Operations on Neutrosophic Vague Graphs

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Abstract: Neutrosophic graph is a mathematical tool to hold with imprecise and unspecified data. In this manuscript, the operations on neutrosophic vague graphs are introduced. Moreover, Cartesian product, lexicographic product, cross product, strong product and composition of neutrosophic vague graphs are investigated. The proposed concepts are demonstrated with suitable examples.

Keywords: Neutrosophic vague graph, Operations of neutrosophic vague graph, Cartesian product, Cross product, Strong product

1. Introduction

In a classical graph, any vertex or edge have two situations, namely, it is either in the graph or it is not in the graph and it is not sufficient to model uncertain optimization problems. Therefore, real-life problems are not suitable to model using classical graphs. Hence the fuzzy set arises, which is an extension of classical set; here the objects have varying membership degrees. Vague sets are regarded as a special case of context-dependent fuzzy sets. At first, vague set theory was investigated by Gau and Buehrer [36] that is an extension of fuzzy set theory. The classical fuzzy set handles only the membership degree, but intuitionistic fuzzy handles independent membership degree and non-membership degree for any element with the only requirement is that the sum of non-membership and membership degree values is not greater than one [16].

On the other hand, to hold this indeterminate and inconsistent information, the neutrosophic set is introduced by F. Smarandache and has been studied extensively (see [31]-[35]). Neutrosophic set and related notions have weird applications in many different fields. In the definition of neutrosophic set, the indeterminacy value is quantified explicitly and truth-membership, false-membership and indeterminacy-membership are stated as exactly independent provided sum of these values belonging to 0 and 3. Neutrosophic soft rough graphs with applications are established in [10]. Neutrosophic soft relations and neutrosophic refined relations with their properties are studied in [15, 20]. Single valued neutrosophic graph are studied in [17, 18]. Some types of neutrosophic graphs and co-neutrosophic graphs are discussed in [23]. Neutrosophic vague set is first initiated in [11]. Al-Quran and Hassan in [7] introduced the notion of neutrosophic vague soft expert set as a generalization of neutrosophic vague set and soft expert set in order to revise the application in decision-making in real-life problems. Intuitionistic bipolar neutrosophic set and its application to graphs are established in [28]. Further, neutrosophic vague graphs are investigated in

[27]. Motivated by the articles [11, 27, 28, 29], we introduce the concept of operations on neutrosophic vague graphs. The main contributions in this manuscript are given below:

- Operations on neutrosophic vague graphs are established. In Section 2, basic definitions regarding to neutrosophic vague graphs are explained with an example.
- In Section 3, Cartesian product, lexicographic product, cross product, strong product and composition of neutrosophic vague graph are illustrated with examples. Finally, a conclusion is elaborated with future direction.

2. Preliminaries

In this section, basic definitions and example are given, which is used to prove the main results.

Definition 2.1 [36] A vague set $\mathbb{A}$ on a non empty set $\mathbb{X}$ is a pair $(\mathbb{T}_{\mathbb{A}}, \mathbb{F}_{\mathbb{A}})$, where $\mathbb{T}_{\mathbb{A}}: \mathbb{X} \to [0,1]$ and $\mathbb{F}_{\mathbb{A}}: \mathbb{X} \to [0,1]$ are true membership and false membership functions, respectively, such that

$$0 \leq \mathbb{T}_{\mathbb{A}}(x) + \mathbb{F}_{\mathbb{A}}(x) \leq 1 \text{ for every } x \in \mathbb{X}.$$

Let $\mathbb{X}$ and $\mathbb{Y}$ be two non-empty sets. A vague relation R of $\mathbb{X}$ to $\mathbb{Y}$ is a vague set R on $\mathbb{X} \times \mathbb{Y}$ that is $R = (\mathbb{T}_R, \mathbb{F}_R)$, where $\mathbb{T}_R: \mathbb{X} \times \mathbb{Y} \to [0,1]$, $\mathbb{F}_R: \mathbb{X} \times \mathbb{Y} \to [0,1]$ and satisfies the condition:

$$0 \leq \mathbb{T}_R(x,y) + \mathbb{F}_R(x,y) \leq 1 \text{ for any } x,y \in \mathbb{X}.$$

Definition 2.2 [12] Let $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$ be a graph. A pair $\mathbb{G} = (\mathbb{J}, \mathbb{K})$ is called a vague graph on $\mathbb{G}^*$, where $\mathbb{J} = (\mathbb{T}_{\mathbb{J}}, \mathbb{F}_{\mathbb{J}})$ is a vague set on $\mathbb{V}$ and $\mathbb{K} = (\mathbb{T}_{\mathbb{K}}, \mathbb{F}_{\mathbb{K}})$ is a vague set on $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ such that for each $xy \in \mathbb{E}$,

$$\mathbb{T}_{\mathbb{K}}(xy) \leq \min\{\mathbb{T}_{\mathbb{J}}(x), \mathbb{T}_{\mathbb{J}}(y)\} \text{ and } \mathbb{F}_{\mathbb{K}}(xy) \geq \max\{\mathbb{F}_{\mathbb{J}}(x), \mathbb{F}_{\mathbb{J}}(y)\}.$$

Definition 2.3 [31] A Neutrosophic set $\mathbb{A}$ is contained in another neutrosophic set $\mathbb{B}$, (i.e) $\mathbb{A} \subseteq \mathbb{B}$ if $\forall x \in \mathbb{X}, \mathbb{T}_{\mathbb{A}}(x) \leq \mathbb{T}_{\mathbb{B}}(x), \mathbb{I}_{\mathbb{A}}(x) \geq \mathbb{I}_{\mathbb{B}}(x)$and $\mathbb{F}_{\mathbb{A}}(x) \geq \mathbb{F}_{\mathbb{B}}(x)$.

Definition 2.4 [20, 31] Let $\mathbb{X}$ be a space of points (objects), with generic elements in $\mathbb{X}$ denoted by x. A single valued neutrosophic set $\mathbb{A}$ in $\mathbb{X}$ is characterised by truth-membership function $\mathbb{T}_{\mathbb{A}}(x)$, indeterminacy-membership function $\mathbb{I}_{\mathbb{A}}(x)$ and falsity-membership-function $\mathbb{F}_{\mathbb{A}}(x)$,
For each point x in $\mathbb{X}$, $\mathbb{T}_{\mathbb{A}}(x), \mathbb{I}_{\mathbb{A}}(x), \mathbb{F}_{\mathbb{A}}(x) \in [0,1]$. Also

$$\mathbb{A} = \{\langle x, \mathbb{T}_{\mathbb{A}}(x), \mathbb{I}_{\mathbb{A}}(x), \mathbb{F}_{\mathbb{A}}(x)\rangle\} \text{ and } 0 \leq \mathbb{T}_{\mathbb{A}}(x), +\mathbb{I}_{\mathbb{A}}(x) + \mathbb{F}_{\mathbb{A}}(x) \leq 3.$$

Definition 2.5 [6, 18] A neutrosophic graph is defined as a pair $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$ where

(i) $\mathbb{V} = \{v_1, v_2, \ldots, v_n\}$ such that $\mathbb{T}_1 : \mathbb{V} \to [0,1]$, $\mathbb{I}_1 : \mathbb{V} \to [0,1]$ and $\mathbb{F}_1 : \mathbb{V} \to [0,1]$ denote the degree of truth-membership function, indeterminacy function and falsity-membership function, respectively, and

$$0 \leq \mathbb{T}_1(v) + \mathbb{I}_1(v) + \mathbb{F}_1(v) \leq 3,$$

(ii) $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ where $\mathbb{T}_2 : \mathbb{E} \to [0,1]$, $\mathbb{I}_2 : \mathbb{E} \to [0,1]$ and $\mathbb{F}_2 : \mathbb{E} \to [0,1]$ are such that

$$\mathbb{T}_2(uv) \leq \min\{\mathbb{T}_1(u), \mathbb{T}_1(v)\},$$
$$\mathbb{I}_2(uv) \leq \min\{\mathbb{I}_1(u), \mathbb{I}_1(v)\},$$
$$\mathbb{F}_2(uv) \leq \max\{\mathbb{F}_1(u), \mathbb{F}_1(v)\}$$
$$\text{and } 0 \leq \mathbb{T}_2(uv) + \mathbb{I}_2(uv) + \mathbb{F}_2(uv) \leq 3, \forall uv \in \mathbb{E}.$$

Definition 2.6 [11] A Neutrosophic Vague Set $\mathbb{A}_{NV}$ (NVS in short) on the universe of discourse $\mathbb{X}$ written as

$$\mathbb{A}_{NV} = \{\langle x, \widehat{\mathbb{T}}_{\mathbb{A}_{NV}}(x), \widehat{\mathbb{I}}_{\mathbb{A}_{NV}}(x), \widehat{\mathbb{F}}_{\mathbb{A}_{NV}}(x)\rangle, x \in \mathbb{X}\},$$

whose truth-membership, indeterminacy membership and falsity-membership function are defined as

$$\widehat{\mathbb{T}}_{\mathbb{A}_{NV}}(x) = [\mathbb{T}^-(x), \mathbb{T}^+(x)], \widehat{\mathbb{I}}_{\mathbb{A}_{NV}}(x) = [\mathbb{I}^-(x), \mathbb{I}^+(x)] \text{and } \widehat{\mathbb{F}}_{\mathbb{A}_{NV}}(x) = [\mathbb{F}^-(x), \mathbb{F}^+(x)],$$

where $\mathbb{T}^+(x) = 1 - \mathbb{F}^-(x), \mathbb{F}^+(x) = 1 - \mathbb{T}^-(x)$, and $0 \le \mathbb{T}^-(x) + \mathbb{I}^-(x) + \mathbb{F}^-(x) \le 2$.

Definition 2.7 [11] The complement of NVS $\mathbb{A}_{NV}$ is denoted by $\mathbb{A}^c_{NV}$ and it is defined by

$$\widehat{\mathbb{T}}^c_{\mathbb{A}_{NV}}(x) = [1 - \mathbb{T}^+(x), 1 - \mathbb{T}^-(x)],$$
$$\widehat{\mathbb{I}}^c_{\mathbb{A}_{NV}}(x) = [1 - \mathbb{I}^+(x), 1 - \mathbb{I}^-(x)],$$
$$\widehat{\mathbb{F}}^c_{\mathbb{A}_{NV}}(x) = [1 - \mathbb{F}^+(x), 1 - \mathbb{F}^-(x)].$$

Definition 2.8 [11] Let $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ be two NVSs of the universe $\mathbb{U}$. If for all $u_i \in \mathbb{U}$,

$$\widehat{\mathbb{T}}_{\mathbb{A}_{NV}}(u_i) \le \widehat{\mathbb{T}}_{\mathbb{B}_{NV}}(u_i), \widehat{\mathbb{I}}_{\mathbb{A}_{NV}}(u_i) \ge \widehat{\mathbb{I}}_{\mathbb{B}_{NV}}(u_i), \widehat{\mathbb{F}}_{\mathbb{A}_{NV}}(u_i) \ge \widehat{\mathbb{F}}_{\mathbb{B}_{NV}}(u_i),$$

then the NVS, $\mathbb{A}_{NV}$ are included in $\mathbb{B}_{NV}$, denoted by $\mathbb{A}_{NV} \subseteq \mathbb{B}_{NV}$ where $1 \le i \le n$.

Definition 2.9 [11] The union of two NVSs , $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$, is a NVSs, $\mathbb{D}_{NV}$, written as $\mathbb{D}_{NV} = \mathbb{A}_{NV} \cup \mathbb{B}_{NV}$ whose truth-membership function, indeterminacy-membership function and false-membership function are related to those of $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ by

$$\widehat{\mathbb{T}}_{\mathbb{D}_{NV}}(x) = [\max(\mathbb{T}^-_{\mathbb{A}_{NV}}(x), \mathbb{T}^-_{\mathbb{B}_{NV}}(x)), \max(\mathbb{T}^+_{\mathbb{A}_{NV}}(x), \mathbb{T}^+_{\mathbb{B}_{NV}}(x))]$$
$$\widehat{\mathbb{I}}_{\mathbb{D}_{NV}}(x) = [\min(\mathbb{I}^-_{\mathbb{A}_{NV}}(x), \mathbb{I}^-_{\mathbb{B}_{NV}}(x)), \min(\mathbb{I}^+_{\mathbb{A}_{NV}}(x), \mathbb{I}^+_{\mathbb{B}_{NV}}(x))]$$
$$\widehat{\mathbb{F}}_{\mathbb{D}_{NV}}(x) = [\min(\mathbb{F}^-_{\mathbb{A}_{NV}}(x), \mathbb{F}^-_{\mathbb{B}_{NV}}(x)), \min(\mathbb{F}^+_{\mathbb{A}_{NV}}(x), \mathbb{F}^+_{\mathbb{B}_{NV}}(x))].$$

Definition 2.10 [11] The intersection of two NVSs, $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ is a NVSs, $\mathbb{D}_{NV}$, written as $\mathbb{D}_{NV} = \mathbb{A}_{NV} \cap \mathbb{B}_{NV}$, whose truth-membership function, indeterminacy-membership function and false-membership function are related to those of $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ by

$$\widehat{\mathbb{T}}_{\mathbb{D}_{NV}}(x) = [\min(\mathbb{T}^-_{\mathbb{A}_{NV}}(x), \mathbb{T}^-_{\mathbb{B}_{NV}}(x)), \min(\mathbb{T}^+_{\mathbb{A}_{NV}}(x), \mathbb{T}^+_{\mathbb{B}_{NV}}(x))]$$
$$\widehat{\mathbb{I}}_{\mathbb{D}_{NV}}(x) = [\max(\mathbb{I}^-_{\mathbb{A}_{NV}}(x), \mathbb{I}^-_{\mathbb{B}_{NV}}(x)), \max(\mathbb{I}^+_{\mathbb{A}_{NV}}(x), \mathbb{I}^+_{\mathbb{B}_{NV}}(x))]$$
$$\widehat{\mathbb{F}}_{\mathbb{D}_{NV}}(x) = [\max(\mathbb{F}^-_{\mathbb{A}_{NV}}(x), \mathbb{F}^-_{\mathbb{B}_{NV}}(x)), \max(\mathbb{F}^+_{\mathbb{A}_{NV}}(x), \mathbb{F}^+_{\mathbb{B}_{NV}}(x))].$$

Definition 2.11 [27] Let $G^* = (R, S)$ be a graph. A pair $\mathbb{G} = (\mathbb{A}, \mathbb{B})$ is called a neutrosophic vague graph (NVG) on G^* or a neutrosophic vague graph where $\mathbb{A} = (\widehat{\mathbb{T}}_{\mathbb{A}}, \widehat{\mathbb{I}}_{\mathbb{A}}, \widehat{\mathbb{F}}_{\mathbb{A}})$ is a neutrosophic vague set on R and $\mathbb{B} = (\widehat{\mathbb{T}}_{\mathbb{B}}, \widehat{\mathbb{I}}_{\mathbb{B}}, \widehat{\mathbb{F}}_{\mathbb{B}})$is a neutrosophic vague set $S \subseteq R \times R$ where

(1) $R = \{v_1, v_2, \dots, v_n\}$ such that $\mathbb{T}^-_{\mathbb{A}}: R \to [0,1], \mathbb{I}^-_{\mathbb{A}}: R \to [0,1], \mathbb{F}^-_{\mathbb{A}}: R \to [0,1]$ which satisfies the condition $\mathbb{F}^-_{\mathbb{A}} = [1 - \mathbb{T}^+_{\mathbb{A}}]$,

$\mathbb{T}^+_{\mathbb{A}}: R \to [0,1], \mathbb{I}^+_{\mathbb{A}}: R \to [0,1], \mathbb{F}^+_{\mathbb{A}}: R \to [0,1]$ which satisfies the condition $\mathbb{F}^+_{\mathbb{A}} = [1 - \mathbb{T}^-_{\mathbb{A}}]$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i \in R$, and

$$0 \le \mathbb{T}^-_{\mathbb{A}}(v_i) + \mathbb{I}^-_{\mathbb{A}}(v_i) + \mathbb{F}^-_{\mathbb{A}}(v_i) \le 2$$
$$0 \le \mathbb{T}^+_{\mathbb{A}}(v_i) + \mathbb{I}^+_{\mathbb{A}}(v_i) + \mathbb{F}^+_{\mathbb{A}}(v_i) \le 2.$$

(2) $S \subseteq R \times R$ where

$$\mathbb{T}^-_{\mathbb{B}}: R \times R \to [0,1],\ \mathbb{I}^-_{\mathbb{B}}: R \times R \to [0,1],\ \mathbb{F}^-_{\mathbb{B}}: R \times R \to [0,1]$$
$$\mathbb{T}^+_{\mathbb{B}}: R \times R \to [0,1],\ \mathbb{I}^+_{\mathbb{B}}: R \times R \to [0,1],\ \mathbb{F}^+_{\mathbb{B}}: R \times R \to [0,1]$$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_iv_j \in S$, respectively and such that,

$$0 \le \mathbb{T}^-_{\mathbb{B}}(v_iv_j) + \mathbb{I}^-_{\mathbb{B}}(v_iv_j) + \mathbb{F}^-_{\mathbb{B}}(v_iv_j) \le 2$$
$$0 \le \mathbb{T}^+_{\mathbb{B}}(v_iv_j) + \mathbb{I}^+_{\mathbb{B}}(v_iv_j) + \mathbb{F}^+_{\mathbb{B}}(v_iv_j) \le 2.$$

such that

$$\mathbb{T}^{-}_{\mathbb{B}}(v_i v_j) \le \min\{\mathbb{T}^{-}_{\mathbb{A}}(v_i), \mathbb{T}^{-}_{\mathbb{A}}(v_j)\}$$
$$\mathbb{I}^{-}_{\mathbb{B}}(v_i v_j) \le \min\{\mathbb{I}^{-}_{\mathbb{A}}(v_i), \mathbb{I}^{-}_{\mathbb{A}}(v_j)\}$$
$$\mathbb{F}^{-}_{\mathbb{B}}(v_i v_j) \le \max\{\mathbb{F}^{-}_{\mathbb{A}}(v_i), \mathbb{F}^{-}_{\mathbb{A}}(v_j)\}$$

and similarly

$$\mathbb{T}^{+}_{\mathbb{B}}(v_i v_j) \le \min\{\mathbb{T}^{+}_{\mathbb{A}}(v_i), \mathbb{T}^{+}_{\mathbb{A}}(v_j)\}$$
$$\mathbb{I}^{+}_{\mathbb{B}}(v_i v_j) \le \min\{\mathbb{I}^{+}_{\mathbb{A}}(v_i), \mathbb{I}^{+}_{\mathbb{A}}(v_j)\}$$
$$\mathbb{F}^{+}_{\mathbb{B}}(v_i v_j) \le \max\{\mathbb{F}^{+}_{\mathbb{A}}(v_i), \mathbb{F}^{+}_{\mathbb{A}}(v_j)\}.$$

Example 2.12 Consider a neutrosophic vague graph $G = (R, S)$ such that $\mathbb{A} = \{a, b, c\}$ and $\mathbb{B} = \{ab, bc, ca\}$ are defined by

$$\hat{a} = T[0.5,0.6], I[0.4,0.3], F[0.4,0.5], \qquad \hat{b} = T[0.4,0.6], I[0.7,0.3], F[0.4,0.6],$$
$$\hat{c} = T[0.4,0.4], I[0.5,0.3], F[0.6,0.6]$$
$$a^{-} = (0.5,0.4,0.4), b^{-} = (0.4,0.7,0.4), c^{-} = (0.4,0.5,0.6)$$

$$\mathbf{a}^{+} = (\mathbf{0.6}, \mathbf{0.3}, \mathbf{0.5}),\ \mathbf{b}^{+} = (\mathbf{0.6}, \mathbf{0.3}, \mathbf{0.6}),\ \mathbf{c}^{+} = (\mathbf{0.4}, \mathbf{0.3}, \mathbf{0.6}).$$

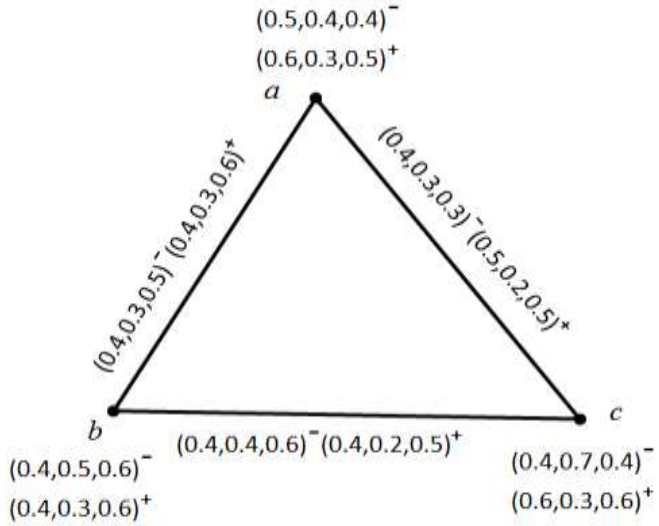


Figure 1: NEUTROSOPHIC VAGUE GRAPH

3. Operations on Neutrosophic Vague Graphs

In this section, the results on operations of neutrosophic vague graphs with example are established.

Definition 3.1 The Cartesian product of two NVGs G_1 and G_2 is denoted by the pair $G_1 \times G_2 = (R_1 \times R_2, S_1 \times S_2)$ and defined as

$$T^{-}_{A_1 \times A_2}(kl) = T^{-}_{A_1}(k) \wedge T^{-}_{A_2}(l)$$
$$I^{-}_{A_1 \times A_2}(kl) = I^{-}_{A_1}(k) \wedge I^{-}_{A_2}(l)$$
$$F^{-}_{A_1 \times A_2}(kl) = F^{-}_{A_1}(k) \vee F^{-}_{A_2}(l)$$
$$T^{+}_{A_1 \times A_2}(kl) = T^{+}_{A_1}(k) \wedge T^{+}_{A_2}(l)$$
$$I^{+}_{A_1 \times A_2}(kl) = I^{+}_{A_1}(k) \wedge I^{+}_{A_2}(l)$$
$$F^{+}_{A_1 \times A_2}(kl) = F^{+}_{A_1}(k) \vee F^{+}_{A_2}(l),$$

for all $(k, l) \in R_1 \times R_2$.

The membership value of the edges in $G_1 \times G_2$ can be calculated as,

$$(1)\ T^-_{B_1\times B_2}(kl_1)(kl_2) = T^-_{A_1}(k) \wedge T^-_{B_2}(l_1l_2)$$
$$T^+_{B_1\times B_2}(kl_1)(kl_2) = T^+_{A_1}(k) \wedge T^+_{B_2}(l_1l_2),$$

$$(2)\ I^-_{B_1\times B_2}(kl_1)(kl_2) = I^-_{A_1}(k) \wedge I^-_{B_2}(l_1l_2)$$
$$I^+_{B_1\times B_2}(kl_1)(kl_2) = I^+_{A_1}(k) \wedge I^+_{B_2}(l_1l_2),$$

$$(3)\ F^-_{B_1\times B_2}(kl_1)(kl_2) = F^-_{A_1}(k) \vee F^-_{B_2}(l_1l_2)$$
$$F^+_{B_1\times B_2}(kl_1)(kl_2) = F^+_{A_1}(k) \vee F^+_{B_2}(l_1l_2),$$

for all $k \in R_1, l_1l_2 \in S_2$.

$$(4)\ T^-_{B_1\times B_2}(k_1l)(k_2l) = T^-_{A_2}(l) \wedge T^-_{B_2}(k_1k_2)$$
$$T^+_{B_1\times B_2}(k_1l)(k_2l) = T^+_{A_2}(l) \wedge T^+_{B_2}(k_1k_2),$$

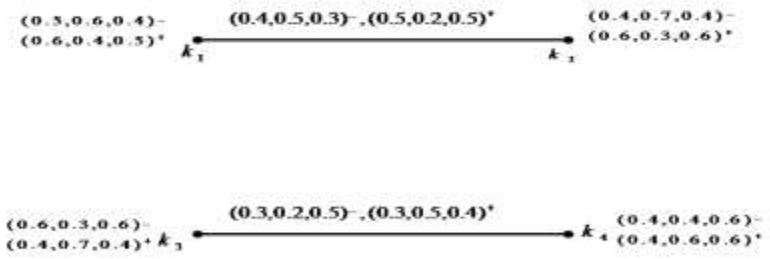
$$(5)\ I^-_{B_1\times B_2}(k_1l)(k_2l) = I^-_{A_2}(l) \wedge I^-_{B_2}(k_1k_2)$$
$$I^+_{B_1\times B_2}(k_1l)(k_2l) = I^+_{A_2}(l) \wedge I^+_{B_2}(k_1k_2),$$

$$(6)\ F^-_{B_1\times B_2}(k_1l)(k_2l) = F^-_{A_2}(l) \vee F^-_{B_2}(k_1k_2)$$
$$F^+_{B_1\times B_2}(k_1l)(k_2l) = F^+_{A_2}(l) \vee F^+_{B_2}(k_1k_2),$$

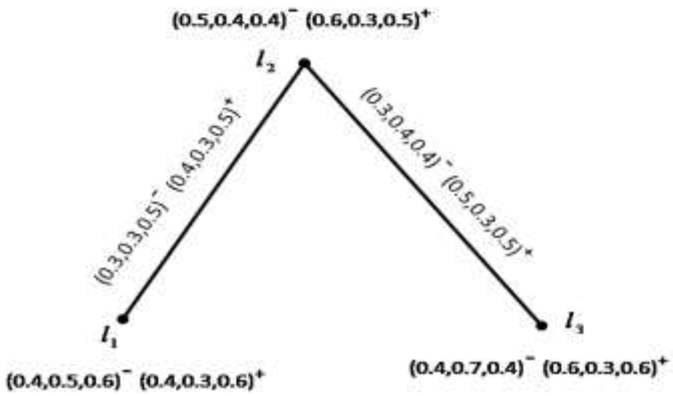
for all $k_1k_2 \in S_1, l \in R_2$.

Example 3.2 Consider $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ are two NVGs of $G = (R, S)$, as represented in Figure 2, now we get $G_1 \times G_2$ as follows see Figure 3.

$$\hat{k}_1 = T[0.5,0.6], I[0.6,0.4], F[0.4,0.5], \hat{k}_2 = T[0.4,0.6], I[0.7,0.3], F[0.4,0.6],$$
$$\hat{k}_3 = T[0.6,0.4], I[0.3,0.7], F[0.6,0.4], \hat{k}_4 = T[0.4,0.4], I[0.4,0.6], F[0.6,0.6]$$
$$\hat{l}_1 = T[0.4,0.4], I[0.5,0.3], F[0.6,0.6], \hat{l}_2 = T[0.5,0.6], I[0.4,0.3], F[0.4,0.5],$$
$$\hat{l}_3 = T[0.4,0.6], I[0.7,0.3], F[0.4,0.6]$$
$$k_1^- = (0.5,0.6,0.4), k_2^- = (0.4,0.7,0.4), k_3^- = (0.6,0.3,0.6), k_4^- = (0.4,0.4,0.6)$$
$$k_1^+ = (0.6,0.4,0.5), k_2^+ = (0.6,0.3,0.6), k_3^+ = (0.4,0.7,0.4), k_4^- = (0.4,0.6,0.6)$$
$$l_1^- = (0.4,0.5,0.6), l_2^- = (05,0.4,0.4), l_3^- = (0.4,0.7,0.4)$$
$$\mathbf{l_1^+} = (\mathbf{0.4}, \mathbf{0.3}, \mathbf{0.6}), \mathbf{l_2^+} = (\mathbf{0.6}, \mathbf{0.3}, \mathbf{0.5}), \mathbf{l_3^+} = (\mathbf{0.6}, \mathbf{0.3}, \mathbf{0.6}).$$

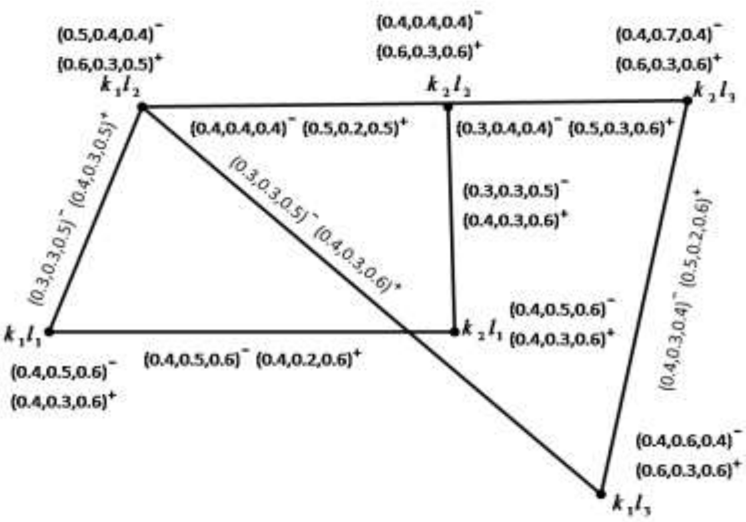


G_1



G_2

Figure 2: NEUTROSOPHIC VAGUE GRAPH



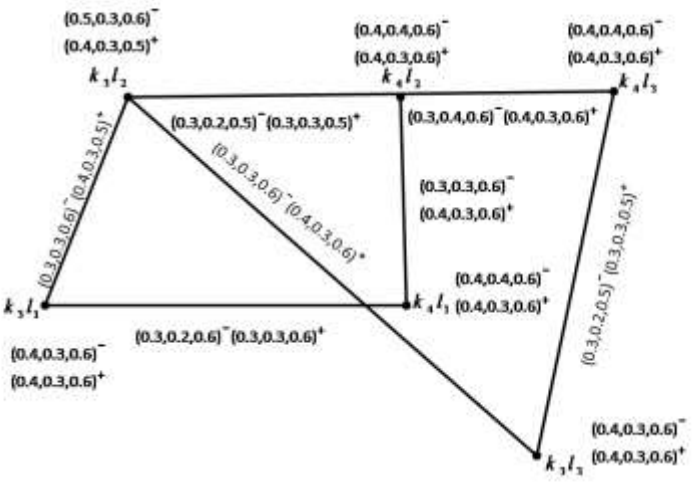


Figure 3: CARTESIAN PRODUCT OF NEUTROSOPHIC VAGUE GRAPH

Theorem 3.3 The Cartesian product $G_1 \times G_2 = (R_1 \times R_2, S_1 \times S_2)$ of two NVG G_1 and G_2 is also the NVG of $G_1 \times G_2$.

Proof. We consider two cases.

Case 1: for $k \in R_1, l_1 l_2 \in S_2$,

$$\begin{aligned}\widehat{T}_{(B_1 \times B_2)}((kl_1)(kl_2)) &= \widehat{T}_{A_1}(k) \wedge \widehat{T}_{B_2}(l_1 l_2)\\ &\leq \widehat{T}_{A_1}(k) \wedge [\widehat{T}_{A_2}(l_1) \wedge \widehat{T}_{A_2}(l_2)]\\ &= [\widehat{T}_{A_1}(k) \wedge \widehat{T}_{A_2}(l_1)] \wedge [\widehat{T}_{A_1}(k) \wedge \widehat{T}_{A_2}(l_2)]\\ &= \widehat{T}_{(A_1 \times A_2)}(k, l_1) \wedge \widehat{T}_{(A_1 \times A_2)}(k, l_2)\end{aligned}$$

$$\begin{aligned}\hat{I}_{(B_1 \times B_2)}((kl_1)(kl_2)) &= \hat{I}_{A_1}(k) \wedge \hat{I}_{B_2}(l_1 l_2)\\ &\leq \hat{I}_{A_1}(k) \wedge [\hat{I}_{A_2}(l_1) \wedge \hat{I}_{A_2}(l_2)]\\ &= [\hat{I}_{A_1}(k) \wedge \hat{I}_{A_2}(l_1)] \wedge [\hat{I}_{A_1}(k) \wedge \hat{I}_{A_2}(l_2)]\\ &= \hat{I}_{(A_1 \times A_2)}(k, l_1) \wedge \hat{I}_{(A_1 \times A_2)}(k, l_2)\end{aligned}$$

$$\begin{aligned}\widehat{F}_{(B_1 \times B_2)}((kl_1)(kl_2)) &= \widehat{F}_{A_1}(k) \vee \widehat{F}_{B_2}(l_1 l_2)\\ &\leq \widehat{F}_{A_1}(k) \vee [\widehat{F}_{A_2}(l_1) \vee \widehat{F}_{A_2}(l_2)]\\ &= [\widehat{F}_{A_1}(k) \vee \widehat{F}_{A_2}(l_1)] \vee [\widehat{F}_{A_1}(k) \vee \widehat{F}_{A_2}(l_2)]\\ &= \widehat{F}_{(A_1 \times A_2)}(k, l_1) \vee \widehat{F}_{(A_1 \times A_2)}(k, l_2)\end{aligned}$$

for all $kl_1, kl_2 \in G_1 \times G_2$.

Case 2: for $k \in R_2, l_1 l_2 \in S_1$.

$$\begin{aligned}\widehat{T}_{(B_1 \times B_2)}((l_1 k)(l_2 k)) &= \widehat{T}_{A_2}(k) \wedge \widehat{T}_{B_1}(l_1 l_2)\\ &\leq \widehat{T}_{A_2}(k) \wedge [\widehat{T}_{A_1}(l_1) \wedge \widehat{T}_{A_1}(l_2)]\\ &= [\widehat{T}_{A_2}(k) \wedge \widehat{T}_{A_1}(l_1)] \wedge [\widehat{T}_{A_2}(k) \wedge \widehat{T}_{A_1}(l_2)]\\ &= \widehat{T}_{(A_1 \times A_2)}(l_1, k) \wedge \widehat{T}_{(A_1 \times A_2)}(l_2, k)\end{aligned}$$

$$\hat{I}_{(B_1 \times B_2)}((l_1 k)(l_2 k)) = \hat{I}_{A_2}(k) \wedge \hat{I}_{B_1}(l_1 l_2)$$

$$
\begin{aligned}
&\leq \hat{I}_{A_2}(k) \wedge [\hat{I}_{A_1}(l_1) \wedge \hat{I}_{A_1}(l_2)] \\
&= [\hat{I}_{A_2}(k) \wedge \hat{I}_{A_1}(l_1)] \wedge [\hat{I}_{A_2}(k) \wedge \hat{I}_{A_1}(l_2)] \\
&= \hat{I}_{(A_1 \times A_2)}(l_1, k) \wedge \hat{I}_{(A_1 \times A_2)}(l_2, k)
\end{aligned}
$$

$$
\begin{aligned}
\hat{F}_{(B_1 \times B_2)}((l_1 k)(l_2 k)) &= \hat{F}_{A_2}(k) \vee \hat{F}_{B_1}(l_1 l_2) \\
&\leq \hat{F}_{A_2}(k) \vee [\hat{F}_{A_1}(l_1) \vee \hat{F}_{A_1}(l_2)] \\
&= [\hat{F}_{A_2}(k) \vee \hat{F}_{A_1}(l_1)] \vee [\hat{F}_{A_2}(k) \vee \hat{F}_{A_1}(l_2)] \\
&= \hat{F}_{(A_1 \times A_2)}(l_1, k) \vee \hat{F}_{(A_1 \times A_2)}(l_2, k)
\end{aligned}
$$

for all $l_1 k, l_2 k \in G_1 \times G_2$.

Definition 3.4 The Cross product of two NVGs G_1 and G_2 is denoted by the pair $G_1 \star G_2 = (R_1 \star R_2, S_1 \star S_2)$ and is defined as

$$
\begin{aligned}
\text{(i)}\, &T^-_{A_1 \star A_2}(kl) = T^-_{A_1}(k) \wedge T^-_{A_2}(l) \\
&I^-_{A_1 \star A_2}(kl) = I^-_{A_1}(k) \wedge I^-_{A_2}(l) \\
&F^-_{A_1 \star A_2}(kl) = F^-_{A_1}(k) \vee F^-_{A_2}(l) \\
&T^+_{A_1 \star A_2}(kl) = T^+_{A_1}(k) \wedge T^+_{A_2}(l) \\
&I^+_{A_1 \star A_2}(kl) = I^+_{A_1}(k) \wedge I^+_{A_2}(l) \\
&F^+_{A_1 \star A_2}(kl) = F^+_{A_1}(k) \vee F^+_{A_2}(l),
\end{aligned}
$$

for all $k, l \in R_1 \star R_2$.

$$
\begin{aligned}
\text{(ii)}\, &T^-_{(B_1 \star B_2)}(k_1 l_1)(k_2 l_2) = T^-_{B_1}(k_1 k_2) \wedge T^-_{B_2}(l_1 l_2) \\
&I^-_{(B_1 \star B_2)}(k_1 l_1)(k_2 l_2) = I^-_{B_1}(k_1 k_2) \wedge I^-_{B_2}(l_1 l_2) \\
&F^-_{(B_1 \star B_2)}(k_1 l_1)(k_2 l_2) = F^-_{B_1}(k_1 k_2) \vee F^-_{B_2}(l_1 l_2) \\
\text{(iii)}\, &T^+_{(B_1 \star B_2)}(k_1 l_1)(k_2 l_2) = T^+_{B_1}(k_1 k_2) \wedge T^+_{B_2}(l_1 l_2) \\
&I^+_{(B_1 \star B_2)}(k_1 l_1)(k_2 l_2) = I^+_{B_1}(k_1 k_2) \wedge I^+_{B_2}(l_1 l_2) \\
&F^+_{(B_1 \star B_2)}(k_1 l_1)(k_2 l_2) = F^+_{B_1}(k_1 k_2) \vee F^+_{B_2}(l_1 l_2),
\end{aligned}
$$

for all $k_1 k_2 \in S_1, l_1 l_2 \in S_2$.

Example 3.5 Consider $\mathbf{G_1} = (\mathbf{R_1}, \mathbf{S_1})$ and $\mathbf{G_2} = (\mathbf{R_2}, \mathbf{S_2})$ as two NVG of $\mathbf{G} = (\mathbf{R}, \mathbf{S})$ respectively, (see Figure 2). We obtain the cross product of $\mathbf{G_1} \star \mathbf{G_2}$ as follows (see Figure 4).

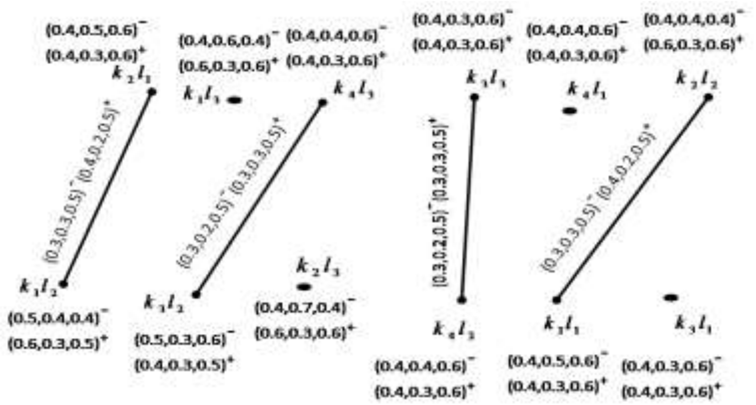


Figure 4: CROSS PRODUCT OF NEUTROSOPHIC VAGUE GRAPH

Theorem 3.6 The cross product $G_1 \star G_2 = (R_1 \star R_2, S_1 \star S_2)$ of two NVG G_1 and G_2 is an the NVG of $G_1 \star G_2$.

Proof. For all $k_1l_1, k_2l_2 \in G_1 \star G_2$

$$\begin{aligned}\hat{T}_{(B_1\star B_2)}((k_1l_1)(k_2l_2)) &= \hat{T}_{B_1}(k_1k_2) \wedge \hat{T}_{B_2}(l_1l_2)\\ &\leq [\hat{T}_{A_1}(k_1) \wedge \hat{T}_{A_1}(k_2)] \wedge [\hat{T}_{A_2}(l_1) \wedge \hat{T}_{A_2}(l_2)]\\ &= [\hat{T}_{A_1}(k_1) \wedge \hat{T}_{A_2}(l_1)] \wedge [\hat{T}_{A_1}(k_2) \wedge \hat{T}_{A_2}(l_2)]\\ &= \hat{T}_{(A_1\star A_2)}(k_1l_1) \wedge \hat{T}_{(A_1\star A_2)}(k_2, l_2)\end{aligned}$$

$$\begin{aligned}\hat{I}_{(B_1\star B_2)}((k_1l_1)(k_2l_2)) &= \hat{I}_{B_1}(k_1k_2) \wedge \hat{I}_{B_2}(l_1l_2)\\ &\leq [\hat{I}_{A_1}(k_1) \wedge \hat{I}_{A_1}(k_2)] \wedge [\hat{I}_{A_2}(l_1) \wedge \hat{I}_{A_2}(l_2)]\\ &= [\hat{I}_{A_1}(k_1) \wedge \hat{I}_{A_2}(l_1)] \wedge [\hat{I}_{A_1}(k_2) \wedge \hat{I}_{A_2}(l_2)]\\ &= \hat{I}_{(A_1\star A_2)}(k_1l_1) \wedge \hat{I}_{(A_1\star A_2)}(k_2, l_2)\end{aligned}$$

$$\begin{aligned}\hat{F}_{(B_1\star B_2)}((k_1l_1)(k_2l_2)) &= \hat{F}_{B_1}(k_1k_2) \vee \hat{F}_{B_2}(l_1l_2)\\ &\leq [\hat{F}_{A_1}(k_1) \vee \hat{F}_{A_1}(k_2)] \vee [\hat{F}_{A_2}(l_1) \vee \hat{F}_{A_2}(l_2)]\\ &= [\hat{F}_{A_1}(k_1) \vee \hat{F}_{A_2}(l_1)] \vee [\hat{F}_{A_1}(k_2) \vee \hat{F}_{A_2}(l_2)]\\ &= \hat{F}_{(A_1\star A_2)}(k_1l_1) \vee \hat{F}_{(A_1\star A_2)}(k_2, l_2).\end{aligned}$$

This completes the proof.

Definition 3.7 The lexicographic product of two NVGs G_1 and G_2 is denoted by the pair $G_1 \bullet G_2 = (R_1 \bullet R_2, S_1 \bullet S_2)$ and defined as

$$\begin{aligned}&(i) T^-_{(A_1\bullet A_2)}(kl) = T^-_{A_1}(k) \wedge T^-_{A_2}(l)\\ &I^-_{(A_1\bullet A_2)}(kl) = I^-_{A_1}(k) \wedge I^-_{A_2}(l)\\ &F^-_{(A_1\bullet A_2)}(kl) = F^-_{A_1}(k) \vee F^-_{A_2}(l)\\ &T^+_{(A_1\bullet A_2)}(kl) = T^+_{A_1}(k) \wedge T^+_{A_2}(l)\\ &I^+_{(A_1\bullet A_2)}(kl) = I^+_{A_1}(k) \wedge I^+_{A_2}(l)\\ &F^+_{(A_1\bullet A_2)}(kl) = F^+_{A_1}(k) \vee F^+_{A_2}(l),\end{aligned}$$

for all $kl \in R_1 \times R_2$

$$\begin{aligned}&(ii) T^-_{(B_1\bullet B_2)}(kl_1)(kl_2) = T^-_{A_1}(k) \wedge T^-_{B_2}(l_1l_2)\\ &I^-_{(B_1\bullet B_2)}(kl_1)(kl_2) = I^-_{A_1}(k) \wedge I^-_{B_2}(l_1l_2)\\ &F^-_{(B_1\bullet B_2)}(kl_1)(kl_2) = F^-_{A_1}(k) \vee F^-_{B_2}(l_1l_2)\\ &T^+_{(B_1\bullet B_2)}(kl_1)(kl_2) = T^+_{A_1}(k) \wedge T^+_{B_2}(l_1l_2)\\ &I^+_{(B_1\bullet B_2)}(kl_1)(kl_2) = I^+_{A_1}(k) \wedge I^+_{B_2}(l_1l_2)\\ &F^+_{(B_1\bullet B_2)}(kl_1)(kl_2) = F^+_{A_1}(k) \vee F^+_{B_2}(l_1l_2),\end{aligned}$$

for all $k \in R_1, l_1l_2 \in S_2$.

$$\begin{aligned}&(iii) T^-_{(B_1\bullet B_2)}(k_1l_1)(k_2l_2) = T^-_{B_1}(k_1k_2) \wedge T^-_{B_2}(l_1l_2)\\ &I^-_{(B_1\bullet B_2)}(k_1l_1)(k_2l_2) = I^-_{B_1}(k_1k_2) \wedge I^-_{B_2}(l_1l_2)\\ &F^-_{(B_1\bullet B_2)}(k_1l_1)(k_2l_2) = F^-_{B_1}(k_1k_2) \vee F^-_{B_2}(l_1l_2)\\ &T^+_{(B_1\bullet B_2)}(k_1l_1)(k_2l_2) = T^+_{B_1}(k_1k_2) \wedge T^+_{B_2}(l_1l_2)\\ &I^+_{(B_1\bullet B_2)}(k_1l_1)(k_2l_2) = I^+_{B_1}(k_1k_2) \wedge I^+_{B_2}(l_1l_2)\\ &F^+_{(B_1\bullet B_2)}(k_1l_1)(k_2l_2) = F^+_{B_1}(k_1k_2) \vee F^+_{B_2}(l_1l_2), \quad \text{for all } k_1k_2 \in S_1, l_1l_2 \in S_2.\end{aligned}$$

Example 3.8 The lexicographic product of NVG $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ shown in Figure 2 is defined as $G_1 \bullet G_2 = (R_1 \bullet R_2, S_1 \bullet S_2)$ and is presented in Figure 5.

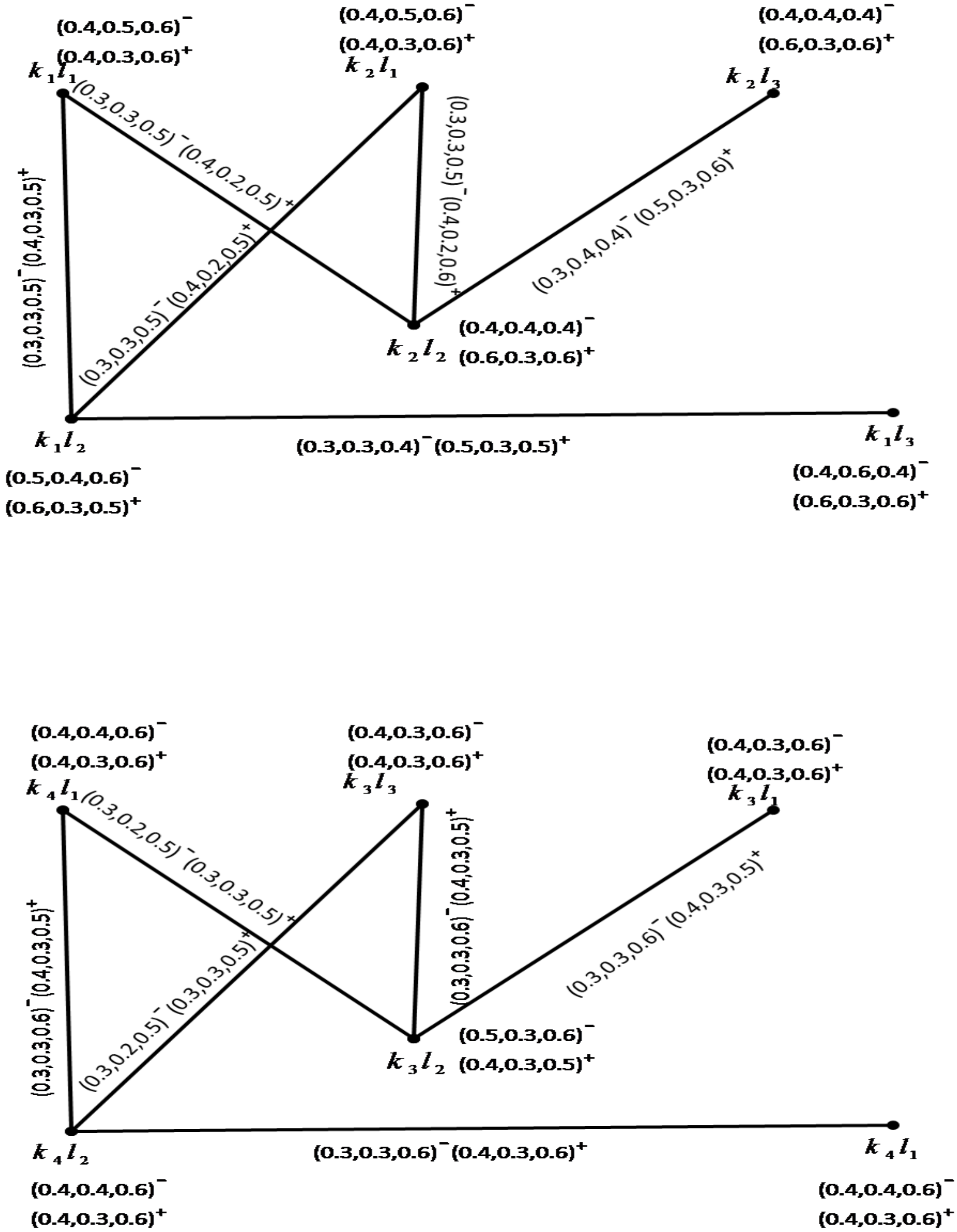


Figure 5: LEXICOGRAPHIC PRODUCT OF NEUTROSOPHIC VAGUE GRAPH

Theorem 3.9 The lexicographic product $G_1 \bullet G_2 = (R_1 \bullet R_2, S_1 \bullet S_2)$ of two NVG G_1 and G_2 is the NVG of $G_1 \bullet G_2$.

Proof. We have two cases.

Case 1: For $k \in R_1, l_1l_2 \in S_2$,

$$\begin{aligned}\widehat{T}_{(B_1\bullet B_2)}((kl_1)(kl_2)) &= \widehat{T}_{A_1}(k) \wedge \widehat{T}_{B_2}(l_1l_2)\\ &\leq \widehat{T}_{A_1}(k) \wedge [\widehat{T}_{A_2}(l_1) \wedge \widehat{T}_{A_2}(l_2)]\\ &= [\widehat{T}_{A_1}(k) \wedge \widehat{T}_{A_2}(l_1)] \wedge [\widehat{T}_{A_1}(k) \wedge \widehat{T}_{A_2}(l_2)]\\ &= \widehat{T}_{(A_1\bullet A_2)}(k, l_1) \wedge \widehat{T}_{(A_1\bullet A_2)}(k, l_2)\end{aligned}$$

$$\begin{aligned}\hat{I}_{(B_1\bullet B_2)}((kl_1)(kl_2)) &= \hat{I}_{A_1}(k) \wedge \hat{I}_{B_2}(l_1l_2)\\ &\leq \hat{I}_{A_1}(k) \wedge [\hat{I}_{A_2}(l_1) \wedge \hat{I}_{A_2}(l_2)]\\ &= [\hat{I}_{A_1}(k) \wedge \hat{I}_{A_2}(l_1)] \wedge [\hat{I}_{A_1}(k) \wedge \hat{I}_{A_2}(l_2)]\\ &= \hat{I}_{(A_1\bullet A_2)}(k, l_1) \wedge \hat{I}_{(A_1\bullet A_2)}(k, l_2)\end{aligned}$$

$$\begin{aligned}\widehat{F}_{(B_1\bullet B_2)}((kl_1)(kl_2)) &= \widehat{F}_{A_1}(k) \vee \widehat{F}_{B_2}(l_1l_2)\\ &\leq \widehat{F}_{A_1}(k) \vee [\widehat{F}_{A_2}(l_1) \vee \widehat{F}_{A_2}(l_2)]\\ &= [\widehat{F}_{A_1}(k) \vee \widehat{F}_{A_2}(l_1)] \vee [\widehat{F}_{A_1}(k) \vee \widehat{F}_{A_2}(l_2)]\\ &= \widehat{F}_{(A_1\bullet A_2)}(k, l_1) \vee \widehat{F}_{(A_1\bullet A_2)}(k, l_2)\end{aligned}$$

for all $kl_1, kl_2 \in S_1 \times S_2$.

Case 2: For all $k_1l_1 \in S_1, k_2l_2 \in S_2$,

$$\begin{aligned}\widehat{T}_{(B_1\bullet B_2)}((k_1l_1)(k_2l_2)) &= \widehat{T}_{B_1}(k_1k_2) \wedge \widehat{T}_{B_2}(l_1l_2)\\ &\leq [\widehat{T}_{A_1}(k_1) \wedge \widehat{T}_{A_1}(k_2)] \wedge [\widehat{T}_{A_2}(l_1) \wedge \widehat{T}_{A_2}(l_2)]\\ &= [\widehat{T}_{A_1}(k_1) \wedge \widehat{T}_{A_2}(l_1)] \wedge [\widehat{T}_{A_1}(k_2) \wedge \widehat{T}_{A_2}(l_2)]\\ &= \widehat{T}_{(A_1\bullet A_2)}(k_1l_1) \wedge \widehat{T}_{(A_1\bullet A_2)}(k_2, l_2)\end{aligned}$$

$$\begin{aligned}\hat{I}_{(B_1\bullet B_2)}((k_1l_1)(k_2l_2)) &= \hat{I}_{B_1}(k_1k_2) \wedge \hat{I}_{B_2}(l_1l_2)\\ &\leq [\hat{I}_{A_1}(k_1) \wedge \hat{I}_{A_1}(k_2)] \wedge [\hat{I}_{A_2}(l_1) \wedge \hat{I}_{A_2}(l_2)]\\ &= [\hat{I}_{A_1}(k_1) \wedge \hat{I}_{A_2}(l_1)] \wedge [\hat{I}_{A_1}(k_2) \wedge \hat{I}_{A_2}(l_2)]\\ &= \hat{I}_{(A_1\bullet A_2)}(k_1l_1) \wedge \hat{I}_{(A_1\bullet A_2)}(k_2, l_2)\end{aligned}$$

$$\begin{aligned}\widehat{F}_{(B_1\bullet B_2)}((k_1l_1)(k_2l_2)) &= \widehat{F}_{B_1}(k_1k_2) \vee \widehat{F}_{B_2}(l_1l_2)\\ &\leq [\widehat{F}_{A_1}(k_1) \vee \widehat{F}_{A_1}(k_2)] \vee [\widehat{F}_{A_2}(l_1) \vee \widehat{F}_{A_2}(l_2)]\\ &= [\widehat{F}_{A_1}(k_1) \vee \widehat{F}_{A_2}(l_1)] \vee [\widehat{F}_{A_1}(k_2) \vee \widehat{F}_{A_2}(l_2)]\\ &= \widehat{F}_{(A_1\bullet A_2)}(k_1l_1) \vee \widehat{F}_{(A_1\bullet A_2)}(k_2, l_2)\end{aligned}$$

for all $k_1, l_1 \in k_2, l_2 \in R_1 \bullet R_2$.

Definition 3.10 The strong product of two NVG G_1 and G_2 is denoted by the pair $G_1 \boxtimes G_2 = (R_1 \boxtimes R_2, S_1 \boxtimes S_2)$ and defined as

$$\begin{aligned}&(i) T^-_{(A_1\boxtimes A_2)}(kl) = T^-_{A_1}(k) \wedge T^-_{A_2}(l)\\ &\quad I^-_{(A_1\boxtimes A_2)}(kl) = I^-_{A_1}(k) \wedge I^-_{A_2}(l)\\ &\quad F^-_{(A_1\boxtimes A_2)}(kl) = F^-_{A_1}(k) \vee F^-_{A_2}(l)\end{aligned}$$

$$T^{+}_{(A_1 \boxtimes A_2)}(kl) = T^{+}_{A_1}(k) \wedge T^{+}_{A_2}(l)$$
$$I^{+}_{(A_1 \boxtimes A_2)}(kl) = I^{+}_{A_1}(k) \wedge I^{+}_{A_2}(l)$$
$$F^{+}_{(A_1 \boxtimes A_2)}(kl) = F^{+}_{A_1}(k) \vee F^{+}_{A_2}(l)$$

for all $kl \in R_1 \boxtimes R_2$

$$\text{(ii)} T^{-}_{(B_1 \boxtimes B_2)}(kl_1)(kl_2) = T^{-}_{A_1}(k) \wedge T^{-}_{B_2}(l_1l_2)$$
$$I^{-}_{(B_1 \boxtimes B_2)}(kl_1)(kl_2) = I^{-}_{A_1}(k) \wedge I^{-}_{B_2}(l_1l_2)$$
$$F^{-}_{(B_1 \boxtimes B_2)}(kl_1)(kl_2) = F^{-}_{A_1}(k) \vee F^{-}_{B_2}(l_1l_2)$$
$$T^{+}_{(B_1 \boxtimes B_2)}(kl_1)(kl_2) = T^{+}_{A_1}(k) \wedge T^{+}_{B_2}(l_1l_2)$$
$$I^{+}_{(B_1 \boxtimes B_2)}(kl_1)(kl_2) = I^{+}_{A_1}(k) \wedge I^{+}_{B_2}(l_1l_2)$$
$$F^{+}_{(B_1 \boxtimes B_2)}(kl_1)(kl_2) = F^{+}_{A_1}(k) \vee F^{+}_{B_2}(l_1l_2),$$

for all $k \in R_1, l_1l_2 \in S_2$.

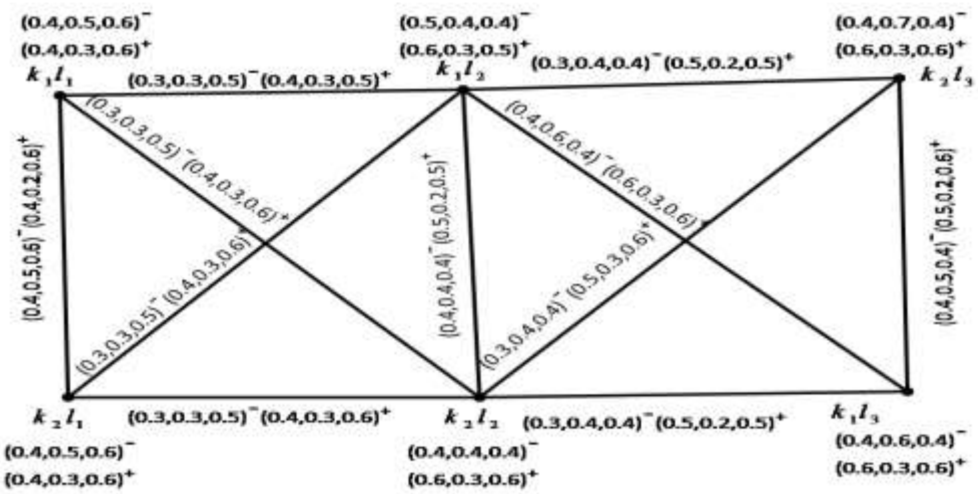
$$\text{(iii)} T^{-}_{B_1 \boxtimes B_2}(k_1l)(k_2l) = T^{-}_{A_2}(l) \wedge T^{-}_{B_2}(k_1k_2)$$
$$I^{-}_{B_1 \boxtimes B_2}(k_1l)(k_2l) = I^{-}_{A_2}(l) \wedge I^{-}_{B_2}(k_1k_2)$$
$$F^{-}_{B_1 \boxtimes B_2}(k_1l)(k_2l) = F^{-}_{A_2}(l) \vee F^{-}_{B_2}(k_1k_2)$$
$$T^{+}_{B_1 \boxtimes B_2}(k_1l)(k_2l) = T^{+}_{A_2}(l) \wedge T^{+}_{B_2}(k_1k_2)$$
$$I^{+}_{B_1 \boxtimes B_2}(k_1l)(k_2l) = I^{+}_{A_2}(l) \wedge I^{+}_{B_2}(k_1k_2)$$
$$F^{+}_{B_1 \boxtimes B_2}(k_1l)(k_2l) = F^{+}_{A_2}(l) \vee F^{+}_{B_2}(k_1k_2),$$

for all $k_1k_2 \in S_1, l \in R_2$.

$$\text{(iv)} T^{-}_{(B_1 \boxtimes B_2)}(k_1l_1)(k_2l_2) = T^{-}_{B_1}(k_1k_2) \wedge T^{-}_{B_2}(l_1l_2)$$
$$I^{-}_{(B_1 \boxtimes B_2)}(k_1l_1)(k_2l_2) = I^{-}_{B_1}(k_1k_2) \wedge I^{-}_{B_2}(l_1l_2)$$
$$F^{-}_{(B_1 \boxtimes B_2)}(k_1l_1)(k_2l_2) = F^{-}_{B_1}(k_1k_2) \vee F^{-}_{B_2}(l_1l_2)$$
$$T^{+}_{(B_1 \boxtimes B_2)}(k_1l_1)(k_2l_2) = T^{+}_{B_1}(k_1k_2) \wedge T^{+}_{B_2}(l_1l_2)$$
$$I^{+}_{(B_1 \boxtimes B_2)}(k_1l_1)(k_2l_2) = I^{+}_{B_1}(k_1k_2) \wedge I^{+}_{B_2}(l_1l_2)$$
$$F^{+}_{(B_1 \boxtimes B_2)}(k_1l_1)(k_2l_2) = F^{+}_{B_1}(k_1k_2) \vee F^{N}_{B_2}(l_1l_2),$$

for all $k_1k_2 \in S_1, l_1l_2 \in S_2$.

Example 3.11 The strong product of NVG $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ shown in Figure 2 is defined as $G_1 \boxtimes G_2 = (S_1 \boxtimes S_2, T_1 \boxtimes T_2)$ and is presented in Figure 6.



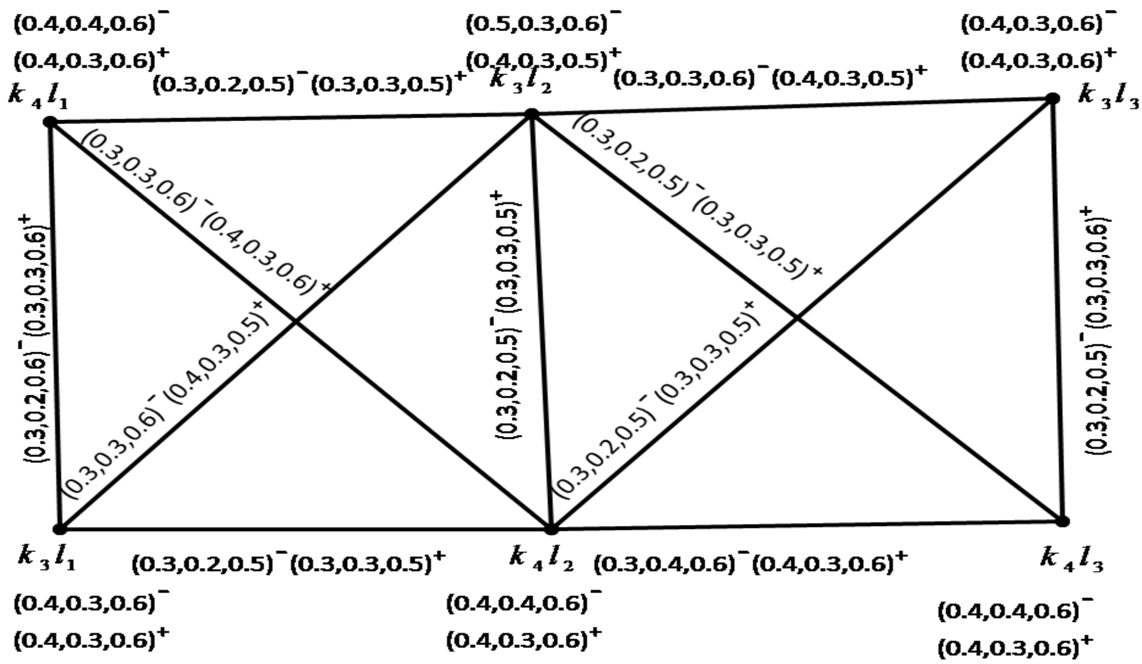


Figure 6: STRONG PRODUCT OF NEUTROSOPHIC VAGUE GRAPH

Theorem 3.12 The strong product $G_1 \boxtimes G_2 = (R_1 \boxtimes R_2, S_1 \boxtimes S_2)$ of two NVG G_1 and G_2 is a NVG of $G_1 \boxtimes G_2$.

Proof. There are three cases:

Case 1: for $k \in R_1, l_1l_2 \in S_2$,

$$\begin{aligned}
\widehat{T}_{(B_1 \boxtimes B_2)}((kl_1)(kl_2)) &= \widehat{T}_{A_1}(k) \wedge \widehat{T}_{B_2}(l_1l_2) \\
&\leq \widehat{T}_{A_1}(k) \wedge [\widehat{T}_{A_2}(l_1) \wedge \widehat{T}_{A_2}(l_2)] \\
&= [\widehat{T}_{A_1}(k) \wedge \widehat{T}_{A_2}(l_1)] \wedge [\widehat{T}_{A_1}(k) \wedge \widehat{T}_{A_2}(l_2)] \\
&= \widehat{T}_{(A_1 \boxtimes A_2)}(k, l_1) \wedge \widehat{T}_{(A_1 \boxtimes A_2)}(k, l_2)
\end{aligned}$$

$$\begin{aligned}
\hat{I}_{(B_1 \boxtimes B_2)}((kl_1)(kl_2)) &= \hat{I}_{A_1}(k) \wedge \hat{I}_{B_2}(l_1l_2) \\
&\leq \hat{I}_{A_1}(k) \wedge [\hat{I}_{A_2}(l_1) \wedge \hat{I}_{A_2}(l_2)] \\
&= [\hat{I}_{A_1}(k) \wedge \hat{I}_{A_2}(l_1)] \wedge [\hat{I}_{A_1}(k) \wedge \hat{I}_{A_2}(l_2)] \\
&= \hat{I}_{(A_1 \boxtimes A_2)}(k, l_1) \wedge \hat{I}_{(A_1 \boxtimes A_2)}(k, l_2)
\end{aligned}$$

$$\begin{aligned}
\widehat{F}_{(B_1 \boxtimes B_2)}((kl_1)(kl_2)) &= \widehat{F}_{A_1}(k) \vee \widehat{F}_{B_2}(l_1l_2) \\
&\leq \widehat{F}_{A_1}(k) \vee [\widehat{F}_{A_2}(l_1) \vee \widehat{F}_{A_2}(l_2)] \\
&= [\widehat{F}_{A_1}(k) \vee \widehat{F}_{A_2}(l_1)] \vee [\widehat{F}_{A_1}(k) \vee \widehat{F}_{A_2}(l_2)] \\
&= \widehat{F}_{(A_1 \boxtimes A_2)}(k, l_1) \vee \widehat{F}_{(A_1 \boxtimes A_2)}(k, l_2),
\end{aligned}$$

for all $kl_1, kl_2 \in R_1 \boxtimes R_2$.

Case 2: for $k \in R_2, l_1l_2 \in S_1$,

$$\widehat{T}_{(B_1 \boxtimes B_2)}((l_1k)(l_2k)) = \widehat{T}_{A_2}(k) \wedge \widehat{T}_{B_1}(l_1l_2)$$

$$\leq \hat{T}_{A_2}(k) \wedge [\hat{T}_{A_1}(l_1) \wedge \hat{T}_{A_1}(l_2)]$$
$$= [\hat{T}_{A_2}(k) \wedge \hat{T}_{A_1}(l_1)] \wedge [\hat{T}_{A_2}(k) \wedge \hat{T}_{A_1}(l_2)]$$
$$= \hat{T}_{(A_1 \boxtimes A_2)}(l_1, k) \wedge \hat{T}_{(A_1 \boxtimes A_2)}(l_2, k)$$

$$\hat{I}_{(B_1 \boxtimes B_2)}((l_1k)(l_2k)) = \hat{I}_{A_2}(k) \wedge \hat{I}_{B_1}(l_1l_2)$$
$$\leq \hat{I}_{A_2}(k) \wedge [\hat{I}_{A_1}(l_1) \wedge \hat{I}_{A_1}(l_2)]$$
$$= [\hat{I}_{A_2}(k) \wedge \hat{I}_{A_1}(l_1)] \wedge [\hat{I}_{A_2}(k) \wedge \hat{I}_{A_1}(l_2)]$$
$$= \hat{I}_{(A_1 \boxtimes A_2)}(l_1, k) \wedge \hat{I}_{(A_1 \boxtimes A_2)}(l_2, k)$$

$$\hat{F}_{(B_1 \boxtimes B_2)}((l_1k)(l_2k)) = \hat{F}_{A_2}(k) \vee \hat{F}_{B_1}(l_1l_2)$$
$$\leq \hat{F}_{A_2}(k) \vee [\hat{F}_{A_1}(l_1) \vee \hat{F}_{A_1}(l_2)]$$
$$= [\hat{F}_{A_2}(k) \vee \hat{F}_{A_1}(l_1)] \vee [\hat{F}_{A_2}(k) \vee \hat{F}_{A_1}(l_2)]$$
$$= \hat{F}_{(A_1 \boxtimes A_2)}(l_1, k) \vee \hat{F}_{(A_1 \boxtimes A_2)}(l_2, k)$$

for all $l_1k, l_2k \in R_1 \boxtimes R_2$.

Case 3: for $k_1, k_2 \in S_1, l_1l_2 \in S_2$

$$\hat{T}_{(B_1 \boxtimes B_2)}((k_1l_1)(k_2l_2)) = \hat{T}_{B_1}(k_1k_2) \wedge \hat{T}_{B_2}(l_1l_2)$$
$$\leq [\hat{T}_{A_1}(k_1) \wedge \hat{T}_{A_1}(k_2)] \wedge [\hat{T}_{A_2}(l_1) \wedge \hat{T}_{A_2}(l_2)]$$
$$= [\hat{T}_{A_1}(k_1) \wedge \hat{T}_{A_2}(l_1)] \wedge [\hat{T}_{A_1}(k_2) \wedge \hat{T}_{A_2}(l_2)]$$
$$= \hat{T}_{(A_1 \boxtimes A_2)}(k_1l_1) \wedge \hat{T}_{(A_1 \boxtimes A_2)}(k_2, l_2)$$

$$\hat{I}_{(B_1 \boxtimes B_2)}((k_1l_1)(k_2l_2)) = \hat{I}_{B_1}(k_1k_2) \wedge \hat{I}_{B_2}(l_1l_2)$$
$$\leq [\hat{I}_{A_1}(k_1) \wedge \hat{I}_{A_1}(k_2)] \wedge [\hat{I}_{A_2}(l_1) \wedge \hat{I}_{A_2}(l_2)]$$
$$= [\hat{I}_{A_1}(k_1) \wedge \hat{I}_{A_2}(l_1)] \wedge [\hat{I}_{A_1}(k_2) \wedge \hat{I}_{A_2}(l_2)]$$
$$= \hat{I}_{(A_1 \boxtimes A_2)}(k_1l_1) \wedge \hat{I}_{(A_1 \boxtimes A_2)}(k_2, l_2)$$

$$\hat{F}_{(B_1 \boxtimes B_2)}((k_1l_1)(k_2l_2)) = \hat{F}_{B_1}(k_1k_2) \vee \hat{F}_{B_2}(l_1l_2)$$
$$\leq [\hat{F}_{A_1}(k_1) \vee \hat{F}_{A_1}(k_2)] \vee [\hat{F}_{A_2}(l_1) \vee \hat{F}_{A_2}(l_2)]$$
$$= [\hat{F}_{A_1}(k_1) \vee \hat{F}_{A_2}(l_1)] \vee [\hat{F}_{A_1}(k_2) \vee \hat{F}_{A_2}(l_2)]$$
$$= \hat{F}_{(A_1 \boxtimes A_2)}(k_1l_1) \vee \hat{F}_{(A_1 \boxtimes A_2)}(k_2, l_2),$$

for all $l_1k_1, l_2k_1 \in R_1 \boxtimes R_2$.

Definition 3.13 The composition of two NVG G_1 and G_2 is denoted by the pair $G_1 \circ G_2 = (R_1 \boxtimes R_2, S_1 \circ S_2)$ and defined as

$$\text{(i)}T^-_{(A_1 \circ A_2)}(kl) = T^-_{A_1}(k) \wedge T^-_{A_2}(l)$$
$$I^-_{(A_1 \circ A_2)}(kl) = I^-_{A_1}(k) \wedge I^-_{A_2}(l)$$
$$F^-_{(A_1 \circ A_2)}(kl) = F^-_{A_1}(k) \vee F^-_{A_2}(l)$$
$$T^+_{(A_1 \circ A_2)}(kl) = T^+_{A_1}(k) \wedge T^+_{A_2}(l)$$
$$I^+_{(A_1 \circ A_2)}(kl) = I^+_{A_1}(k) \wedge I^+_{A_2}(l)$$
$$F^+_{(A_1 \circ A_2)}(kl) = F^+_{A_1}(k) \vee F^+_{A_2}(l)$$

for all $kl \in R_1 \circ R_2$.

$$\text{(ii)}T^-_{(B_1 \circ B_2)}(kl_1)(kl_2) = T^-_{A_1}(k) \wedge T^-_{B_2}(l_1l_2)$$

$$I^-_{(B_1\circ B_2)}(kl_1)(kl_2) = I^-_{A_1}(k) \wedge I^-_{B_2}(l_1l_2)$$
$$F^-_{(B_1\circ B_2)}(kl_1)(kl_2) = F^-_{A_1}(k) \vee F^-_{B_2}(l_1l_2)$$
$$T^+_{(B_1\circ B_2)}(kl_1)(kl_2) = T^+_{A_1}(k) \wedge T^+_{B_2}(l_1l_2)$$
$$I^+_{(B_1\circ B_2)}(kl_1)(kl_2) = I^+_{A_1}(k) \wedge I^+_{B_2}(l_1l_2)$$
$$F^+_{(B_1\circ B_2)}(kl_1)(kl_2) = F^+_{A_1}(k) \vee F^+_{B_2}(l_1l_2),$$

for all $k \in R_1, l_1l_2 \in S_2$.

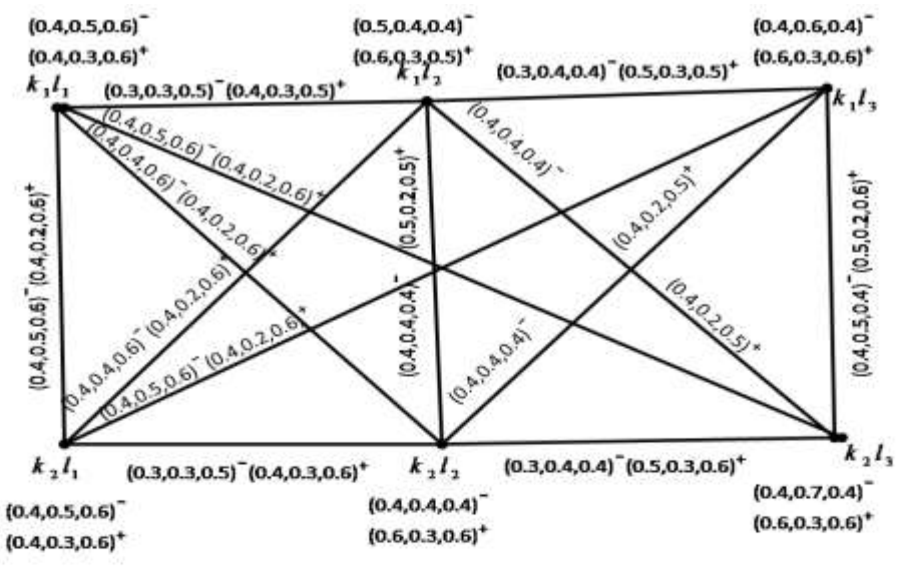
$$\text{(iii)}\,T^-_{B_1\circ B_2}(k_1l)(k_2l) = T^-_{A_2}(l) \wedge T^-_{B_2}(k_1k_2)$$
$$I^-_{B_1\circ B_2}(k_1,l)(k_2,l) = I^-_{A_2}(l) \wedge I^-_{B_2}(k_1k_2)$$
$$F^-_{B_1\circ B_2}(k_1,l)(k_2,l) = F^-_{A_2}(l) \vee F^-_{B_2}(k_1k_2)$$
$$T^+_{B_1\circ B_2}(k_1,l)(k_2,l) = T^+_{A_2}(l) \wedge T^+_{B_2}(k_1k_2)$$
$$I^+_{B_1\circ B_2}(k_1,l)(k_2,l) = I^+_{A_2}(l) \wedge I^+_{B_2}(k_1k_2)$$
$$F^+_{B_1\circ B_2}(k_1,l)(k_2,l) = F^+_{A_2}(l) \vee F^+_{B_2}(k_1k_2),$$

for all $k_1k_2 \in S_1, l \in R_2$.

$$\text{(iv)}\,T^-_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) = T^-_{B_1}(k_1k_2) \wedge T^-_{A_2}(l_1) \wedge T^-_{A_2}(l_2)$$
$$I^-_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) = I^-_{B_1}(k_1k_2) \wedge I^-_{A_2}(l_1) \wedge I^-_{A_2}(l_2)$$
$$F^-_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) = F^-_{B_1}(k_1k_2) \vee F^-_{A_2}(l_1) \vee F^-_{A_2}(l_2)$$
$$T^+_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) = T^-_{B_1}(k_1k_2) \wedge T^+_{A_2}(l_1) \wedge T^+_{A_2}(l_2)$$
$$I^+_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) = I^+_{B_1}(k_1k_2) \wedge I^+_{A_2}(l_1) \wedge I^+_{A_2}(l_2)$$
$$F^+_{(B_1\circ B_2)}(k_1l_1)(k_2l_2) = F^+_{B_1}(k_1k_2) \vee F^+_{A_2}(l_1) \vee F^+_{A_2}(l_2),$$

for all $k_1k_2 \in S_1, l_1l_2 \in S_2$.

Example 3.14 The composition of NVG $G_1 = (R_1, S_1)$ and $G_2 = (R_2, S_2)$ shown in Figure 2 is defined as $G_1 \circ G_2 = (R_1 \circ R_2, S_1 \circ S_2)$ and is presented in Figure 7.



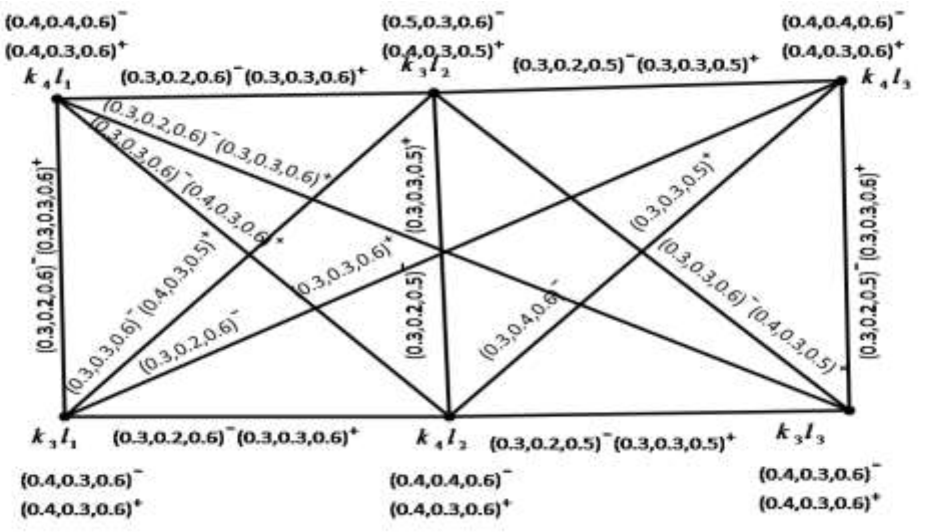


Figure 7: COMPOSITION OF NEUTROSOPHIC VAGUE GRAPH

Theorem 3.15 Composition $G_1 \circ G_2 = (R_1 \circ R_2, S_1 \circ S_2)$ of two NVG G_1 and G_2 is the NVG of $G_1 \circ G_2$.

Proof. We divide the proof into three cases:

Case:1 For $k \in R_1, l_1l_2 \in S_2$,

$$\begin{aligned}\widehat{T}_{(B_1\circ B_2)}\big((kl_1)(kl_2)\big) &= \widehat{T}_{A_1}(k) \wedge \widehat{T}_{B_2}(l_1l_2)\\ &\le \widehat{T}_{A_1}(k) \wedge [\widehat{T}_{A_2}(l_1) \wedge \widehat{T}_{A_2}(l_2)]\\ &= [\widehat{T}_{A_1}(k) \wedge \widehat{T}_{A_2}(l_1)] \wedge [\widehat{T}_{A_1}(k) \wedge \widehat{T}_{A_2}(l_2)]\\ &= \widehat{T}_{(A_1\circ A_2)}(k, l_1) \wedge \widehat{T}_{(A_1\circ A_2)}(k, l_2)\end{aligned}$$

$$\begin{aligned}\hat{I}_{(B_1\circ B_2)}((kl_1)(kl_2)) &= \hat{I}_{A_1}(k) \wedge \hat{I}_{B_2}(l_1l_2)\\ &\le \hat{I}_{A_1}(k) \wedge [\hat{I}_{A_2}(l_1) \wedge \hat{I}_{A_2}(l_2)]\\ &= [\hat{I}_{A_1}(k) \wedge \hat{I}_{A_2}(l_1)] \wedge [\hat{I}_{A_1}(k) \wedge \hat{I}_{A_2}(l_2)]\\ &= \hat{I}_{(A_1\circ A_2)}(k, l_1) \wedge \hat{I}_{(A_1\circ A_2)}(k, l_2)\end{aligned}$$

$$\begin{aligned}\widehat{F}_{(B_1\circ B_2)}((kl_1)(kl_2)) &= \widehat{F}_{A_1}(k) \vee \widehat{F}_{B_2}(l_1l_2)\\ &\le \widehat{F}_{A_1}(k) \vee [\widehat{F}_{A_2}(l_1) \vee \widehat{F}_{A_2}(l_2)]\\ &= [\widehat{F}_{A_1}(k) \vee \widehat{F}_{A_2}(l_1)] \vee [\widehat{F}_{A_1}(k) \vee \widehat{F}_{A_2}(l_2)]\\ &= \widehat{F}_{(A_1\circ A_2)}(k, l_1) \vee \widehat{F}_{(A_1\circ A_2)}(k, l_2)\end{aligned}$$

for all $kl_1, kl_2 \in R_1 \circ R_2$.

Case 2: for $k \in R_2, l_1l_2 \in S_1$,

$$\begin{aligned}\widehat{T}_{(B_1\circ B_2)}((l_1k)(l_2k)) &= \widehat{T}_{A_2}(k) \wedge \widehat{T}_{B_1}(l_1l_2)\\ &\le \widehat{T}_{A_2}(k) \wedge [\widehat{T}_{A_1}(l_1) \wedge \widehat{T}_{A_1}(l_2)]\\ &= [\widehat{T}_{A_2}(k) \wedge \widehat{T}_{A_1}(l_1)] \wedge [\widehat{T}_{A_2}(k) \wedge \widehat{T}_{A_1}(l_2)]\\ &= \widehat{T}_{(A_1\circ A_2)}(l_1, k) \wedge \widehat{T}_{(A_1\circ A_2)}(l_2, k)\end{aligned}$$

$$\hat{\mathrm{I}}_{(B_1\circ B_2)}((l_1k)(l_2k)) = \hat{\mathrm{I}}_{A_2}(k) \wedge \hat{\mathrm{I}}_{B_1}(l_1l_2)$$
$$\leq \hat{\mathrm{I}}_{A_2}(k) \wedge [\hat{\mathrm{I}}_{A_1}(l_1) \wedge \hat{\mathrm{I}}_{A_1}(l_2)]$$
$$= [\hat{\mathrm{I}}_{A_2}(k) \wedge \hat{\mathrm{I}}_{A_1}(l_1)] \wedge [\hat{\mathrm{I}}_{A_2}(k) \wedge \hat{\mathrm{I}}_{A_1}(l_2)]$$
$$= \hat{\mathrm{I}}_{(A_1\circ A_2)}(l_1,k) \wedge \hat{\mathrm{I}}_{(A_1\circ A_2)}(l_2,k)$$

$$\hat{F}_{(B_1\circ B_2)}((l_1k)(l_2k)) = \hat{F}_{A_2}(k) \vee \hat{F}_{B_1}(l_1l_2)$$
$$\leq \hat{F}_{A_2}(k) \vee [\hat{F}_{A_1}(l_1) \vee \hat{F}_{A_1}(l_2)]$$
$$= [\hat{F}_{A_2}(k) \vee \hat{F}_{A_1}(l_1)] \vee [\hat{F}_{A_2}(k) \vee \hat{F}_{A_1}(l_2)]$$
$$= \hat{F}_{(A_1\circ A_2)}(l_1,k) \vee \hat{F}_{(A_1\circ A_2)}(l_2,k), \quad \text{for all } l_1k, l_2k \in R_1 \circ R_2.$$

Case 3: For $k_1k_2 \in S_1, l_1, l_2 \in R_2$ such that $l_1 \neq l_2$,

$$\hat{T}_{(B_1\circ B_2)}((k_1l_1)(k_2l_2)) = \hat{T}_{B_1}(k_1,k_2) \wedge \hat{T}_{A_2}(l_1) \wedge \hat{T}_{A_2}(l_2)$$
$$\leq [\hat{T}_{A_1}(k_1) \wedge \hat{T}_{A_1}(k_2)] \wedge [\hat{T}_{A_2}(l_1) \wedge \hat{T}_{A_2}(l_2)]$$
$$= [\hat{T}_{A_1}(k_1) \wedge \hat{T}_{A_2}(l_1)] \wedge [\hat{T}_{A_1}(k_2) \wedge \hat{T}_{A_2}(l_2)]$$
$$= \hat{T}_{(A_1\circ A_2)}(k_1l_1) \wedge \hat{T}_{(A_1\circ A_2)}(k_2l_2)$$

$$\hat{\mathrm{I}}_{(B_1\circ B_2)}((k_1l_1)(k_2l_2)) = \hat{\mathrm{I}}_{B_1}(k_1,k_2) \wedge \hat{\mathrm{I}}_{A_2}(l_1) \wedge \hat{\mathrm{I}}_{A_2}(l_2)$$
$$\leq [\hat{\mathrm{I}}_{A_1}(k_1) \wedge \hat{\mathrm{I}}_{A_1}(k_2)] \wedge [\hat{\mathrm{I}}_{A_2}(l_1) \wedge \hat{\mathrm{I}}_{A_2}(l_2)]$$
$$= [\hat{\mathrm{I}}_{A_1}(k_1) \wedge \hat{\mathrm{I}}_{A_2}(l_1)] \wedge [\hat{\mathrm{I}}_{A_1}(k_2) \wedge \hat{\mathrm{I}}_{A_2}(l_2)]$$
$$= \hat{\mathrm{I}}_{(A_1\circ A_2)}(k_1l_1) \wedge \hat{\mathrm{I}}_{(A_1\circ A_2)}(k_2l_2)$$

$$\hat{F}_{(B_1\circ B_2)}((k_1l_1)(k_2l_2)) = \hat{F}_{B_1}(k_1,k_2) \vee \hat{F}_{A_2}(l_1) \vee \hat{F}_{A_2}(l_2)$$
$$\leq [\hat{F}_{A_1}(k_1) \vee \hat{F}_{A_1}(k_2)] \vee [\hat{F}_{A_2}(l_1) \vee \hat{F}_{A_2}(l_2)]$$
$$= [\hat{F}_{A_1}(k_1) \vee \hat{F}_{A_2}(l_1)] \vee [\hat{F}_{A_1}(k_2) \vee \hat{F}_{A_2}(l_2)]$$
$$= \hat{F}_{(A_1\circ A_2)}(k_1l_1) \vee \hat{F}_{(A_1\circ A_2)}(k_2l_2), \quad \text{for all } k_1l_1, k_2l_2 \in R_1 \circ R_2.$$

Conclusion

Graph theory is an extremely useful tool in studying and modeling several applications in computer science, engineering, genetics, decision-making, economics, etc. An extension of intuitionistic fuzzy graph is regarded as a single-valued neutrosophic graph which is very useful to formulate the appropriate real life situation. In this research article, the operations on neutrosophic vague graphs have been established. Moreover, Cartesian product, lexicographic product, cross product, strong product and composition of neutrosophic vague graph have been investigated and the given concepts are demonstrated through examples. Furthermore, in future, we are able to investigate the domination number and isomorphic properties of the NVGs.

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