Neutrosophic Sets and Systems, Vol. 35, 2020
University of New Mexico

A Novel Approach for Pairwise Separation Axioms on Bi-Soft Topology Using Neutrosophic Sets and An Output Validation in Real Life Application

Chinnadurai V[1], Sindhu M P[2]

[1] Department of Mathematics, Annamalai University, Annamalainagar, Tamilnadu, India; kv.chinnaduraimaths@gmail.com
[2] Department of Mathematics, Karpagam College of Engineering, Coimbatore, Tamilnadu, India; sindhu.mp@kce.ac.in

Correspondence: kv.chinnaduraimaths@gmail.com

Abstract: The set that lightens the vagueness stage more energetically than fuzzy sets are neutrosophic sets. Bi-soft topological space is a space which goes for two different topologies with certain parameters. This work carries out, construction of such type of topology on neutrosophic. Besides by means of this, separation axioms are extended to pairwise separation axioms by using neutrosophic and to analyze the relationship among the class of such spaces. Here some of their properties are discussed with illustrative examples. In addition to it, we initiate the matrix form of neutrosophic soft sets in such space. Here problems deal to take a decision in life by the choice of two different groups. The aim of this decision making problem is to determine the unique thing or person from the universe by giving marks depending on parameters. Step by step process of solving the problem is explained in algorithm, also formulae given to determine their values with illustrative examples.

Keywords: Neutrosophic sets (NSs); neutrosophic soft sets (NSSs); neutrosophic soft topological spaces (NSTSs); neutrosophic soft $T_{i=0,1,2,3,4}$ -spaces (NS $T_{i=0,1,2,3,4}$ -spaces); neutrosophic bitopological spaces (NBTSs); neutrosophic bi-soft topological spaces (NBSTSs); pairwise neutrosophic soft $T_{i=0,1,2,3,4}$ -spaces (pairwise NS $T_{i=0,1,2,3,4}$ -spaces); decision making (DM).

1. Introduction

Zadeh [54], evaluated a fuzzy set (1965) to explore the situations like risky, unclear, erratic and distortion occurs in our life cycle. Fuzzy sets simplify classical sets and are unique cases of the membership functions. It has been used in a spacious collection of domains. This set extended to develop intuitionistic fuzzy set (IFS) theory (1986) by Atanassov [47]. Smarandache [7] originated a set which forecast the indeterminancy part along with truth and false statements, called NS (1998), such as blending of network arises to unpredictable states. It is a dynamic structure which postulates the concept of all other sets introduced before. Later, he generalized the NS on IFS [8] and recently proposed his work on attributes valued set, plithogenic set (PS) [9]. In day-to-day life decisions taken to diagnostic the problems either positive or negative even not both. Such types of problems are key role in all fields and so most of the researchers studied DM problem. In recent times various works have been done on these sets by Salma and Alblowi [33] and on extension of neutrosophic analysis on DM by Abdel et al. [1-6].

Soft set (1999) is a broad mathematical gadget which accord with a group of objects based on fairly accurate descriptions with orientation to elements of a parameter set was projected by Molodtsov [46]. Topological structure on this set explored by Shabir & Naz [38] as soft topological spaces (2011). Anon this thought were developed by Ali et al. [35, 40], Bayramov and Gunduz [22, 29], Cagman et al. [37], Chen [43], Feng et al. [41], Hussain and Ahmad [36], Maji et al. [44, 45], Min [39], Nazmul and Samanta [32], Pie and Miao [42], Tantawy et al. [26], Varol and Aygun [31], Zorlutuna et al. [34]. Maji [30] presented the binding of neutrosophic with soft set termed as NSSs (2013). Bera & Mahapatra [23] defined such type of set on topological structure, named as NSTSs (2017). Using these concepts, Deli & Broumi [27], Bera & Mahapatra [10, 24], on separation axioms by Cigdem et al. [20, 21] have done some research works. Mostly DM applied on these sets related to fuzzy with multicriteria by Chinnadurai et al. [13, 14 & 19], Abishek et al. [12], Muhammad et al. [16], Mehmood et al. [17], Evanzalin Ebenanjar et al. [18] and Faruk [25].

Kelly [55] imported the approach of a set equipped with two topologies, named as bitopological space (BTS) (1963), which is the generic system of topological space. Also it was carried out by Lane [53], Patty [52], Kalaiselvi and Sindhu [15] and pairwise concepts by Kim [51], Singal and Asha [50], Lal [48], Reilly [49]. Naz, Shabir and Ali [28] introduced bi-soft topological spaces (BSTSs) (2015) and studied the separation axioms on it. Taha and Alkan [11] presented BTS on neutrosophic structure as NBTSs (2019) which is engaged with two neutrosophic topologies (NTs).

The intension of this paper is to initiate the idea of NS on BSTS and to study some essential properties of such spaces. Also, the pairwise concept on separation axioms implemented in NBSTS. In addition, the NSSs referred as matrix form on NBSTS. As real life application, decisions made to select the one among the universe based on its parameters by considering two different groups as neutrosophic soft topologies (NSTs).

The arrangements made in this paper are as follows. Some basic definitions related to NS are in segment 2. The results of NBSTS are proved and disproved by counter examples in segment 3. The bonding among the pairwise separation axioms on NBSTS are stated with illustrative examples in segment 4. In segment 5, the method of evaluating DM problems are described in algorithm and formula specified to calculate the scores of universe set, to choose the best among them with illustrative examples. Finally, concluded with future work in segment 6.

2. Preamble

In this segment, we evoke few primary definitions associated to NSS, NSTS, BSTS and NBSTS.

Definition 2.1 [30] Let V be the set of universe and E be a set of parameters. Let $NS(V)$ denote the set of all NSs of V. Then a estimated function of NSS K over V is a set defined by a mapping $f_K : E \to NS(V)$. The NSS is a parameterized family of the set $NS(V)$ which can be written as a set of ordered pairs,

$$K = \left\{\left(e, \left\{\langle v, T_{f_K(e)}(v), I_{f_K(e)}(v), F_{f_K(e)}(v)\rangle : v \in V\right\}\right) : e \in E\right\}$$

where $T_{f_K(e)}(v), I_{f_K(e)}(v), F_{f_K(e)}(v) \in [0,1]$ respectively called the truth-membership, indeterminacy-membership and false-membership functions of $f_K(e)$ and the inequality $0 \le T_{f_K(e)}(v) + I_{f_K(e)}(v) + F_{f_K(e)}(v) \le 3$ is obvious.

Definition 2.2 [23] Let *NSS*(*V*) denote the set of all NSSs of *V* through all $e \in E$ and $\tau_u \subset NSS(V,E)$. Then τ_u is called NST on (*V, E*) if the following conditions are satisfied.

(i) $\phi_u, 1_u \in \tau_u$, where null NSS $\phi_u = \{(e, \{\langle v,0,0,1 \rangle\rangle : v \in V\}) : e \in E\}$ and absolute NSS $1_u = \{(e, \{\langle v,1,1,0 \rangle\rangle : v \in V\}) : e \in E\}$.

(ii) the intersection of any finite number of members of τ_u belongs to τ_u.

(iii) the union of any collection of members of τ_u belongs to τ_u.

Then the triplet (*V, E,* τ_u) is called a NSTS.

Every member of τ_u is called τ_u-open NSS, whose complement is τ_u-closed NSS.

Definition 2.3 [21] Let *NSS*(*V, E*) denote the family of all NSSs over *V*. The NSS $u_e^{(\alpha,\beta,\gamma)}$ is called a NSP, for every $u \in V, 0 < \alpha, \beta, \gamma \le 1, e \in E$ and is defined as follows:

$$u_{(e)}^{(\alpha,\beta,\gamma)}(e')(v) = \begin{cases} (\alpha,\beta,\gamma), & if\ e' = e\ and\ v = u \\ (0,0,1), & if\ e' \ne e\ and\ v \ne u \end{cases}$$

Obviously, every NSS is the union of its NSPs.

Definition 2.4 [11] Let (*V,* τ_{u1}) and (*V,* τ_{u2}) be the two different NTs on *V*. Then (*V,* τ_{u1} , τ_{u2}) is called a NBTS.

3. NBSTS

In this segment, the conception of NBSTS is defined and some key resources of topology are studied on it. The theoretical results are supported by some significant descriptive examples.

Definition 3.1 The quadruple (*V, E,* τ_{u1} , τ_{u2}) is called a NBSTS over (*V, E*), where τ_{u1} and τ_{u2} are NSTs independently satisfy the axioms of NSTS.

The elements of τ_{u1} are τ_{u1}-neutrosophic soft open sets (τ_{u1}-NSOSs) and the complement of it are τ_{u1}-neutrosophic soft closed sets (τ_{u1}-NSOSs).

Example 3.2 Let $V = \{v_1, v_2\}$, $E = \{e_1, e_2\}$ and $\tau_{u1} = \{\phi_u, 1_u, K_1\}$ and $\tau_{u2} = \{\phi_u, 1_u, L_1, L_2\}$ where K_1, L_1, L_2 are NSSs over (*V, E*), defined as follows

$f_{K_1}(e_1) = \{< v_1, (1,1,0) >, < v_2, (0,0,1) >\}$,

$f_{K_1}(e_2) = \{< v_1, (0,0,1) >, < v_2, (1,0,1) >\}$

and

$f_{L_1}(e_1) = \{< v_1, (1,0,1) >, < v_2, (0,0,1) >\}$,

$f_{L_1}(e_2) = \{< v_1, (0,1,0) >, < v_2, (1,1,0) >\}$;

$f_{L_2}(e_1) = \{< v_1, (0,1,0) >, < v_2, (1,1,0) >\}$,

$f_{L_2}(e_2) = \{< v_1, (1,0,1) >, < v_2, (0,0,1) >\}$.

Thus τ_{u1} and τ_{u2} are NSTs on (*V, E*) and so (*V, E,* τ_{u1} , τ_{u2}) is a NBSTS over (*V, E*).

Example 3.3 Let the neutrosophic soft indiscrete (trivial) topology τ_{u1} = $\{\phi_u, 1_u\}$ and neutrosophic soft discrete topology $\tau_{u2} = NSS(V, E)$.
Then (*V, E,* τ_{u1} , τ_{u2}) is a NBSTS over (*V, E*).

Definition 3.4 Let (*V, E,* τ_{u1} , τ_{u2}) be a NBSTS over (*V, E*), where τ_{u1} and τ_{u2} are NSTs on (*V, E*) and $P, Q \in NSS(V, E)$ be any two arbitrary NSSs. Suppose $\tau_{v1} = \{P \cap K_i / K_i \in \tau_{u1}\}$ and $\tau_{v2} = \{Q \cap L_i / L_i \in \tau_{u2}\}$. Then τ_{v1} and τ_{v2} are also NSTs on (*V, E*). Thus (*V, E,* τ_{v1} , τ_{v2}) is a NBST subspace of (*V, E,* τ_{u1} , τ_{u2}).

Theorem 3.5 Let (*V, E,* τ_{u1} , τ_{u2}) be a NBSTS over (*V, E*), where $\tau_{u1}(e)$ and $\tau_{u2}(e)$ are defined as

$\tau_{u1}(e) = \{ f_K(e) / K \in \tau_{u1}\}$

$\tau_{u2}(e) = \{ f_L(e) / L \in \tau_{u2}\}$ for each $e \in E$.

Then (*V, E,* $\tau_{u1}(e)$, $\tau_{u2}(e)$) is a NBTS over (*V, E*).

Proof. Follows from the fact that τ_{u1} and τ_{u2} are NTs on *V*.

Example 3.6 Let $V = \{v_1, v_2, v_3\}$, $E = \{e_1, e_2\}$ and τ_{u1} = $\{\phi_u, 1_u, K_1, K_2\}$ and τ_{u2} = $\{\phi_u, 1_u, L_1, L_2, L_3, L_4\}$ where $K_1, K_2, L_1, L_2, L_3, L_4$ are NSSs over (*V, E*), defined as follows

$f_{K_1}(e_1) = \{< v_1, (1,.5,.4) >, < v_2, (.6,.6,.6) >, < v_3, (.3,.4,.9) >\}$,

$f_{K_1}(e_2) = \{< v_1, (.8,.4,.5) >, < v_2, (.7,.7,.3) >, < v_3, (.7,.5,.6) >\}$;

$f_{K_2}(e_1) = \{< v_1, (.8,.5,.1) >, < v_2, (.8,.6,.5) >, < v_3, (.5,.6,.4) >\}$,

$f_{K_2}(e_2) = \{< v_1, (.9,.7,.1) >, < v_2, (.9,.9,.2) >, < v_3, (.8,.6,.3) >\}$

and

$f_{L_1}(e_1) = \{< v_1, (.3,.7,.6) >, < v_2, (.4,.3,.8) >, < v_3, (.6,.4,.5) >\}$,

$f_{L_1}(e_2) = \{< v_1, (.4,.6,.8) >, < v_2, (.3,.7,.2) >, < v_3, (.3,.3,.7) >\}$;

$f_{L_2}(e_1) = \{< v_1, (.6,.6,.8) >, < v_2, (.2,.9,.3) >, < v_3, (.1,.2,.4) >\}$,

$f_{L_2}(e_2) = \{< v_1, (.7,.9,.5) >, < v_2, (.4,.2,.3) >, < v_3, (.5,.5,.4) >\}$;

$f_{L_3}(e_1) = \{< v_1, (.6,.7,.6) >, < v_2, (.4,.9,.3) >, < v_3, (.6,.4,.4) >\}$,

$f_{L_3}(e_2) = \{< v_1, (.7,.9,.5) >, < v_2, (.4,.7,.2) >, < v_3, (.5,.5,.4) >\}$;

$f_{L_4}(e_1) = \{< v_1, (.3,.6,.8) >, < v_2, (.2,.3,.8) >, < v_3, (.1,.2,.5) >\}$,

$f_{L_4}(e_2) = \{< v_1, (.4,.6,.8) >, < v_2, (.3,.2,.3) >, < v_3, (.3,.3,.7) >\}$

Thus τ_{u1} and τ_{u2} are NSTs on (*V, E*) and so (*V, E,* τ_{u1}, τ_{u2}) is a NBSTS over (*V, E*).

Now,

$$\tau_{u1}(e_1) = \left\{ \begin{matrix} \phi_u, 1_u, \{< v_1, (1,.5,.4) >, < v_2, (.6,.6,.6) >, < v_3, (.3,.4,.9) >\}, \\ \{< v_1, (.8,.5,.1) >, < v_2, (.8,.6,.5) >, < v_3, (.5,.6,.4) >\} \end{matrix} \right\},$$

$$\tau_{u2}(e_1) = \left\{ \begin{matrix} \phi_u, 1_u, \{< v_1, (.3,.7,.6) >, < v_2, (.4,.3,.8) >, < v_3, (.6,.4,.5) >\}, \\ \{< v_1, (.6,.6,.8) >, < v_2, (.2,.9,.3) >, < v_3, (.1,.2,.4) >\}, \\ \{< v_1, (.6,.7,.6) >, < v_2, (.4,.9,.3) >, < v_3, (.6,.4,.4) >\}, \\ \{< v_1, (.3,.6,.8) >, < v_2, (.2,.3,.8) >, < v_3, (.1,.2,.5) >\} \end{matrix} \right\}$$

and

$$\tau_{u1}(e_2) = \left\{ \begin{matrix} \phi_u, 1_u, \{< v_1, (.8,.4,.5) >, < v_2, (.7,.7,.3) >, < v_3, (.7,.5,.6) >\}, \\ \{< v_1, (.9,.7,.1) >, < v_2, (.9,.9,.2) >, < v_3, (.8,.6,.3) >\} \end{matrix} \right\},$$

$$\tau_{u2}(e_2) = \left\{ \begin{matrix} \phi_u, 1_u, \{< v_1, (.4,.6,.8) >, < v_2, (.3,.7,.2) >, < v_3, (.3,.3,.7) >\}, \\ \{< v_1, (.7,.9,.5) >, < v_2, (.4,.2,.3) >, < v_3, (.5,.5,.4) >\}, \\ \{< v_1, (.7,.9,.5) >, < v_2, (.4,.7,.2) >, < v_3, (.5,.5,.4) >\}, \\ \{< v_1, (.4,.6,.8) >, < v_2, (.3,.2,.3) >, < v_3, (.3,.3,.7) >\} \end{matrix} \right\}$$

are NTs on *V*.

Thus (*V, E,* $\tau_{u1}(e), \tau_{u2}(e)$) is a NBTS over (*V, E*).

Definition 3.7 Let (*V, E,* τ_{u1}, τ_{u2}) be a NBSTS over (*V, E*). Then the supremum NST is $\tau_{u1} \vee \tau_{u2}$, which is the smallest NST on *V* that contains $\tau_{u1} \cup \tau_{u2}$.

Example 3.8 Let us consider 3.5 example, where τ_{u1} and τ_{u2} are NSTs on (*V, E*).

Then,

$$K_1 \cup L_1 = P = \left\{ \begin{matrix} f_P(e_1) = \{< v_1, (.3,.7,.4) >, < v_2, (.6,.6,.6) >, < v_3, (.6,.4,.5) >\} \\ f_P(e_2) = \{< v_1, (.8,.6,.5) >, < v_2, (.7,.7,.2) >, < v_3, (.7,.5,.6) >\} \end{matrix} \right.$$

and

$$K_1 \vee L_1 = Q = \left\{ \begin{matrix} f_Q(e_1, e_1) = \{< v_1, (.3,.7,.4) >, < v_2, (.6,.6,.6) >, < v_3, (.6,.4,.5) >\} \\ f_Q(e_1, e_2) = \{< v_1, (.8,.4,.5) >, < v_2, (.7,.7,.3) >, < v_3, (.7,.5,.4) >\} \\ f_Q(e_2, e_1) = \{< v_1, (1,.6,.4) >, < v_2, (.6,.7,.2) >, < v_3, (.3,.4,.7) >\} \\ f_Q(e_2, e_2) = \{< v_1, (.8,.6,.5) >, < v_2, (.7,.7,.2) >, < v_3, (.7,.5,.6) >\} \end{matrix} \right.$$

Thus $K_1 \vee L_1$ is the smallest NSS on *V* that contains $K_1 \cup L_1$.

Theorem 3.9 If (V, E, τ_{u1}, τ_{u2}) is a NBSTS over (V, E), then $\tau_{u1} \cap \tau_{u2}$ is a NST over (V, E).
Proof. Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E).

(i) Since $\phi_u, 1_u \in \tau_{u1}$ and $\phi_u, 1_u \in \tau_{u1}$, it follows that $\phi_u, 1_u \in \tau_{u1} \cap \tau_{u2}$.

(ii) Suppose that $\{K_i / i \in I\}$ is a family of NSSs in $\tau_{u1} \cap \tau_{u2}$.
Then $K_i \in \tau_{u1}$ and $K_i \in \tau_{u2}$ for all $i \in I$.

Thus $\bigcup_{i\in I} K_i \in \tau_{u1}$ and $\bigcup_{i\in I} K_i \in \tau_{u2}$.

Therefore $\bigcup_{i\in I} K_i \in \tau_{u1} \cap \tau_{u2}$.

(iii) Let $K, L \in \tau_{u1} \cap \tau_{u2}$.
Then $K, L \in \tau_{u1}$ and $K, L \in \tau_{u2}$.
Since $K \cap L \in \tau_{u1}$ and $K \cap L \in \tau_{u2}$, we have $K \cap L \in \tau_{u1} \cap \tau_{u2}$.
Hence $\tau_{u1} \cap \tau_{u2}$ is a NST over (V, E).

Remark 3.10 If (V, E, τ_{u1}, τ_{u2}) is a NBSTS over (V, E), then $\tau_{u1} \cup \tau_{u2}$ need not be a NST over (V, E).

Example 3.11 Let us consider 3.5 example where τ_{u1} and τ_{u2} are NSTs on (V, E).
Then,

$$K_1 \cup L_1 = P = \begin{cases} f_P(e_1) = \{< v_1, (0.3, 0.7, 0.4) >, < v_2, (0.6, 0.6, 0.6) >, < v_3, (0.6, 0.4, 0.5) >\} \\ f_P(e_2) = \{< v_1, (0.8, 0.6, 0.5) >, < v_2, (0.7, 0.7, 0.2) >, < v_3, (0.7, 0.5, 0.6) >\} \end{cases}$$

Thus $K_1 \cup L_1 \notin \tau_{u1} \cup \tau_{u2}$.
Hence $\tau_{u1} \cup \tau_{u2}$ is not a NST over (V, E).

4. Neutrosophic bi-soft separation axioms

In this segment, the separation of NBSTS is explored. The pairwise NS $T_{i=0,1,2,3,4}$-spaces on NBSTS are introduced and the relationships among them are examined with relevant examples.

Definition 4.1 A NBSTS (V, E, τ_{u1}, τ_{u2}) over (V, E) is called a pairwise NS T_0-space, if $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ are distinct NSPs then there exist τ_{u1}-NSOS K and τ_{u2}-NSOS L such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in K \;\; ; \;\; u_{(e)}^{(\alpha,\beta,\gamma)} \cap L = \phi_u$$

$$\text{or } \; v_{(e')}^{(\alpha,\beta,\gamma)} \in L \;\; ; \;\; v_{(e')}^{(\alpha,\beta,\gamma)} \cap K = \phi_u .$$

Example 4.2 Consider neutrosophic soft indiscrete (trivial) topology $\tau_{u1}=\{\phi_u,1_u\}$ and neutrosophic soft discrete topology $\tau_{u2}=NSS(U,E)$.
Thus (*V, E,* τ_{u1},τ_{u2}) is a pairwise NS T_0-space.

Theorem 4.3 Let (*V, E,* τ_{u1}, τ_{u2}) be a NBSTS over (*V, E*). If (*V, E,* τ_{u1},τ_{u2}) is a pairwise NS T_0-space then (*V, E,* $\tau_{u1}\vee\tau_{u2}$) is a NS T_0-space.
Proof. Let (*V, E,* τ_{u1},τ_{u2}) be a NBSTS over (*V, E*).
Suppose that (*V, E,* τ_{u1},τ_{u2}) is a pairwise NS T_0-space.

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs.

Then there exist τ_{u1}-NSOS *K* and τ_{u2}-NSOS *L* such that

$$u_{(e)}^{(\alpha,\beta,\gamma)}\in K \;\; ; \;\; u_{(e)}^{(\alpha,\beta,\gamma)}\bigcap L=\phi_u$$

$$\text{or} \;\; v_{(e')}^{(\alpha,\beta,\gamma)}\in L \;\; ; \;\; v_{(e')}^{(\alpha,\beta,\gamma)}\bigcap K=\phi_u$$

In either case $K,L\in\tau_{u1}\vee\tau_{u2}$.
Hence (*V, E,* $\tau_{u1}\vee\tau_{u2}$) is a NS T_0-space.

Theorem 4.4 Let (*V, E,* τ_{u1},τ_{u2}) be a NBSTS over (*V, E*). If (*V, E,* τ_{u1},τ_{u2}) is a pairwise NS T_0-space then (*V, E,* τ_{v1}, τ_{v2}) is also a pairwise NS T_0-space.
Proof. Let (*V, E,* τ_{u1},τ_{u2}) be a NBSTS over (*V, E*).

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs and $P,Q\in NSS(U,E)$.

Suppose that (*V, E,* τ_{u1},τ_{u2}) is a pairwise NS T_0-space.
Then there exist τ_{u1}-NSOS *K* and τ_{u2}-NSOS *L* such that

$$u_{(e)}^{(\alpha,\beta,\gamma)}\in K \;\; ; \;\; u_{(e)}^{(\alpha,\beta,\gamma)}\bigcap L=\phi_u$$

$$\text{or} \;\; v_{(e')}^{(\alpha,\beta,\gamma)}\in L \;\; ; \;\; v_{(e')}^{(\alpha,\beta,\gamma)}\bigcap K=\phi_u$$

Now $u_{(e)}^{(\alpha,\beta,\gamma)}\in P$ and $u_{(e)}^{(\alpha,\beta,\gamma)}\in K$.

Then $u_{(e)}^{(\alpha,\beta,\gamma)}\in P\bigcap K$, where $K\in\tau_{u1}$.

Consider $u_{(e)}^{(\alpha,\beta,\gamma)}\bigcap L=\phi_u$.

$$\Rightarrow u_{(e)}^{(\alpha,\beta,\gamma)}\bigcap L\bigcap Q=\phi_u\bigcap Q.$$

$$\Rightarrow u_{(e)}^{(\alpha,\beta,\gamma)}\bigcap(Q\bigcap L)=\phi_u.$$

Thus $u_{(e)}^{(\alpha,\beta,\gamma)}\in P\bigcap K$; $u_{(e)}^{(\alpha,\beta,\gamma)}\bigcap(Q\bigcap L)=\phi_u$, where $P\bigcap K\in\tau_{v1}$, $Q\bigcap L\in\tau_{v2}$.

Or if $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$; $v_{(e')}^{(\alpha,\beta,\gamma)} \cap K = \phi_u$, it can be proved that

$v_{(e')}^{(\alpha,\beta,\gamma)} \in Q \cap L$; $v_{(e')}^{(\alpha,\beta,\gamma)} \cap (Q \cap L) = \phi_u$, where $P \cap K \in \tau_{v1}$, $Q \cap L \in \tau_{v2}$.

Hence (*V, E,* τ_{v1}, τ_{v2}) is also a pairwise NS T_0 -space.

Definition 4.5 A NBSTS (*V, E,* τ_{u1}, τ_{u2}) over (*V, E*) is called pairwise NS T_1 -space, if $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ are distinct NSPs then there exist τ_{u1} -NSOS *K* and τ_{u2} -NSOS *L* such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in K \quad ; \quad u_{(e)}^{(\alpha,\beta,\gamma)} \cap L = \phi_u$$

and $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$; $v_{(e')}^{(\alpha,\beta,\gamma)} \cap K = \phi_u$.

Example 4.6 Let $V = \{v_1, v_2\}$, E = {*e*}, and $v_{1(e)}^{(.2,.3,.7)}$ and $v_{2(e)}^{(.9,.4,.1)}$ be NSPs. Let $\tau_{u1} = \{\phi_u, 1_u, K\}$ and $\tau_{u2} = \{\phi_u, 1_u, L\}$ where *K* and *L* are NSSs over (*V, E*), defined as

$$K = v_{1(e)}^{(.2,.3,.7)} = f_K(e) = \{< v_1, (.2,.3,.7) >, < v_2, (0,0,1) >\}$$

and

$$L = v_{2(e)}^{(.9,.4,.1)} = f_L(e) = \{< v_1, (0,0,1) >, < v_2, (.9,.4,.1) >\} .$$

Thus (*V, E,* τ_{u1}, τ_{u2}) is a NBSTS over (*V, E*).
Hence (*V, E,* τ_{u1}, τ_{u2}) is a pairwise NS T_1 -space, also a pairwise NS T_0 -space.

Theorem 4.7 Every pairwise NS T_1 -space is also a pairwise NS T_0 -space.
Proof. Follows from the Definitions 4.1 and 4.3.

Remark 4.8 The converse of the 4.7 theorem is not true, which is shown in the following example.

Example 4.9 Let $V = \{v_1, v_2\}$, $E = \{e_1, e_2\}$, and $v_{1(e_1)}^{(.2,.5,.7)}$, $v_{1(e_2)}^{(.2,.8,.2)}$, $v_{2(e_1)}^{(.2,.7,.5)}$ and $v_{2(e_2)}^{(.1,.1,.9)}$ be NSPs. Let $\tau_{u1} = \{\phi_u, 1_u, K_1, K_2, K_3\}$ and $\tau_{u2} = \{\phi_u, 1_u, L_1, L_2\}$ where K_1, K_2, K_3, L_1, L_2 are NSSs over (*V, E*), defined as

$$K_1 = v_{1(e_1)}^{(.2,.5,.7)} = \begin{cases} f_{K_1}(e_1) = \{< v_1, (.2,.5,.7) >, < v_2, (0,0,1) >\} \\ f_{K_1}(e_2) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \end{cases} ;$$

$$K_2 = v_{2(e_2)}^{(.1,.1,.9)} = \begin{cases} f_{K_2}(e_1) = \{< v_1,(0,0,1) >,< v_2,(0,0,1) >\} \\ f_{K_2}(e_2) = \{< v_1,(0,0,1) >,< v_2,(.1,.1,.9) >\} \end{cases} ;$$

$$K_3 = K_1 \cup K_2$$

and

$$L_1 = v_{2(e_1)}^{(.2,.7,.5)} = \begin{cases} f_{L_1}(e_1) = \{< v_1,(0,0,1) >,< v_2,(.2,.7,.5) >\} \\ f_{L_1}(e_2) = \{< v_1,(0,0,1) >,< v_2,(0,0,1) >\} \end{cases} ;$$

$$L_2 = \left\{ v_{1(e_1)}^{(.2,.5,.7)}, v_{1(e_2)}^{1(.2,.8,.2)}, v_{2(e_1)}^{(.2,.7,.5)}, v_{2(e_2)}^{(.1,.1,.9)} \right\} = \begin{cases} f_{L_2}(e_1) = \{< v_1,(.2,.5,.7) >,< v_2,(.2,.7,.5) >\} \\ f_{L_2}(e_2) = \{< v_1,(.2,.8,.2) >,< v_2,(.1,.1,.9) >\} \end{cases}$$

Thus (V, E, τ_{u1}, τ_{u2}) is a NBSTS over (V, E).
Hence (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_0-space, but not a pairwise NS T_1-space since for NSPs $v_{1(e_1)}^{(.2,.5,.7)}$ and $v_{2(e_2)}^{(.1,.1,.9)}$, (V, E, τ_{u1}, τ_{u2}) is not a pairwise NS T_1-space.

Theorem 4.10 Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E). If (V, E, τ_{u1}) or (V, E, τ_{u2}) is not a NS T_0-space, then (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_0-space but not a pairwise NS T_1-space.

Proof. Let $K \in \tau_{u1}$ and $L \in \tau_{u2}$, also $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs.

Suppose (V, E, τ_{u1}) is a NS T_0-space and (V, E, τ_{u2}) is not a NS T_0-space.

Then, $u_{(e)}^{(\alpha,\beta,\gamma)} \in K$; $u_{(e)}^{(\alpha,\beta,\gamma)} \cap L = \phi_u$

and $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$; $v_{(e')}^{(\alpha,\beta,\gamma)} \cap K \neq \phi_u$

Thus by Definitions 4.1 and 4.3,
(V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_0-space but not a pairwise NS T_1-space.

Theorem 4.11 Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E). Then (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_1-space if and only if (V, E, τ_{u1}) and (V, E, τ_{u2}) are NS T_1-spaces.
Proof. Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E).

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs.

Suppose that (V, E, τ_{u1}) and (V, E, τ_{u2}) are NS T_1-spaces.
Then there exist τ_{u1}-NSOS K and τ_{u2}-NSOS L such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in K \ ; \ u_{(e)}^{(\alpha,\beta,\gamma)} \cap L = \phi_u$$

and $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$; $v_{(e')}^{(\alpha,\beta,\gamma)} \cap K = \phi_u$

In either case the result follows immediately.
Thus (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_1-space.
Conversely, assume that (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_1-space.

Then there exist some τ_{u1}-NSOS K_1 and τ_{u2}-NSOS L_1 such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in K_1 \;\; ; \;\; u_{(e)}^{(\alpha,\beta,\gamma)} \bigcap L_1 = \phi_u$$

and $v_{(e')}^{(\alpha,\beta,\gamma)} \in L_1 \;\; ; \;\; v_{(e')}^{(\alpha,\beta,\gamma)} \bigcap K_1 = \phi_u$

Also there exist some τ_{u1}-NSOS K_2 and τ_{u2}-NSOS L_2 such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in K_2 \;\; ; \;\; u_{(e)}^{(\alpha,\beta,\gamma)} \bigcap L_2 = \phi_u$$

and $v_{(e')}^{(\alpha,\beta,\gamma)} \in L_2 \;\; ; \;\; v_{(e')}^{(\alpha,\beta,\gamma)} \bigcap K_2 = \phi_u$

Hence (*V*, *E*, τ_{u1}) and (*V*, *E*, τ_{u2}) are NS T_1-spaces.

Theorem 4.12 Let (*V*, *E*, τ_{u1}, τ_{u2}) be a NBSTS over (*V*, *E*). If (*V*, *E*, τ_{u1}, τ_{u2}) is a pairwise NS T_1-space then (*V*, *E*, $\tau_{u1} \vee \tau_{u2}$) is a NS T_1-space.

Proof. Let (*V*, *E*, τ_{u1}, τ_{u2}) be a NBSTS over (*V*, *E*).

Suppose that (*V*, *E*, τ_{u1}, τ_{u2}) is a pairwise NS T_1-space.

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs.

Then there exist τ_{u1}-NSOS *K* and τ_{u2}-NSOS *L* such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in K \;\; ; \;\; u_{(e)}^{(\alpha,\beta,\gamma)} \bigcap L = \phi_u$$

and $v_{(e')}^{(\alpha,\beta,\gamma)} \in L \;\; ; \;\; v_{(e')}^{(\alpha,\beta,\gamma)} \bigcap K = \phi_u$

In either case $K, L \in \tau_{u1} \vee \tau_{u2}$.

Hence (*V*, *E*, $\tau_{u1} \vee \tau_{u2}$) is a NS T_1-space.

Theorem 4.13 Let (*V*, *E*, τ_{u1}, τ_{u2}) be a NBSTS over (*V*, *E*). If (*V*, *E*, τ_{u1}, τ_{u2}) is a pairwise NS T_1-space then (*V*, *E*, τ_{v1}, τ_{v2}) is also a pairwise NS T_1-space.

Proof. Let (*V*, *E*, τ_{u1}, τ_{u2}) be a NBSTS over (*V*, *E*).

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs and $P, Q \in NSS(U, E)$.

Suppose that (*V*, *E*, τ_{u1}, τ_{u2}) is a pairwise NS T_1-space.

Then there exist τ_{u1}-NSOS *K* and τ_{u2}-NSOS *L* such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in K \;\; ; \;\; u_{(e)}^{(\alpha,\beta,\gamma)} \bigcap L = \phi_u$$

and $v_{(e')}^{(\alpha,\beta,\gamma)} \in L \;\; ; \;\; v_{(e')}^{(\alpha,\beta,\gamma)} \bigcap K = \phi_u$

Now $u_{(e)}^{(\alpha,\beta,\gamma)} \in P$ and $u_{(e)}^{(\alpha,\beta,\gamma)} \in K$.

Then $u_{(e)}^{(\alpha,\beta,\gamma)} \in P \bigcap K$, where $K \in \tau_{u1}$.

Consider $u_{(e)}^{(\alpha,\beta,\gamma)} \cap L = \phi_u$.

$\Rightarrow u_{(e)}^{(\alpha,\beta,\gamma)} \cap L \cap Q = \phi_u \cap Q$.

$\Rightarrow u_{(e)}^{(\alpha,\beta,\gamma)} \cap (Q \cap L) = \phi_u$.

Thus $u_{(e)}^{(\alpha,\beta,\gamma)} \in P \cap K$; $u_{(e)}^{(\alpha,\beta,\gamma)} \cap (Q \cap L) = \phi_u$, where $P \cap K \in \tau_{v1}$, $Q \cap L \in \tau_{v2}$.

Further if $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$; $v_{(e')}^{(\alpha,\beta,\gamma)} \cap K = \phi_u$, it can be proved that

$v_{(e')}^{(\alpha,\beta,\gamma)} \in Q \cap L$; $v_{(e')}^{(\alpha,\beta,\gamma)} \cap (Q \cap L) = \phi_u$, where $P \cap K \in \tau_{v1}$, $Q \cap L \in \tau_{v2}$.

Hence (*V, E,* τ_{v1}, τ_{v2}) is also a pairwise NS T_1-space.

Theorem 4.14 Let (*V, E,* τ_{u1}, τ_{u2}) be a NBSTS over (*V, E*). For each pair of distinct NSPs $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$, $u_{(e)}^{(\alpha,\beta,\gamma)}$ is a τ_{u2}-NSCS and $v_{(e')}^{(\alpha,\beta,\gamma)}$ is a τ_{u1}-NSCS, then (*V, E,* τ_{u1}, τ_{u2}) is a pairwise NS T_1-space.

Proof. Let (*V, E,* τ_{u1}, τ_{u2}) be a NBSTS over (*V, E*).

Suppose that for each pair of distinct NSPs $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$, $u_{(e)}^{(\alpha,\beta,\gamma)}$ is a τ_{u2}-NSCS.

Then $\left(u_{(e)}^{(\alpha,\beta,\gamma)}\right)^c$ is a τ_{u2}-NSOS.

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs.

(i.e.,) $u_{(e)}^{(\alpha,\beta,\gamma)} \cap v_{(e')}^{(\alpha,\beta,\gamma)} = \phi_u$.

Thus

$$v_{(e')}^{(\alpha,\beta,\gamma)} \in \left(u_{(e)}^{(\alpha,\beta,\gamma)}\right)^c \text{ and } u_{(e)}^{(\alpha,\beta,\gamma)} \cap \left(u_{(e)}^{(\alpha,\beta,\gamma)}\right)^c = \phi_u \tag{1}$$

Similarly assume that for each NSP, $v_{(e')}^{(\alpha,\beta,\gamma)}$ is a τ_{u1}-NSCS.

Then $\left(v_{(e')}^{(\alpha,\beta,\gamma)}\right)^c$ is a τ_{u1}-NSOS such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in \left(v_{(e')}^{(\alpha,\beta,\gamma)}\right)^c \text{ and } v_{(e')}^{(\alpha,\beta,\gamma)} \cap \left(v_{(e')}^{(\alpha,\beta,\gamma)}\right)^c = \phi_u \tag{2}$$

From (1) and (2),

$u_{(e)}^{(\alpha,\beta,\gamma)} \in \left(v_{(e')}^{(\alpha,\beta,\gamma)}\right)^c$; $u_{(e)}^{(\alpha,\beta,\gamma)} \cap \left(u_{(e)}^{(\alpha,\beta,\gamma)}\right)^c = \phi_u$

and $v_{(e')}^{(\alpha,\beta,\gamma)} \in \left(u_{(e)}^{(\alpha,\beta,\gamma)}\right)^c$; $v_{(e')}^{(\alpha,\beta,\gamma)} \cap \left(v_{(e')}^{(\alpha,\beta,\gamma)}\right)^c = \phi_u$

Hence (*V, E,* τ_{u1}, τ_{u2}) is a pairwise NS T_1-space.

Definition 4.15 A NBSTS (*V, E,* τ_{u1}, τ_{u2}) over (*V, E*) is called pairwise NS T_2-space or pairwise NS Hausdorff space, if $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ are distinct NSPs then there exist τ_{u1}-NSOS *K* and τ_{u2}-NSOS *L* such that $u_{(e)}^{(\alpha,\beta,\gamma)} \in K$, $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$ and $K \bigcap L = \phi_u$.

Example 4.16 Let *V* = $\{v_1, v_2\}$, *E* = $\{e_1, e_2\}$, and $v_{1(e_1)}^{(.2,.5,.7)}$, $v_{1(e_2)}^{(.2,.8,.2)}$, $v_{2(e_1)}^{(.2,.7,.5)}$ and $v_{2(e_2)}^{(.1,.1,.9)}$ be NSPs. Let τ_{u1} = { $\phi_u, 1_u, K_1, K_2, K_3$ } and τ_{u2} = { $\phi_u, 1_u, L_1, L_2, L_3$ } where $K_1, K_2, K_3, L_1, L_2, L_3$ are NSSs over (*V, E*), defined as follows

$$K_1 = v_{1(e_1)}^{(.2,.5,.7)} = \begin{cases} f_{K_1}(e_1) = \{<v_1,(.2,.5,.7)>, <v_2,(0,0,1)>\} \\ f_{K_1}(e_2) = \{<v_1,(0,0,1)>, <v_2,(0,0,1)>\} \end{cases};$$

$$K_2 = v_{2(e_2)}^{(.1,.1,.9)} = \begin{cases} f_{K_2}(e_1) = \{<v_1,(0,0,1)>, <v_2,(0,0,1)>\} \\ f_{K_2}(e_2) = \{<v_1,(0,0,1)>, <v_2,(.1,.1,.9)>\} \end{cases};$$

$$K_3 = K_1 \bigcup K_2$$

and

$$L_1 = v_{2(e_1)}^{(.2,.7,.5)} = \begin{cases} f_{L_1}(e_1) = \{<v_1,(0,0,1)>, <v_2,(.2,.7,.5)>\} \\ f_{L_1}(e_2) = \{<v_1,(0,0,1)>, <v_2,(0,0,1)>\} \end{cases};$$

$$L_2 = v_{1(e_2)}^{(.2,.8,.2)} = \begin{cases} f_{L_2}(e_1) = \{<v_1,(0,0,1)>, <v_2,(0,0,1)>\} \\ f_{L_2}(e_2) = \{<v_1,(.2,.8,.2)>, <v_2,(0,0,1)>\} \end{cases};$$

$$L_3 = L_1 \bigcup L_2$$

Then (*V, E,* τ_{u1}, τ_{u2}) is a NBSTS over (*V, E*).
Hence (*V, E,* τ_{u1}, τ_{u2}) is a pairwise NS T_2-space.

Theorem 4.17 Let (*V, E,* τ_{u1}, τ_{u2}) be a NBSTS over (*V, E*). If (*V, E,* τ_{u1}, τ_{u2}) is a pairwise NS T_2-space then (*V, E,* $\tau_{u1} \vee \tau_{u2}$) is a NS T_2-space.
Proof. Let (*V, E,* τ_{u1}, τ_{u2}) be a NBSTS over (*V, E*).
Suppose that (*V, E,* τ_{u1}, τ_{u2}) is a pairwise NS T_2-space.

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs.

Then there exist τ_{u1}-NSOS *K* and τ_{u2}-NSOS *L* such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in K, \; v_{(e')}^{(\alpha,\beta,\gamma)} \in L \text{ and } K \bigcap L = \phi_u.$$

In either case $K, L \in \tau_{u1} \vee \tau_{u2}$.
Hence (*V, E,* $\tau_{u1} \vee \tau_{u2}$) is a NS T_2-space.

Theorem 4.18 Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E). If (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_2-space then (V, E, τ_{v1}, τ_{v2}) is also a pairwise NS T_2-space.
Proof. Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E).

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs and $P, Q \in NSS(U,E)$.

Suppose that (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_2-space.
Then there exist τ_{u1}-NSOS K and τ_{u2}-NSOS L such that

$u_{(e)}^{(\alpha,\beta,\gamma)} \in K$, $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$ and $K \cap L = \phi_u$.

Now $u_{(e)}^{(\alpha,\beta,\gamma)} \in P$ and $u_{(e)}^{(\alpha,\beta,\gamma)} \in K$; $v_{(e')}^{(\alpha,\beta,\gamma)} \in Q$ and $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$

Then $u_{(e)}^{(\alpha,\beta,\gamma)} \in P \cap K$, $v_{(e')}^{(\alpha,\beta,\gamma)} \in Q \cap L$ where $K \in \tau_{u1}$, $L \in \tau_{u2}$.

Consider $K \cap L = \phi_u$.

$$\Rightarrow (P \cap K) \cap (L \cap Q) = P \cap \phi_u \cap Q.$$
$$\Rightarrow (P \cap K) \cap (Q \cap L) = \phi_u..$$

Thus $u_{(e)}^{(\alpha,\beta,\gamma)} \in P \cap K$, $v_{(e')}^{(\alpha,\beta,\gamma)} \in Q \cap L$ and $(P \cap K) \cap (Q \cap L) = \phi_u$.

Hence (V, E, τ_{v1}, τ_{v2}) is also a pairwise NS T_2-space.

Theorem 4.19 Every pairwise NS T_2-space is also a pairwise NS T_1-space.
Proof. Follows from Definitions 4.3 and 4.15.

Theorem 4.20 Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E). (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_2-space if and only if for any two distinct NSPs $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$, there exist τ_{u1}-NSOS K containing $u_{(e)}^{(\alpha,\beta,\gamma)}$ but not $v_{(e')}^{(\alpha,\beta,\gamma)}$ such that $v_{(e')}^{(\alpha,\beta,\gamma)} \notin \overline{K}$.

Proof. Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E).

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs.

Suppose that (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_2-space.
Then there exist τ_{u1}-NSOS K and τ_{u2}-NSOS L such that

$u_{(e)}^{(\alpha,\beta,\gamma)} \in K$, $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$ and $K \cap L = \phi_u$.

Since $u_{(e)}^{(\alpha,\beta,\gamma)} \cap v_{(e')}^{(\alpha,\beta,\gamma)} = \phi_u$ and $K \cap L = \phi_u$, $v_{(e')}^{(\alpha,\beta,\gamma)} \notin K$.

Thus $v_{(e')}^{(\alpha,\beta,\gamma)} \notin \overline{K}$.

Conversely, assume that for any two distinct NSPs $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$, there exist τ_{u1}-NSOS K containing $u_{(e)}^{(\alpha,\beta,\gamma)}$ but not $v_{(e')}^{(\alpha,\beta,\gamma)}$ such that $v_{(e')}^{(\alpha,\beta,\gamma)} \notin \overline{K}$.

Then $v_{(e')}^{(\alpha,\beta,\gamma)} \notin \left(\overline{K}\right)^c$.

Thus $\overline{K}$ and $\left(\overline{K}\right)^c$ are disjoint τ_{u1}-NSOS and τ_{u1}-NSOS containing $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ respectively.

Theorem 4.21 Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E) and (V, E, τ_{u1}, τ_{u2}) be a pairwise NS T_1-space for every NSP $u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}$. If there exist τ_{u2}-NSOS L such that $u_{(e)}^{(\alpha,\beta,\gamma)} \in L \subseteq \overline{L} \subseteq K$, then ($V$, E, τ_{u1}, τ_{u2}) is a pairwise NS T_2-space.

Proof. Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E) and let it be a pairwise NS T_1-space

Suppose that $u_{(e)}^{(\alpha,\beta,\gamma)} \cap v_{(e')}^{(\alpha,\beta,\gamma)} = \phi_u$.

Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ be a τ_{u1}-NSCS and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be a τ_{u2}-NSCS.

Then $\left(v_{(e')}^{(\alpha,\beta,\gamma)}\right)^c$ is a τ_{u2}-NSOS such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in \left(v_{(e')}^{(\alpha,\beta,\gamma)}\right)^c \in \tau_{u2}$$

Then there exist a τ_{u2}-NSOS L such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in L \subseteq \overline{L} \subseteq \left(v_{(e')}^{(\alpha,\beta,\gamma)}\right)^c.$$

Thus $\left(v_{(e')}^{(\alpha,\beta,\gamma)}\right)^c \in \left(\overline{L}\right)^c$, $u_{(e)}^{(\alpha,\beta,\gamma)} \in L$ and $L \cap \left(\overline{L}\right)^c = \phi_u$.

Hence (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_2-space.

Remark 4.22 Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E). For any NSS K over (V, E), $\left(\overline{K}\right)^{\tau_{u2}}$ denotes the NS closure of K with respect to τ_{u2}-NST over (V, E).

Theorem 4.23 Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E). Then the following are equilavent:

(1) (V, E, τ_{u1}, τ_{u2}) is a pairwise NS Hausdorff space over (V, E).

(2) If $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ are distinct NSPs, there exist τ_{u1}-NSOS K such that

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in K \text{ and } v_{(e')}^{(\alpha,\beta,\gamma)} \in \left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c.$$

Proof. (1) $\Rightarrow$ (2). Suppose that (V, E, τ_{u1}, τ_{u2}) is a pairwise NS Hausdorff space over (V, E).
Then there exist τ_{u1}-NSOS K and τ_{u2}-NSOS L such that

$u_{(e)}^{(\alpha,\beta,\gamma)} \in K$, $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$ and $K \cap L = \phi_u$.

So that $K \subseteq L^c$.

Since $\left(\overline{K}\right)^{\tau_{u2}}$ is the smallest τ_{u2}-NSCS that contains K and L^c is a τ_{u2}-NSCS, then $\left(\overline{K}\right)^{\tau_{u2}} \subseteq L^c$

$$\Rightarrow L \subseteq \left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c .$$

Thus $v_{(e')}^{(\alpha,\beta,\gamma)} \in L \subseteq \left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c$.

Hence $v_{(e')}^{(\alpha,\beta,\gamma)} \in \left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c$.

(2) $\Rightarrow$ (1). Let $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ be any two distinct NSPs.

By assumption, there exist τ_{u1}-NSOS K such that $u_{(e)}^{(\alpha,\beta,\gamma)} \in K$ and $v_{(e')}^{(\alpha,\beta,\gamma)} \in \left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c$.

As $\left(\overline{K}\right)^{\tau_{u2}}$ is a τ_{u2}-NSCS so $L = \left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c \in \tau_{u2}$.

Now $u_{(e)}^{(\alpha,\beta,\gamma)} \in K$, $v_{(e')}^{(\alpha,\beta,\gamma)} \in L$ and

$$K \cap L = K \cap \left(\overline{K}\right)^{\tau_{u2}} .$$

$$\supseteq K \cap \left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c \qquad (\because K \subseteq \left(\overline{K}\right)^{\tau_{u2}})$$

$$= \phi_u .$$

Thus $K \cap L = \phi_u$.

Hence (V, E, τ_{u1} , τ_{u2}) is a pairwise NS Hausdorff space over (V, E).

Definition 4.24 Let $NSS(V, E)$ be the family of all NSSs over the universe V and $u \in V$. Then $u_E^{(\alpha,\beta,\gamma)}$ denotes the NSS over (V, E) for which $u_{(e)}^{(\alpha,\beta,\gamma)} = \left\{u^{(\alpha,\beta,\gamma)}\right\}$, for all $e \in E$.

Corollary 4.25 Let (V, E, τ_{u1} , τ_{u2}) be a pairwise NS T_2-space over (V, E). Then for each NSP $u_{(e)}^{(\alpha,\beta,\gamma)}$,

$$u_E^{(\alpha,\beta,\gamma)} = \bigcap \left\{\left(\overline{K}\right)^{\tau_{u2}} : u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}\right\}.$$

Proof. Let (V, E, τ_{u1} , τ_{u2}) be a pairwise NS T_2-space over (V, E) and $u_{(e)}^{(\alpha,\beta,\gamma)}$ be a NSP.

Then there exist a NSOS $u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}$.

If $u_{(e)}^{(\alpha,\beta,\gamma)}$ and $v_{(e')}^{(\alpha,\beta,\gamma)}$ are distinct NSPs, by 4.24 theorem, there exist τ_{u1}-NSOS K such that

$u_{(e)}^{(\alpha,\beta,\gamma)} \in K$ and $v_{(e')}^{(\alpha,\beta,\gamma)} \in \left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c$.

$$\Rightarrow v_{(e')}^{(\alpha,\beta,\gamma)} \notin f_{\left(\overline{K}\right)^{\tau_{u2}}}(e').$$

$$\Rightarrow v_{(e')}^{(\alpha,\beta,\gamma)} \notin \bigcap_{u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}} \left(f_{\left(\overline{K}\right)^{\tau_{u2}}}(e')\right) \quad \text{for all } e' \in E.$$

Thus

$$\bigcap\left\{\left(\overline{K}\right)^{\tau_{u2}} : u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}\right\} \subseteq u_E^{(\alpha,\beta,\gamma)} \tag{1}$$

Also it is obvious that $u_{(e)}^{(\alpha,\beta,\gamma)} \in K \subseteq \left(\overline{K}\right)^{\tau_{u2}}$.

Thus

$$u_E^{(\alpha,\beta,\gamma)} \subseteq \bigcap\left\{\left(\overline{K}\right)^{\tau_{u2}} : u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}\right\} \tag{2}$$

Hence from (1) and (2),

$$u_E^{(\alpha,\beta,\gamma)} = \bigcap\left\{\left(\overline{K}\right)^{\tau_{u2}} : u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}\right\}.$$

Corollary 4.26 Let (*V*, *E*, τ_{u1}, τ_{u2}) be a pairwise NS T_2-space over (*V*, *E*). Then for each NSP $u_{(e)}^{(\alpha,\beta,\gamma)}$, $\left(u_E^{(\alpha,\beta,\gamma)}\right)^c \in \tau_{ui}$ for *i* = 1, 2.

Proof. Let (*V*, *E*, τ_{u1}, τ_{u2}) be a pairwise NS T_2-space over (*V*, *E*) and $u_{(e)}^{(\alpha,\beta,\gamma)}$ be a NSP.

By 4.25 corollary,

$$\left(u_E^{(\alpha,\beta,\gamma)}\right)^c = \bigcup\left\{\left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c : u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}\right\}$$

Since $\left(\overline{K}\right)^{\tau_{u2}}$ is a τ_{u2}-NSCS, then $\left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c \in \tau_{u2}$.

By the axioms of a NS topological space,

$$\bigcup\left\{\left(\left(\overline{K}\right)^{\tau_{u2}}\right)^c : u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}\right\} \in \tau_{u2}.$$

Thus $\left(u_E^{(\alpha,\beta,\gamma)}\right)^c \in \tau_{u2}$.

Similarly it can be proved that, $\left(u_E^{(\alpha,\beta,\gamma)}\right)^c \in \tau_{u1}$.

Hence $\left(u_E^{(\alpha,\beta,\gamma)}\right)^c \in \tau_{ui}$ for *i* = 1, 2.

Definition 4.27 A NBSTS $(V, E, \tau_{u1}, \tau_{u2})$ over (V, E) is called pairwise NS regular space, if K is a τ_{u1}-NSCS and $u_{(e)}^{(\alpha,\beta,\gamma)} \cap K = \phi_u$ then there exist τ_{u2}-NSOSs L_1 and L_2 such that $u_{(e)}^{(\alpha,\beta,\gamma)} \in L_1$, $K \subseteq L_2$ and $L_1 \cap L_2 = \phi_u$.

A NBSTS $(V, E, \tau_{u1}, \tau_{u2})$ over (V, E) is called pairwise NS T_3-space, if it is both a pairwise NS regular space and a pairwise NS T_1-space.

Theorem 4.28 Let $(V, E, \tau_{u1}, \tau_{u2})$ be a NBSTS over (V, E). Then $(V, E, \tau_{u1}, \tau_{u2})$ is a pairwise NS T_3-space if and only if for every $u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}$, there exists $L \in \tau_{u2}$ such that $u_{(e)}^{(\alpha,\beta,\gamma)} \in L \subseteq \overline{L} \subseteq K$.

Proof. Let $(V, E, \tau_{u1}, \tau_{u2})$ be a NBSTS over (V, E).

Suppose that $(V, E, \tau_{u1}, \tau_{u2})$ is a pairwise NS T_3-space and $u_{(e)}^{(\alpha,\beta,\gamma)} \in K \in \tau_{u1}$.

Since $(V, E, \tau_{u1}, \tau_{u2})$ is a pairwise NS T_3-space for the NSP $u_{(e)}^{(\alpha,\beta,\gamma)}$ and τ_{u1}-NSCS K^c, there exist τ_{u2}-NSOSs L_1 and L_2 such that $u_{(e)}^{(\alpha,\beta,\gamma)} \in L_1$, $K^c \subseteq L_2$ and $L_1 \cap L_2 = \phi_u$.

Thus $u_{(e)}^{(\alpha,\beta,\gamma)} \in L_1 \subseteq (L_2)^c \subseteq K$.

Since $(L_2)^c$ is a τ_{u2}-NSCS, $\overline{L_1} \subseteq (L_2)^c$.

Hence $u_{(e)}^{(\alpha,\beta,\gamma)} \in L_1 \subseteq \overline{L_1} \subseteq K$.

Conversely, let $u_{(e)}^{(\alpha,\beta,\gamma)} \cap K = \phi_u$ and K be a τ_{u1}-NSCS.

Thus $u_{(e)}^{(\alpha,\beta,\gamma)} \in K^c$.

From the condition of the theorem,

$$u_{(e)}^{(\alpha,\beta,\gamma)} \in L \subseteq \overline{L} \subseteq K^c.$$

Then $u_{(e)}^{(\alpha,\beta,\gamma)} \in L$, $K \subseteq \overline{L}^c$ and $L \cap \overline{L}^c = \phi_u$.

Hence $(V, E, \tau_{u1}, \tau_{u2})$ is a pairwise NS T_3-space.

Definition 4.29 A NBSTS $(V, E, \tau_{u1}, \tau_{u2})$ over (V, E) is called pairwise NS normal space, if for every pair of disjoint τ_{u1}-NSCSs K_1 and K_2, there exists disjoint τ_{u2}-NSOSs L_1 and L_2 such that $K_1 \subseteq L_1$ and $K_2 \subseteq L_2$.

A NBSTS $(V, E, \tau_{u1}, \tau_{u2})$ over (V, E) is called pairwise NS T_4-space, if it is both a pairwise NS normal space and a pairwise NS T_1-space.

Theorem 4.30 Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E). Then (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_4-space if and only if for each τ_{u1}-NSCS K and τ_{u1}-NSOS L with $K \subseteq L$, there exists a τ_{u2}-NSOS P such that $K \subseteq P \subseteq \overline{P} \subseteq L$.

Proof. Let (V, E, τ_{u1}, τ_{u2}) be a NBSTS over (V, E).

Suppose that (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_4-space and K be τ_{u1}-NSCS and $K \subseteq L \in \tau_{u1}$.

Then L^c is a τ_{u1}-NSCS and $K \cap L^c = \phi_u$.

Since (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_4-space, there exist τ_{u2}-NSOSs P_1 and P_2 such that $K \subseteq P_1$, $M^c \subseteq P_2$ and $P_1 \cap P_2 = \phi_u$.

Thus $K \subseteq P_1 \subseteq (P_2)^c \subseteq L$.

Since $(P_2)^c$ is a τ_{u2}-NSCS, $\overline{P_1} \subseteq (P_2)^c$.

Hence $K \subseteq P_1 \subseteq \overline{P_1} \subseteq L$.

Conversely, let K_1 and K_2 be any two disjoint τ_{u1}-NSCSs.

Then $K_1 \subseteq (K_2)^c$.

From the condition of the theorem, there exists a τ_{u2}-NSOS P such that $K_1 \subseteq P \subseteq \overline{P} \subseteq (K_2)^c$.

Thus P and $(\overline{P})^c$ are τ_{u2}-NSOSs.

Then $K_1 \subseteq P$, $K_2 \subseteq (\overline{P})^c$ and $P \cap (\overline{P})^c = \phi_u$.

Hence (V, E, τ_{u1}, τ_{u2}) is a pairwise NS T_4-space.

Example 4.31 Let $V = \{v_1, v_2\}$, $E = \{e_1, e_2, e_3\}$, and $v_{1(e_1)}^{(.2,.4,.3)}$, $v_{1(e_2)}^{(.2,.8,.2)}$, $v_{1(e_3)}^{(.2,.5,.7)}$, $v_{2(e_1)}^{(.1,.2,.5)}$, $v_{2(e_2)}^{(.2,.7,.5)}$ and $v_{2(e_3)}^{(.1,.1,.9)}$ be NSPs.

Then $\tau_{u1} = \{\phi_u, 1_u, K_1, K_2, K_3, K_4, K_5, K_6, K_7\}$ and $\tau_{u2} = \{\phi_u, 1_u, L_1, L_2, L_3, L_4, L_5, L_6, L_7\}$ where $K_1, K_2, K_3, K_4, K_5, K_6, K_7, L_1, L_2, L_3, L_4, L_5, L_6, L_7$ are NSSs over (V, E), defined as follows

$$K_1 = \begin{cases} f_{K_1}(e_1) = \{< v_1, (.2,.4,.3) >, < v_2, (1,1,0) >\} \\ f_{K_1}(e_2) = \{< v_1, (1,1,0) >, < v_2, (1,1,0) >\} \\ f_{K_1}(e_3) = \{< v_1, (1,1,0) >, < v_2, (1,1,0) >\} \end{cases};$$

$$K_2 = \begin{cases} f_{K_2}(e_1) = \{< v_1, (1,1,0) >, < v_2, (1,1,0) >\} \\ f_{K_2}(e_2) = \{< v_1, (1,1,0) >, < v_2, (.2,.7,.5) >\} \\ f_{K_2}(e_3) = \{< v_1, (1,1,0) >, < v_2, (1,1,0) >\} \end{cases};$$

$$K_3 = \begin{cases} f_{K_3}(e_1) = \{< v_1, (1,1,0) >, < v_2, (1,1,0) >\} \\ f_{K_3}(e_2) = \{< v_1, (1,1,0) >, < v_2, (1,1,0) >\} \\ f_{K_3}(e_3) = \{< v_1, (.2,.5,.7) >, < v_2, (1,1,0) >\} \end{cases};$$

$K_4 = K_1 \cap K_2$;
$K_5 = K_1 \cap K_3$;
$K_6 = K_2 \cap K_3$;
$K_7 = K_1 \cap K_2 \cap K_3$

and

$$L_1 = \begin{cases} f_{L_1}(e_1) = \{< v_1, (.5, .7, .1) >, < v_2, (0,0,1) >\} \\ f_{L_1}(e_2) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \\ f_{L_1}(e_3) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \end{cases} ;$$

$$L_2 = \begin{cases} f_{L_2}(e_1) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \\ f_{L_2}(e_2) = \{< v_1, (0.0,1) >, < v_2, (.8, .6, .1) >\} \\ f_{L_2}(e_3) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \end{cases} ;$$

$$L_3 = \begin{cases} f_{L_3}(e_1) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \\ f_{L_3}(e_2) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \\ f_{L_3}(e_3) = \{< v_1, (.7, .5, .2) >, < v_2, (0,0,1) >\} \end{cases} ;$$

$L_4 = L_1 \cup L_2$;
$L_5 = L_1 \cup L_3$;
$L_6 = L_2 \cup L_3$;
$L_7 = L_1 \cup L_2 \cup L_3$;

Thus (*V, E*, τ_{u1}, τ_{u2}) is a NBSTS over (*V, E*).

Consider $(\tau_{u1})^c = \{\phi_u, 1_u, (K_1)^c, (K_2)^c, (K_3)^c, (K_4)^c, (K_5)^c, (K_6)^c, (K_7)^c\}$

where $(K_1)^c, (K_2)^c, (K_3)^c, (K_4)^c, (K_5)^c, (K_6)^c, (K_7)^c$ are τ_{u1}-NSCSs over (*V, E*), defined as follows

$$(K_1)^c = \begin{cases} f_{(K_1)^c}(e_1) = \{< v_1, (.3, .6, .2) >, < v_2, (0,0,1) >\} \\ f_{(K_1)^c}(e_2) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \\ f_{(K_1)^c}(e_3) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \end{cases} ;$$

$$(K_2)^c = \begin{cases} f_{(K_2)^c}(e_1) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \\ f_{(K_2)^c}(e_2) = \{< v_1, (0,0,1) >, < v_2, (.5, .3, .2) >\} \\ f_{(K_2)^c}(e_3) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \end{cases} ;$$

$$(K_3)^c = \begin{cases} f_{(K_3)^c}(e_1) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \\ f_{(K_3)^c}(e_2) = \{< v_1, (0,0,1) >, < v_2, (0,0,1) >\} \\ f_{(K_3)^c}(e_3) = \{< v_1, (.7, .5, .2) >, < v_2, (0,0,1) >\} \end{cases} ;$$

$(K_4)^c = (K_1)^c \cup (K_2)^c$;

$(K_5)^c = (K_1)^c \cup (K_3)^c$;

$(K_6)^c = (K_2)^c \cup (K_3)^c$;

$(K_7)^c = (K_1)^c \cup (K_2)^c \cup (K_3)^c$

Hence (*V, E,* τ_{u1} , τ_{u2}) is a pairwise NS T_4 -space, also a pairwise NS T_3 -space.

5. DM Problem in NBSTS

In this segment, measured the output of problem and evaluated the decision on NBSTS.

Definition 5.1 Let *V* be the set of universal set, *E* be its parameter and $\tau_{u1} = \{\phi_u, 1_u, P\}$ and $\tau_{u2} = \{\phi_u, 1_u, Q\}$ be two NSTs. Then NSSs *P* and *Q* in NBSTS (*V, E,* τ_{u1} , τ_{u2}) over (*V, E*) are defined by $k \times l$ matrix where every entries are marks of v_k based on each parameters e_l .

$$[P]_{k\times l} = \begin{bmatrix} <T_{f_P(e_1)}(v_1), F_{f_P(e_1)}(v_1), I_{f_P(e_1)}(v_1)> & <T_{f_P(e_2)}(v_1), F_{f_P(e_2)}(v_1), I_{f_P(e_2)}(v_1)> & \cdots & <T_{f_P(e_l)}(v_1), F_{f_P(e_l)}(v_1), I_{f_P(e_l)}(v_1)> \\ <T_{f_P(e_1)}(v_2), F_{f_P(e_1)}(v_2), I_{f_P(e_1)}(2)> & <T_{f_P(e_2)}(v_2), F_{f_P(e_2)}(v_2), I_{f_P(e_2)}(v_2)> & \cdots & <T_{f_P(e_l)}(v_2), F_{f_P(e_l)}(v_2), I_{f_P(e_l)}(v_2)> \\ \vdots & \vdots & & \vdots \\ <T_{f_P(e_1)}(v_k), F_{f_P(e_1)}(v_k), I_{f_P(e_1)}(v_k)> & <T_{f_P(e_2)}(v_k), F_{f_P(e_2)}(v_k), I_{f_P(e_2)}(v_k)> & \cdots & <T_{f_P(e_l)}(v_k), F_{f_P(e_l)}(v_k), I_{f_P(e_l)}(v_k)> \end{bmatrix}$$

and

$$[Q]_{k\times l} = \begin{bmatrix} <T_{f_Q(e_1)}(v_1), F_{f_Q(e_1)}(v_1), I_{f_Q(e_1)}(v_1)> & <T_{f_Q(e_2)}(v_1), F_{f_Q(e_2)}(v_1), I_{f_Q(e_2)}(v_1)> & \cdots & <T_{f_Q(e_l)}(v_1), F_{f_Q(e_l)}(v_1), I_{f_Q(e_l)}(v_1)> \\ <T_{f_Q(e_1)}(v_2), F_{f_Q(e_1)}(v_2), I_{f_Q(e_1)}(2)> & <T_{f_Q(e_2)}(v_2), F_{f_Q(e_2)}(v_2), I_{f_Q(e_2)}(v_2)> & \cdots & <T_{f_Q(e_l)}(v_2), F_{f_Q(e_l)}(v_2), I_{f_Q(e_l)}(v_2)> \\ \vdots & \vdots & & \vdots \\ <T_{f_Q(e_1)}(v_k), F_{f_Q(e_1)}(v_k), I_{f_Q(e_1)}(v_k)> & <T_{f_Q(e_2)}(v_k), F_{f_Q(e_2)}(v_k), I_{f_Q(e_2)}(v_k)> & \cdots & <T_{f_Q(e_l)}(v_k), F_{f_Q(e_l)}(v_k), I_{f_Q(e_l)}(v_k)> \end{bmatrix}$$

where $v_1, v_2, \ldots, v_k \in V$ and $e_1, e_2, \ldots, e_l \in E$.

Clearly $\tau_{u1} = \{\phi_u, 1_u, [P]_{k\times l}\}$ and $\tau_{u2} = \{\phi_u, 1_u, [Q]_{k\times l}\}$ are also NSTs in NBSTS (*V, E,* τ_{u1} , τ_{u2}) over (*V, E*).

Thus the outcome result (OR) of $v \in V$ is given by the formula

$$OR(v)^e = \left[\frac{\left(T_{f_P(e)}(v) - F_{f_Q(e)}(v)\right) + \left(T_{f_Q(e)}(v) - F_{f_P(e)}(v)\right)}{2}\right]\left[1 - \frac{\left(I_{f_P(e)}(v) + I_{f_Q(e)}(v)\right)}{2}\right] \quad (5.1.1)$$

where $e \in E$.

The Net Result (NR) of each $v_1, v_2, \ldots, v_k \in V$ is

$$NR(v_i)^{e_j} = \sum_{j=1}^{l}\left(R(v_i)^{e_j}\right) \quad (5.1.2)$$

for all *i* = 1 to *k*.

Example 5.2 Let $V = \{v_1, v_2\}$, $E = \{e_1, e_2\}$ and $\tau_{u1} = \{\phi_u, 1_u, [K_1]_{2\times 2}, [K_2]_{2\times 2}, [K_3]_{2\times 2}, [K_4]_{2\times 2}\}$ and

$\tau_{u2} = \{\phi_u, 1_u, [L_1]_{2\times 2}, [L_2]_{2\times 2}\}$ where $[K_1]_{2\times 2}, [K_2]_{2\times 2}, [K_3]_{2\times 2}, [K_4]_{2\times 2}, [L_1]_{2\times 2}, [L_2]_{2\times 2}$ are NSSs over (*V, E*),

defined as follows

$$[K_1]_{2\times 2} = \begin{bmatrix} <.1,.2,.3> & <.4,.5,.6> \\ <.9,.4,.7> & <.2,.6.3> \end{bmatrix},$$

$$[K_2]_{2\times 2} = \begin{bmatrix} <.2,.3,.1> & <.4,.7,.2> \\ <.5,.7,.6> & <.3,.5.2> \end{bmatrix},$$

$$[K_3]_{2\times 2} = \begin{bmatrix} <.2,.3,.1> & <.4,.7,.2> \\ <.9,.7,.6> & <.3,.6.2> \end{bmatrix},$$

$$[K_4]_{2\times 2} = \begin{bmatrix} <.1,.2,.3> & <.4,.5,.6> \\ <.5,.4,.7> & <.2,.5.3> \end{bmatrix}.$$

and

$$[L_1]_{2\times 2} = \begin{bmatrix} <.5,.8,.1> & <.3,.4,.2> \\ <.3,.1,.2> & <.6,.7.2> \end{bmatrix},$$

$$[L_2]_{2\times 2} = \begin{bmatrix} <.4,.7,.2> & <.2,.3,.5> \\ <.2,.1,.6> & <.3,.6.9> \end{bmatrix}.$$

Thus (V, E, τ_{u1}, τ_{u2}) is a NBSTS over (V, E).

Algorithm

Step 1: List the set of things or person $v \in V$ with their parameters $e \in E$.

Step 2: Go through the records of the particulars.

Step 3: Collect the data for each $v \in V$ according to all $e \in E$.

Step 4: Define NSSs.

Step 5: Define two different topologies τ_{u1} and τ_{u2} where each satisfies the condition of NST and so (V, E, τ_{u1}, τ_{u2}) is a NBSTS over (V, E).

Step 6: Form $NSSs \in \tau_{ui=u1,u2}$ matrix with collected data where v_k as rows and e_l as columns.

Step 7: Calculate the OR for all $v \in V$.

Step 8: Calculate the NR for all $v \in V$.

Step 9: Select a highest value among all the calculated NR.

Step 10: If two or more NR are identical, add one more parameter and repeat the process.

Step 11: End the process while we acquire the unique NR of v_k.

Problem 5.3 Let us suppose that there are two groups of women. First group consists of young age women (YAW, aging 20-25), say τ_{u1}, and second group consists of middle age women (MAW, aging 30-35), say τ_{u2}. Our aim is to insist both groups of women to select a saree together according to their desire and choice.

1. Let $V = \{sr_1, sr_2, sr_3, sr_4, sr_5\}$ be the set of sample sarees and selection done by the set of parameters let it be $E = \{c, q, d, p\}$ where is c = colour, q = quality, d = design and p = price.
2. Both groups are analyzing the sarees collections.
3. Data are collected for each sarees according to its paramaters given.
4. Convert these data as NSSs, say YAW and MAW.
5. Let $\tau_{u1} = \{\phi_u, 1_u, YAW\}$ and $\tau_{u2} = \{\phi_u, 1_u, MAW\}$ be two NSTs and so (V, E, τ_{u1}, τ_{u2}) is a NBSTS over (V, E).
6. The matrix form of NSSs YAW and MAW are as follows:

$$[YAW]_{5\times 4} = \begin{bmatrix} <.5,.7,.2> & <.7,.2,.2> & <.4,.2,.7> & <.8,.4,.5> \\ <.9,.3,.4> & <.3,.4,.2> & <.8,.7,.5> & <.9,.3,.6> \\ <.6,.3,.9> & <.7,.4,.8> & <.2,.7,.1> & <.3,.2,.5> \\ <.2,.4,.6> & <.4,.6,.8> & <.5,.3,.1> & <.7,.1,.3> \\ <.7,.5,.2> & <.1,.2,.3> & <.3,.6,.9> & <.1,.3,.4> \end{bmatrix}$$

and

$$[MAW]_{5\times 4} = \begin{bmatrix} <.6,.4,.3> & <.7,.3,.2> & <.3,.6,.2> & <.4,.7,.3> \\ <.8,.4,.2> & <.8,.4,.2> & <.5,.9,.4> & <.4,.4,.4> \\ <.1,.9,.2> & <.6,.7,.3> & <.8,.2,.1> & <.5,.1,.2> \\ <.2,.6,.7> & <.3,.4,.5> & <.7,.2,.1> & <.2,.9,.1> \\ <.3,.1,.2> & <.2,.1,.2> & <.3,.5,.2> & <.2,.4,.8> \end{bmatrix}$$

7. The **Table 5.3.1** is obtained by using the formula (5.1.1),

Table 5.3.1. OR table.

	sr_1	sr_2	sr_3	sr_4	sr_5
c	.105	.3575	−.08	−.225	.21
q	.375	.21	.045	.15	−.085
d	.06	.04	.22	.45	−.1125
p	.09	.0975	.0425	.125	−.2925

8. The **Table 5.3.2** is obtained by using the formula (5.1.2),

Table 5.3.2. NR table.

	sr_1	sr_2	sr_3	sr_4	sr_5
c	.105	.3575	−.08	−.225	.21
q	.375	.21	.045	.15	−.085
d	.06	.04	.22	.45	−.1125
p	.09	.0975	.0425	.125	−.2925
NR	.63	**.705**	.2275	.2	−.28

Thus the second saree has selected by both the categories of women.

Problem 5.4 Consider the situation of problem 5.3.

1. Let $V = \{sr_1, sr_2, sr_3, sr_4, sr_5\}$ be the set of sample sarees and selection done by the set of parameters let it be $E = \{c, q, d, p\}$ where is c = colour, q = quality, d = design and p = price.
2. Both groups are analyzing the sarees collections.
3. Data are collected for each sarees according to its paramaters given.
4. Convert these data as NSSs, say YAW and MAW.
5. Let $\tau_{u1} = \{\phi_u, 1_u, YAW\}$ and $\tau_{u2} = \{\phi_u, 1_u, MAW\}$ be two NSTs and so $(V, E, \tau_{u1}, \tau_{u2})$ is a NBSTS over (V, E).
6. The matrix form of NSSs *YAW* and *MAW* are as follows:

$$[YAW]_{5\times 4} = \begin{bmatrix} <.5,.7,.2> & <.5,.3,.1> & <.4,.2,.7> & <.8,.4,.5> \\ <.9,.3,.4> & <.3,.4,.2> & <.8,.7,.5> & <.9,.3,.6> \\ <.1,.3,.4> & <.7,.4,.8> & <.2,.7,.1> & <.3,.2,.5> \\ <.2,.4,.6> & <.4,.6,.8> & <.7,.2,.2> & <.7,.1,.3> \\ <.7,.5,.2> & <.1,.2,.3> & <.3,.6,.9> & <.6,.3,.9> \end{bmatrix}$$

and

$$[MAW]_{5\times 4} = \begin{bmatrix} <.6,.4,.3> & <.7,.2,.1> & <.3,.6,.2> & <.4,.7,.3> \\ <.8,.4,.2> & <.8,.4,.2> & <.5,.9,.4> & <.4,.4,.4> \\ <.2,.4,.8> & <.6,.7,.3> & <.8,.2,.1> & <.5,.1,.2> \\ <.2,.6,.7> & <.3,.4,.5> & <.7,.3,.2> & <.2,.9,.1> \\ <.3,.1,.2> & <.2,.1,.2> & <.3,.5,.2> & <.1,.9,.2> \end{bmatrix}$$

7. The **Table 5.4.1** is obtained by using the formula (5.1.1),

Table 5.4.1. OR table.

	sr1	sr2	sr3	sr4	sr5
c	.105	.3575	−.2925	−.225	.21
q	.45	.21	.045	.15	−.085
d	.06	.04	.22	.375	−.1125
p	.09	.0975	.0425	.125	−.08

8. The **Table 5.4.2** is obtained by using the formula (5.1.2),

Table 5.4.2. NR table.

	sr1	sr2	sr3	sr4	sr5
c	.105	.3575	−.2925	−.225	.21
q	.45	.21	.045	.15	−.085
d	.06	.04	.22	.375	−.1125
p	.09	.0975	.0425	.125	−.08
NR	**.705**	**.705**	.015	.125	−.0675

Thus first and second sarees have selected by both categories of women.

In this situation, we just add a parameter *f* = fabric in *E* and repeat the process.

4. After adding one more parameter, convert these data as NSSs, say YAW^* and MAW^*.

5. Let $\tau_{u1} = \{\phi_u, 1_u, YAW^*\}$ and $\tau_{u2} = \{\phi_u, 1_u, MAW^*\}$ be two NSTs and so $(V, E, \tau_{u1}, \tau_{u2})$ is a NBSTS over (*V*, *E*).

6. The matrix form of NSSs YAW^* and MAW^* are as follows:

$$[YAW^*]_{5\times5} = \begin{bmatrix} <.5,.7,.2> & <.5,.3,.1> & <.4,.2,.7> & <.8,.4,.5> & <.6,.7,.2> \\ <.9,.3,.4> & <.3,.4,.2> & <.8,.7,.5> & <.9,.3,.6> & <.5,.1,.3> \\ <.1,.3,.4> & <.7,.4,.8> & <.2,.7,.1> & <.3,.2,.5> & <.4,.5,.2> \\ <.2,.4,.6> & <.4,.6,.8> & <.7,.2,.2> & <.7,.1,.3> & <.7,.8,.4> \\ <.7,.5,.2> & <.1,.2,.3> & <.3,.6,.9> & <.6,.3,.9> & <.1,.3,.6> \end{bmatrix}$$

and

$$[MAW^*]_{5\times5} = \begin{bmatrix} <.6,.4,.3> & <.7,.2,.1> & <.3,.6,.2> & <.4,.7,.3> & <.9,.6,.3> \\ <.8,.4,.2> & <.8,.4,.2> & <.5,.9,.4> & <.4,.4,.4> & <.7,.8,.1> \\ <.2,.4,.8> & <.6,.7,.3> & <.8,.2,.1> & <.5,.1,.2> & <.6,.5,.4> \\ <.2,.6,.7> & <.3,.4,.5> & <.7,.3,.2> & <.2,.9,.1> & <.2,.3,.4> \\ <.3,.1,.2> & <.2,.1,.2> & <.3,.5,.2> & <.1,.9,.2> & <.6,.2,.7> \end{bmatrix}$$

7. The **Table 5.4.3** is obtained by using the formula (5.1.1),

Table 5.4.3. OR table after adding a parameter.

	sr1	sr2	sr3	sr4	sr5
c	.105	.3575	−.2925	−.225	.21
q	.45	.21	.045	.15	−.085
d	.06	.04	.22	.375	−.1125
p	.09	.0975	.0425	.125	−.08
f	.175	.22	.1	.0225	−.225

8. The **Table 5.4.4** is obtained by using the formula (5.1.2),

Table 5.4.4. NR table after adding a parameter.

	sr1	sr2	sr3	sr4	sr5
c	.105	.3575	−.2925	−.225	.21
q	.45	.21	.045	.15	−.085
d	.06	.04	.22	.375	−.1125
p	.09	.0975	.0425	.125	−.08
f	.175	.22	.1	.0225	−.225
NR	.88	**.925**	.115	.1475	−.2925

Thus the second saree has selected by both categories of women.

Problem 5.5 Consider the situation that there are six students on the main stage for Quiz Finale. There are two teams, each team consists of three students, one is Winner (*W*) and other is Runner (*R*). Let *FA*1 and *FA*2 be two final authorities to judge the event. Our problem is to find the best player in the winning team whose teammates are not mentioned here.

1. Let $V = \{st_1, st_2, st_3, st_4, st_5, st_6\}$ be the set of students and judgement is based on the set of parameters let it be *E* = {*ra*, *eff*, *ca*, *mr*, *gp*} where *ra* = right answers, *eff* = effectiveness, *ca* = complex analysis, *mr* = memory, *gp* = grasping power.

2. First of all these final authorities will go through the records of the students.
3. They will collect student's data according to their paramaters given.
4. These data are converted into two different NSSs, say *FA*1 and *FA*2.
5. Let $\tau_{u1} = \{\phi_u, 1_u, FA1\}$ and $\tau_{u2} = \{\phi_u, 1_u, FA2\}$ be two NSTs and so ($V, E, \tau_{u1}, \tau_{u2}$) is a NBSTS over ($V, E$).
6. The matrix form of NSSs *FA*1 and *FA*2 are as follows:

$$[FA1]_{6\times5} = \begin{bmatrix} <.4,.2,.7> & <.6,.3,.1> & <.2,.4,.8> & <.2,.9,.1> & <.6,.5,.3> \\ <.7,.3,.2> & <.8,.6,.1> & <.5,.4,.3> & <.9,.7,.2> & <.2,.7,.5> \\ <.3,.6,.6> & <.3,.5,.4> & <.6,.4,.2> & <.1,.2,.3> & <.5,.4,.6> \\ <.2,.6,.3> & <.7,.5,.4> & <.8,.6,.1> & <.4,.2,.7> & <.7,.3,.4> \\ <.6,.5,.4> & <.9,.2,.1> & <.7,.3,.4> & <.3,.5,.4> & <.4,.1,.4> \\ <.7,.3,.4> & <.6,.7,.2> & <.8,.9,.6> & <.3,.5,.4> & <.3,.5,.2> \end{bmatrix}$$

and

$$[FA2]_{6\times5} = \begin{bmatrix} <.4,.7,.3> & <.2,.3,.4> & <.5,.7,.2> & <.3,.4,.2> & <.2,.7,.1> \\ <.5,.1,.2> & <.6,.7,.3> & <.9,.3,.4> & <.7,.4,.8> & <.7,.2,.2> \\ <.7,.8,.1> & <.9,.3,.6> & <.1,.3,.4> & <.4,.6,.8> & <.3,.6,.9> \\ <.2,.6,.7> & <.7,.4,.8> & <.2,.4,.6> & <.1,.2,.3> & <.8,.4,.5> \\ <.5,.9,.4> & <.1,.2,.3> & <.7,.5,.2> & <.4,.2,.7> & <.3,.2,.5> \\ <.8,.4,.5> & <.9,.6,.3> & <.5,.3,.1> & <.1,.3,.4> & <.7,.3,.2> \end{bmatrix}$$

7. The **Table 5.5.1** is obtained by using the formula (5.1.1),

Table 5.5.1. OR table.

	st1	**st2**	**st3**	**st4**	**st5**	**st6**
ra	.11	.32	.045	−.12	.045	.195
eff	.105	.175	.24	.055	.24	.175
ca	.0675	.2275	.0325	.075	.24	.12
mr	.035	.135	−.018	−.02	−.13	.24
gp	.08	.055	−.175	.195	−.085	.18

8. The **Table 5.5.2** is obtained by using the formula (5.1.2),

Table 5.5.2. NR table.

	st1	**st2**	**st3**	**st4**	**st5**	**st6**
ra	.11	.32	.045	−.12	.045	.195
eff	.105	.175	.24	.055	.24	.175
ca	.0675	.2275	.0325	.075	.24	.12
mr	.035	.135	−.018	−.02	−.13	.24
gp	.08	.055	−.175	.195	−.085	.18
NR	.3975	**.9125**	−.0375	.005	.31	**.91**

Here both st_2 and st_6 got high score from judges, so they both does not belongs to *R*.

Case (i). If $st_2 \in W$ and $st_6 \in R$, then the best player award goes to st_2.

Case (ii). If $st_2 \in R$ and $st_6 \in W$, then the best player award goes to st_6.

Case (ii). If $st_2 \in W$ and $st_6 \in W$, then we just add a parameter ld = leadership.

4. After adding one more parameter, convert these data as NSSs, say $FA1^*$ and $FA2^*$.

5. Let $\tau_{u1} = \{\phi_u, 1_u, FA1^*\}$ and $\tau_{u2} = \{\phi_u, 1_u, FA2^*\}$ be two NSTs and so ($V, E, \tau_{u1}, \tau_{u2}$) is a NBSTS over ($V, E$).

6. The matrix form of NSSs $FA1^*$ and $FA2^*$ are as follows:

$$[FA1^*]_{6\times 6} = \begin{bmatrix} <.4,.2,.7> & <.6,.3,.1> & <.2,.4,.8> & <.2,.9,.1> & <.6,.5,.3> & <.3,.2,.4> \\ <.7,.3,.2> & <.8,.6,.1> & <.5,.4,.3> & <.9,.7,.2> & <.2,.7,.5> & <.9,.1,.1> \\ <.3,.6,.6> & <.3,.5,.4> & <.6,.4,.2> & <.1,.2,.3> & <.5,.4,.6> & <.7,.5,.3> \\ <.2,.6,.3> & <.7,.5,.4> & <.8,.6,.1> & <.4,.2,.7> & <.7,.3,.4> & <.8,.2,.1> \\ <.6,.5,.4> & <.9,.2,.1> & <.7,.3,.4> & <.3,.5,.4> & <.4,.1,.4> & <.3,.1,.5> \\ <.7,.3,.4> & <.6,.7,.2> & <.8,.9,.6> & <.3,.5,.4> & <.3,.5,.2> & <.6,.2,.6> \end{bmatrix}$$

and

$$[FA2^*]_{6\times 6} = \begin{bmatrix} <.4,.7,.3> & <.2,.3,.4> & <.5,.7,.2> & <.3,.4,.2> & <.2,.7,.1> & <.7,.5,.4> \\ <.5,.1,.2> & <.6,.7,.3> & <.9,.3,.4> & <.7,.4,.8> & <.7,.2,.2> & <.6,.9,.1> \\ <.7,.8,.1> & <.9,.3,.6> & <.1,.3,.4> & <.4,.6,.8> & <.3,.6,.9> & <.8,.4,.4> \\ <.2,.6,.7> & <.7,.4,.8> & <.2,.4,.6> & <.1,.2,.3> & <.8,.4,.5> & <.3,.4,.5> \\ <.5,.9,.4> & <.1,.2,.3> & <.7,.5,.2> & <.4,.2,.7> & <.3,.2,.5> & <.7,.3,.2> \\ <.8,.4,.5> & <.9,.6,.3> & <.5,.3,.1> & <.1,.3,.4> & <.7,.3,.2> & <.5,.9,.4> \end{bmatrix}$$

7. The **Table 5.5.3** is obtained by using the formula (5.1.1),

Table 5.5.3. OR table after adding a parameter.

	st1	**st2**	**st3**	**st4**	**st5**	**st6**
ra	.11	.32	.045	−.12	.045	.195
eff	.105	.175	.24	.055	.24	.175
ca	.0675	.2275	.0325	.075	.24	.12
mr	.035	.135	−.018	−.02	−.13	.24
gp	.08	.055	−.175	.195	−.085	.18
ld	.065	.352	−.055	.175	.12	.0675

8. The **Table 5.5.4** is obtained by using the formula (5.1.2),

Table 5.5.4. NR table after adding a parameter.

	st1	**st2**	**st3**	**st4**	**st5**	**st6**
ra	.11	.32	.045	−.12	.045	.195
eff	.105	.175	.24	.055	.24	.175
ca	.0675	.2275	.0325	.075	.24	.12
mr	.035	.135	−.018	−.02	−.13	.24
gp	.08	.055	−.175	.195	−.085	.18

Id	.065	.352	$-.055$	.175	.12	.0675
NR	.4625	**1.2375**	$-.0925$	.18	.43	.9775

Thus the best player award goes to st_2.

6. Conclusion

The main involvement of this paper is to preface the definition of NBSTSs and the study of some important properties of such spaces including separation axioms and the relationship between $T_{i=0,1,2,3,4}$-spaces. The key of this paper is to apply NBSTS in real life problems to take a decision, which might be positive or negative. In our problems two different types of NSTs are combined together to choose a unique decision according to the algorithm and calculation made by the formulae given here. Subsequently, NBSTS can be built up to pairwise NS separated sets, pairwise NS connected spaces, pairwise NS connected sets, pairwise NS disconnected spaces, pairwise NS disconnected sets and so on. We look forward to encourage this type of NBSTS will find a way to other types of topological structures. In future, some case studies which we mention in this paper need to develop on multicriteria DM also.

References

1. Abdel-Basset, M.; El-hoseny, M.; Gamal, A.; Smarandache, F. A novel model for evaluation Hospital medical care systems based on plithogenic sets. Artificial intelligence in medicine **2019**, 100, 101710.
2. Abdel-Basset, M.; Manogaran, G.; Gamal, A.; Chang, V. A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT. IEEE Internet of Things Journal **2019**.
3. Abdel-Basset, M.; Mohamed, R.; Zaied, A. E. N. H.; Smarandache, F. A hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics. Symmetry **2019**, 11(7), 903.
4. Abdel-Basset, M.; Nabeeh, N. A.; El-Ghareeb, H. A.; Aboelfetouh, A. Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. Enterprise Information Systems **2019**, 1-21.
5. Abdel-Baset, M.; Chang, V.; Gamal, A. Evaluation of the green supply chain management practices: A novel neutrosophic approach. Computers in Industry **2019**, 108, 210-220.
6. Abdel-Basset, M.; Mohamed, M.; Smarandache, F. An Extension of Neutrosophic AHP–SWOT Analysis for Strategic Planning and Decision Making. Symmetry **2018**, 10, 116.
7. Smarandache, F. Neutrosophy, Neutrosophic Probability, Set and Logic. American Research Press, Rehoboth, USA **1998,** 105.
8. Smarandache, F. Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. International Journal of Pure and Applied Mathematics **2005**, 24, 287-297.
9. Smarandache, F. Plithogeny, Plithogenic Set, Logic, Probability, and Statistics; Infinity Study **2017**, 141.
10. Bera, T.; Mahapatra, N.K. On Neutrosophic Soft Topological Space. Neutrosophic Sets and Systems **2018**, 19, 3 15.
11. Taha, Y. O.; Alkan, O. Neutrosophic Bitopological Spaces. Neutrosophic Sets and Systems **2019**, 30, 88-97.
12. Abishek, G.; Saurabh, S.; Rakesh, K. B. On Parametric Diverdence Measure of Neutrosophic Sets with its Application in Decision-Making Models. Neutrosophic Sets and Systems **2019**, 29, 101-120.
13. Chinnadurai, V.; Smarandache, F.; Bobin, A. Multi-Aspect Decision-Making Process in Equity Investment Using Neutrosophic Soft Matrices. Neutrosophic Sets and Systems **2020**, 31, 224-241.
14. Chinnadurai, V.; Swaminathan, A.; Bobin, A.; Kathar Kani, B. Generalised Interval Valued Intuitionistic Fuzzy Soft Matrices and Their Application to Multicriteria Decision Making. Recent Trends in Pure and Applied Mathematics **2019**, AIP Conf. Proc. 2177, 020019-1 – 020019-11.
15. Kalaiselvi, S.; Sindhu, M. P. $\tau_1\tau_2$-fb-open Sets in Fine-Bitopological Spaces. International Journal of Multidisciplinary Research and Modern Education **2016,** 2(2), 435-441.

16. Muhammad, Riaz.; Khalid, Nazeem.; Iqra, Zareef.; Deeba, Afzal. Neutrosophic N-soft sets with TOPSIS method for Mutiple Attribute Decision Making, Neutrosophic Sets and Systems **2020**, 32, 146-170.
17. Mehmood, A.; Nadeem, F.; Nordo, G.; Zamir, M.; Park, C.; Kalsoom, H.; Jabeen, S.; Khan, M.I.; Generalized Neutrosophic Separation Axioms in Neutrosophic Soft Topological Spaces, Neutrosophic Sets and Systems **2020**, 32, 38-51.
18. Evanzalin Ebenanjar, P.; Jude Immaculate, H.; Sivaranjani, K. Introduction to neutrosophic soft topological spatial region, Neutrosophic Sets and Systems **2020**, 31, 297-304.
19. Chinnadurai, V.; Swaminathan, A.; Bobin, A.; Kathar Kani, B. Generalised Interval Valued Intuitionistic Fuzzy Soft Matrices and Their Application to Multicriteria Decision Making. Recent Trends in Pure and Applied Mathematics **2019**, AIP Conf. Proc. 2177, 020019-1 − 020019-11.
20. Taha, Y.O.; Cigdem, G.A.; Sadi, B. A new approach to operations on neutrosophic soft sets and to neutrosophic soft topological spaces. Communications in Mathematics and Applications **2019**, 10(3), 481-493.
21. Taha, Y.O.; Cigdem, G.A.; Sadi. B. Separation axioms on neutrosophic soft topological spaces. Turkish Journal of Mathematics **2019**, 43, 498-510.
22. Bayramov, S.; Gunduz, C. A new approach to separability and compactness in soft topological spaces. TWMS Journal of Pure and Applied Mathematics **2018**, 9, 82-93.
23. Bera, T.; Mahapatra, N.K. Introduction to neutrosophic soft topological space. OPSEARCH **2017**, 54, 841-867, DOI 10.1007/s12597-017-0308-7.
24. Bera. T.; Mahapatra, N.K. On neutrosophic soft function. Annals of Fuzzy Mathematics and Informatics **2016**, 12, 101-119.
25. Faruk, K. Neutrosophic Soft Sets with Applications in Decision Making. International Journal of Information Science and Intelligent Systems **2015**, 4(2), 1-20.
26. Tantawy, O.; EL-Sheikh, S. A.; Hamde. S. Separation axioms on soft topological spaces. Annals of Fuzzy Mathematics and Informatics **2015**, 1-15.
27. Deli, I.; Broumi, S. Neutrosophic soft relations and some properties. Annals of Fuzzy Mathematics and Informatics **2015**, 9, 169-182.
28. Naz, M.; Ali, M.I.; Shabir, M. Separation Axioms in Bi-Soft Topological Spaces. arXiv:1509.00866v1[math.GN] **2015**, 1-22.
29. Bayramov, S.; Gunduz, C. On intuitionistic fuzzy soft topological spaces. TWMS Journal of Pure and Applied Mathematics **2014**, 5, 66-79.
30. Maji, P.K. Neutrosophic soft set. Annals of Fuzzy Mathematics and Informatics **2013**, 5, 157-168.
31. Varol, B. P.; Aygun, H. On Soft Hausdorff Spaces, Annals of Fuzzy Mathematics and Informatics **2013**, 5(1), 15-24.
32. Nazmul, S.K.; Samanta, S. K. Neighborhood Properties of Soft Topological spaces. Annals of Fuzzy Mathematics and Informatics **2012**, 1-16.
33. Salma, A.A.; Alblowi, S.A. Neutrosophic set and neutrosophic topological spaces. IOSR Journal of Applied Mathematics **2012**; 3, 31-35.
34. Zorlutuna, I.; Akdag, M.; Min, W.K.; Atmaca, S. Remarks on soft topological spaces. Annals of Fuzzy Mathematics and Informatics **2012**, 3(2), 171-185.
35. Ali, M.I.; Shabir, M.; Naz, M. Algebraic structures of soft sets associated with new operations. Computers and Mathematics with Applications **2011**, 61, 2647–2654.
36. Hussain, S.; Ahmad, B. Some properties of soft topological spaces. Computers and Mathematics with Applications **2011**, 62, 4058-4067.
37. Cagman, N.; Karatas, S.; Enginoglu, S. Soft topology. Computers and Mathematics with Applications **2011**, 62, 351-358.
38. Shabir, M.; Naz, M. On soft topological spaces, Computers and Mathematics with Applications **2011**, 61, 1786-1799.
39. Min, W. K. A note on soft topological spaces. Computers and Mathematics with Applications **2011**, 62, 3524-3528.
40. Ali, M.I.; Feng, F.; Liu, X.Y.; Min, W.K.; Shabir, M. On some new operations in soft set theory, Computers and Mathematics with Applications **2009**, 57, 1547–1553.

41. Feng, F.; Jun, Y.B.; Zhao, X.Z. Soft semirings. Computers and Mathematics with Applications **2009**, 57, 1547-1553.
42. Pie, D.; Miao, D. From soft sets to information systems. IEEE International Conference on Granular Computing **2005**, 2, 617–621.
43. Chen, D. The parametrization reduction of soft sets and its applications. Computers and Mathematics with Applications **2005**, 49, 757-763.
44. Maji, P.K.; Biswas, R.; Roy, A.R. Soft set theory. Computers and Mathematics with Applications **2003**, 45, 555-562.
45. Maji, P.K.; Roy, A. R.; Biswas, R. An application of soft sets in a decision making problem. Computers and Mathematics with Applications **2002**, 44, 1077-1083.
46. Molodtsov, D. Soft set theory-first results. Computers and Mathematics with Applications **1999**, 37, 19-31.
47. Atanassov, K. Intuitionistic fuzzy sets. Fuzzy Sets and Systems **1986**, 20, 87-96.
48. Lal, S. Pairwise concepts in bitopological spaces. Journal of the Australian Mathematical Society (Series A) **1978**, 26, 241-250.
49. Reilly, I. L. On bitopological separation properties. Nanta Mathematica **1972**, 5, 14-25.
50. Singal, M.K.; Asha, R.S. Some more separation axioms in bitopological spaces. Annales de la Société scientifique de Bruxelles **1970**, 84, 207-230.
51. Kim, Y. W. Pairwise compactness. Publications Mathématiques de l'IHÉS **1968,** 15, 87-90.
52. Patty, C. W. Bitopological spaces. Duke Mathematical Journal **1967**, 34, 387-392.
53. Lane, E. P. Bitopological spaces and quasi-uniform spaces. Proceedings of the London Mathematical Society **1967**, 17, 241-256.
54. Zadeh, L.A. Fuzzy sets. Information and Control **1965**, 8, 338-353.
55. Kelly, J. C. Bitopological spaces. Proceedings of the London Mathematical Society **1963**, 13, 71-89.

Received: Apr 22, 2020. Accepted: July 12 2020