &= rmax\{(\kappa_I)_{\delta}^M(t_1), (\kappa_I)_{\delta'}^M(t_1)\} \\
&= rmax\{\kappa_I(t_1). \delta, \kappa_I(t_1). \delta'\} \\
&= \kappa_I(t_1). \delta' \\
((\kappa_I)_{\delta}^M \cup (\kappa_I)_{\delta'}^M)(t_1) &= (\kappa_I)_{\delta'}^M(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_F)_{\delta}^M \cup (\kappa_F)_{\delta'}^M)(t_1) &= rmax\{(\kappa_F)_{\delta}^M(t_1), (\kappa_F)_{\delta'}^M(t_1)\} \\
&= rmax\{\kappa_F(t_1). \delta, \kappa_F(t_1). \delta'\} \\
&= \kappa_F(t_1). \delta' \\
((\kappa_F)_{\delta}^M \cup (\kappa_F)_{\delta'}^M)(t_1) &= (\kappa_F)_{\delta'}^M(t_1)
\end{aligned}$$

and

$$\begin{aligned} ((v_T)_{\delta}^M \cup (v_T)_{\delta'}^M)(t_1) &= \min\{(v_T)_{\delta}^M(t_1), (v_T)_{\delta'}^M(t_1)\} \\ &= \min\{v_T(t_1) \cdot \delta, v_T(t_1) \cdot \delta'\} \\ &= v_T(t_1) \cdot \delta \\ ((v_T)_{\delta}^M \cup (v_T)_{\delta'}^M)(t_1) &= (v_T)_{\delta}^M(t_1), \end{aligned}$$

$$\begin{aligned} ((v_I)_{\delta}^M \cup (v_I)_{\delta'}^M)(t_1) &= \min\{(v_I)_{\delta}^M(t_1), (v_I)_{\delta'}^M(t_1)\} \\ &= \min\{v_I(t_1) \cdot \delta, v_I(t_1) \cdot \delta'\} \\ &= v_I(t_1) \cdot \delta \\ ((v_I)_{\delta}^M \cup (v_I)_{\delta'}^M)(t_1) &= (v_I)_{\delta}^M(t_1), \end{aligned}$$

$$\begin{aligned} ((v_F)_{\delta}^M \cup (v_F)_{\delta'}^M)(t_1) &= \min\{(v_F)_{\delta}^M(t_1), (v_F)_{\delta'}^M(t_1)\} \\ &= \min\{v_F(t_1) \cdot \delta, v_F(t_1) \cdot \delta'\} \\ &= v_F(t_1) \cdot \delta \\ ((v_F)_{\delta}^M \cup (v_F)_{\delta'}^M)(t_1) &= (v_F)_{\delta}^M(t_1). \end{aligned}$$

Hence $B_{\delta}^M \cup B_{\delta'}^M$ is **NCID** of Y.

3.3 Magnified Translative Interpretation of Neutrosophic Cubic Subalgebra and Neutrosophic Cubic Ideal

In this section, we define the notion of neutrosophic cubic magnified translation **NCMT** and investigate some results.

Definition 3.3.1 Let $B = (\kappa_{T,I,F}, v_{T,I,F})$ be a **NCS** of Y and for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathcal{E}]$ and for all $\delta \in [0,1]$. An object having the form $B_{\delta, \alpha, \beta, \gamma}^{MT} = \{(\kappa_{T,I,F})_{\delta, \alpha, \beta, \gamma}^{MT}, (v_{T,I,F})_{\delta, \alpha, \beta, \gamma}^{MT}\}$ is said to be a **NCMT** of B, when $(\kappa_T)_{\delta, \alpha}^{MT}(t_1) = \delta \cdot \kappa_T(t_1) + \alpha$, $(\kappa_I)_{\delta, \beta}^{MT}(t_1) = \delta \cdot \kappa_I(t_1) + \beta$, $(\kappa_F)_{\delta, \gamma}^{MT}(t_1) = \delta \cdot \kappa_F(t_1) - \gamma$ and $(v_T)_{\delta, \alpha}^{MT}(t_1) = \delta \cdot v_T(t_1) + \alpha$, $(v_I)_{\delta, \beta}^{MT}(t_1) = \delta \cdot v_I(t_1) + \beta$, $(v_F)_{\delta, \gamma}^{MT}(t_1) = \delta \cdot v_F(t_1) - \gamma$ for all $t_1 \in Y$.

Example 3.3.1 Let $Y = \{0, 1, 2\}$ be a BF-algebra as defined in Example 3.2.1. A **NCS** $B = (\kappa_{T,I,F}, v_{T,I,F})$ of Y is defined as

$$\kappa_T(t_1) = \begin{cases} [0.1, 0.3] & \text{if } t_1 = 0 \\ [0.4, 0.7] & \text{if otherwise} \end{cases}$$

$$\kappa_I(t_1) = \begin{cases} [0.2, 0.4] & \text{if } t_1 = 0 \\ [0.5, 0.7] & \text{if otherwise} \end{cases}$$

$$\kappa_F(t_1) = \begin{cases} [0.4, 0.6] & \text{if } t_1 = 0 \\ [0.5, 0.8] & \text{if otherwise} \end{cases}$$

and

$$v_T(t_1) = \begin{cases} 0.1 & \text{if } t_1 = 0 \\ 0.4 & \text{if otherwise} \end{cases}$$

$$v_I(t_1) = \begin{cases} 0.2 & \text{if } t_1 = 0 \\ 0.3 & \text{if otherwise} \end{cases}$$

$$v_F(t_1) = \begin{cases} 0.5 & \text{if } t_1 = 0 \\ 0.7 & \text{if otherwise.} \end{cases}$$

Then B is a neutrosophic cubic subalgebra, for $v_{T,I,F}$ choose $\delta = 0.1, \alpha = 0.02, \beta = 0.03, \gamma = 0.04$ and for $\kappa_{T,I,F}$ choose $\delta = [0.1, 0.4], \alpha = [0.03, 0.07], \beta = [0.04, 0.08], \gamma = [0.02, 0.06]$ then the mapping $B_{(0.1)}^{MT}(\alpha, \beta, \gamma)|Y \rightarrow [0,1]$ is given by

$$(\kappa_T)_{[0.1, 0.4] [0.03, 0.07]}^M(t_1) = \begin{cases} [0.04, 0.19] & \text{if } t_1 = 1 \\ [0.07, 0.35] & \text{if otherwise} \end{cases}$$

$$(\kappa_I)_{[0.1, 0.4] [0.04, 0.08]}^M(t_1) = \begin{cases} [0.06, 0.24] & \text{if } t_1 = 1 \\ [0.09, 0.36] & \text{if otherwise} \end{cases}$$

$$(\kappa_F)_{[0.1, 0.4] [0.02, 0.06]}^M(t_1) = \begin{cases} [0.02, 0.18] & \text{if } t_1 = 1 \\ [0.03, 0.26] & \text{if otherwise} \end{cases}$$

and

$$(v_T)_{0.1, 0.02}^M(t_1) = \begin{cases} 0.03 & \text{if } t_1 = 1 \\ 0.06 & \text{if otherwise} \end{cases}$$

$$(v_I)_{0.1, 0.03}^M(t_1) = \begin{cases} 0.05 & \text{if } t_1 = 1 \\ 0.06 & \text{if otherwise} \end{cases}$$

$$(v_F)_{0.1, 0.04}^M(t_1) = \begin{cases} 0.01 & \text{if } t_1 = 1 \\ 0.03 & \text{if otherwise,} \end{cases}$$

which imply $(\kappa_T)_{[0.1, 0.4] [0.03, 0.07]}^M(t_1) = [0.1, 0.4]. \kappa_T(t_1) + [0.03, 0.07]$, $(\kappa_I)_{[0.1, 0.4] [0.04, 0.08]}^M(t_1) = [0.1, 0.4]. \kappa_I(t_1) + [0.04, 0.08]$, $(\kappa_F)_{[0.1, 0.4] [0.02, 0.06]}^M(t_1) = [0.1, 0.4]. \kappa_F(t_1) - [0.02, 0.06]$ and $(v_T)_{(0.1)(0.02)}^M(t_1) = (0.1). v_T(t_1) + 0.02$, $(v_I)_{(0.1)(0.03)}^M(t_1) = (0.1). v_I(t_1) + 0.03$, $(v_F)_{(0.1)(0.04)}^M(t_1) = (0.1). v_F(t_1) - 0.04$ for all $t_1 \in Y$. Hence B^{MT} is a neutrosophic cubic magnified translation.

Theorem 3.3.1 Let B be a neutrosophic cubic subset of Y such that for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathbb{E}]$ and $\delta \in [0,1]$ and a mapping $B_{\delta, \alpha, \beta, \gamma}^{MT,I,F}|Y \rightarrow [0,1]$ be a NCMT of B . If B is NCSU of Y , then $B_{\delta, \alpha, \beta, \gamma}^{MT,I,F}$ is a NCSU of Y .

Proof. Let B be a neutrosophic cubic subset of Y such that for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathbb{E}]$ and $\delta \in [0,1]$ and a mapping $B_{\delta, \alpha, \beta, \gamma}^{MT,I,F}|Y \rightarrow [0,1]$ be a NCMT of B . Suppose B is a NCSU of Y . Then $\kappa_T(t_1 * t_2) \geq rmin\{\kappa_T(t_1), \kappa_T(t_2)\}$, $\kappa_I(t_1 * t_2) \geq rmin\{\kappa_I(t_1), \kappa_I(t_2)\}$, $\kappa_F(t_1 * t_2) \geq rmin\{\kappa_F(t_1), \kappa_F(t_2)\}$ and $v_T(t_1 * t_2) \leq max\{v_T(t_1), v_T(t_2)\}$, $v_I(t_1 * t_2) \leq max\{v_I(t_1), v_I(t_2)\}$, $v_F(t_1 * t_2) \leq max\{v_F(t_1), v_F(t_2)\}$. Now

$$\begin{aligned} (\kappa_T)_{\delta, \alpha}^{MT}(t_1 * t_2) &= \delta. \kappa_T(t_1 * t_2) + \alpha \\ &\geq \delta. rmin\{\kappa_T(t_1), \kappa_T(t_2)\} + \alpha \end{aligned}$$

$$\begin{aligned}
&= \text{rmin}\{\delta \cdot \kappa_T(t_1) + \alpha, \delta \cdot \kappa_T(t_2) + \alpha\} \\
(\kappa_T)_{\delta \alpha}^{MT}(t_1 * t_2) &= \text{rmin}\{(\kappa_T)_{\delta \alpha}^{MT}(t_1), (\kappa_T)_{\delta \alpha}^{MT}(t_2)\} \\
(\kappa_T)_{\delta \alpha}^{MT}(t_1 * t_2) &\geq \text{rmin}\{(\kappa_T)_{\delta \alpha}^{MT}(t_1), (\kappa_T)_{\delta \alpha}^{MT}(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(\kappa_I)_{\delta \beta}^{MT}(t_1 * t_2) &= \delta \cdot \kappa_I(t_1 * t_2) + \beta \\
&\geq \delta \cdot \text{rmin}\{\kappa_I(t_1), \kappa_I(t_2)\} + \beta \\
&= \text{rmin}\{\delta \cdot \kappa_I(t_1) + \beta, \delta \cdot \kappa_I(t_2) + \beta\} \\
(\kappa_I)_{\delta \beta}^{MT}(t_1 * t_2) &= \text{rmin}\{(\kappa_I)_{\delta \beta}^{MT}(t_1), (\kappa_I)_{\delta \beta}^{MT}(t_2)\} \\
(\kappa_I)_{\delta \beta}^{MT}(t_1 * t_2) &\geq \text{rmin}\{(\kappa_I)_{\delta \beta}^{MT}(t_1), (\kappa_I)_{\delta \beta}^{MT}(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(\kappa_F)_{\delta \gamma}^{MT}(t_1 * t_2) &= \delta \cdot \kappa_F(t_1 * t_2) - \gamma \\
&\geq \delta \cdot \text{rmin}\{\kappa_F(t_1), \kappa_F(t_2)\} - \gamma \\
&= \text{rmin}\{\delta \cdot \kappa_F(t_1) - \gamma, \delta \cdot \kappa_F(t_2) - \gamma\} \\
(\kappa_F)_{\delta \gamma}^{MT}(t_1 * t_2) &= \text{rmin}\{(\kappa_F)_{\delta \gamma}^{MT}(t_1), (\kappa_F)_{\delta \gamma}^{MT}(t_2)\} \\
(\kappa_F)_{\delta \gamma}^{MT}(t_1 * t_2) &\geq \text{rmin}\{(\kappa_F)_{\delta \gamma}^{MT}(t_1), (\kappa_F)_{\delta \gamma}^{MT}(t_2)\}
\end{aligned}$$

and

$$\begin{aligned}
(v_T)_{\delta \alpha}^{MT}(t_1 * t_2) &= \delta \cdot v_T(t_1 * t_2) + \alpha \\
&\leq \delta \cdot \max\{v_T(t_1), v_T(t_2)\} + \alpha \\
&= \max\{\delta \cdot v_T(t_1) + \alpha, \delta \cdot v_T(t_2) + \alpha\} \\
(v_T)_{\delta \alpha}^{MT}(t_1 * t_2) &= \max\{(v_T)_{\delta \alpha}^{MT}(t_1), (v_T)_{\delta \alpha}^{MT}(t_2)\} \\
(v_T)_{\delta \alpha}^{MT}(t_1 * t_2) &\leq \max\{(v_T)_{\delta \alpha}^{MT}(t_1), (v_T)_{\delta \alpha}^{MT}(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(v_I)_{\delta \beta}^{MT}(t_1 * t_2) &= \delta \cdot v_I(t_1 * t_2) + \beta \\
&\leq \delta \cdot \max\{v_I(t_1), v_I(t_2)\} + \beta \\
&= \max\{\delta \cdot v_I(t_1) + \beta, \delta \cdot v_I(t_2) + \beta\} \\
(v_I)_{\delta \beta}^{MT}(t_1 * t_2) &= \max\{(v_I)_{\delta \beta}^{MT}(t_1), (v_I)_{\delta \beta}^{MT}(t_2)\} \\
(v_I)_{\delta \beta}^{MT}(t_1 * t_2) &\leq \max\{(v_I)_{\delta \beta}^{MT}(t_1), (v_I)_{\delta \beta}^{MT}(t_2)\},
\end{aligned}$$

$$\begin{aligned}
(v_F)_{\delta \gamma}^{MT}(t_1 * t_2) &= \delta \cdot v_F(t_1 * t_2) - \gamma \\
&\leq \delta \cdot \max\{v_F(t_1), v_F(t_2)\} - \gamma \\
&= \max\{\delta \cdot v_F(t_1) - \gamma, \delta \cdot v_F(t_2) - \gamma\} \\
(v_F)_{\delta \gamma}^{MT}(t_1 * t_2) &= \max\{(v_F)_{\delta \gamma}^{MT}(t_1), (v_F)_{\delta \gamma}^{MT}(t_2)\} \\
(v_F)_{\delta \gamma}^{MT}(t_1 * t_2) &\leq \max\{(v_F)_{\delta \gamma}^{MT}(t_1), (v_F)_{\delta \gamma}^{MT}(t_2)\}.
\end{aligned}$$

Hence **NCMT** $B_{\delta \alpha, \beta, \gamma}^{M T}$ is a **NCSU** of Y .

Theorem 3.3.2 Let B be a **NCS** of Y such that and for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathbb{E}]$ and $\delta \in [0,1]$ and a mapping $B_{\delta \alpha, \beta, \gamma}^{M T}: Y \rightarrow [0,1]$ be a **NCMT** of B . If $B_{\delta \alpha, \beta, \gamma}^{M T}$ is **NCSU** of Y . Then B is a **NCSU** of Y .

Proof. Let B be a neutrosophic cubic subset of Y , where $\alpha, \beta, \gamma \in [0, \mathbb{Y}]$, $\delta \in [0,1]$ and a mapping $B_{\delta \alpha, \beta, \gamma}^{M T}: Y \rightarrow [0,1]$ be a **NCMT** of B . Suppose $B_{\delta \alpha, \beta, \gamma}^{M T} = \{(\kappa_B)_{\delta \alpha, \beta, \gamma}^{M T, I, F}, (v_B)_{\delta \alpha, \beta, \gamma}^{M T, I, F}\}$ is a **NCSU** of Y , then

$$\begin{aligned} & \delta \cdot \kappa_T(t_1 * t_2) + \alpha = (\kappa_T)_{\delta \alpha}^{M T}(t_1 * t_2) \\ & \geq rmin\{(\kappa_T)_{\delta \alpha}^{M T}(t_1), (\kappa_T)_{\delta \alpha}^{M T}(t_2)\} \\ & = rmin\{\delta \cdot \kappa_T(t_1) + \alpha, \delta \cdot \kappa_T(t_2) + \alpha\} \\ & \delta \cdot \kappa_T(t_1 * t_2) + \alpha = \delta \cdot rmin\{\kappa_T(t_2), \kappa_T(t_1)\} + \alpha, \end{aligned}$$

$$\begin{aligned} & \delta \cdot \kappa_I(t_1 * t_2) + \beta = (\kappa_I)_{\delta \beta}^{M T}(t_1 * t_2) \\ & \geq rmin\{(\kappa_I)_{\delta \beta}^{M T}(t_1), (\kappa_I)_{\delta \beta}^{M T}(t_2)\} \\ & = rmin\{\delta \cdot \kappa_I(t_1) + \beta, \delta \cdot \kappa_I(t_2) + \beta\} \\ & \delta \cdot \kappa_I(t_1 * t_2) + \beta = \delta \cdot rmin\{\kappa_I(t_2), \kappa_I(t_1)\} + \beta, \end{aligned}$$

$$\begin{aligned} & \delta \cdot \kappa_F(t_1 * t_2) - \gamma = (\kappa_F)_{\delta \gamma}^{M T}(t_1 * t_2) \\ & \geq rmin\{(\kappa_F)_{\delta \gamma}^{M T}(t_1), (\kappa_F)_{\delta \gamma}^{M T}(t_2)\} \\ & = rmin\{\delta \cdot \kappa_F(t_1) - \gamma, \delta \cdot \kappa_F(t_2) - \gamma\} \\ & \delta \cdot \kappa_F(t_1 * t_2) - \gamma = \delta \cdot rmin\{\kappa_F(t_2), \kappa_F(t_1)\} - \gamma, \end{aligned}$$

and

$$\begin{aligned} & \delta \cdot v_T(t_1 * t_2) + \alpha = (v_T)_{\delta \alpha}^{M T}(t_1 * t_2) \\ & \leq max\{(v_T)_{\delta \alpha}^{M T}(t_1), (v_T)_{\delta \alpha}^{M T}(t_2)\} \\ & = max\{\delta \cdot v_T(t_1) + \alpha, \delta \cdot v_T(t_2) + \alpha\} \\ & \delta \cdot v_T(t_1 * t_2) + \alpha = \delta \cdot max\{v_T(t_2), v_T(t_1)\} + \alpha, \end{aligned}$$

$$\begin{aligned} & \delta \cdot v_I(t_1 * t_2) + \beta = (v_I)_{\delta \beta}^{M T}(t_1 * t_2) \\ & \leq max\{(v_I)_{\delta \beta}^{M T}(t_1), (v_I)_{\delta \beta}^{M T}(t_2)\} \\ & = max\{\delta \cdot v_I(t_1) + \beta, \delta \cdot v_I(t_2) + \beta\} \\ & \delta \cdot v_I(t_1 * t_2) + \beta = \delta \cdot max\{v_I(t_2), v_I(t_1)\} + \beta, \end{aligned}$$

$$\begin{aligned}
& \delta \cdot v_F(t_1 * t_2) - \gamma = (v_F)_{\delta \gamma}^{MT}(t_1 * t_2) \\
& \leq \max\{(v_F)_{\delta \gamma}^{MT}(t_1), (v_F)_{\delta \gamma}^{MT}(t_2)\} \\
& = \max\{\delta \cdot v_F(t_1) - \gamma, \delta \cdot v_F(t_2) - \gamma\} \\
& \delta \cdot v_F(t_1 * t_2) - \gamma = \delta \cdot \max\{v_F(t_2), v_F(t_1)\} - \gamma,
\end{aligned}$$

which imply $\kappa_T(t_1 * t_2) \geq r\min\{\kappa_T(t_1), \kappa_T(t_2)\}$, $\kappa_I(t_1 * t_2) \geq r\min\{\kappa_I(t_1), \kappa_I(t_2)\}$, $\kappa_F(t_1 * t_2) \geq r\min\{\kappa_F(t_1), \kappa_F(t_2)\}$ and $v_T(t_1 * t_2) \leq \max\{v_T(t_1), v_T(t_2)\}$, $v_I(t_1 * t_2) \leq \max\{v_I(t_1), v_I(t_2)\}$, $v_F(t_1 * t_2) \leq \max\{v_F(t_1), v_F(t_2)\}$ for all $t_1, t_2 \in Y$. Hence B is a NCSU of Y .

Theorem 3.3.3 If B is a **NCID** of Y . Then **NCMT** $B_{\delta \alpha, \beta, \gamma}^{MT}$ of B is a **NCID** of Y for all $\kappa_{T,I,F}$, $\alpha, \beta \in [0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathcal{E}]$ and $\delta \in (0,1]$.

Proof. Suppose $B = (\kappa_{T,I,F}, v_{T,I,F})$ is a **NCID** of Y . Then

$$\begin{aligned}
(\kappa_T)_{\delta \alpha}^{MT}(0) &= \delta \cdot \kappa_T(0) + \alpha \\
&\geq \delta \cdot \kappa_T(t_1) + \alpha \\
(\kappa_T)_{\delta \alpha}^{MT}(0) &= (\kappa_T)_{\delta \alpha}^{MT}(t_1), \\
(\kappa_I)_{\delta \beta}^{MT}(0) &= \delta \cdot \kappa_I(0) + \beta \\
&\geq \delta \cdot \kappa_I(t_1) + \beta \\
(\kappa_I)_{\delta \beta}^{MT}(0) &= (\kappa_I)_{\delta \beta}^{MT}(t_1), \\
(\kappa_F)_{\delta \gamma}^{MT}(0) &= \delta \cdot \kappa_F(0) - \gamma \\
&\geq \delta \cdot \kappa_F(t_1) - \gamma \\
(\kappa_F)_{\delta \gamma}^{MT}(0) &= (\kappa_F)_{\delta \gamma}^{MT}(t_1)
\end{aligned}$$

and

$$\begin{aligned}
(v_T)_{\delta \alpha}^{MT}(0) &= \delta \cdot v_T(0) + \alpha \\
&\leq \delta \cdot v_T(t_1) + \alpha \\
(v_T)_{\delta \alpha}^{MT}(0) &= (v_T)_{\delta \alpha}^{MT}(t_1), \\
(v_I)_{\delta \beta}^{MT}(0) &= \delta \cdot v_I(0) + \beta \\
&\leq \delta \cdot v_I(t_1) + \beta \\
(v_I)_{\delta \beta}^{MT}(0) &= (v_I)_{\delta \beta}^{MT}(t_1), \\
(v_F)_{\delta \gamma}^{MT}(0) &= \delta \cdot v_F(0) - \gamma \\
&\leq \delta \cdot v_F(t_1) - \gamma \\
(v_F)_{\delta \gamma}^{MT}(0) &= (v_F)_{\delta \gamma}^{MT}(t_1)
\end{aligned}$$

Now

$$\begin{aligned}
 (\kappa_T)_{\delta \alpha}^{MT}(t_1) &= \delta \cdot \kappa_T(t_1) + \alpha \\
 &\geq \delta \cdot rmin\{\kappa_T(t_1 * t_2), \kappa_T(t_2)\} + \alpha \\
 &= rmin\{\delta \cdot \kappa_T(t_1 * t_2) + \alpha, \delta \cdot \kappa_T(t_2) + \alpha\} \\
 (\kappa_T)_{\delta \alpha}^{MT}(t_1) &= rmin\{(\kappa_T)_{\delta \alpha}^{MT}(t_1 * t_2), (\kappa_T)_{\delta \alpha}^{MT}(t_2)\} \\
 \Rightarrow (\kappa_T)_{\delta \alpha}^{MT}(t_1) &\geq rmin\{(\kappa_T)_{\delta \alpha}^{MT}(t_1 * t_2), (\kappa_T)_{\delta \alpha}^{MT}(t_2)\},
 \end{aligned}$$

$$\begin{aligned}
 (\kappa_I)_{\delta \beta}^{MT}(t_1) &= \delta \cdot \kappa_I(t_1) + \beta \\
 &\geq \delta \cdot rmin\{\kappa_I(t_1 * t_2), \kappa_I(t_2)\} + \beta \\
 &= rmin\{\delta \cdot \kappa_I(t_1 * t_2) + \beta, \delta \cdot \kappa_I(t_2) + \beta\} \\
 (\kappa_I)_{\delta \beta}^{MT}(t_1) &= rmin\{(\kappa_I)_{\delta \beta}^{MT}(t_1 * t_2), (\kappa_I)_{\delta \beta}^{MT}(t_2)\} \\
 \Rightarrow (\kappa_I)_{\delta \beta}^{MT}(t_1) &\geq rmin\{(\kappa_I)_{\delta \beta}^{MT}(t_1 * t_2), (\kappa_I)_{\delta \beta}^{MT}(t_2)\},
 \end{aligned}$$

$$\begin{aligned}
 (\kappa_F)_{\delta \gamma}^{MT}(t_1) &= \delta \cdot \kappa_F(t_1) - \gamma \\
 &\geq \delta \cdot rmin\{\kappa_F(t_1 * t_2), \kappa_F(t_2)\} - \gamma \\
 &= rmin\{\delta \cdot \kappa_F(t_1 * t_2) - \gamma, \delta \cdot \kappa_F(t_2) - \gamma\} \\
 (\kappa_F)_{\delta \gamma}^{MT}(t_1) &= rmin\{(\kappa_F)_{\delta \gamma}^{MT}(t_1 * t_2), (\kappa_F)_{\delta \gamma}^{MT}(t_2)\} \\
 \Rightarrow (\kappa_F)_{\delta \gamma}^{MT}(t_1) &\geq rmin\{(\kappa_F)_{\delta \gamma}^{MT}(t_1 * t_2), (\kappa_F)_{\delta \gamma}^{MT}(t_2)\}
 \end{aligned}$$

and

$$\begin{aligned}
 (v_T)_{\delta \alpha}^{MT}(t_1) &= \delta \cdot v_T(t_1) + \alpha \\
 &\leq \delta \cdot max\{v_T(t_1 * t_2), v_T(t_2)\} + \alpha \\
 &= max\{\delta \cdot v_T(t_1 * t_2) + \alpha, \delta \cdot v_T(t_2) + \alpha\} \\
 (v_T)_{\delta \alpha}^{MT}(t_1) &= max\{(v_T)_{\delta \alpha}^{MT}(t_1 * t_2), (v_T)_{\delta \alpha}^{MT}(t_2)\} \\
 \Rightarrow (v_T)_{\delta \alpha}^{MT}(t_1) &\leq max\{(v_T)_{\delta \alpha}^{MT}(t_1 * t_2), (v_T)_{\delta \alpha}^{MT}(t_2)\},
 \end{aligned}$$

$$\begin{aligned}
 (v_I)_{\delta \beta}^{MT}(t_1) &= \delta \cdot v_I(t_1) + \beta \\
 &\leq \delta \cdot max\{v_I(t_1 * t_2), v_I(t_2)\} + \beta \\
 &= max\{\delta \cdot v_I(t_1 * t_2) + \beta, \delta \cdot v_I(t_2) + \beta\} \\
 (v_I)_{\delta \beta}^{MT}(t_1) &= max\{(v_I)_{\delta \beta}^{MT}(t_1 * t_2), (v_I)_{\delta \beta}^{MT}(t_2)\}
 \end{aligned}$$

$$\Rightarrow (v_I)_{\delta \beta}^{MT}(t_1) \leq \max\{(v_I)_{\delta \beta}^{MT}(t_1 * t_2), (v_I)_{\delta \beta}^{MT}(t_2)\},$$

$$\begin{aligned} (v_F)_{\delta \gamma}^{MT}(t_1) &= \delta \cdot v_F(t_1) - \gamma \\ &\leq \delta \cdot \max\{v_F(t_1 * t_2), v_F(t_2)\} - \gamma \\ &= \max\{\delta \cdot v_F(t_1 * t_2) - \gamma, \delta \cdot v_F(t_2) - \gamma\} \\ (v_F)_{\delta \gamma}^{MT}(t_1) &= \max\{(v_F)_{\delta \gamma}^{MT}(t_1 * t_2), (v_F)_{\delta \gamma}^{MT}(t_2)\} \\ \Rightarrow (v_F)_{\delta \gamma}^{MT}(t_1) &\leq \max\{(v_F)_{\delta \gamma}^{MT}(t_1 * t_2), (v_F)_{\delta \gamma}^{MT}(t_2)\}, \end{aligned}$$

for all $t_1, t_2 \in Y$ and all for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathbb{E}]$ and $\delta \in (0,1]$. Hence $B_{\delta \alpha, \beta, \gamma}^{MT}$ of B is a **NCID** of Y .

Theorem 3.3.3 If B is a neutrosophic cubic set of Y such that **NCMT** $B_{\delta \alpha, \beta, \gamma}^{MT}$ of B is a **NCID** of Y for all for $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathbb{E}]$ and $\delta \in (0,1]$, then B is a **NCID** of Y .

Proof. Suppose **NCMT** $B_{\delta \alpha, \beta, \gamma}^{MT}$ is a **NCID** of Y for some $\kappa_{T,I,F}$, $\alpha, \beta \in [[0,0], 7]$ and $\gamma \in [[0,0], \mathbb{Y}]$, where for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$ and $\gamma \in [0, \mathbb{E}]$ and $\delta \in (0,1]$ and $t_1, t_2 \in Y$. Then

$$\begin{aligned} \delta \cdot \kappa_T(0) + \alpha &= (\kappa_T)_{\delta \alpha}^{MT}(0) \\ &\geq (\kappa_T)_{\delta \alpha}^{MT}(t_1) \\ \delta \cdot \kappa_T(0) + \alpha &= \delta \cdot \kappa_T(t_1) + \alpha, \\ \delta \cdot \kappa_I(0) + \beta &= (\kappa_I)_{\delta \beta}^{MT}(0) \\ &\geq (\kappa_I)_{\delta \beta}^{MT}(t_1) \\ \delta \cdot \kappa_I(0) + \beta &= \delta \cdot \kappa_I(t_1) + \beta, \\ \delta \cdot \kappa_F(0) - \gamma &= (\kappa_F)_{\delta \gamma}^{MT}(0) \\ &\geq (\kappa_F)_{\delta \gamma}^{MT}(t_1) \end{aligned}$$

$$\delta \cdot \kappa_F(0) - \gamma = \delta \cdot \kappa_F(t_1) - \gamma,$$

and

$$\begin{aligned} \delta \cdot v_T(0) + \alpha &= (v_T)_{\delta \alpha}^{MT}(0) \\ &\leq (v_T)_{\delta \alpha}^{MT}(t_1) \\ \delta \cdot v_T(0) + \alpha &= \delta \cdot v_T(t_1) + \alpha, \\ \delta \cdot v_I(0) + \beta &= (v_I)_{\delta \beta}^{MT}(0) \\ &\leq (v_I)_{\delta \beta}^{MT}(t_1) \\ \delta \cdot v_I(0) + \beta &= \delta \cdot v_I(t_1) + \beta, \end{aligned}$$

$$\begin{aligned}\delta \cdot v_F(0) - \gamma &= (v_F)_{\delta \gamma}^{MT}(0) \\ &\leq (v_F)_{\delta \gamma}^{MT}(t_1) \\ \delta \cdot v_F(0) - \gamma &= \delta \cdot v_F(t_1) - \gamma,\end{aligned}$$

which imply $\kappa_T(0) \geq \kappa_T(t_1)$, $\kappa_I(0) \geq \kappa_I(t_1)$, $\kappa_F(0) \geq \kappa_F(t_1)$ and $v_T(0) \leq v_T(t_1)$, $v_I(0) \leq v_I(t_1)$, $v_F(0) \leq v_F(t_1)$. Now, we have

$$\begin{aligned}\delta \cdot \kappa_T(t_1) + \alpha &= (\kappa_T)_{\delta \alpha}^{MT}(t_1) \\ &\geq rmin\{(\kappa_T)_{\delta \alpha}^{MT}(t_1 * t_2), (\kappa_T)_{\delta \alpha}^{MT}(t_2)\} \\ &= rmin\{\delta \cdot \kappa_T(t_1 * t_2) + \alpha, \delta \cdot \kappa_T(t_2) + \alpha\} \\ \delta \cdot \kappa_T(t_1) + \alpha &= \delta \cdot rmin\{\kappa_T(t_1 * t_2), \kappa_T(t_2)\} + \alpha,\end{aligned}$$

$$\begin{aligned}\delta \cdot \kappa_I(t_1) + \beta &= (\kappa_I)_{\delta \beta}^{MT}(t_1) \\ &\geq rmin\{(\kappa_I)_{\delta \beta}^{MT}(t_1 * t_2), (\kappa_I)_{\delta \beta}^{MT}(t_2)\} \\ &= rmin\{\delta \cdot \kappa_I(t_1 * t_2) + \beta, \delta \cdot \kappa_I(t_2) + \beta\} \\ \delta \cdot \kappa_I(t_1) + \beta &= \delta \cdot rmin\{\kappa_I(t_1 * t_2), \kappa_I(t_2)\} + \beta,\end{aligned}$$

$$\begin{aligned}\delta \cdot \kappa_F(t_1) - \gamma &= (\kappa_F)_{\delta \gamma}^{MT}(t_1) \\ &\geq rmin\{(\kappa_F)_{\delta \gamma}^{MT}(t_1 * t_2), (\kappa_F)_{\delta \gamma}^{MT}(t_2)\} \\ &= rmin\{\delta \cdot \kappa_F(t_1 * t_2) - \gamma, \delta \cdot \kappa_F(t_2) - \gamma\} \\ \delta \cdot \kappa_F(t_1) - \gamma &= \delta \cdot rmin\{\kappa_F(t_1 * t_2), \kappa_F(t_2)\} - \gamma\end{aligned}$$

and

$$\begin{aligned}\delta \cdot v_T(t_1) + \alpha &= (v_T)_{\delta \alpha}^{MT}(t_1) \\ &\leq max\{(v_T)_{\delta \alpha}^{MT}(t_1 * t_2), (v_T)_{\delta \alpha}^{MT}(t_2)\} \\ &= max\{\delta \cdot v_T(t_1 * t_2) + \alpha, \delta \cdot v_T(t_2) + \alpha\} \\ \delta \cdot v_T(t_1) + \alpha &= \delta \cdot max\{v_T(t_1 * t_2), v_T(t_2)\} + \alpha,\end{aligned}$$

$$\begin{aligned}\delta \cdot v_I(t_1) + \beta &= (v_I)_{\delta \beta}^{MT}(t_1) \\ &\leq max\{(v_I)_{\delta \beta}^{MT}(t_1 * t_2), (v_I)_{\delta \beta}^{MT}(t_2)\} \\ &= max\{\delta \cdot v_I(t_1 * t_2) + \beta, \delta \cdot v_I(t_2) + \beta\} \\ \delta \cdot v_I(t_1) + \beta &= \delta \cdot max\{v_I(t_1 * t_2), v_I(t_2)\} + \beta,\end{aligned}$$

$$\begin{aligned}\delta \cdot v_F(t_1) - \gamma &= (v_F)_{\delta \gamma}^{MT}(t_1) \\ &\leq max\{(v_F)_{\delta \gamma}^{MT}(t_1 * t_2), (v_F)_{\delta \gamma}^{MT}(t_2)\} \\ &= max\{\delta \cdot v_F(t_1 * t_2) - \gamma, \delta \cdot v_F(t_2) - \gamma\}\end{aligned}$$

$$\delta \cdot v_F(t_1) - \gamma = \delta \cdot \max\{v_F(t_1 * t_2), v_F(t_2)\} - \gamma$$

which imply $\kappa_T(t_1) \geq r\min\{\kappa_T(t_1 * t_2), \kappa_T(t_2)\}$, $\kappa_I(t_1) \geq r\min\{\kappa_I(t_1 * t_2), \kappa_I(t_2)\}$, $\kappa_F(t_1) \geq r\min\{\kappa_F(t_1 * t_2), \kappa_F(t_2)\}$ and $v_T(t_1) \leq \max\{v_T(t_1 * t_2), v_T(t_2)\}$, $v_I(t_1) \leq \max\{v_I(t_1 * t_2), v_I(t_2)\}$, $v_F(t_1) \leq \max\{v_F(t_1 * t_2), v_F(t_2)\}$ for all $t_1, t_2 \in Y$. Hence B is a **NCID** of Y .

Theorem 3.3.4 Intersection of any two **NCMT** of a **NCID** B of Y is a **NCID** of Y .

Proof. Suppose $B_{\delta, \alpha, \beta, \gamma}^{M T}$ and $B_{\delta', \alpha', \beta', \gamma'}^{M T}$ are two **NCMTs** of **NCID** B of Y , where for $B_{\alpha, \beta, \gamma}^{M T}$, for $\kappa_{T, I, F}$, $\alpha, \beta \in [0, 0], \gamma \in [0, 0], \gamma \in [0, \Gamma]$, for $v_{T, I, F}$, $\alpha, \beta \in [0, \Gamma], \gamma \in [0, \varepsilon]$ and for $B_{\alpha', \beta', \gamma'}^{M T}$, for $\kappa_{T, I, F}$, $\alpha', \beta' \in [0, 0], \gamma' \in [0, 0], \gamma' \in [0, \Gamma]$, for $v_{T, I, F}$, $\alpha', \beta' \in [0, \Gamma], \gamma' \in [0, \varepsilon]$. Assume $\alpha \leq \alpha'$, $\beta \leq \beta'$, $\gamma \leq \gamma'$ and $\delta = \delta'$. Then by Theorem 3.3.3, $B_{\delta, \alpha, \beta, \gamma}^{M T}$ and $B_{\delta', \alpha', \beta', \gamma'}^{M T}$ are **NCIDs** of Y . So

$$\begin{aligned} ((\kappa_T)_{\delta, \alpha}^{M T} \cap (\kappa_T)_{\delta', \alpha'}^{M T})(t_1) &= r\min\{(\kappa_T)_{\delta, \alpha}^{M T}(t_1), (\kappa_T)_{\delta', \alpha'}^{M T}(t_1)\} \\ &= r\min\{\delta \cdot \kappa_T(t_1) + \alpha, \delta' \cdot \kappa_T(t_1) + \alpha'\} \\ &= \delta \cdot \kappa_T(t_1) + \alpha \\ ((\kappa_T)_{\delta, \alpha}^{M T} \cap (\kappa_T)_{\delta', \alpha'}^{M T})(t_1) &= (\kappa_T)_{\delta, \alpha}^{M T}(t_1), \end{aligned}$$

$$\begin{aligned} ((\kappa_I)_{\delta, \beta}^{M T} \cap (\kappa_I)_{\delta', \beta'}^{M T})(t_1) &= r\min\{(\kappa_I)_{\delta, \beta}^{M T}(t_1), (\kappa_I)_{\delta', \beta'}^{M T}(t_1)\} \\ &= r\min\{\delta \cdot \kappa_I(t_1) + \beta, \delta' \cdot \kappa_I(t_1) + \beta'\} \\ &= \delta \cdot \kappa_I(t_1) + \beta \\ ((\kappa_I)_{\delta, \beta}^{M T} \cap (\kappa_I)_{\delta', \beta'}^{M T})(t_1) &= (\kappa_I)_{\delta, \beta}^{M T}(t_1), \end{aligned}$$

$$\begin{aligned} ((\kappa_F)_{\delta, \gamma}^{M T} \cap (\kappa_F)_{\delta', \gamma'}^{M T})(t_1) &= r\min\{(\kappa_F)_{\delta, \gamma}^{M T}(t_1), (\kappa_F)_{\delta', \gamma'}^{M T}(t_1)\} \\ &= r\min\{\delta \cdot \kappa_F(t_1) - \gamma, \delta' \cdot \kappa_F(t_1) - \gamma'\} \\ &= \delta' \cdot \kappa_F(t_1) - \gamma' \\ ((\kappa_F)_{\delta, \gamma}^{M T} \cap (\kappa_F)_{\delta', \gamma'}^{M T})(t_1) &= (\kappa_F)_{\delta', \gamma'}^{M T}(t_1) \end{aligned}$$

and

$$\begin{aligned} ((v_T)_{\delta, \alpha}^{M T} \cap (v_T)_{\delta', \alpha'}^{M T})(t_1) &= \max\{(v_T)_{\delta, \alpha}^{M T}(t_1), (v_T)_{\delta', \alpha'}^{M T}(t_1)\} \\ &= \max\{\delta \cdot v_T(t_1) + \alpha, \delta' \cdot v_T(t_1) + \alpha'\} \\ &= \delta' \cdot v_T(t_1) + \alpha' \\ ((v_T)_{\delta, \alpha}^{M T} \cap (v_T)_{\delta', \alpha'}^{M T})(t_1) &= (v_T)_{\delta', \alpha'}^{M T}(t_1), \end{aligned}$$

$$\begin{aligned} ((v_I)_{\delta, \beta}^{M T} \cap (v_I)_{\delta', \beta'}^{M T})(t_1) &= \max\{(v_I)_{\delta, \beta}^{M T}(t_1), (v_I)_{\delta', \beta'}^{M T}(t_1)\} \\ &= \max\{\delta \cdot v_I(t_1) + \beta, \delta' \cdot v_I(t_1) + \beta'\} \\ &= \delta' \cdot v_I(t_1) + \beta' \\ ((v_I)_{\delta, \beta}^{M T} \cap (v_I)_{\delta', \beta'}^{M T})(t_1) &= (v_I)_{\delta', \beta'}^{M T}(t_1), \end{aligned}$$

$$\begin{aligned}
((v_F)_{\delta Y}^{MT} \cap (v_F)_{\delta' Y'}^{MT})(t_1) &= \max\{(v_F)_{\delta Y}^{MT}(t_1), (v_F)_{\delta' Y'}^{MT}(t_1)\} \\
&= \max\{\delta \cdot v_F(t_1) - \gamma, \delta' \cdot v_F(t_1) - \gamma'\} \\
&= \delta \cdot v_F(t_1) - \gamma \\
((v_F)_{\delta Y}^{MT} \cap (v_F)_{\delta' Y'}^{MT})(t_1) &= (v_F)_{\delta Y}^{MT}(t_1).
\end{aligned}$$

Hence $B_{\delta \alpha, \beta, \gamma}^{MT} \cap B_{\delta' \alpha', \beta', \gamma'}^{MT}$ is **NCID** of Y .

Theorem 3.3.5 Union of any two **NCMT** $B_{\delta \alpha, \beta, \gamma}^{MT}$ of a **NCID** B of Y is a **NCID** of Y .

Proof. Suppose $B_{\delta \alpha, \beta, \gamma}^{MT}$ and $B_{\delta' \alpha', \beta', \gamma'}^{MT}$ are two NCMTs of NCID B of Y , where for $B_{\alpha, \beta, \gamma}^{MT}$, for $\kappa_{T,I,F}$, $\alpha, \beta \in [0, 0], \gamma \in [0, 0], \gamma' \in [0, 0], \gamma \neq \gamma'$, for $v_{T,I,F}$, $\alpha, \beta \in [0, \Gamma]$, $\gamma \in [0, \varepsilon]$ and for $B_{\alpha', \beta', \gamma'}^{MT}$, for $\kappa_{T,I,F}$, $\alpha', \beta' \in [0, 0], \gamma' \in [0, 0], \gamma' \neq \gamma$, for $v_{T,I,F}$, $\alpha', \beta' \in [0, \Gamma]$, $\gamma' \in [0, \varepsilon]$. Assume $\alpha \geq \alpha'$, $\beta \geq \beta'$, $\gamma \geq \gamma'$ and $\delta = \delta'$. Then by Theorem 3.3.3, $B_{\delta \alpha, \beta, \gamma}^{MT}$ and $B_{\delta' \alpha', \beta', \gamma'}^{MT}$ are NCIDs of Y . So

$$\begin{aligned}
((\kappa_T)_{\delta \alpha}^{MT} \cup (\kappa_T)_{\delta' \alpha'}^{MT})(t_1) &= rmax\{(\kappa_T)_{\delta \alpha}^{MT}(t_1), (\kappa_T)_{\delta' \alpha'}^{MT}(t_1)\} \\
&= rmax\{\delta \cdot \kappa_T(t_1) + \alpha, \delta' \cdot \kappa_T(t_1) + \alpha'\} \\
&= \delta \cdot \kappa_T(t_1) + \alpha \\
((\kappa_T)_{\delta \alpha}^{MT} \cup (\kappa_T)_{\delta' \alpha'}^{MT})(t_1) &= (\kappa_T)_{\delta \alpha}^{MT}(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_I)_{\delta \beta}^{MT} \cup (\kappa_I)_{\delta' \beta'}^{MT})(t_1) &= rmax\{(\kappa_I)_{\delta \beta}^{MT}(t_1), (\kappa_I)_{\delta' \beta'}^{MT}(t_1)\} \\
&= rmax\{\delta \cdot \kappa_I(t_1) + \beta, \delta' \cdot \kappa_I(t_1) + \beta'\} \\
&= \delta \cdot \kappa_I(t_1) + \beta \\
((\kappa_I)_{\delta \beta}^{MT} \cup (\kappa_I)_{\delta' \beta'}^{MT})(t_1) &= (\kappa_I)_{\delta \beta}^{MT}(t_1),
\end{aligned}$$

$$\begin{aligned}
((\kappa_F)_{\delta Y}^{MT} \cup (\kappa_F)_{\delta' Y'}^{MT})(t_1) &= rmax\{(\kappa_F)_{\delta Y}^{MT}(t_1), (\kappa_F)_{\delta' Y'}^{MT}(t_1)\} \\
&= rmax\{\delta \cdot \kappa_F(t_1) - \gamma, \delta' \cdot \kappa_F(t_1) - \gamma'\} \\
&= \delta' \cdot \kappa_F(t_1) - \gamma' \\
((\kappa_F)_{\delta Y}^{MT} \cup (\kappa_F)_{\delta' Y'}^{MT})(t_1) &= (\kappa_F)_{\delta' Y'}^{MT}(t_1)
\end{aligned}$$

and

$$\begin{aligned}
((v_T)_{\delta \alpha}^{MT} \cup (v_T)_{\delta' \alpha'}^{MT})(t_1) &= min\{(v_T)_{\delta \alpha}^{MT}(t_1), (v_T)_{\delta' \alpha'}^{MT}(t_1)\} \\
&= min\{\delta \cdot v_T(t_1) + \alpha, \delta' \cdot v_T(t_1) + \alpha'\} \\
&= \delta' \cdot v_T(t_1) + \alpha' \\
((v_T)_{\delta \alpha}^{MT} \cup (v_T)_{\delta' \alpha'}^{MT})(t_1) &= (v_T)_{\delta' \alpha'}^{MT}(t_1),
\end{aligned}$$

$$\begin{aligned}
((v_I)_{\delta \beta}^{MT} \cup (v_I)_{\delta' \beta'}^{MT})(t_1) &= min\{(v_I)_{\delta \beta}^{MT}(t_1), (v_I)_{\delta' \beta'}^{MT}(t_1)\} \\
&= min\{\delta \cdot v_I(t_1) + \beta, \delta' \cdot v_I(t_1) + \beta'\} \\
&= \delta' \cdot v_I(t_1) + \beta'
\end{aligned}$$

$$((v_I)_{\delta \beta}^{MT} \cup (v_I)_{\delta' \beta'}^{MT})(t_1) = (v_I)_{\delta' \beta'}^{MT}(t_1),$$

$$\begin{aligned} ((v_F)_{\delta \gamma}^{MT} \cup (v_F)_{\delta' \gamma'}^{MT})(t_1) &= \min\{(v_F)_{\delta \gamma}^{MT}(t_1), (v_F)_{\delta' \gamma'}^{MT}(t_1)\} \\ &= \min\{\delta \cdot v_F(t_1) - \gamma, \delta' \cdot v_F(t_1) - \gamma'\} \\ &= \delta \cdot v_F(t_1) - \gamma \\ ((v_F)_{\delta \gamma}^{MT} \cup (v_F)_{\delta' \gamma'}^{MT})(t_1) &= (v_F)_{\delta \gamma}^{MT}(t_1). \end{aligned}$$

Hence $B_{\delta, \beta, \gamma}^{MT} \cup B_{\delta', \beta', \gamma'}^{MT}$ is NCID of Y.

4. Conclusion

In this paper, we defined neutrosophic cubic translation,, neutrosophic cubic multiplication and neutrosophic cubic magnified translation for neutrosophic cubic set on BF-algebra. We provided the new sort of different conditions for neutrosophic cubic translation, neutrosophic cubic multiplication and neutrosophic cubic magnified translation and proved with examples. Moreover, for better understanding we investigated many results for NCT, NCM and NCMT using the subalgebra and ideals. For future work, translation and multiplication can be applied on neutrosophic cubic soft set and T-neutrosophic cubic set.

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