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Triangular Neutrosophic Based Production Reliability Model of Deteriorating Item with Ramp Type Demand under Shortages and Time Discounting

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Abstract: An economic production quantity model with triangular neutrosophic environment has been developed for deteriorating items with ramp type demand rate and reliability dependent unit production. The main objective of this paper is to determine the most cost effective production to generate better quality items under time discounting. Additionally, it is considered that the deterioration function deals with three parameters Weibull's distribution under finite time horizon. Moreover, it also considered the effect of shortages which are partially backordered and partially lost in sale. Here the reliability of the production process along with the production period is considered as decision variables. A numerical example is studied in both crisp and neutrosophic environment and a comparative analysis is performed here. It is observed that the model performs better in triangular neutrosophic arena rather than crisp domain. Finally, a sensitivity analysis of optimal solution is observed for some parameters and some crucial decision is taken with managerial insight.

Keywords: Ramp-type demand, Finite time horizon, Time-value of money, Reliability, Triangular Neutrosophic number.

1. Introduction

In market economy system, for a single product, many items are produced by the different manufacturing companies. The manufacturers are trying to give wide variety of option to the customer to gain competitive advantages over their competitors. But customers choose those items which have high reliability i.e. better in quality, and lower in cost. The companies require advanced planning many years prior to the sale target date in order to minimize the total cost and maximize the profit. Thus the facts like variation in the reliability of the production process, demand rate of an item, deterioration and shortages are in growing interest. In case of classical EPQ model the basic assumptions are that the production set-up cost is fixed and the item produced are of perfect quality. All the manufacturing sectors want to produce perfect quality item, but in reality the product quality are not always perfect because there may be machine breakdown, labor problem, etc. The product quality is directly affected by the reliability of production process. In addition to that, the classical models also consider an ideal case that the demand and quality of the items remains unaffected by time and replenishment is done instantaneously. However in reality these assumptions do not hold. The inventories are often replenished periodically at certain production rate. Even if the items are purchased it takes days to sell the item so the items remained stored and hence the item deteriorates and their value reduces with time. Cheng [1] proposed a general equation for relationship between production set up cost and process reliability and flexibility. Later it was used by (Leung [2]; Bag et al. [3]) in their respective models studied on fuzzy random demand with flexibility and reliability on production process. Sarkar [4] analyzed an EMQ model with reliability in an imperfect production process. Many researchers (like Gomez et al. [5]; Cai et al. [6]) worked for production quality, tracking production control, etc. Pan and Li [7] worked with stochastic production system for deteriorating item with some environmental constrains. Rathore [8] explored a production reliability model with advertisement related demand. The paper considers reliability in unit production cost in order to identify the product quality with minimum total cost.

Traditionally in inventory models, the researchers have assumed constant demand pattern in their deterministic models, but in reality demand has specific patterns which depicts the real scenarios in market. There are various types of demand rates such as linear or quadratic function of time, exponentially increasing or decreasing, price and stock dependent, etc. If the demand is linearly dependent on time i.e., demand as well as the vending increases and decreases in growth and decline phase respectively. Researchers have manifested these demands in their respective papers (Hariga [9], Bose et al. [10], etc). Demand of the item depending on price and stocking amount of the items with optimal replenishment policy for non-instantaneous deteriorating items with partial backlogging was discussed by Wu et al. [11]. Alfares [12] worked on stock dependent demand. Chung and Wee [13] organized an inventory model for stock dependent selling rate with deterioration under replenishment plan. Pal et al. [14] has developed a inventory model with price and stock depended demand rate for deteriorating item under inflation and delay in payment. In this field, some remarkable researches were done by Yang et al. [15]. It was observed that for seasonal and fashionable products the nature of demand is increasing-steady-decreasing. But for newly launched fashion goods and cosmetics, garments, etc. the demand rate increases linearly with time and then it become constant. Thus to understand the concept of such a demand, the ramp type function of time was introduced. (Skouri et al. [16], Luo [17], Manna and Chaudhari [18]) worked with ramp type demand rate with time dependent deterioration. Pal et al. [19] considered the EOQ model with ramp type demand under finite time horizon.

As the effect of deterioration cannot be ignored so many researchers worked on it (Skouri et al. [20], Jaggi et al. [21], etc.). Generally, deterioration means spoilage or damage obsolescence, etc. which cannot be used further for its original purpose. Medicine, blood banks, etc. are difficult to preserve and they have some expiry date i.e., products maximum life time is time bounded. Electronic products become obsolete as technology changes; new fashion depreciates the clothing value over time; all these are also considered as deterioration. It has been observed that the delinquency in the life expectancy drugs, deterioration of roasted ground coffee, corn seeds, frozen food, pasteurized milk, refrigerated meat, ice creams, and leakage failure of the batteries can be expressed in terms of Weibull's distribution. Wu [22] presented an inventory model with ramp type demand and Weibull's distribution deterioration under partial backlogging. Many researcher such as Skouri et al. [23], Sharma and Chaudhury [24], etc. worked with this type of deterioration. Mandal [25] discussed an inventory model with Weibull's distributed deterioration with ramp type demand rate. A common characteristic in most of these models are that they does not allows shortages. Widyadana et al. [26] developed an EOQ model for deteriorating items with planned backorder level. Wee et al. al. [27] worked with shortages and finite time horizon for deteriorating items. Yang [28] developed an inventory model with deterioration as three parameter Weibull's distribution in two ware house system. Recently Pal and Chakraborty [29] have worked on non- instantaneous deteriorating items under shortage, Rahaman et al. [30] worked on arbitrary ordered generalized EPQ model with and without deterioration. In this paper shortages is also considered where the part of the unsatisfied demand are backordered and part of the sales are lost.

As the amount of the money available at the present time is worth more than that of the same amount in the future due to its potential earning capacity. So it is necessary to consider the effect of time value of money in today's inventory where forecasting is required. To consider the effect of time value of money, a finite time horizon for planning the replenishment cycle is considered. From the last few decades we have observed that the economic situation of most countries has changes so it would be unrealistic to ignore the effect of time value of money. Hariga [31] developed the effect of inflation and time value of money for time dependent demand. Hou [32] considered a model for deteriorating items and stock-dependent demand rate with shortages and time discounting. Dash et al. [33] worked on EPQ model for declined quadratic demand with time value of money and shortages. Thus the paper considers time value of money specially when investment and forecasting are considered.

In this current century, vagueness theory plays a crucial role in different Öeld of mathematical modeling and engineering problems. The theory of impreciseness was first invented by Zadeh [34]. Difference between crisp set and fuzzy set is shown briefly in this article by considering membership gradation and its formulation. Demonstration of triangular [35], trapezoidal [36], pentagonal [37] fuzzy number has already been developed by the researchers. In 1983 and later in 1986 Attasonov [38, 39] manifested a remarkable idea of intuitionistic fuzzy set where membership and non-membership functions are both considered together. Further, triangular intuitionistic [40, 41], trapezoidal intuitionistic [42] number has been introduced in this intuitionistic fuzzy research arena. After that, in 1998 Smarandache [43] established an amazing concept of neutrosophic fuzzy set where three disjunctive kinds of membership functions has been considered namely i) truthness ii) falseness iii) indeterminacy. Due to the presence of hesitation factor in fuzzy arena, neutrosophic number becomes more logical and scientific significance in research work. In this current era, researchers from different arena are focusing on neutrosophic concept and developed lots of interesting articles in this domain. Illustration of triangular, trapezoidal neutrosophic number has been introduced day by day and recently in 2018 Chakraborty et.al [44, 45] classifies different form of triangular and trapezoidal neutrosophic number and de-neutrosophication technique for crispification. Further, bipolarization of triangular bipolar number has been developed by

Chakraborty et.al [46] and also Maity et.al [47] manifested the concept of heptagonal dense fuzzy number related EOQ based model in 2018. Recently, Mullai [48] introduced EOQ model in neutrosophic domain and Mondal et.al [49] manifested optimization of EOQ Model with limited storage capacity by neutrosophic Geometric Programming application. Also, Majumdar et.al [50] focused on EPQ Model of deteriorating Items under partial trade credit financing and demand declining market in neutrosophic environment. Some useful articles [51-58] are also developed by the researchers in the neutrosophic arena recently. As developments goes on, some researchers [59-62] have extended the idea of neutrosophic set into plithogenic set and applied it in MCDM, MADM and optimization technique supply chain based model. Currently, several researchers from distinct fields focused on triangular neutrosophic number related to operation research models. As uncertainty prevails in various parameters such as inflation, holding cost, purchase cost so we have developed an EPQ under ramp type demand and considered the hesitation in those parameters by considering those parameter as neutrosophic number. Finally we compare the model in crisp and neutrosophic domain and observe that the model works better in neutrosophic arena.

Previously the researchers have worked on ramp type demand with two parameter Weibull's distribution as deterioration. But in this paper we have considered ramp type demand with three parameter Weibull's distribution. In addition the model assumes that the product qualities are never perfect and it is the function of reliability of the production process so the production of items depend on the reliability of the items i.e., if the items are highly reliable then there is more demand in the market and hence its production should be more in order to fulfill the demand. In this model we also have considered finite planning horizon to observe the effect of time value of money under shortage. The shortage items are partially backlogged or partially lost in sales, which cannot be ignored. Also under this complicated scenario no work has been done by considering holding cost, purchase cost and inflation as triangular neutrosophic number.

The rest of the paper is organized as follows: In section 2 we have presented some assumptions and notations and some definition of neutrosophic number that we have used in this paper. In this section we have defined few terminologies related to triangular neutrosophic number and also have formulated the model. In section 3 we have analyzed and optimized of the model. In Section 4 we have discussed the de-neutrosophication of the triangular neutrosophic number. In section 5 we present the numerical example and its mathematical analysis which is shown graphically. It is observed that the model works better in neutrosophic domain. In section 6 we present sensitivity analysis of some parameters. Finally in section 7 a concluding remark is stated along with its future extension.

2. Mathematical formulation of the inventory model

In this model we have considered ramp type demand with deterioration as three parameter Weibull distributions, shortages, lost in sales under the influence of time discounting in finite planning horizon. The finite time horizon has been considered to evaluate the effect of inflation on the total cost for a finite period. The paper also considered reliability in production of items. The proposed model is graphically shown in figure-1.

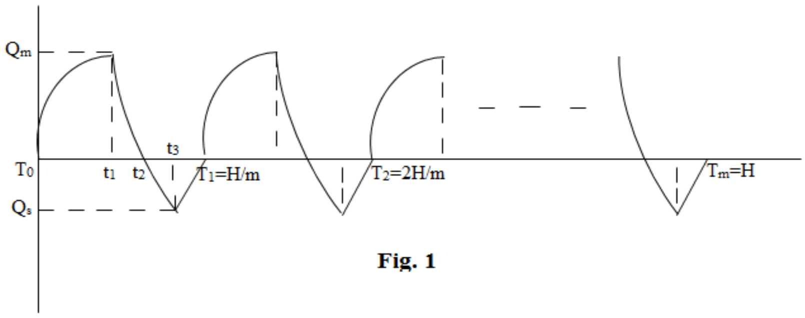

Fig. 1

The production process starts from t=0 and ends $t=t_1$. The production has occurred along with the demand in the market and at $t=t_1$ the inventory level is maximum, Q_m. From $t=t_1$ to $t=t_2$ the inventory level decreases and at time $t=t_2$, the inventory level reaches zero. Now during $[t_2,t_3]$ the model undergoes shortage with partial backlog and partial lost in sales. Only the backlogged items are replaced by the next replenishment. During $[t_3,T_1]$ production resumes to overcome the shortage (i.e., for backlogged items). Thus the total number of backlogged items is replaced in the next replenishment and the cycle repeats.

Notations

The notations used in this paper are as follows:

G Demand rate,

P Production rate,

p Unit production cost,

$\varrho(t)$ Time distribution for deterioration of the item,

k Discount rate,

h Inventory carrying cost per unit item per unit time,

d Deterioration cost per unit per unit time,

S Set-up cost for one replenishment cycle.

c_1 Purchase cost per unit item,

c_2 Shortage cost,

c_3 Penalty cost of a lost sale including loss of profit,

r Production process reliability (a decision variable)

B Fraction of backorder ($0<B\leq 1$),

T Replenishment cycle,

H Finite Planning horizon,

m No. of replenishment during the planning horizon i.e., $m=(H/T)$,

T_j Time between start and end of j^{th} replenishment cycle i.e., $T_0=0,T_1=T,T_2=2T,\ldots,T_m=mT=H$,

Q_m Maximum quantity of inventory,

Q_s Maximum quantity of inventory after shortage.

Assumptions

The assumptions which are considered in this model are as follows.

1. A ramp type demand rate G=f(t) is a function of time $f(t) = R[t - (t - \mu)H(t - \mu)], R > 0$ and $H(t)$ is a Heaviside function $H(t - \mu) = \begin{cases} 1 & if\ t \geq \mu \\ 0 & if\ t < \mu \end{cases}$

2. A function of three parameter Weibull's distribution of time is used to represent deterioration of the item is $\rho(t) = \alpha\beta(t - \gamma)^{\beta - 1}, 0 < \alpha < 1, \beta \geq 1, -\infty < \gamma < \infty$ actually in this model $T_j < \gamma < T_{j+1}, i = 0,1,2, \ldots, m, where\ \alpha\ (0 < \alpha < 1)$ is a scaling parameter, β is the shape parameter and γ is the location parameter i.e., items shelf-time and t is the time of deterioration.

3. Deterioration begins as it reaches the inventory.

4. One item is considered in the prescribed time cycle.

5. Demand during shortage is partially lost and partially backordered.

6. Time discounting effect is considered under finite time horizon.

7 Production rate is greater than demand rate so P=σf(t) is the production rate where σ >1.

8. μ is less than production time.

9. The unit production cost is inversely proportional to the demand rate (G) and directly proportional to production reliability (r), so the unit production cost is $p = aG^{-b}r^{c}$, where b(>1) is called price elasticity and a,c (>0) are scaling parameters.

10. The reliability r means, r% of all the item produced are of acceptable quality that can fulfill the demand.

Few assumptions taken above are the basic assumption used in classical inventory model for deteriorating item with shortages. The first assumption states that the demand rate linearly increases with time when t<μ and then become steady i.e., constant at and after t≥μ. We can see this type of demand in newly launched items like fashionable products, electronic items, etc. The demand increases with time during the initial stage i.e., [0,μ]. After some time the demand become constant, this continues for some period i.e., in the time interval [μ,T_1]. Then the cycle ends. Again the next cycle starts with another new brand item and it will follow the same pattern of demand and production i.e., increasing and then steady and then stops. The finite time horizon has been considered to evaluate the effect of the time value of money on the total cost. Thus to understand the concept of value of future money in present date (which actually decreases due to time discounting rate) we need to consider a finite time horizon where its effect will be observed. The last assumption is mainly based on the unit variable production which is dependent on demand and process reliability. When the demand of an item increases then the production/purchase cost per unit item decreases and hence the unit production cost reduces which is inversely proportional to demand. Again the reliability of the produced items increases by using high quality raw material, technologically advanced machinery, quality control inspections, etc. Thus to produce high reliable product the production cost per unit item increases.

3. Neutrosophic number and its De-neutrosophication technique

Definition 3.1 (Neutrosophic Set [5]) A set $\tilde{S}$ in the universal discourse X, it is said to be a neutrosophic set if $\tilde{S} = \{\langle x; [\pi_{\tilde{S}}(x), \theta_{\tilde{S}}(x), \eta_{\tilde{S}}(x)]\rangle : x \in X\}, where\ \pi_{\tilde{S}}(x): X \to]-0,1+[$ is called the truth membership function, $\theta_{\tilde{S}}(x): X \to]-0,1+[$ is called the hesitation membership function, and $\eta_{\tilde{S}}(x)$: $X \to]-0,1+[$ is called the false membership function of the decision maker, where $\pi_{\tilde{S}}(x), \theta_{\tilde{S}}(x), \eta_{\tilde{S}}(x)$ satisfies the following condition: $0 \le Sup\{\pi_{\tilde{S}}(x)\} + Sup\{\theta_{\tilde{S}}(x)\} + Sup\{\eta_{\tilde{S}}(x)\} \le 3$.

Definition 3.2 (Single-Valued Neutrosophic Set) A Neutrosophic set $\tilde{S}$ in the above definition 2.1 is also known as single-Valued Neutrosophic Set $sig(\tilde{S})$ if x is a single-valued independent variable.
$sig(\tilde{S}) = \{< x; [\pi_{sig(\tilde{S})}(x), \theta_{sig(\tilde{S})}(x), \eta_{sig(\tilde{S})}(x)]\rangle : x \in X\}$, where $\pi_{sig(\tilde{S})}(x), \theta_{sig(\tilde{S})}(x), \eta_{sig(\tilde{S})}(x)$ represent the concept of truth, hesitation and falsity memberships function respectively.

Definition 3.2.1: (Neutro-normal) Let us consider three points, for which p,q,r for which, $\pi_{sig(\tilde{S})}(p) = 1$, $\theta_{sig(\tilde{S})}(q) = 1, \eta_{sig(\tilde{S})}(r) = 1$ then the $sig(\tilde{S})$ is defined as neutro-normal.

Definition 3.2.2: (Neutro-convex) A $sig(\tilde{S})$ is called neutro-convex if the following condition holds:

$$(i)\pi_{sig(\tilde{S})}(\lambda\alpha + (1-\lambda)\beta) \ge min(\pi_{sig(\tilde{S})}(\alpha), \pi_{sig(\tilde{S})}(\beta))$$
$$(ii)\theta_{sig(\tilde{S})}(\lambda\alpha + (1-\lambda)\beta) \ge min(\theta_{sig(\tilde{S})}(\alpha), \theta_{sig(\tilde{S})}(\beta)),$$
$$(iii)\eta_{sig(\tilde{S})}(\lambda\alpha + (1-\lambda)\beta) \ge min(\eta_{sig(\tilde{S})}(\alpha), \eta_{sig(\tilde{S})}(\beta))$$

$where\ \alpha, \beta \in R, and\ \lambda \in [0,1]$

Definition 3.3 (Triangular Single Valued Neutrosophic Number) A triangular Single Valued Neutrosophic Number $(\tilde{S})$ is defined as $\tilde{S} =< (m_1, m_2, m_3: \mu), (n_1, n_2, n_3: \vartheta), (p_1, p_2, p_3: \zeta) >, where\ \mu, \vartheta, \zeta \in [0,1]$. Here the truth membership function$\pi_{\tilde{S}}: R \to [0, \mu]$, the hesitation membership function $\theta_{\tilde{S}}: R \to [\vartheta, 1]$ and the falsity membership function $\eta_{\tilde{S}}: R \to [\zeta, 1]$ are defined as follows:

$$\pi_{\tilde{S}}(x) = \begin{cases} \delta_{\tilde{S}l}(x), & m_1 \le x < m_2 \\ \mu, & x = m_2 \\ \delta_{\tilde{S}r}(x), & m_2 < x \le m_3 \\ 0, & otherwise \end{cases} \quad \theta_{\tilde{S}(x)} = \begin{cases} \varepsilon_{\tilde{S}l}(x), n_1 \le x < n_2 \\ \vartheta, \quad x = n_2 \\ \varepsilon_{\tilde{S}r}(x), n_2 < x \le n_3 \\ 1, \quad otherwise \end{cases} \quad \eta_{\tilde{S}}(x) = \begin{cases} l_{\tilde{S}l}(x), p_1 \le x < p_2 \\ \vartheta, \quad x = p_2 \\ l_{\tilde{S}r}(x), p_2 < x \le p_3 \\ 1, \quad otherwise \end{cases}$$

De-neutrosophication of triangular single valued neutrosophic number: In this model we have applied removal area technique to evaluate the de-neutrosophication value of triangular single valued neutrosophic number
$\tilde{S} =< (m_1, m_2, m_3: \mu), (n_1, n_2, n_3: \vartheta), (p_1, p_2, p_3: \zeta) >$ as done by (Chakraborty, et. al.). The de-neutrosophic form of $\tilde{S}$ is given as $neuD_{\tilde{S}} = \left(\frac{m_1+2m_2+m_3+n_1+2n_2+n_3+p_1+2p_2+p_3}{12}\right)$

4. Proposed model

Thus the inventory level for the proposed model at any time t over [0,T] is described mathematically by the following equations:

$$\frac{dQ(t)}{dt}+\rho(t)Q(t)=rP-G=(r\sigma-1)Rt, \quad 0\le t\le \mu \tag{1}$$

$$\frac{dQ(t)}{dt}+\rho(t)Q(t)=(r\sigma-1)R\mu, \qquad \mu\le t\le t_1 \tag{2}$$

$$\frac{dQ(t)}{dt}+\rho(t)Q(t)=-G=-R\mu, \qquad t_1\le t\le t_2 \tag{3}$$

$$\frac{dQ(t)}{dt}=-BG=-BR\mu, \qquad t_2\le t\le t_3 \tag{4}$$

$$\frac{dQ(t)}{dt}=rP-G=rK-R\mu=(r\sigma-1)R\mu, t_3\le t\le T_1 \tag{5}$$

with boundary conditions

$$Q(0)=0, Q(\mu)=I, Q(t_1)=Q_m, Q(t_2)=0, Q(t_3)=-Q_s \text{ and } Q(T_1)=0,$$

$$where\ I=(r\sigma-1)R[\left(\frac{\mu^2}{2}\right)+\left(\frac{\alpha\gamma}{\beta+1}\right)(\mu-\gamma)^{\beta+1}+\left(\frac{\alpha}{\beta+2}\right)(\mu-\gamma)^{\beta+2}+\frac{(-1)^{\beta}\gamma^{\beta+2}}{(\beta+1)(\beta+2)}]$$

4.1 Mathematical Analysis of the proposed model

From the above differential equations [1, 2, 3, 4, 5] and using the assumptions and the boundary conditions we obtain the inventory level of the proposed inventory model as follows:

$$Q(t)=(r\sigma-1)R[\left(\frac{t^2}{2}\right)-\left(\frac{\alpha t^2}{2}\right)(t-\gamma)^{\beta}+\left(\frac{\alpha\gamma}{\beta+1}\right)(t-\gamma)^{\beta+1}+\left(\frac{\alpha}{\beta+2}\right)(t-\gamma)^{\beta+2}+\left(\frac{(-1)^{\{\beta\}}\gamma^{\beta+2}}{(\beta+1)(\beta+2)}\right)]$$

$$Q(t)=(r\sigma-1)R[t\mu-\left(\frac{\mu^2}{2}\right)+\mu\alpha(t-\gamma)^{\beta}\left(\left(\frac{t-\gamma}{\beta+1}\right)-t+\left(\frac{\mu}{2}\right)\right)-\left(\frac{\alpha}{(\beta+1)(\beta+2)}\right)\{(\mu-\gamma)^{\beta+2}-(-1)^{\beta}\gamma^{\beta+2}\}] \tag{6}$$

$$Q(t)=R\mu[t_1-t+\left(\frac{\alpha}{\beta+1}\right)\{(t_1-\gamma)^{\beta+1}-(t-\gamma)^{\beta+1}\}+\alpha(t-t_1)(t-\gamma)^{\beta}]+Q_m\left(1-\alpha(t-\gamma)^{\beta}+\alpha(t_1-\gamma)^{\beta}\right), t_1\le t\le t_2 \tag{7}$$

$$Q(t)=-BR\mu(t-t_2),\ \ t_2\le t\le t_3 \tag{8}$$

$$Q(t)=(r\sigma-1)R\mu(t-t_3)-Q_s,\ \ t_3\le t\le T_1 \tag{9}$$

Now using $Q(t_2)=0$ and eq.(6) we get the maximum amount inventory Q_m,

$$Q_m=R\mu[t_2-t_1+\left(\frac{\alpha}{\beta+1}\right)(t_2-\gamma)^{\beta+1}-\alpha(t_1-\gamma)^{\beta}(\left(\frac{t^1-\gamma}{\beta+1}\right)+t_2-t_1)] \tag{10}$$

Now using eq.(8), eq.(9) and the relation $Q(t_3)=-Q_s$ we get the maximum shortages in the inventory level,

$$Q_s=BR\mu(t_3-t_2) \tag{11}$$

Inventory carrying cost or holding cost:

$$HC=h\left[\int_0^{\mu}Q(t)dt+\int_{\mu}^{t_1}Q(t)dt+\int_{t_1}^{t_2}Q(t)dt\right]$$

$$=h[(r\sigma-1)R\{\left(\frac{\mu^4}{6}\right)-\left(\frac{\alpha\beta\mu(\mu-\gamma)^{\beta+3}}{2(\beta+2)(\beta+3)}\right)(\left(\frac{\gamma(\beta+5)}{\beta+1}\right)+\mu+\left(\frac{((-1)^{\beta}\alpha\mu\gamma^{\beta+2})}{(\beta+1)(\beta+2)}\right)\left(\mu-\left(\frac{\gamma}{\beta+3}\right)\right)$$
$$+\left(\frac{\alpha\mu\gamma^2(\mu-\gamma)^{\beta+1}}{2(\beta+1)}\right)+\left(\frac{\mu t_1}{2}\right)(t_1-\mu)-\left(\frac{\alpha\beta\mu(t_1-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right)+R\mu\{-\left(\frac{(t_2-t_1)^2}{2}\right)$$

$$+\left(\frac{\alpha\mu\left(\left(\frac{\mu}{2}\right)-\gamma\right)}{\beta+1}\right)\left[(t_1-\gamma)^{\beta+1}-(\mu-\gamma)^{\beta+1}\right]+\left(\frac{(\alpha(\mu-\gamma)^{\beta+2})}{\beta+2}\right)\left(\mu-\left(\frac{t_1}{\beta+1}\right)\right)+\left(\frac{(-1)^{\beta}\alpha\gamma^{(\beta+2)}}{(\beta+1)(\beta+2)}\right)(t_1-\mu)\}+$$

$$\left(\frac{\alpha(t_1-\gamma)(t_2-\gamma)}{\beta+1}\right)\left[(t_1-\gamma)^{\beta}-(t_2-\gamma)^{\beta}\right]+Q_m\left(t_2-t_1-\left(\frac{\alpha(t_2-\gamma)^{\beta+1}}{\beta+1}\right)+\alpha(t_1-\gamma)^{\beta}\left(\left(\frac{t_1-\gamma}{\beta+1}\right)-t_2-t_1\right)\right)\}] \quad (12)$$

Production cost: The unit production cost depends on demand and process reliability. When the demand of an item increases then the production/purchase cost of the item decreases hence the unit production cost reduces i.e., production / purchase cost varies inversely with demand. The process reliability level r means only r% of the produced items is of acceptable quality which can be used to meet demand.
The unit production cost $p = aD^{-b}r^{c}$ *where* $a, b, c > 0$ *and* $b \neq 2$.

The cost of production in $[t, t+dt]$ *is* $Kpdt = \sigma D.aD^{-b}r^{c}dt = \left(\frac{\sigma ar^{c}}{D^{b-1}}\right)dt$.

Since the production occurs $[0,t_1]$ and $[t_3,T_1]$ so the production cost (PDC) is given as follows.

Production cost (PDC)= $\int_0^{\mu}\left(\frac{\sigma ar^{c}}{D^{b-1}}\right)dt+\int_{\mu}^{t_1}\left(\frac{\sigma ar^{c}}{D^{b-1}}\right)dt+\int_{t_3}^{T_1}\left(\frac{\sigma ar^{c}}{D^{b-1}}\right)dt$

$$=\sigma ar^{c}\left[\int_0^{\mu}(Rt)^{1-b}dt+\int_{\mu}^{t_1}(R\mu)^{1-b}dt+\int_{t_3}^{T_1}(R\mu)^{1-b}dt\right]$$

$$=\left(\frac{\sigma ar^{c}R^{1-b}}{2-b}\right)\left[(b-1)\mu^{2-b}+(2-b)\mu^{1-b}(t_1+T_1-t_3)\right], b\neq 2 \quad (13)$$

Deterioration cost: The total no. of deteriorated items in $[0,T_1]$ is same as deterioration in $[0,t_2]$ as there is no deterioration of items during the period $[t_2,T_1]$.
D_1=Total no. of deteriorated items in $[0,t_2]$
=r×Production in $[0,\mu]$+r×Production in $[\mu,t_1]$-Demand in $[0,\mu]$-Demand in $[\mu,t_2]$

$$=r\sigma\int_0^{\mu}Rtdt+r\sigma\int_{\mu}^{t_1}R\mu dt-\int_0^{\mu}Rtdt-\int_{\mu}^{t_2}R\mu dt$$

$$=\left(\frac{1}{2}\right)Rr\mu\sigma(2t_1-\mu)-\left(\frac{1}{2}\right)R\mu(2t_2-\mu)$$

$$\therefore Deterioration\ cost\ (DC)=(dD_1)=\left(\frac{R\mu d}{2}\right)(r\sigma(2t_1-\mu)-(2t_2-\mu)) \quad (14)$$

Purchase cost: Since there is shortages in our model so the producer has to purchase raw material not only during $[0,t_1]$ but also in $[t_3, T_1]$. So we have to calculate purchase cost during the above two period.

$$PC=c_1\sigma\left(\int_0^{\mu}Rtdt+r\sigma\int_{\mu}^{t_1}R\mu dt+\int_{t_3}^{T_1}R\mu dt\right)=c_1\sigma R\mu\left(t_1+T_1-t_3-\left(\frac{\mu}{2}\right)\right) \quad (15)$$

Shortage cost: Since the model undergoes shortages so we observe shortages during $[t_2, T_1]$.

$$SC=c_2\int_{t_2}^{t_3}-Q(t)dt+c_2\int_{t_3}^{T_1}-Q(t)dt=\left(\frac{c^2R\mu}{2}\right)\left[B(t_3-t_2)^2+(r\sigma-1)(T_1-t_3)^2\right] \quad (16)$$

Lost cost: Due to urgency of demand the consumer opt to another shop so there is a chance for loss in sale during the shortages period $[t_2, t_3]$. Thus the lost cost for one replenishment interval is (LC).

$$LC=c_3(1-B)\int_{t_2}^{t_3}R\mu dt=c_3(1-B)R\mu(t_3-t_2) \quad (17)$$

The present value of ***total cost*** is (TC):

$$TC = (DC + PC + HC + LC + PDC + SC)\sum_{i=1}^{m} e^{-(i-1)kT} \approx (DC + PC + LC + SC + PDC + HC)\left(\frac{1 - e^{-kmT}}{1 - e^{-kT}}\right)$$

$$= R\mu\Bigg[\left(\frac{d}{2}\right)\left(r\sigma(2t_1 - \mu) - (2t_2 - \mu)\right) + c_1\sigma\left(t_1 + T_1 - t_3 - \frac{\mu}{2}\right) + c_3(1 - B)(t_3 - t_2) + \left(\frac{c_2}{2}\right)[B(t_3 - t_2)^2 +$$

$$(r\sigma - 1)(T_1 - t_3)^2] + \left(\frac{\sigma a r^{c_R - b}}{2-b}\right)[(b-1)\mu^{1-b} + (2-b)\mu^{-b}(t_1 + T_1 - t_3)] + h\Bigg[(r\sigma - 1)\Bigg\{\xi + \left(\frac{t_1}{2}\right)(t_1 - \mu) -$$

$$\left(\frac{\alpha\{\mu\beta(t_1-\gamma)^{\beta+2} - t_1(\mu-\gamma)^{\beta+2} + (-1)^{(\beta)}\alpha\gamma^{\beta+2}t_1\}}{\mu(\beta+1)(\beta+2)}\right) + \frac{\left(\alpha\left(\frac{\mu}{2}-\gamma\right)\right)(t_1-\gamma)^{\beta+1}}{\beta+1}\Bigg\} - \left(\frac{(t_2-t_1)^2}{2}\right) + \left(\frac{\alpha[(t_1-\gamma)^{\beta+1}(t_2-\gamma)-(t_1-\gamma)(t_2-\gamma)^{\beta+1}]}{\beta+1}\right) +$$

$$Q_m\left(t_2 - t_1 - \left(\frac{\alpha(t_2-\gamma)^{\beta+1}}{\beta+1}\right) + \alpha(t_1 - \gamma)^{\beta}\left(\left(\frac{t_1-\gamma}{\beta+1}\right) - t_2 - t_1\right)\right)\Bigg]\Bigg]\left(\frac{1-e^{-kmT}}{1-e^{-kT}}\right) \tag{18}$$

$Where$

$$b \neq 2, \qquad \xi = \frac{\mu^3}{6} - \frac{\alpha\beta(\mu-\gamma)^{\beta+3}}{2(\beta+2)(\beta+3)}\left(\frac{\gamma(\beta+5)}{\beta+1} + \mu\right) + \frac{(-1)^{\beta}\alpha\gamma^{\beta+2}}{(\beta+1)(\beta+2)}(\mu - \frac{\gamma}{\beta+3} - 1)$$
$$+ \frac{\alpha(\mu-\gamma)^{\beta+1}(\gamma^2 + 2\gamma - \mu)}{2(\beta+1)} + \frac{\alpha(\mu-\gamma)^{\beta+2}}{\beta+2}$$

We observe that TC is a function of t_1,t_2,t_3 and m. But for the sake of simplicity we simplified t_2 and t_3 in terms of t_1 and r.

Considering eq.(7), eq.(8) and the condition $Q(t_1)=Q_m$ we get t_2 in terms of t_1, and r. Expanding the exponential terms and neglecting the second and higher order terms of α and after simplifying the above two equations we get,

$$t_2 = (r\sigma - 1)[\frac{\mu}{2} - \frac{\alpha}{\mu(\beta+1)(\beta+2)}\{(\mu-\gamma)^{\beta+2} - (-1)^{\beta}\gamma^{\beta+2}\}] + r\sigma[t_1 + \frac{\alpha(t_1-\gamma)^{\beta+1}}{\beta+1}] \tag{19}$$

Also considering (11), and $Q(T_1)=0$, we get t_3 in terms of t_1,and r.

$$BP\mu(t_3 - t_2) = (\gamma - 1)R\mu(T_1 - t_3)$$

$$t_3 = \frac{1}{B + r\sigma - 1}((r\sigma - 1)T_1 + Bt_2) \tag{20}$$

Thus the total cost TC is function of t_1, r and m.

Optimization process

The following technique is derived to obtain the optimal value of t_1, r and m.

Step 1: Start by choosing a discrete value of m, a positive integer number.

Step 2: Take the partial derivative of total cost $TC(t_1, r, m)$ with respect to t_1 and r and equate it to zero, the necessary condition for optimality is $\frac{\partial TC(t_1,r,m)}{\partial t_1} = 0 \ and \ \frac{\partial TC(t_1,r,m)}{\partial r} = 0$.

Step 3: For different values of m, Obtain the optimum value of the time taken t_1^* and reliability r^* from the above two equation. Then substituting the value of t_1^*, r^* and m in equation [18] and obtain $TC(t_1^*,r^*,m)$

Step 4: Repeat step 2 and step 3 for different values of m and obtain the $TC(t_1^*,r^*,m)$. The minimum value of TC is obtained for optimum value of m^*. Thus (t_1^*,r^*,m^*) and $TC(t_1^*,r^*,m^*)$ are the optimal solution of our model. It satisfies the following condition:

$$\Delta TC(t_1^*, r^*, m^* - 1) < 0 < \Delta TC(t_1^*, r^*, m^* + 1)$$

Where $\Delta TC(t_1^*, r^*, m^*) = TC(t_1^*, r^*, m^* + 1) - TC(t_1^*, r^*, m^*)$

Step 5: To confirm that the objective function is convex, the derived value of $TC(t_1^*, r^*, m^*)$ must satisfy the sufficient condition:

$$\begin{pmatrix} \frac{\partial^2 TC(t_1,r)}{\partial t_1^2} & \frac{\partial^2 TC(t_1,r)}{\partial r \partial t_1} \\ \frac{\partial^2 TC(t_1,r)}{\partial t_1 \partial r} & \frac{\partial^2 TC(t_1,r)}{\partial r^2} \end{pmatrix} > 0 \; and \; \frac{\partial^2 TC(t_1,r)}{\partial t_1^2} > 0 \; or \; \frac{\partial^2 TC(t_1,r)}{\partial r^2} > 0 \qquad (21)$$

Since TC^* is very complicated with high powers so it is not possible to show the analytic validity of eq.(21). For this reason the above inequality is assessed by a numerical example.

4.2 Effect of Neutrosophication of parameter in proposed inventory model

Neutrosophic number actually deals with the conception of three different kinds of membership function related with real life scenario. It consists of truth, hesitation and falseness of an imprecise number. In this model we have considered purchase cost (c_1), holding cost (h) and inflation (k) as neutrosophic fuzzy number since in reality all the parameters are uncertain and contains a dilemma in decision maker's mind. So we try to manifest the model by introducing neutrosophication in the above cost and rates, and thus observe the effect of the above by comparing it with crisp model. The neutrosophic form of holding cost, purchase cost and inflation are represented by $\tilde{h}$, $\tilde{c_1}$ and $\tilde{k}$. Thus

$$\tilde{h} = < (h_1 - \varepsilon_1, h_1, h_1 + \varepsilon_2 : \mu), (h_2 - \varepsilon_1, h_2, h_2 + \varepsilon_2 : \vartheta), (h_3 - \varepsilon_1, h_3, h_3 + \varepsilon_2 : \zeta) >,$$

$$\tilde{c_1} = < (c_{11} - \varepsilon_1, c_{11}, c_{11} + \varepsilon_2 : \mu), (c_{12} - \varepsilon_1, c_{12}, c_{12} + \varepsilon_2 : \vartheta), (c_{13} - \varepsilon_1, c_{13}, c_{13} + \varepsilon_2 : \zeta) >,$$

$$\tilde{k} = < (k_1 - \varepsilon_1, k_1, k_1 + \varepsilon_2 : \mu), (k_2 - \varepsilon_1, k_2, k_2 + \varepsilon_2 : \vartheta), (k_3 - \varepsilon_1, k_3, k_3 + \varepsilon_2 : \zeta) >$$

$where\ \mu, \vartheta, \zeta \in [0,1]\ and\ 0 < \varepsilon_1, \varepsilon_2 < 1.$

This neutrosophic fuzzy number is implemented in this model and thus the total cost obtain using this neutrosophic number is

$$TC_{neu}(\tilde{h},\ \tilde{c_1},\ \tilde{k}) = R\mu \Bigg[\left(\frac{d}{2}\right)\left(r\sigma(2t_1 - \mu) - (2t_2 - \mu)\right) + \tilde{c_1}\sigma\left(t_1 + T_1 - t_3 - \frac{\mu}{2}\right) + c_3(1 - B)(t_3 - t_2) +$$

$$\left(\frac{c_2}{2}\right)[B(t_3 - t_2)^2 + (r\sigma - 1)(T_1 - t_3)^2] + \left(\frac{\sigma a r^{c} R^{-b}}{2-b}\right)[(b-1)\mu^{1-b} + (2-b)\mu^{-b}(t_1 + T_1 - t_3)] + \tilde{h}\Bigg[(r\sigma - 1)\Bigg\{\xi +$$

$$\left(\frac{t_1}{2}\right)(t_1 - \mu) - \left(\frac{\alpha\{\mu\beta(t_1-\gamma)^{\beta+2} - t_1(\mu-\gamma)^{\beta+2} + (-1)^{(\beta)}\alpha\gamma^{\beta+2}t_1\}}{\mu(\beta+1)(\beta+2)}\right) + \frac{\left(\alpha\left(\frac{\mu}{2}-\gamma\right)\right)(t_1-\gamma)^{\beta+1}}{\beta+1}\Bigg\} - \left(\frac{(t_2-t_1)^2}{2}\right) +$$

$$\left(\frac{\alpha[(t_1-\gamma)^{\beta+1}(t_2-\gamma)-(t_1-\gamma)(t_2-\gamma)^{\beta+1}]}{\beta+1}\right)+Q_m\left(t_2-t_1-\left(\frac{\alpha(t_2-\gamma)^{\beta+1}}{\beta+1}\right)+\alpha(t_1-\gamma)^{\beta}\left(\left(\frac{t_1-\gamma}{\beta+1}\right)-t_2-t_1\right)\right)\Bigg]\Bigg]\left(\frac{1-e^{-\tilde{k}mT}}{1-e^{-\tilde{k}T}}\right) \tag{22}$$

Using removal area technique (Chakraborty et. al. [3]) the de- neutrosophic numbers are

$$\widetilde{h_{neuD}}=\frac{h_1+h_2+h_3}{3}-\frac{\varepsilon_1+\varepsilon_2}{4},\widetilde{(c_1)_{neuD}}=\frac{c_{11}+c_{12}+c_{13}}{3}-\frac{\varepsilon_1+\varepsilon_2}{4},and\ \widetilde{k_{neuD}}=\frac{k_1+k_2+k_3}{3}-\frac{\varepsilon_1+\varepsilon_2}{4}.$$

So we substitute the value of $h_{neuD}, (c_1)_{neuD}$ and k_{neuD} and obtain the total cost in neutrosophic domain.

Thus by de-neutrosophication we get

$$TC_{neu}(\tilde{h},\tilde{c_1},\tilde{k})=R\mu\Bigg[\left(\frac{d}{2}\right)\left(r\sigma(2t_1-\mu)-(2t_2-\mu)\right)+\widetilde{(c_1)_{neuD}}\sigma\left(t_1+T_1-t_3-\frac{\mu}{2}\right)+c_3(1-B)(t_3-t_2)+\left(\frac{c_2}{2}\right)[B(t_3-t_2)^2+(r\sigma-1)(T_1-t_3)^2]+\left(\frac{\sigma a r^{c}{}_R{}^{-b}}{2-b}\right)[(b-1)\mu^{1-b}+(2-b)\mu^{-b}(t_1+T_1-t_3)]+\widetilde{h_{neuD}}\Bigg[(r\sigma-1)\left\{\xi+\left(\frac{t_1}{2}\right)(t_1-\mu)-\left(\frac{\alpha\{\mu\beta(t_1-\gamma)^{\beta+2}-t_1(\mu-\gamma)^{\beta+2}+(-1)^{(\beta)}\alpha\gamma^{\beta+2}t_1\}}{\mu(\beta+1)(\beta+2)}\right)+\frac{\left(\alpha\left(\frac{\mu}{2}-\gamma\right)\right)(t_1-\gamma)^{\beta+1}}{\beta+1}\right\}-\left(\frac{(t_2-t_1)^2}{2}\right)+\left(\frac{\alpha[(t_1-\gamma)^{\beta+1}(t_2-\gamma)-(t_1-\gamma)(t_2-\gamma)^{\beta+1}]}{\beta+1}\right)+Q_m\left(t_2-t_1-\left(\frac{\alpha(t_2-\gamma)^{\beta+1}}{\beta+1}\right)+\alpha(t_1-\gamma)^{\beta}\left(\left(\frac{t_1-\gamma}{\beta+1}\right)-t_2-t_1\right)\right)\Bigg]\Bigg]\left(\frac{1-e^{-\widetilde{k_{neuD}}mT}}{1-e^{-\widetilde{k_{neuD}}T}}\right) \tag{23}$$

5. Numerical Example

The model is illustrated by an example. A new brand item follows the demand rate as ramp type function of time where the produced items are directly affected by reliability(r) of production process. The manufacturer maintains the production rate 1.3 times the demand rate where demand factor is considered as 12 unit per cycle. Also the items deteriorate with time is in the form of $\alpha\beta(t-\gamma)^{\beta-1}$, (where γ = 0.6 unit and α=0.001,β=1) which cost 1$ per unit time. The purchase cost of the raw material of the item is 3.5$ per unit item and 100$ is used for setting up for the production cycle. To hold the item in store the retailer has to pay 0.4$ per unit item. During shortages, which cost 3.2$, let 0.75 fraction of stock demand get backordered as the rest sales are lost. The cost for penalty (lost in sell) is 15$. The model is considered under 15 years of planning horizon with various replenishment cycle i.e., m=2,3,4,5 and discounting rate of inflation as 12%.

Therefore, the data considered to illustrate the models are as follows:

$$c_1=3.5,c_2=3.2,c_3=15,h=0.4,d=1,B=0.75,H=15,T=H/m,\mu=1.2,\sigma=1.3,\alpha=0.001,\beta=1,\gamma=0.6,a=3,b=0.8,c=2,k=0.12,R=12,S=100.$$

Table 1: Optimal solution of inventory model for different replenishment

m	T in year	t_1^* in year	t_2^* in year	t_3^* in year	reliability (r^*)	TC^*
2	7.5	7.175	7.452	7.454	0.799	806.54
3	5	4.571	4.883	4.894	0.828	738.13
4	3.75	3.249	3.579	3.603	0.864	717.58
5*	3*	2.451*	2.789*	2.83*	0.909*	715.26*

From the table 1 it is observed that the optimal solution is obtained (i.e., total cost is minimum) if we consider short replenishment cycles. This is realistic because if we decrease the time of the production then it produces less items and hence the total cost of the inventory decreases. It is also observed the better quality items are produced at shorter replenishment cycle i.e., the reliability (r) of the items increases in shorter production or replenishment cycle. This occurs because if we take small cycle then at the end of each cycle their is maintenance in production system happens regularly and thus the reliability of the items increases.

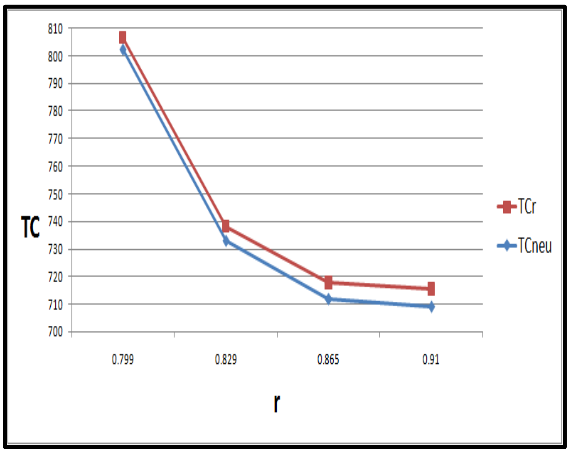

Figure 2: Graphical presentation of production cycle vs total cost

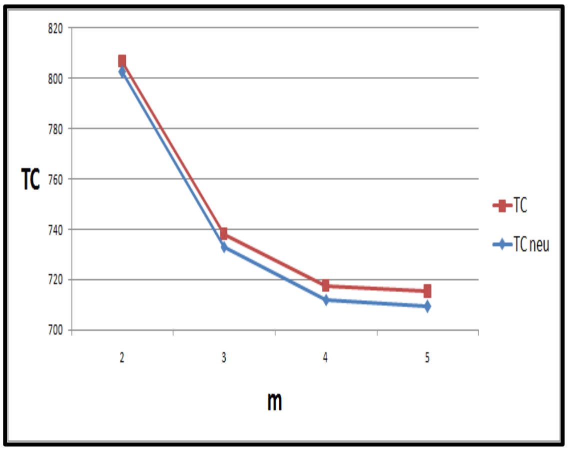

Figure 3: Graphical presentation of reliability vs total cost.

We observe from the figure 2 that for smaller production cycle (i.e., for large value of m), the optimal total cost (TC) decreases with optimal cost at m = 5.

In figure 3 we observe that the as reliability (r) increases then the total cost (TC) decreases. This holds because as reliability increases the demand of the item in the market increases as a result the cost per unit item decreases and hence the total cost decreases.

The above result is desirable because in the competitive market the business strategies of the manufacturer is to work in small cycle and producing highly reliable items at less cost.

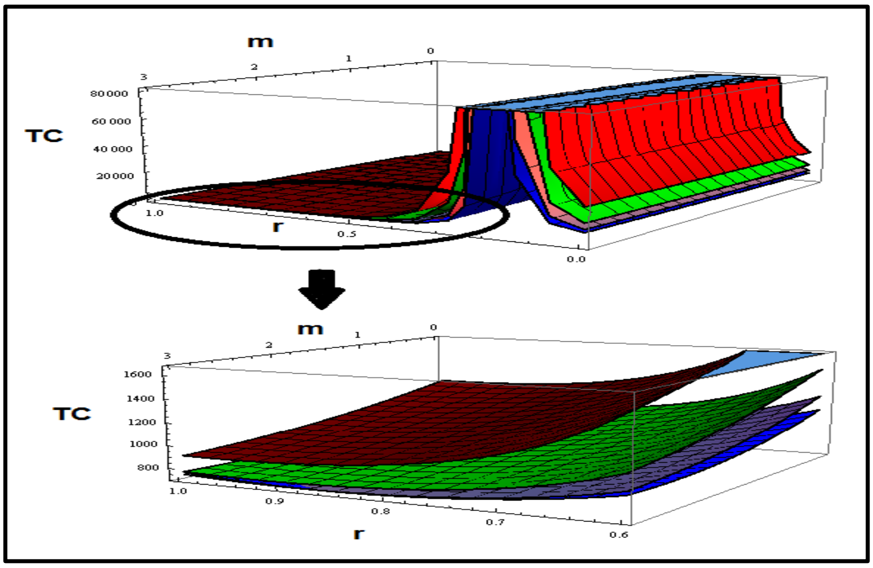

Figure 4: Graphical representation of total cost vs reliability and production time

Figure 4 gives the 3-dimensional plot of the total cost, reliability and no. of replenishment cycle in crisp model. In this figure we observe that reliability (r) increases for large value of m where the total cost (TC) decreases, i.e., highly reliable items are produced during small replenishment cycle at less cost, which is desirable in producer-oriented EPQ model. This is obvious as, in small cycle, the machinery gets upgraded and ameliorated eventually at the end of each cycle, and hence better quality of items are produced at much faster rate and thus cost per unit items decreases and hence the total costing of the inventory decreases.

In reality few parameters are uncertain and thus there is a dilemma in decision maker's mind. Thus instead of considering the model in crisp domain let us consider the model in neutrosophic domain and examine the same example as above. Here we have considered purchase cost (c_1), holding cost (h) and inflation (k) as triangular neutrosophic fuzzy number. Thus the neutrosophic numbers of the above parameters are $k_1 = 0.125, k_2 = 0.118, k_3 = 0.132, h_1 = 0.38, h_2 = 0.4, h_3 = 0.42, c_{11} = 2.5, c_{12} = 2.45. c_{13} = 2.55, \varepsilon_1 = 0.005, \varepsilon_2 = 0.007$.

$$Then, \widetilde{c_1} =< (2.495,2.5,2.507), (2.445,2.45,2.457), (2.545,2.55,2.557) >,$$

$$\tilde{h} =< (0.375,0.38,0.387), (0.395,0.4,0.407), (0.415,0.42,0.427) > and$$

$$\tilde{k} =< (0.12,0.125,0.132), (0.113,0.118,0.125), (0.127,0.132,0.139) >.$$

Thus we obtained table 2 under neutrosophic arena for the optimal solution of the model for different replenishment cycle.

Table 2: Optimal time and cost of inventory model under neutrosophic domain

m	T in year	t_1^* in year	t_2^* in year	t_3^* in year	reliability (r^*)	TC^*
2	7.5	7.172	7.448	7.451	0.799	802.45
3	5	4.569	4.886	4.897	0.829	733
4	3.75	3.247	3.58	3.605	0.865	711.87
5^*	3^*	2.449^*	2.789^*	2.83^*	0.91^*	709.11^*

Thus if we compare table 1 and table 2 it is observed that the total cost (TC) decreases if we consider the model in neutrosophic arena. This is desirable as few parameters has hesitation factor in decision maker's mind and thus this model under neutrosophic domain gives us better result.

6. Sensitivity Analysis

The retailer should be aware of the effect in the total cost for any changes in the parameter. In order to examine the implications of these changes, the sensitivity analysis will be helpful for decision-making. Using the numerical example as given in the preceding section, we perform the sensitivity analysis by changing few crisp parameters by -10%, -5%, 5% and 10% by taking one parameter at time and keeping the other parameter fixed. As per Table 1 we observe that optimal solution is obtained when we consider small replenishment cycle. So we perform the sensitivity analysis for m=5.

Table 3. Sensitivity analysis of some parameters

Parameters	Change (%)	t_1^* in year	t_2^* in year	t_3^* in year	reliability (r^*)	TC^*	% change of TC^*
	-10	2.642	2.933	2.44	0.878	677.95	-5.5
c_1	-5	2.557	2.872	2.894	0.892	696.78	-2.65
	5	2.313	2.677	2.748	0.932	733.29	2.46
	10	2.122	2.517	2.64	0.968	750.71	4.72
	-10	2.398	2.776	2.825	0.93	711.92	-0.47
h	-5	2.425	2.761	2.808	0.91	713.67	-0.22
	5	2.475	2.796	2.834	0.9	716.7	0.2
	10	2.498	2.803	2.838	0.892	718.01	0.38
	-10	2.451	2.789	2.83	0.909	687.65	-4.02
S	-5	2.451	2.789	2.83	0.909	701.45	-1.97
	5	2.451	2.789	2.83	0.909	729.06	1.89
	10	2.451	2.789	2.83	0.909	742.87	3.72
	-10	2.617	2.903	2.918	0.975	681.84	-4.9
σ	-5	2.533	2.844	2.871	0.939	698.61	-2.38
	5	2.364	2.728	2.786	0.883	731.72	2.25
	10	2.264	2.66	2.742	0.865	747.95	4.37
	-10	2.398	2.771	2.819	0.923	681.47	-4.96
μ	-5	2.425	2.781	2.825	0.916	698.63	-2.38
	5	2.475	2.798	2.836	0.903	731.35	2.2
	10	2.498	2.805	2.841	0.897	746.92	4.24
	-10	2.451	2.789	2.83	0.909	750.84	4.74
k	-5	2.451	2.789	2.83	0.909	732.63	2.37
	5	2.451	2.789	2.83	0.909	698.69	-2.37
	10	2.451	2.789	2.83	0.909	682.87	-4.74
	-10	2.41	2.781	2.828	0.926	671.72	-6.48
R	-5	2.431	2.785	2.829	0.917	693.48	-3.14
	5	2.471	2.794	2.832	0.901	737.05	2.96
	10	2.49	2.799	2.834	0.894	758.86	5.74

From the above table 3 it is observed that the model is highly sensitive to purchase cost, demand rate factor (R), moderately sensitive to setup cost, σ, μ, inflation (k) and less sensitive to holding cost. It is also noted that the model is insensitive to the shortage cost, lost in sale cost and deterioration cost. That means deterioration is not going to affect the model as much.

(i) The model is highly sensitive to purchase cost i.e., if we increase purchase cost (c_1), the total cost increases. It is also noted that as the purchase cost increases, the reliability increases and production time decreases which means if we buy good quality raw material then we have better quality of finished good at less manufacturing time. Again the total cost TC increases with increase in demand factor R. This is obvious because if demand increases means more items are produced and hence the production time and production cost also increases which leads to increase in total cost.

(ii) The model is moderately sensitive to set up cost (S), σ, μ, inflation (k). Investing more money for upgradation of machineries, i.e., by increasing in set up cost (S), the total cost increases. It is noted that in our model the set up cost does not depends on reliability and production time. Again with the increase in production rate (σ) and production time (μ), the total cost increases. This is true because, if production time increases then more items are produced also if we increase the production rate then we have more finished good at less manufacturing time and thus in both the case the total cost increases. Also the toal cost decreases with increase in inflation (k). This is obvious because with the increase in inflation the time value of money increases and thus the total cost decreases in present day.

(iii) It is noticed that as the holding cost (h) is a less sensitive parameter. With the increase in holding cost, the total cost increases. It is also observed that the production time also increases with increase in holding cost. It means that the items has to be held for longer time with high value of holding cost then obviously the total cost will increase.

It has been observed that there are various parameters which are very less sensitive hence it is not included in the table.

7. Concluding remarks

This paper developed an EPQ model for deteriorating item with reliability in production process and ramp type demand rate under crisp and neutrosophic domain. The model also considers shortages where part of the items gets backlogged and part of the sales are lost. The model coincides with practical situations since we have considered the effect of time value of money under finite time horizon. Also the model optimizes by considering the reliability of production process, as the reliability of production process increases, the total cost decreases. This model is cost effective because highly reliable items are obtained at less cost and which is desirable in managerial point of view. It is also observed that the highly reliable items are produced in small cycles. The paper also compares the model under two different environment, crisp and neutrosophic, and it is observed that the model works better in neutrosophic domain as compare to crisp environment. In this paper we have done sensitivity analysis in crisp environment to illustrate our example and we have noted that the minimum value of total cost is obtained for short replenishment cycle. This work could be extended by considering multi-layer supply chain lot sizing model with manufacturer end, retailer end under neutrosophic environment. Also we can extend this same model and can compare the model with neutrosophic number and hybrid plithogenic decision-making method.

Further, in the forthcoming research, people can fruitfully execute and apply the idea of triangular neutrosophic into distinct research arenas like structural modeling, diagnostic problems, realistic modeling, recruitment based problems, pattern recognition etc.

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