Neutrosophic Sets and Systems, Vol. 35, 2020
University of New Mexico

Basic operations on hypersoft sets and hypersoft point

Mujahid Abbas [1], Ghulam Murtaza [2,*] and Florentin Smarandache [3]

[1] Department of Mathematics, Government College University, Lahore 54000, Pakistan and Department of Mathematics and Applied Mathematics,, University of Pretoria, Lynnwood Road, Pretoria 0002, South Africa; abbas.mujahid@gmail.com,

[2]Department of Mathematics, University of Management Technology, Lahore, 54000, Pakistan; ghulammurtaza@umt.edu.pk

[2] University of New Mexico, USA; fsmarandache@gmail.com; smarand@unm.edu

* Correspondence: ghulammurtaza@umt.edu.pk

Abstract: The aim of this paper is to initiat formal study of hypersoft sets. We first, present basic operations like union, intersection and difference of hypersoft sets; basic ingrediants for topological structures on the collection of hypersoft sets. Moreover we introduce hypersoft points in different envorinments like fuzzy hypersoft set, intuitionistic fuzzy hypersoft set, neutrosophic hypersoft, plithogenic hypersoft set, and give some basic properties of hypersoft points in these envorinments. We expect that this will constitue an appropriate framework of hypersoft functions and the study of hypersoft function spaces. Examples are provided to explain the newly defined concepts.

Keywords: soft set; hypersoft set; set operations on hypersoft sets; hypersoft point; fuzzy hypersoft set; intuitionistic fuzzy hypersoft set; neutrosophic hypersoft; plithogenic hypersoft set.

---

1. Introduction

Molodtsov [16] defined soft set as a mathematical tool to deal with uncertainties associated with real world problems. Soft set theory has application in decision making, demand analysis, forecasting, information sciences and other disciplines (see for example, [13, 14, 15, 17, 18, 19, 20, 21, 22, 23]). Plithogenic and neutrosophic hypersoft sets theory is being applied successfully in decision making problems (see, [2, 3, 4, 5, 6, 7, 8, 9,10,11,12]).

By definition, a soft set can be identified by a pair (F, A), where F stands for a multivalued function defined on the set of parameters A.

Smarandache [1] extended the notion of a soft set to the hypersoft set by replacing the function F with a multi-argument function defined on the Cartesian product of n different set of parameters. This concept is more flexible than soft set and more suitable in the context of decision making problems.

We expect the notion of hypersoft set will attract the attention of researchers working on soft set theory and its diverse applications. The purpose of this paper is to initiate a formal investigation in this new area of research.

As a first step, we present the basic operations like union, intersection and difference of hypersoft sets. Moreover we introduce hypersoft points and some basic properties of these points which may provide the foundation for the hypersoft functions and hence the hypersoft fixed point theory.

2. Operations on hypersoft sets

In this section, we define basic operations on hypersoft sets. Smarandache defined the hypersoft set in the following manner:

Definition 1 *[1] Let U be a universe of discourse, $P(U)$ the power set U and $E_1, E_2, \dots, E_n$ the pairwise disjoint sets of parameters. Let A_i be the nonempty subset of E_i for each $i = 1,2,\dots,n$. A hypersoft set can be identified by the pair $(F, A_1 \times A_2 \times \cdots \times A_n)$, where:*

$$F: A_1 \times A_2 \times \cdots \times A_n \to P(U).$$

For sake of simplicity, we write the symbols $\mathbf{E}$ for $E_1 \times E_2 \times \cdots \times E_n$, $\mathbf{A}$ for $A_1 \times A_2 \times \cdots \times A_n$ and $\boldsymbol{\alpha}$ for an element of the set $\mathbf{A}$. We also suppose that none of the set A_i is empty.

Definition 2 *[1] A hypersoft set;*
on a crisp universe of discourse U_C is called Crisp Hypersoft set (or simply "hypersoft set");
on a fuzzy universe of discourse U_F is called Fuzzy Hypersoft set.
on a Intuitionistic Fuzzy universe of discourse U_{IF} is called Intuitionistic Fuzzy Hypersoft set;
on a Neutrosophic universe of discourse U_N is called Neutrosophic Hypersoft Set;
on a Plithogenic universe of discourse U_P is called Plithogenic Hypersoft Set.

The nature of $F(\boldsymbol{\alpha})$ is determined by the nature of universe of discourse. Therefore $P(U)$ depends upon the nature of universe. We denote $\mathcal{H}(U_*, \mathbf{E})$ by the family of all *-hypersoft sets over $(U_*, \mathbf{E})$, where $*$ can take any value in the set $\{C, F, IF, N, P\}$, where symbols C, F, IF, N, P denote Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Plithogenic sets, respectively.

The following are the basic operations on *-hypersoft set.

Definition 3 *Let U_* be a universe of discourse and $\boldsymbol{A}$ a subset of $\boldsymbol{E}$. Then $(F, \boldsymbol{A})$ is called*
1. *a null *-hypersoft set if for each parameter $\boldsymbol{\alpha} \in \boldsymbol{A}$, $F(\boldsymbol{\alpha})$ is an 0_*. We will denote it by $\Phi_{\boldsymbol{A}}$.*
2. *an absolute *-hypersoft set if for each parameter $\boldsymbol{\alpha} \in \boldsymbol{A}$, $F(\boldsymbol{\alpha}) = U_*$. We will denote it by $\widetilde{U}_{\boldsymbol{A}}$.*

Remark 1 *We consider $0_C = \varnothing$ for empty set, $0_F = \{\frac{x}{0}, x \in U_F\}$ for null fuzzy set, $0_{IF} = \{\frac{x}{<0,1>}, x \in U_{IF}\}$ for null intuitionistic fuzzy set, $0_N = \{\frac{x}{<0,1,1>}, x \in U_N\}$ for null neutrosophic set. However, in case of plithogenic set, we have the following notations:*

- *Null plithgenic crisp set*

$$0_{PC} = \{x(0,0,\dots,0), \text{for all} x \in U_P\}.$$

- *Universal plithgenic crisp set*

$$1_{PC} = \{x(1,1,\dots,1), \text{for all} x \in U_P\}.$$

Note that null plithgenic fuzzy set will be same as null plithgenic crisp set and universal plithgenic fuzzy set will be the same as universal plithgenic crisp set.

- *Null plithgenic intuitionistic fuzzy set*

$$0_{PIF} = \{x((0,1),(0,1),\ldots,(0,1)), \text{forall} x \in U_P\}.$$

- *Universal plithgenic intuitionistic fuzzy set*

$$1_{PIF} = \{x((1,0),(1,0),\ldots,(1,0)), \text{forall} x \in U_P\}.$$

- *Null plithgenic neutrosophic set*

$$0_{PN} = \{x((0,1,1),(0,1,1),\ldots,(0,1,1)), \text{forall} x \in U_P\}.$$

- *Universal plithgenic neutrosophic set*

$$1_{PN} = \{x((1,0,0),(1,0,0),\ldots,(1,0,0)), \text{forall} x \in U\}.$$

Definition 4 *Let $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ be two *-hypersoft sets over U_*. Then union of $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ is denoted by $(H,\boldsymbol{C}) = (F,\boldsymbol{A}) \tilde{\cup} (G,\boldsymbol{B})$ with $\boldsymbol{C} = C_1 \times C_2 \times \cdots \times C_n$, where $C_i = A_i \cup B_i$ for $i = 1,2,\ldots,n$, and H is defined by*

$$H(\boldsymbol{\alpha}) = \begin{cases} F(\boldsymbol{\alpha}), & \text{if} \boldsymbol{\alpha} \in \mathbf{A} - \mathbf{B} \\ G(\boldsymbol{\alpha}), & \text{if} \boldsymbol{\alpha} \in \mathbf{B} - \mathbf{A} \\ F(\boldsymbol{\alpha}) \cup_* G(\boldsymbol{\alpha}), & \text{if} \boldsymbol{\alpha} \in \mathbf{A} \cap \mathbf{B}, \\ 0_*, & \text{else,} \end{cases}$$

where $\boldsymbol{\alpha} = (c_1, c_2, \ldots, c_n) \in C$.

Remark 2 *Note that, in the case of union of two hypersoft sets the set of parameters is a Cartesian product of sets of parameters whereas in the case of union of two soft sets the set of parameter is just the union of sets of parameters.*

Definition 5 *Let $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ be two *-hypersoft sets over U_*. Then intersection of $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ is denoted by $(H,\boldsymbol{C}) = (F,\boldsymbol{A}) \tilde{\cap} (G,\boldsymbol{B})$, where $\boldsymbol{C} = C_1 \times C_2 \times \cdots \times C_n$ is such that $C_i = A_i \cap B_i$ for $i = 1,2,\ldots,n$ and H is defined as*

$$H(\boldsymbol{\alpha}) = F(\boldsymbol{\alpha}) \cap_* G(\boldsymbol{\alpha}),$$

*where $\boldsymbol{\alpha} = (c_1, c_2, \ldots, c_n) \in \boldsymbol{C}$. If C_i is an empty set for some i, then $(F,\boldsymbol{A}) \tilde{\cap} (G,\boldsymbol{B})$ is defined to be a null *-hypersoft set.*

Definition 6 *Let $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ be two *-hypersoft sets over U_*. Then $(F,\boldsymbol{A})$ is called a *-hypersoft subset of $(G,\boldsymbol{B})$ if $\boldsymbol{A} \subseteq \boldsymbol{B}$, and $F(\boldsymbol{\alpha}) \subseteq_* G(\boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in \boldsymbol{A}$. We denote this by $(F,\boldsymbol{A}) \tilde{\subseteq} (G,\boldsymbol{B})$. Thus $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ are said to equal if $(F,\boldsymbol{A}) \tilde{\subseteq} (G,\boldsymbol{B})$ and $(F,\boldsymbol{A}) \tilde{\supseteq} (G,\boldsymbol{B})$.*

Definition 7 *Let $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$ be two *-hypersoft sets over U_*. Then *-hypersoft difference of $(F,\boldsymbol{A})$ and $(G,\boldsymbol{B})$, denoted by $(H,\boldsymbol{C}) = (F,\boldsymbol{A}) \tilde{\setminus} (G,\boldsymbol{B})$, where $\boldsymbol{C} = C_1 \times C_2 \times \cdots \times C_n$ is such that $C_i = A_i \cap B_i$ for $i = 1,2,\ldots,n$, and H is defined by*

$$H(\boldsymbol{\alpha}) = F(\boldsymbol{\alpha}) \setminus_* G(\boldsymbol{\alpha}),$$

where $\boldsymbol{\alpha} = (c_1, c_2, \ldots, c_n) \in \mathbf{C}$. If C_i is an empty set for some i then $(F,\mathbf{A}) \tilde{\setminus} (G,\mathbf{B})$ is defined to be $(F,\mathbf{A})$.

Definition 8 *The complement of a *-hypersoft set $(F,\boldsymbol{A})$ is denoted as $(F,\boldsymbol{A})^c$ and is defined by $(F,\boldsymbol{A})^c = (F^c,\boldsymbol{A})$ where $F^c(\boldsymbol{\alpha})$ is the *-complemet of $F(\boldsymbol{\alpha})$ for each $\boldsymbol{\alpha} \in \boldsymbol{A}$.*

Example 1 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11}, a_{12}\}, E_2 = \{a_{21}, a_{22}\}, E_3 = \{a_{31}, a_{32}\}.$$

Suppose that

$$A_1 = \{a_{11}, a_{12}\}, A_2 = \{a_{21}, a_{22}\}, A_3 = \{a_{31}\}, \text{and}$$
$$B_1 = \{a_{11}\}, B_2 = \{a_{21}, a_{22}\}, B_3 = \{a_{31}, a_{32}\}$$

that is,. $A_i, B_i \subseteq E_i$ for each $i = 1,2,3$.

Let the crisp hypersoft sets $(F, \mathbf{A})$ and $(G, \mathbf{B})$ be defined by

$$(F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{x_1, x_2\}), ((a_{11}, a_{22}, a_{31}), \{x_2\}),$$
$$((a_{12}, a_{21}, a_{31}), \{x_3, x_4\}), ((a_{12}, a_{22}, a_{31}), \{x_1, x_4\})\}.$$

and

$$(G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_2, x_3\}), ((a_{11}, a_{22}, a_{31}), \{x_2\}),$$
$$((a_{11}, a_{21}, a_{32}), \{x_1, x_4\}), ((a_{11}, a_{22}, a_{32}), \{x_3, x_4\})\}.$$

We have excluded those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is an empty set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is an empty set).

Then the union and intersections of $(F, \mathbf{A})$ and $(G, \mathbf{B})$ are given by:

$$(F, \mathbf{A}) \tilde{\cup} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_1, x_2, x_3\}), ((a_{11}, a_{22}, a_{31}), \{x_2\}),$$
$$((a_{12}, a_{21}, a_{31}), \{x_3, x_4\}), ((a_{12}, a_{22}, a_{31}), \{x_1, x_4\}),$$
$$((a_{11}, a_{21}, a_{32}), \{x_1, x_4\}), ((a_{11}, a_{22}, a_{32}), \{x_3, x_4\}),$$
$$((a_{12}, a_{21}, a_{32}), 0_C), ((a_{12}, a_{22}, a_{32}), 0_C)\};$$

and

$$(F, \mathbf{A}) \tilde{\cap} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_2\}), ((a_{11}, a_{22}, a_{31}), \{x_2\})\}.$$

The differences $(F, \mathbf{A}) \tilde{\setminus} (G, \mathbf{B})$ and $(G, \mathbf{B}) \tilde{\setminus} (F, \mathbf{A})$ are the following

$$(F, \mathbf{A}) \tilde{\setminus} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_1\}), ((a_{11}, a_{22}, a_{31}), 0_C)\};$$
$$(G, \mathbf{B}) \tilde{\setminus} (F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{x_3\}), ((a_{11}, a_{22}, a_{31}), 0_C)\}.$$

Example 2 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11}, a_{12}\}, E_2 = \{a_{21}, a_{22}\}, E_3 = \{a_{31}, a_{32}\}.$$

Suppose that

$$A_1 = \{a_{11}, a_{12}\}, A_2 = \{a_{21}, a_{22}\}, A_3 = \{a_{31}\}, \text{and}$$
$$B_1 = \{a_{11}\}, B_2 = \{a_{21}, a_{22}\}, B_3 = \{a_{31}, a_{32}\}$$

are subsets of E_i for each $i = 1,2,3$, that is,. $A_i, B_i \subseteq E_i$ for each i.

Let the fuzzy hypersoft sets $(F, \mathbf{A})$ and $(G, \mathbf{B})$ be defined by

$$(F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_1}{0.5}, \tfrac{x_2}{0.7}\}), ((a_{11}, a_{22}, a_{31}), \{\tfrac{x_2}{0.3}\}),$$
$$((a_{12}, a_{21}, a_{31}), \{\tfrac{x_3}{0.8}, \tfrac{x_4}{0.9}\}), ((a_{12}, a_{22}, a_{31}), \{\tfrac{x_1}{0.5}, \tfrac{x_4}{0.4}\})\}.$$

and

$$(G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_2}{0.2}, \tfrac{x_3}{0.9}\}), ((a_{11}, a_{22}, a_{31}), \{\tfrac{x_2}{0.6}\}),$$
$$((a_{11}, a_{21}, a_{32}), \{\tfrac{x_1}{0.4}, \tfrac{x_4}{0.7}\}), ((a_{11}, a_{22}, a_{32}), \{\tfrac{x_3}{0.2}, \tfrac{x_4}{0.8}\})\}.$$

We have excluded those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is a null fuzzy set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is a null fuzzy set).

Then the union and intersections of $(F, \mathbf{A})$ and $(G, \mathbf{B})$ are given by:

$(F,\mathbf{A})\ \tilde{\cup}\ (G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{0.5},\frac{x_2}{0.7},\frac{x_3}{0.9}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{0.6}\}),$
$((a_{12},a_{21},a_{31}),\{\frac{x_3}{0.8},\frac{x_4}{0.9}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{0.5},\frac{x_4}{0.4}\}),$
$((a_{11},a_{21},a_{32}),\{\frac{x_1}{0.4},\frac{x_4}{0.7}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{0.2},\frac{x_4}{0.8}\}),$
$((a_{12},a_{21},a_{32}),0_F),((a_{12},a_{22},a_{32}),0_F)\};$

and

$$(F,\mathbf{A})\ \tilde{\cap}\ (G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{0.2}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{0.3}\})\}.$$

The differences $(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B})$ and $(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A})$ are the following

$(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{0.5},\frac{x_2}{0.5}\}),((a_{11},a_{22},a_{31}),0_F)\};$
$(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_3}{0.9}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{0.3}\})\}.$

Example 3 Let $\mathrm{U} = \{\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11},a_{12}\}, E_2 = \{a_{21},a_{22}\}, E_3 = \{a_{31},a_{32}\}.$$

Suppose that

$A_1 = \{a_{11},a_{12}\}, A_2 = \{a_{21},a_{22}\}, A_3 = \{a_{31}\}$, and
$B_1 = \{a_{11}\}, B_2 = \{a_{21},a_{22}\}, B_3 = \{a_{31},a_{32}\}$

that is,. $A_i, B_i \subseteq E_i$ for each $i = 1,2,3$.

Let the intuitionistic fuzzy hypersoft sets $(F,\mathbf{A})$ and $(G,\mathbf{B})$ be defined by

$(F,\mathbf{A}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{\langle 0.5,0.3\rangle},\frac{x_2}{\langle 0.7,0.2\rangle}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{\langle 0.3,0.5\rangle}\}),$

$((a_{12},a_{21},a_{31}),\{\frac{x_3}{\langle 0.8,0.1\rangle},\frac{x_4}{\langle 0.1,0.5\rangle}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{\langle 0.5,0.3\rangle},\frac{x_4}{\langle 0.4,0.2\rangle}\})\}.$

and

$(G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{\langle 0.2,0.6\rangle},\frac{x_3}{\langle 0.8,0.1\rangle}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{\langle 0.6,0.3\rangle}\}),$

$((a_{11},a_{21},a_{32}),\{\frac{x_1}{\langle 0.4,0.5\rangle},\frac{x_4}{\langle 0.7,0.2\rangle}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{\langle 0.4,0.2\rangle},\frac{x_4}{\langle 0.1,0.8\rangle}\})\}.$

We have excluded all those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is a null intuitionistic fuzzy set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is a null intuitionistic fuzzy set).

The union and intersections of $(F,\mathbf{A})$ and $(G,\mathbf{B})$ are given by:

$(F,\mathbf{A})\ \tilde{\cup}\ (G,\mathbf{B})$

$= \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{\langle 0.5,0.3\rangle},\frac{x_2}{\langle 0.7,0.2\rangle},\frac{x_3}{\langle 0.8,0.1\rangle}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{\langle 0.6,0.3\rangle}\}),$

$((a_{12},a_{21},a_{31}),\{\frac{x_3}{\langle 0.8,0.1\rangle},\frac{x_4}{\langle 0.1,0.5\rangle}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{\langle 0.5,0.3\rangle},\frac{x_4}{\langle 0.4,0.2\rangle}\}),$

$((a_{11},a_{21},a_{32}),\{\frac{x_1}{\langle 0.4,0.5\rangle},\frac{x_4}{\langle 0.7,0.2\rangle}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{\langle 0.4,0.2\rangle},\frac{x_4}{\langle 0.1,0.8\rangle}\}),$

$((a_{12},a_{21},a_{32}),0_{IF}),((a_{12},a_{22},a_{32}),0_{IF})\};$

and

$$(F,\mathbf{A})\ \tilde{\cap}\ (G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{\langle 0.2,0.6\rangle}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{\langle 0.3,0.5\rangle}\})\}.$$

The differences $(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B})$ and $(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A})$ are the following

$$(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.3>},\frac{x_2}{<0.6,0.2>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.6>}\})\};$$

$$(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.2,0.7>},\frac{x_3}{<0.8,0.1>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.5,0.3>}\})\}.$$

Example 4 Let $\mathrm{U} = \{\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11},a_{12}\}, E_2 = \{a_{21},a_{22}\}, E_3 = \{a_{31},a_{32}\}.$$

Suppose that

$$A_1 = \{a_{11},a_{12}\}, A_2 = \{a_{21},a_{22}\}, A_3 = \{a_{31}\}, \text{and}$$
$$B_1 = \{a_{11}\}, B_2 = \{a_{21},a_{22}\}, B_3 = \{a_{31},a_{32}\}$$

that is,. $A_i, B_i \subseteq E_i$ for each $i = 1,2,3$.

Let the neutrosophic hypersoft sets $(F,\mathbf{A})$ and $(G,\mathbf{B})$ be defined by

$$(F,\mathbf{A}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>},\frac{x_2}{<0.7,0.3,0.2>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.2,0.5>}\}),$$

$$((a_{12},a_{21},a_{31}),\{\frac{x_3}{<0.8,0.4,0.1>},\frac{x_4}{<0.1,0.5,0.5>}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>},\frac{x_4}{<0.4,0.3,0.2>}\})\}.$$

and

$$(G,\mathbf{B}) = \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.2,0.5,0.6>},\frac{x_3}{<0.8,0.6,0.1>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.6,0.2,0.3>}\}),$$

$$((a_{11},a_{21},a_{32}),\{\frac{x_1}{<0.4,0.3,0.5>},\frac{x_4}{<0.7,0.3,0.2>}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{<0.4,0.4,0.2>},\frac{x_4}{<0.1,0.3,0.8>}\})\}.$$

We have excluded those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is a null intuitionistic fuzzy set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is a null intuitionistic fuzzy set).

The union and intersections of $(F,\mathbf{A})$ and $(G,\mathbf{B})$ are given by:

$$(F,\mathbf{A}) \tilde{\cup} (G,\mathbf{B})$$

$$= \{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>},\frac{x_2}{<0.7,0.3,0.2>},\frac{x_3}{<0.8,0.6,0.1>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.6,0.2,0.3>}\}),$$

$$((a_{12},a_{21},a_{31}),\{\frac{x_3}{<0.8,0.4,0.1>},\frac{x_4}{<0.1,0.5,0.5>}\}),((a_{12},a_{22},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>},\frac{x_4}{<0.4,0.3,0.2>}\}),$$

$$((a_{11},a_{21},a_{32}),\{\frac{x_1}{<0.4,0.3,0.5>},\frac{x_4}{<0.7,0.3,0.2>}\}),((a_{11},a_{22},a_{32}),\{\frac{x_3}{<0.4,0.4,0.2>},\frac{x_4}{<0.1,0.3,0.8>}\}),$$

$$((a_{12},a_{21},a_{32}),0_N),((a_{12},a_{22},a_{32}),0_N)\};$$

and

$$(F,\mathbf{A}) \tilde{\cap} (G,\mathbf{B})$$

$$= \{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.2,0.5,0.6>}\}),((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.2,0.5>}\})\}.$$

The differences $(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B})$ and $(G,\mathbf{B})\tilde{\setminus}(F,\mathbf{A})$ are the following

$$(F,\mathbf{A})\tilde{\setminus}(G,\mathbf{B})$$

$= \{((a_{11}, a_{21}, a_{31}), \{\frac{x_1}{<0.5,0.2,0.3>}, \frac{x_2}{<0.6,0.15,0.2>}\}), ((a_{11}, a_{22}, a_{31}), \{\frac{x_2}{<0.3,0.4,0.6>}\})\};$

$(G, \mathbf{B})\tilde{\setminus}(F, \mathbf{A})$

$= \{((a_{11}, a_{21}, a_{31}), \{\frac{x_2}{<0.2,0.15,0.7>}, \frac{x_3}{<0.8,0.6,0.1>}\}), ((a_{11}, a_{22}, a_{31}), \{\frac{x_2}{<0.5,0.4,0.3>}\})\}.$

Remark 3 *There are four types of plithogenic hypersoft sets namely: plithogenic crisp hypersoft set, plithogenic fuzzy hypersoft set, plithogenic intuitionistic fuzzy hypersoft set, plithogenic neutrosophic hypersoft set. Here we discuss only plithogenic crisp hypersoft point whereas examples for other types of sets can be constructed in the similar way.*

Example 5 Let $U = \{x_1, x_2, x_3, x_4\}$. Define the attributes sets by:

$$E_1 = \{a_{11}, a_{12}\}, E_2 = \{a_{21}, a_{22}\}, E_3 = \{a_{31}, a_{32}\}.$$

Suppose that

$A_1 = \{a_{11}, a_{12}\}, A_2 = \{a_{21}, a_{22}\}, A_3 = \{a_{31}\}$, and

$B_1 = \{a_{11}\}, B_2 = \{a_{21}, a_{22}\}, B_3 = \{a_{31}, a_{32}\}$

that is,. $A_i, B_i \subseteq E_i$ for each $i = 1,2,3$.

Let the plithogenic crisp hypersoft sets $(F, \mathbf{A})$ and $(G, \mathbf{B})$ be defined by

$(F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{x_1(1,0,1), x_2(1,1,1)\}), ((a_{11}, a_{22}, a_{31}), \{x_2(0,0,1)\}),$

$((a_{12}, a_{21}, a_{31}), \{x_3(1,1,0), x_4(1,1,1)\}), ((a_{12}, a_{22}, a_{31}), \{x_1(1,0,1), x_4(0,1,0)\})\}.$

and

$(G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_2(1,1,1), x_3(1,1,0)\}), ((a_{11}, a_{22}, a_{31}), \{x_2(0,1,0)\}),$

$((a_{11}, a_{21}, a_{32}), \{x_1(0,1,1), x_4(1,1,1)\}), ((a_{11}, a_{22}, a_{32}), \{x_3(1,1,1), x_4(1,1,1)\})\}.$

We have excluded all those $\boldsymbol{\alpha} \in \mathbf{A}$ for which $F(\boldsymbol{\alpha})$ is a null plithogenic crisp set (similarly for those $\boldsymbol{\beta} \in \mathbf{B}$ for which $G(\boldsymbol{\beta})$ is a null plithogenic crisp set).

The union and intersections of $(F, \mathbf{A})$ and $(G, \mathbf{B})$ are given by:

$(F, \mathbf{A}) \tilde{\cup} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_1(1,0,1), x_2(1,1,1), x_3(1,1,0)\}),$

$((a_{11}, a_{22}, a_{31}), \{x_2(0,1,1)\}), ((a_{12}, a_{21}, a_{31}), \{x_3(1,1,0), x_4(1,1,1)\}),$

$((a_{12}, a_{22}, a_{31}), \{x_1(1,0,1), x_4(0,1,0)\}),$

$((a_{11}, a_{21}, a_{32}), \{x_1(0,1,1), x_4(1,1,1)\}), ((a_{11}, a_{22}, a_{32}), \{x_3(1,1,1), x_4(1,1,1)\}),$

$((a_{12}, a_{21}, a_{32}), 0_{PC}), ((a_{12}, a_{22}, a_{32}), 0_{PC})\};$

and

$$(F, \mathbf{A}) \tilde{\cap} (G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_2(1,1,1)\}), ((a_{11}, a_{22}, a_{31}), 0_{PC})\}.$$

The differences $(F, \mathbf{A})\tilde{\setminus}(G, \mathbf{B})$ and $(G, \mathbf{B})\tilde{\setminus}(F, \mathbf{A})$ are the following

$(F, \mathbf{A})\tilde{\setminus}(G, \mathbf{B}) = \{((a_{11}, a_{21}, a_{31}), \{x_1(1,0,1)\}), ((a_{11}, a_{22}, a_{31}), \{x_2(0,0,1)\})\};$

$(G, \mathbf{B})\tilde{\setminus}(F, \mathbf{A}) = \{((a_{11}, a_{21}, a_{31}), \{x_3(1,1,0)\}), ((a_{11}, a_{22}, a_{31}), \{x_2(0,1,0)\})\}.$

Proposition 1 *Let (F, A) be a *-hypersoft set over U_*. Then the following holds;*

1. $(F, \mathbf{A}) \tilde{\cup} \Phi_{\mathbf{A}} = (F, \mathbf{A});$

2. $(F, \mathbf{A}) \tilde{\cap} \Phi_{\mathbf{A}} = \Phi_{\mathbf{A}}$;
3. $(F, \mathbf{A}) \tilde{\cup} \tilde{U}_{\mathbf{A}} = \tilde{U}_{\mathbf{A}}$;
4. $(F, \mathbf{A}) \tilde{\cap} \tilde{U}_{\mathbf{A}} = (F, \mathbf{A})$;
5. $\tilde{U}_{\mathbf{A}} \tilde{\setminus} (F, \mathbf{A}) = (F, \mathbf{A})^c$;

6. $(F, \mathbf{A}) \tilde{\cup} (F, \mathbf{A})^c = \tilde{U}_{\mathbf{A}}$;
7. $(F, \mathbf{A}) \tilde{\cap} (F, \mathbf{A})^c = \Phi_{\mathbf{A}}$.

Proof. We will prove only (i), (ii) and (v) and proofs of remaining are similar.

(i) By the definition of union, we have

$$(F, \mathbf{A}) \tilde{\cup} \Phi_{\mathbf{A}} = (H, \mathbf{C}),$$

where $\mathbf{C} = \mathbf{A}$ and $H(\boldsymbol{\alpha}) = F(\boldsymbol{\alpha}) \cup_* 0_* = F(\boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in \mathbf{C}$. Hence $(H, \mathbf{C}) = (F, \mathbf{A})$.

(ii) By the definition of intersection, we obtain that

$$(F, \mathbf{A}) \tilde{\cap} \Phi_{\mathbf{A}} = (H, \mathbf{C}),$$

where $\mathbf{C} = \mathbf{A}$ and $H(\boldsymbol{\alpha}) = F(\boldsymbol{\alpha}) \cap_* 0_* = 0_*$ for all $\boldsymbol{\alpha} \in \mathbf{C}$. Hence $(H, \mathbf{C}) = \Phi_{\mathbf{A}}$.

(v) By the definition of difference, we get

$$\tilde{U}_{\mathbf{A}} \tilde{\setminus} (F, \mathbf{A}) = (H, \mathbf{C}),$$

where $\mathbf{C} = \mathbf{A}$ and $H(\boldsymbol{\alpha}) = U \setminus_* F(\boldsymbol{\alpha}) = F^c(\boldsymbol{\alpha})$ for all $\boldsymbol{\alpha} \in \mathbf{C}$. Hence $(H, \mathbf{C}) = (F, \mathbf{A})^c$.

3. Hypersoft point

In this section, we define hypersoft point in different frameworks and study some basic properties of such points in each setup.

3.1 Crisp hypersoft point

Definition 9 *Let $\boldsymbol{A} \subseteq \boldsymbol{E}$, $\boldsymbol{\alpha} \in \boldsymbol{A}$, and $x \in U$. A hypersoft set $(F, \boldsymbol{A})$ is said to be a hypersoft point if $F(\boldsymbol{\alpha}')$ is an empty set for every $\boldsymbol{\alpha}' \in \boldsymbol{A} \setminus \{\boldsymbol{\alpha}\}$ and $F(\boldsymbol{\alpha})$ is a singleton set. We will denote hypersoft point $(F, \boldsymbol{A})$ simply by $P^{(\boldsymbol{\alpha}, x)}$.*

Definition 10 *A hypersoft set $(F, \boldsymbol{A})$ is said to be an empty hypersoft point if $F(\boldsymbol{\alpha})$ is an empty set for each $\boldsymbol{\alpha} \in \boldsymbol{A}$. We will denote an empty hypersoft set, corresponding to $\boldsymbol{\alpha}$, by $P^{(\boldsymbol{\alpha}, \emptyset)}$.*

As a matter of fact if $(F, \mathbf{A})$ is a null hypersoft set then for every $\boldsymbol{\alpha} \in \mathbf{A}$ it may be regarded as empty hypersoft set $P^{(\boldsymbol{\alpha}, \emptyset)}$.

Definition 11 *A hypersoft point $P^{(\boldsymbol{\alpha}, x)}$ is said to belong to a hypersoft set $(G, \boldsymbol{A})$ if $P^{(\boldsymbol{\alpha}, x)} \tilde{\subseteq} (G, \boldsymbol{A})$. We write it as $P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (G, \boldsymbol{A})$.*

It is straightforward to check that the hypersoft union of hypersoft points of a hypersoft set $(G, \mathbf{A})$ returns the hypersoft set $(G, \mathbf{A})$, that is,

$$(G, \mathbf{A}) = \tilde{\cup} \{P^{(\boldsymbol{\alpha}, x)} : P^{(\boldsymbol{\alpha}, x)} \tilde{\in} (G, \mathbf{A})\}.$$

We illustrate the above observation through the following example.

Example 6 Let $U = \{x_1, x_2, x_3, x_4\}$, and $(F, \mathbf{A})$ be as given in the example 1. Then the hypersoft points of $(F, \mathbf{A})$ are the following:

$$P_1^{((a_{11},a_{21},a_{31}),x_1)} = \{((a_{11}, a_{21}, a_{31}), \{x_1\})\};$$
$$P_2^{((a_{11},a_{21},a_{31}),x_2)} = \{((a_{11}, a_{21}, a_{31}), \{x_2\})\};$$
$$P_3^{((a_{11},a_{22},a_{31}),x_2)} = \{((a_{11}, a_{22}, a_{31}), \{x_2\})\};$$
$$P_4^{((a_{12},a_{21},a_{31}),x_3)} = \{((a_{12}, a_{21}, a_{31}), \{x_3\})\};$$
$$P_5^{((a_{12},a_{21},a_{31}),x_4)} = \{((a_{12}, a_{21}, a_{31}), \{x_4\})\};$$
$$P_6^{((a_{12},a_{22},a_{31}),x_1)} = \{((a_{12}, a_{22}, a_{31}), \{x_1\})\};$$
$$P_7^{((a_{12},a_{22},a_{31}),x_4)} = \{((a_{12}, a_{22}, a_{31}), \{x_4\})\}.$$

Moreover

$$(F, \mathbf{A}) = P_1^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup} P_2^{((a_{11},a_{21},a_{31}),x_2)} \tilde{\cup} P_3^{((a_{11},a_{22},a_{31}),x_2)}$$
$$\tilde{\cup} P_4^{((a_{12},a_{21},a_{31}),x_3)} \tilde{\cup} P_5^{((a_{12},a_{21},a_{31}),x_4)} \tilde{\cup} P_6^{((a_{12},a_{22},a_{31}),x_1)} \tilde{\cup} P_7^{((a_{12},a_{22},a_{31}),x_4)}.$$

Proposition 2 *Let $(F, \boldsymbol{A}), (F_1, \boldsymbol{A})$ and $(F_2, \boldsymbol{A})$ be hypersoft sets over U. Then the following hold:*

1. *If $(F, \boldsymbol{A})$ is not a null hypersoft set, then $(F, \boldsymbol{A})$ contains at least one nonempty hypersoft point.*
2. *$(F_1, \boldsymbol{A}) \tilde{\subseteq} (F_2, \boldsymbol{A})$ if and only if $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ implies that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \boldsymbol{A})$.*
3. *$P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\cup} (F_2, \boldsymbol{A})$ if and only if $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ or $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \boldsymbol{A})$.*
4. *$P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\cap} (F_2, \boldsymbol{A})$ if and only if $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ and $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \boldsymbol{A})$.*
5. *$P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\setminus} (F_2, \boldsymbol{A})$ if and only if $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ and $P^{(\boldsymbol{\alpha},x)} \tilde{\notin} (F_2, \boldsymbol{A})$.*

Proof. We will prove (1), (2) and (3). Proofs of (4) and (5) are similar to that of (3).

(1) Suppose that $(F, \mathbf{A})$ is not a null hypersoft set, that is, $F(\boldsymbol{\alpha}) \neq \varnothing$ for some $\boldsymbol{\alpha} \in \mathbf{A}$. Now if $\boldsymbol{\alpha}_0 \in \mathbf{A}$ is such that $F(\boldsymbol{\alpha}_0) \neq \varnothing$, then for $x \in F(\boldsymbol{\alpha}_0)$, there will be a hypersoft point $P^{(\boldsymbol{\alpha}_0,x)}$ such that $P^{(\boldsymbol{\alpha}_0,x)} \tilde{\in} (F, \mathbf{A})$.

(2) Suppose that $(F_1, \mathbf{A}) \tilde{\subseteq} (F_2, \mathbf{A})$ and $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A})$. By the definition 11, we have

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_1, \mathbf{A}).$$

Thus

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_1, \mathbf{A}) \tilde{\subseteq} (F_2, \mathbf{A})$$

implies that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A})$.

Conversely suppose that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A})$ which implies that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A})$. By the definition 11, we obtain that

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_2, \mathbf{A}) \text{forall} P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A}).$$

Thus we have

$$(F_1, \mathbf{A}) = \tilde{\cup} \{P^{(\boldsymbol{\alpha},x)} : P^{(\boldsymbol{\alpha},x)} \tilde{\in} (G, \mathbf{A})\} \tilde{\subseteq} (F_2, \mathbf{A}).$$

(3) Suppose that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A}) \tilde{\cup} (F_2, \mathbf{A})$. It follows from the definition 11 that

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_1, \mathbf{A}) \tilde{\cup} (F_2, \mathbf{A}),$$

which implies that $x \in F_1(\boldsymbol{\alpha}) \cup_C F_2(\boldsymbol{\alpha})$. Thus $x \in F_1(\boldsymbol{\alpha})$ or $F_2(\boldsymbol{\alpha})$. Hence we have

$$P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A}) \text{or} P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A}).$$

Conversely suppose that $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A})$ or $P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A})$. This implies that $x \in F_1(\boldsymbol{\alpha})$ or $F_2(\boldsymbol{\alpha})$. Thus $x \in F_1(\boldsymbol{\alpha}) \cup_C F_2(\boldsymbol{\alpha})$ and so we have

$$P^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (F_1, \mathbf{A}) \tilde{\cup} (F_2, \mathbf{A}).$$

3.2 Fuzzy hypersoft point

Definition 12 *Let $\boldsymbol{A} \subseteq \boldsymbol{E}$, $\boldsymbol{\alpha} \in \boldsymbol{A}$, and $x \in U_F$. A fuzzy hypersoft set $(F, \boldsymbol{A})$ is said to be a fuzzy hypersoft point if $F(\boldsymbol{\alpha}')$ is a null fuzzy set for every $\boldsymbol{\alpha}' \in \boldsymbol{A}\backslash\{\boldsymbol{\alpha}\}$ and $F(\boldsymbol{\alpha})(y) = 0$ for all $y \neq x$. We will denote $(F, \boldsymbol{A})$ simply by $FP^{(\boldsymbol{\alpha},x)}$.*

Definition 13 *A fuzzy hypersoft set $(F, \boldsymbol{A})$ is said to be a null fuzzy hypersoft point if $F(\boldsymbol{\alpha})$ is a null fuzzy set for each $\boldsymbol{\alpha} \in \boldsymbol{A}$. We denote a null fuzzy hypersoft set, corresponding to $\boldsymbol{\alpha}$, by $FP^{(\boldsymbol{\alpha},0_F)}$.*

Note that if $(F, \mathbf{A})$ is a null fuzzy hypersoft set then for every $\boldsymbol{\alpha} \in \mathbf{A}$, it can be regarded as null fuzzy hypersoft set $FP^{(\boldsymbol{\alpha},0_F)}$.

Definition 14 *A fuzzy hypersoft point $FP^{(\boldsymbol{\alpha},x)}$ is said to belong to a fuzzy hypersoft set $(G, \boldsymbol{A})$ if $FP^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (G, \boldsymbol{A})$. We write it as $FP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G, \boldsymbol{A})$.*

It is straightforward to check that the fuzzy hypersoft union of fuzzy hypersoft points of a fuzzy hypersoft set $(G, \mathbf{A})$ returns the fuzzy hypersoft set $(G, \mathbf{A})$, that is,

$$(G, \mathbf{A}) = \tilde{\cup} \{FP^{(\boldsymbol{\alpha},x)} : FP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G, \mathbf{A})\}.$$

We illustrate this observation through the following example.

Example 7 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$, and $(\mathrm{F}, \mathbf{A})$ be as given in the example 2. Then some of the fuzzy hypersoft points of $(\mathrm{F}, \mathbf{A})$ are given as:

$$FP_1^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_1}{0.5}\})\right\};$$

$$FP_2^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_1}{0.2}\})\right\};$$

$$FP_3^{((a_{11},a_{21},a_{31}),x_2)} = \left\{((a_{11}, a_{21}, a_{31}), \{\tfrac{x_2}{0.7}\})\right\};$$

$$FP_4^{((a_{11},a_{22},a_{31}),x_2)} = \left\{((a_{11}, a_{22}, a_{31}), \{\tfrac{x_2}{0.3}\})\right\};$$

$$FP_5^{((a_{12},a_{21},a_{31}),x_3)} = \left\{((a_{12}, a_{21}, a_{31}), \{\tfrac{x_3}{0.8}\})\right\};$$

$$FP_6^{((a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12}, a_{21}, a_{31}), \{\tfrac{x_4}{0.6}\})\right\};$$

$$FP_7^{((a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12}, a_{21}, a_{31}), \{\tfrac{x_4}{0.9}\})\right\};$$

$$FP_8^{((a_{12},a_{22},a_{31}),x_1)} = \left\{((a_{12}, a_{22}, a_{31}), \{\tfrac{x_1}{0.5}\})\right\};$$

$$FP_9^{((a_{12},a_{22},a_{31}),x_4)} = \left\{((a_{12}, a_{22}, a_{31}), \{\tfrac{x_4}{0.4}\})\right\}.$$

Moreover we have

$$\begin{aligned}(F, \mathbf{A}) &= FP_1^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup} FP_2^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup} FP_3^{((a_{11},a_{21},a_{31}),x_2)} \\ &\tilde{\cup} FP_4^{((a_{11},a_{22},a_{31}),x_2)} \tilde{\cup} FP_5^{((a_{12},a_{21},a_{31}),x_3)} \tilde{\cup} FP_6^{((a_{12},a_{21},a_{31}),x_4)} \\ &\tilde{\cup} FP_7^{((a_{12},a_{21},a_{31}),x_4)} \tilde{\cup} FP_8^{((a_{12},a_{22},a_{31}),x_1)} \tilde{\cup} FP_9^{((a_{12},a_{22},a_{31}),x_4)}.\end{aligned}$$

Proposition 3 *Let $(F, \boldsymbol{A}), (F_1, \boldsymbol{A})$ and $(F_2, \boldsymbol{A})$ be fuzzy hypersoft sets over U. Then the following hold:*

1. If $(F, \mathbf{A})$ is not a null fuzzy hypersoft set, then $(F, \mathbf{A})$ contains at least one nonnull fuzzy hypersoft point.
2. $(F_1, \mathbf{A}) \tilde{\subseteq} (F_2, \mathbf{A})$ if and only if $FP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \mathbf{A})$ implies that $FP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \mathbf{A})$.

3. $FP^{(\mathbf{\alpha},x)} \tilde{\in} (F_1,\mathbf{A}) \tilde{\cup} (F_2,\mathbf{A})$ if and only if $FP^{(\mathbf{\alpha},x)} \tilde{\in} (F_1,\mathbf{A})$ or $FP^{(\mathbf{\alpha},x)} \tilde{\in} (F_2,\mathbf{A})$.
4. $FP^{(\mathbf{\alpha},x)} \tilde{\in} (F_1,\mathbf{A}) \tilde{\cap} (F_2,\mathbf{A})$ if and only if $FP^{(\mathbf{\alpha},x)} \tilde{\in} (F_1,\mathbf{A})$ and $FP^{(\mathbf{\alpha},x)} \tilde{\in} (F_2,\mathbf{A})$.

5. $FP^{(\mathbf{\alpha},x)} \tilde{\in} (F_1,\mathbf{A}) \tilde{\setminus} (F_2,\mathbf{A})$ if and only if $FP^{(\mathbf{\alpha},x)} \tilde{\in} (F_1,\mathbf{A})$ and $FP^{(\mathbf{\alpha},x)} \tilde{\notin} (F_2,\mathbf{A})$.

The proof of above proposition is similar as in the case of crisp hypersoft point.

3.3 Intuitionistic fuzzy hypersoft point

Definition 15 *Let $\boldsymbol{A} \subseteq \boldsymbol{E}$, $\boldsymbol{\alpha} \in \boldsymbol{A}$, and $x \in U_{IF}$. An intuitionistic fuzzy hypersoft set $(F,\boldsymbol{A})$ is said to be an intuitionistic fuzzy hypersoft point if $F(\boldsymbol{\alpha}')$ is a null intuitionistic fuzzy set for every $\boldsymbol{\alpha}' \in \boldsymbol{A}\setminus\{\boldsymbol{\alpha}\}$ and $F(\boldsymbol{\alpha})(y) =< 0,1 >$ for all $y \neq x$. We will denote $(F,\boldsymbol{A})$ simply by $IFP^{(\boldsymbol{\alpha},x)}$.*

Definition 16 *An intuitionistic fuzzy hypersoft set $(F,\boldsymbol{A})$ is said to be a null intuitionistic fuzzy hypersoft point if $F(\boldsymbol{\alpha})$ is a null intuitionistic fuzzy set for each $\boldsymbol{\alpha} \in \boldsymbol{A}$. We will denote a null intuitionistic fuzzy hypersoft set, corresponding to $\boldsymbol{\alpha}$, by $IFP^{(\boldsymbol{\alpha},0_{IF})}$.*

If $(F,\mathbf{A})$ is a null intuitionistic fuzzy hypersoft set, then for every $\mathbf{\alpha} \in \mathbf{A}$ it can be regarded as null intuitionistic fuzzy hypersoft set $IFP^{(\mathbf{\alpha},0_{IF})}$.

Definition 17 *An intuitionistic fuzzy hypersoft point $IFP^{(\boldsymbol{\alpha},x)}$ is said to belong to an intuitionistic fuzzy hypersoft set $(G,\boldsymbol{A})$ if $IFP^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (G,\boldsymbol{A})$. We write it as $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G,\boldsymbol{A})$.*

It is straightforward to check that the intuitionistic fuzzy hypersoft union of intuitionistic fuzzy hypersoft points of an intuitionistic fuzzy hypersoft set $(G,\mathbf{A})$ gives the intuitionistic fuzzy hypersoft set $(G,\mathbf{A})$, that is,

$$(G,\mathbf{A}) = \tilde{\cup}\, \{IFP^{(\mathbf{\alpha},x)} : IFP^{(\mathbf{\alpha},x)} \tilde{\in} (G,\mathbf{A})\}.$$

We illustrate this observation through the following example.

Example 8 Let $\mathrm{U} = \{\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4\}$, and $(\mathrm{F},\mathbf{A})$ be as given in the example 3. Then some of the intuitionistic fuzzy hypersoft points of $(\mathrm{F},\mathbf{A})$ are the following:

$$IFP_1^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.3>}\})\right\};$$

$$IFP_2^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.2,0.3>}\})\right\};$$

$$IFP_3^{((a_{11},a_{21},a_{31}),x_2)} = \left\{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.7,0.2>}\})\right\};$$

$$IFP_4^{((a_{11},a_{22},a_{31}),x_2)} = \left\{((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.5>}\})\right\};$$

$$IFP_5^{((a_{12},a_{21},a_{31}),x_3)} = \left\{((a_{12},a_{21},a_{31}),\{\frac{x_3}{<0.8,0.1>}\})\right\};$$

$$IFP_6^{((a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12},a_{21},a_{31}),\{\frac{x_4}{<0.1,0.6>}\})\right\};$$

$$IFP_7^{(a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12},a_{21},a_{31}),\{\frac{x_4}{<0.1,0.5>}\})\right\};$$

$$IFP_8^{((a_{12},a_{22},a_{31}),x_1)} = \left\{((a_{12},a_{22},a_{31}),\{\frac{x_1}{<0.5,0.3>}\})\right\};$$

$$IFP_9^{((a_{12},a_{22},a_{31}),x_4)} = \left\{((a_{12},a_{22},a_{31}),\{\frac{x_4}{<0.4,0.2>}\})\right\}.$$

Moreover we have

$$(F,\mathbf{A}) = IFP_1^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup}\, IFP_2^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup}\, IFP_3^{((a_{11},a_{21},a_{31}),x_2)}$$
$$\tilde{\cup}\, IFP_4^{((a_{11},a_{22},a_{31}),x_2)} \tilde{\cup}\, IFP_5^{((a_{12},a_{21},a_{31}),x_3)} \tilde{\cup}\, IFP_6^{((a_{12},a_{21},a_{31}),x_4)}$$
$$\tilde{\cup}\, IFP_7^{((a_{12},a_{21},a_{31}),x_4)} \tilde{\cup}\, IFP_8^{((a_{12},a_{22},a_{31}),x_1)} \tilde{\cup}\, IFP_9^{((a_{12},a_{22},a_{31}),x_4)}.$$

Proposition 4 *Let* $(F,\boldsymbol{A}),(F_1,\boldsymbol{A})$ *and* $(F_2,\boldsymbol{A})$ *be intuitionistic fuzzy hypersoft sets over* U. *Then the following hold:*

1. If $(F,\mathbf{A})$ is not a null intuitionistic fuzzy hypersoft set then $(F,\mathbf{A})$ contains at least one nonnull intuitionistic fuzzy hypersoft point.
2. $(F_1,\boldsymbol{A}) \tilde{\subseteq} (F_2,\boldsymbol{A})$ *if and only if* $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ *implies that* $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2,\boldsymbol{A})$.
3. $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\cup} (F_2,\boldsymbol{A})$ *if and only if* $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ *or* $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2,\boldsymbol{A})$.
4. $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\cap} (F_2,\boldsymbol{A})$ *if and only if* $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ *and* $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2,\boldsymbol{A})$.
5. $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})\tilde{\setminus}(F_2,\boldsymbol{A})$ *if and only if* $IFP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ *and* $IFP^{(\boldsymbol{\alpha},x)} \tilde{\notin} (F_2,\boldsymbol{A})$.

The proof of above proposition is similar as in the case of crisp hypersoft point.

3.4 Neutrosophic hypersoft point

Definition 18 *Let* $\boldsymbol{A} \subseteq \boldsymbol{E}$ *and* $\boldsymbol{\alpha} \in \boldsymbol{A}$, $x \in U_N$. *A neutrosophic hypersoft set* $(F,\boldsymbol{A})$ *is said to be a neutrosophic fuzzy hypersoft point if* $F(\boldsymbol{\alpha}')$ *is a null neutrosophic set for every* $\boldsymbol{\alpha}' \in \boldsymbol{A}\setminus\{\boldsymbol{\alpha}\}$ *and* $F(\boldsymbol{\alpha})(y) =< 0,1,1 >$ *for all* $y \neq x$. *We will denote* $(F,\boldsymbol{A})$ *simply by* $NP^{(\boldsymbol{\alpha},x)}$.

Definition 19 *A neutrosophic hypersoft set* $(F,\boldsymbol{A})$ *is said to be a null neutrosophic hypersoft point if* $F(\boldsymbol{\alpha})$ *is a null neutrosophic set for each* $\boldsymbol{\alpha} \in \boldsymbol{A}$. *We will denote a null neutrosophic hypersoft set, corresponding to* $\boldsymbol{\alpha}$, *by* $NP^{(\boldsymbol{\alpha},0_N)}$.

Its a matter of fact that if $(F,\mathbf{A})$ is a null neutrosophic hypersoft set then for every $\boldsymbol{\alpha} \in \mathbf{A}$ it can be regarded as null neutrosophic hypersoft set $NP^{(\boldsymbol{\alpha},0_N)}$.

Definition 20 *A neutrosophic hypersoft point* $NP^{(\boldsymbol{\alpha},x)}$ *is said to belong to a neutrosophic hypersoft set* $(G,\boldsymbol{A})$ *if* $NP^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (G,\boldsymbol{A})$. *We write it as* $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G,\boldsymbol{A})$.

It is straightforward to check that the neutrosophic hypersoft union of neutrosophic hypersoft points of a neutrosophic hypersoft set $(G,\mathbf{A})$ returns the neutrosophic hypersoft set $(G,\mathbf{A})$, that is,

$$(G,\mathbf{A}) = \tilde{\cup}\, \{NP^{(\boldsymbol{\alpha},x)}: NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G,\mathbf{A})\}.$$

We illustrate this observation through the following example.

Example 9 Let $\mathrm{U} = \{\mathrm{x}_1,\mathrm{x}_2,\mathrm{x}_3,\mathrm{x}_4\}$, and $(\mathrm{F},\mathbf{A})$ be as given in the example 4. Some of the neutrosophic hypersoft points of $(\mathrm{F},\mathbf{A})$ are the following:

$$NP_1^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>}\})\right\};$$

$$NP_2^{((a_{11},a_{21},a_{31}),x_1)} = \left\{((a_{11},a_{21},a_{31}),\{\frac{x_1}{<0.2,0.2,0.3>}\})\right\};$$

$$NP_3^{((a_{11},a_{21},a_{31}),x_2)} = \left\{((a_{11},a_{21},a_{31}),\{\frac{x_2}{<0.7,0.3,0.2>}\})\right\};$$

$$NP_4^{((a_{11},a_{22},a_{31}),x_2)} = \left\{((a_{11},a_{22},a_{31}),\{\frac{x_2}{<0.3,0.2,0.5>}\})\right\};$$

$$NP_5^{((a_{12},a_{21},a_{31}),x_3)} = \left\{((a_{12},a_{21},a_{31}),\{\frac{x_3}{<0.8,0.4,0.1>}\})\right\};$$

$$NP_6^{((a_{12},a_{21},a_{31}),x_4)} = \left\{((a_{12},a_{21},a_{31}),\{\frac{x_4}{<0.1,0.5,0.5>}\})\right\};$$

$$NP_7^{((a_{12},a_{22},a_{31}),x_1)} = \left\{((a_{12},a_{22},a_{31}),\{\frac{x_1}{<0.5,0.2,0.3>}\})\right\};$$

$$NP_8^{((a_{12},a_{22},a_{31}),x_4)} = \left\{((a_{12},a_{22},a_{31}),\{\frac{x_4}{<0.4,0.3,0.2>}\})\right\}.$$

Moreover we have

$$(F,\mathbf{A}) = NP_1^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup}\, NP_2^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup}\, NP_3^{((a_{11},a_{21},a_{31}),x_2)}$$
$$\tilde{\cup}\, NP_4^{((a_{11},a_{22},a_{31}),x_2)} \tilde{\cup}\, NP_5^{((a_{12},a_{21},a_{31}),x_3)} \tilde{\cup}\, NP_6^{((a_{12},a_{21},a_{31}),x_4)}$$
$$\tilde{\cup}\, NP_7^{((a_{12},a_{22},a_{31}),x_1)} \tilde{\cup}\, NP_8^{((a_{12},a_{22},a_{31}),x_4)}.$$

Proposition 5 *Let $(F,\boldsymbol{A}),(F_1,\boldsymbol{A})$ and $(F_2,\boldsymbol{A})$ be neutrosophic hypersoft sets over U. Then the following hold:*

1. *If $(F,\boldsymbol{A})$ is not a null neutrosophic hypersoft set then $(F,\boldsymbol{A})$ contains at least one nonnull neutrosophic hypersoft point.*
2. *$(F_1,\boldsymbol{A}) \tilde{\subseteq} (F_2,\boldsymbol{A})$ if and only if $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ implies that $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2,\boldsymbol{A})$.*
3. *$NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\cup} (F_2,\boldsymbol{A})$ if and only if $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ or $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2,\boldsymbol{A})$.*
4. *$NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\cap} (F_2,\boldsymbol{A})$ if and only if $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ and $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2,\boldsymbol{A})$.*
5. *$NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A}) \tilde{\setminus} (F_2,\boldsymbol{A})$ if and only if $NP^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1,\boldsymbol{A})$ and $NP^{(\boldsymbol{\alpha},x)} \tilde{\notin} (F_2,\boldsymbol{A})$.*

The proof of above proposition is similar as in the case of crisp hypersoft point.

3.5 Plithogenic hypersoft point

There may be four types of plithogenic hypersoft points namely: plithogenic crisp hypersoft point, plithogenic fuzzy hypersoft point, plithogenic intuitionistic fuzzy hypersoft point, plithogenic neutrosophic hypersoft point. But in this section we discuss only plithogenic crisp hypersoft point whereas other concepts and examples can be given in the similar way.

Definition 21 *Let $\boldsymbol{A} \subseteq \boldsymbol{E}$, $\boldsymbol{\alpha} \in \boldsymbol{A}$, and $x \in U_P$. A plithogenic crisp hypersoft set $(F,\boldsymbol{A})$ is said to be a plithogenic crisp hypersoft point if $F(\boldsymbol{\alpha}')$ is a null plithogenic crisp set for every $\boldsymbol{\alpha}' \in \boldsymbol{A}\setminus\{\boldsymbol{\alpha}\}$ and $F(\boldsymbol{\alpha})(y)(\mathbf{0})$ for all $y \neq x$. We will denote $(F,\boldsymbol{A})$ simply by $P_cP^{(\boldsymbol{\alpha},x)}$.*

Definition 22 *A plithogenic crisp hypersoft set $(F,\boldsymbol{A})$ is said to be a null plithogenic crisp hypersoft point if $F(\boldsymbol{\alpha})$ is a null plithogenic crisp set for each $\boldsymbol{\alpha} \in \boldsymbol{A}$. We will denote a null plithogenic crisp hypersoft set, corresponding to $\boldsymbol{\alpha}$, by $P_cP^{(\boldsymbol{\alpha},0_{PC})}$.*

Note that if $(F,\mathbf{A})$ is a null plithogenic crisp hypersoft set, then for every $\boldsymbol{\alpha} \in \mathbf{A}$ it can be regarded as a null plithogenic crisp hypersoft set $P_cP^{(\boldsymbol{\alpha},0_{PC})}$.

Definition 23 *A plithogenic crisp hypersoft point $P_cP^{(\boldsymbol{\alpha},x)}$ is said to belong to a plithogenic crisp hypersoft set $(G,\boldsymbol{A})$ if $P_cP^{(\boldsymbol{\alpha},x)} \tilde{\subseteq} (G,\boldsymbol{A})$. We write it as $P_cP^{(\boldsymbol{\alpha},x)} \tilde{\in} (G,\boldsymbol{A})$.*

It is straightforward to check that the plithogenic crisp hypersoft union of plithogenic crisp hypersoft points of a plithogenic crisp hypersoft set $(G, \mathbf{A})$ gives back the plithogenic crisp hypersoft set $(G, \mathbf{A})$, that is,

$$(G, \mathbf{A}) = \tilde{\cup} \{P_c P^{(\boldsymbol{\alpha},x)} : P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (G, \mathbf{A})\}.$$

We illustrate this observation through the following example.

Example 10 Let $\mathrm{U} = \{\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4\}$, and $(\mathrm{F}, \mathbf{A})$ be as given in the example 5. Then some of the plithogenic crisp hypersoft points of $(\mathrm{F}, \mathbf{A})$ are the following:

$$P_c P_1^{((a_{11},a_{21},a_{31}),x_1)} = \{((a_{11}, a_{21}, a_{31}), \{x_1(1,0,1)\})\};$$
$$P_c P_2^{((a_{11},a_{21},a_{31}),x_2)} = \{((a_{11}, a_{21}, a_{31}), \{x_2(1,1,1)\})\};$$
$$P_c P_3^{((a_{11},a_{22},a_{31}),x_2)} = \{((a_{11}, a_{22}, a_{31}), \{x_2(0,0,1)\})\};$$
$$P_c P_4^{((a_{12},a_{21},a_{31}),x_3)} = \{((a_{12}, a_{21}, a_{31}), \{x_3(1,1,0)\})\};$$
$$P_c P_5^{((a_{12},a_{21},a_{31}),x_4)} = \{((a_{12}, a_{21}, a_{31}), \{x_4(1,1,1)\})\};$$
$$P_c P_6^{((a_{12},a_{22},a_{31}),x_1)} = \{((a_{12}, a_{22}, a_{31}), \{x_1(1,0,1)\})\};$$
$$P_c P_7^{((a_{12},a_{22},a_{31}),x_4)} = \{((a_{12}, a_{22}, a_{31}), \{x_4(0,1,0)\})\}.$$

Moreover we have

$$(F, \mathbf{A}) = P_c P_1^{((a_{11},a_{21},a_{31}),x_1)} \tilde{\cup} P_c P_2^{((a_{11},a_{21},a_{31}),x_2)}$$
$$\tilde{\cup} P_c P_3^{((a_{11},a_{22},a_{31}),x_2)} \tilde{\cup} P_c P_4^{((a_{12},a_{21},a_{31}),x_3)} \tilde{\cup} P_c P_5^{((a_{12},a_{21},a_{31}),x_4)}$$
$$\tilde{\cup} P_c P_6^{((a_{12},a_{22},a_{31}),x_1)} \tilde{\cup} P_c P_7^{((a_{12},a_{22},a_{31}),x_4)}.$$

Proposition 6 *Let $(F, \boldsymbol{A}), (F_1, \boldsymbol{A})$ and $(F_2, \boldsymbol{A})$ be plithogenic crisp hypersoft sets over U. Then the following hold:*

1. *If $(F, \boldsymbol{A})$ is not a null plithogenic crisp hypersoft set then $(F, \boldsymbol{A})$ contains at least one nonnull plithogenic crisp hypersoft point.*
2. $(F_1, \boldsymbol{A}) \tilde{\subseteq} (F_2, \boldsymbol{A})$ *if and only if* $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ *implies that* $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \boldsymbol{A})$.
3. $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\cup} (F_2, \boldsymbol{A})$ *if and only if* $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ *or* $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \boldsymbol{A})$.
4. $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\cap} (F_2, \boldsymbol{A})$ *if and only if* $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ *and* $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_2, \boldsymbol{A})$.
5. $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A}) \tilde{\setminus} (F_2, \boldsymbol{A})$ *if and only if* $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\in} (F_1, \boldsymbol{A})$ *and* $P_c P^{(\boldsymbol{\alpha},x)} \tilde{\notin} (F_2, \boldsymbol{A})$.

The proof of above proposition is similar as in the case of crisp hypersoft point.

4. Conclusions

In this paper, we have initiated the concept of hypersoft point that will lead to define Cartesian product and then function on *-hypersoft sets. As a future work, one may carry out the study of *-hypersoft topological spaces. Once the functions on *-hypersoft sets are defined, this may lead to the study of fixed point results in this new framework.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Smarandache, F., 2018. Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set. Neutrosophic Sets and Systems, p.168.

2. Rana, S., Qayyum, M., Saeed, M., & Smarandache, F. (2019). Plithogenic Fuzzy Whole Hypersoft Set: Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique. Neutrosophic Sets and Systems, 28(1), 5.
3. Gayen, S., Smarandache, F., Jha, S., Singh, M. K., Broumi, S., & Kumar, R. (2020). Introduction to Plithogenic Subgroup. In Neutrosophic Graph Theory and Algorithms (pp. 213-259). IGI Global.
4. Abdel-Basset, M., & Mohamed, R. (2020). A novel plithogenic TOPSIS-CRITIC model for sustainable supply chain risk management. Journal of Cleaner Production, 247, 119586.
5. Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., & Smarandache, F. (2019). A Hybrid Plithogenic Decision-Making Approach with Quality Function Deployment for Selecting Supply Chain Sustainability Metrics. Symmetry, 11(7), 903.
6. Abdel-Basset, M., Nabeeh, N. A., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. Enterprise Information Systems, 1-21.
7. Nabeeh, N. A., Abdel-Basset, M., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Neutrosophic multi-criteria decision making approach for iot-based enterprises. IEEE Access, 7, 59559-59574.
8. Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, 77, 438-452.
9. Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2019). A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. Journal of medical systems, 43(2), 38.
10. Saqlain, M., Jafar, N., Moin, S., Saeed, M., & Broumi, S. (2020). Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets. *Neutrosophic Sets and Systems*, *32*(1), 20.
11. Saqlain, M., Moin, S., Jafar, M. N., Saeed, M., & Smarandache, F. (2020). Aggregate Operators of Neutrosophic Hypersoft Set. *Neutrosophic Sets and Systems*, *32*(1), 18.
12. Saqlain, M., Jafar, M. N., & Riaz, M. (2020). A New Approach of Neutrosophic Soft Set with Generalized Fuzzy TOPSIS in Application of Smart Phone Selection. *Neutrosophic Sets and Systems*, *32*(1), 19.
13. F. Feng, Y. B. Jun, X.Y. Liu, L.F. Li, An adjustable approach to fuzzy soft set based decision making, J. Comput. Appl. Math. 234.1 (2009)10–20.
14. F. Feng, Y. B. Jun, X. Zhao, Soft semirings. Comput. Math. Appl. 56 (10), 2621–2628 (2008).
15. F. Feng, X. Liu, Soft rough sets with applications to demand analysis. In: Int. Workshop Intell. Syst. Appl., ISA 2009, 23-24 May 2009, Wuhan, China. IEEE, pp. 1–4 (2009).
16. T. Herawan, M.M. Deris, On multi-soft sets construction in information systems. In: Emerging Intelligent Computing Technology and Applications with Aspects of Artificial Intelligence: 5th Int. Conf. Intell. Comput., ICIC 2009 Ulsan, South Korea, September 16-19, 2009. Springer, U. T. H. O. Malaysia, (2009) 101–110.
17. T. Herawan, A.N.M. Rose, M.M. Deris, Soft set theoretic approach for dimensionality reduction. In: Database Theory and Application: International Conference, DTA 2009, Jeju Island, Korea, December 10-12, 2009. Springer, (2009) 171–178.
18. Y. K. Kim, W. K. Min, Full soft sets and full soft decision systems. J. Int. Fuzzy Syst. 26.2 (2014) 925–933.
19. D. Molodtsov, Soft set theory—First results, Comput. Math. Appl. 37.4-5 (1999) 19–31.
20. M. M. Mushrif, S. Sengupta, A. K. Ray, Texture classification using a novel, soft-set theory based classification algorithm. Lect. Notes Comput. Sci. 3851 (2006) 246–254.
21. A. R. Roy, P.K. Maji, A fuzzy soft set theoretic approach to decision making problems, J. Comput. Appl. Math. 203.2 (2007) 412–418.
22. P. Zhu, Q. Wen, Probabilistic soft sets. In: 2010 IEEE international conference on granular computing, pp. 635-638.
23. Y. Zou, Z. Xiao, Data analysis approaches of soft sets under incomplete information. Knowl.-Based Syst. 21.8 (2008) 941–945.

Received: Apr 15, 2020. Accepted: July 5 2020