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Introduction to Combined Plithogenic Hypersoft Sets

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Abstract: Plithogenic Hypersoft sets was introduced by Florentin Smarandache, who has extended crisp sets, fuzzy sets, intuitionistic sets, neutrosophic sets to plithogenic sets. The plithogenic sets considers the degree of appurtenance of the elements with respect to the attribute system. Smarandache has presented the classification of the plithogenic hypersoft sets and the applications of plithogenic fuzzy whole hypersoft sets in multi attribute decision making. Inspired by these research works, the concept of combined plithogenic hypersoft sets is introduced in this article. The representations of the degree of appurtenance of the elements determines the type of plithogenic hypersoft set, if it takes a combination of values then the new archetype of plithogenic hypersoft sets gets emerged into decision making scenario. This research work is put forth to project the realistic perception of the experts in the construction process of optimal conclusions.

Keywords: Plithogenic hypersoft set, combined plithogenic hypersoft set, decision making, multi attribute system.

1. Introduction

Classical set theory deals with the sets consisting of elements with membership values 0 or 1. The degree of belongingness of an element in a set has been extended to [0,1] by Zadeh [1] in the name of fuzzy sets, which is gaining momentum since its introduction. Sets comprising of quantitative elements can be defined by conventional concepts of sets, but the elements of qualitative nature can be treated only by fuzzy concepts and its membership value states the degree of confidence of its presence in the set. Atanassov [2] investigated on the degree of its absence in the set, by defining non-membership values. This paved way for the intuitionistic fuzzy sets, which consists of degree of membership, non-membership and hesitation. Fuzzy sets and intuitionistic fuzzy sets are extensively applied in decision making process. But still the human perception was not completely reflected in these two kinds of sets. This gap was filled by Florentine Smarandache [3-5] who introduced neutrosophic fuzzy sets, comprising of degree of truth membership, indeterminacy and degree of false membership. Smarandache has represented each of the three function in a more generalized and independent manner. Neutrosophic sets have extensive application in decision making at recent times. Abdel- Basset et al [6-7] has developed neutrosophic decision making models to solve transition difficulties of IoT-based enterprises and to evaluate green supply chain management practices.

Smarandache also extended the classical sets, fuzzy sets, intuitionistic fuzzy sets and neutrosophic fuzzy sets to plithogenic sets which is a quintuple (P, a, V, d, c) consisting of a set P, the attribute a, the range of attribute values V, degree of appurtenance d, and the degree of contradiction c. The nature of d determines the type of plithogenic sets. Smarandache presented an elaborate discussion on the genesis of plithogenic sets in his research work [8]. Abdel-Basset et al [9-11] has developed decision making models with incorporation of plithogenic sets to evaluate green supply chain management practices and intelligent Medical Decision Support Model Based on Soft Computing and IoT was also built; a hybrid plithogenic decision-making approach with quality function deployment for selecting supply chain sustainability metrics was also framed. These plithogenic decision making models are highly robust and feasible.

Molodtsov [12] introduced and applied soft sets in decision making which was extended to fuzzy soft sets predominantly by Maji [13]. The comprehensive outlook of hypersoft sets was made by Smarandache [14] which took the different forms of fuzzy sets in the course of time. Shazia Rana et al [15] in their recent work on application of plithogenic fuzzy whole hypersoft set in multi attribute decision making introduced the matrix representation of plithogenic hypersoft set and plithogenic fuzzy whole hypersoft set which adds to the compatibility of this decision making technique. The validation of the proposed decision making model with a numerical example in this work has inspired to introduce combined plithogenic hypersoft set.

The paper is organized as follows; section 2 presents a brief description of combined plithogenic hypersoft sets; section 3 comprises the application of combined plithogenic hypersoft sets in decision making based on the technique of Shazia Rana et al [15]; section 4 discusses the results and the last section concludes with the future extension of the proposed concept.

2. Combined plithogenic hypersoft sets

This section comprises of the direct narration and representation of the combined plithogenic hypersoft sets based on the preliminaries discussed by Smarandache [14] and Shazia Rana et al [15] to avoid the repetition of the elementary definitions. Smarandache presented the classification of plithogenic hypersoft sets and the categorization was purely based on the nature of degree of appurtenance. Based on his discussion, let us consider the following example to explain the need of combined plithogenic hypersoft sets

Let U be the universe of discourse that consists of pollution mitigation methods say

$U = \{M_1, M_2, M_3, M_4, M_5\}$ and the set $\mathscr{M} = \{M1, M4\} \subset U$.

The attributes are a_1 = Cost efficiency, a_2 = Eco-compatibility, a_3 = Time efficacy, a_4 = Profit yield. If the pollution abatement methods are supposed to fulfill these attributes, then in realistic perspective the possible attribute values are taken as follows,

Cost efficiency = A_1 = {low, medium, high}, Eco-compatibility = A_2 = {very high, high}, Time efficacy = A_3 = {less, more}, Profit yield = A_4 ={maximum, minimum}.

Suppose a manufacturing firm has decided to implement a pollution control method, then the above attributes and its values are considered for making optimal decision with the possible range of values of attributes. By usual consideration,

Let the function be: $G: A_1 \times A_2 \times A_3 \times A_4 \longrightarrow P(U)$

Let's assume: G ({low, high, more, maximum}) = $\{M_1, M_4\}$.

The degree of appurtenance of an element x to the set $\mathcal{M}$, with respect to each attribute value a is d_x^0(a) that is the deciding factor of the nature of plithogenic hypersoft set.

In the context of decision making with the expert's opinion, then d_x^0(a) is the resultant of the expert's perception. Sometimes the expert's outlook may be a combination of certain, fuzzy, intuitionistic and neutrosophic, which is expressed as follows

G({low, high, more, maximum}) = { M_1 (1,0.8,0.7,(0.4,0.5)),

M_4 (1,0.9,(0.8,0.1,0.1),(0.5,0.6)) }.

This is the pragmatic reflection of the person's perception in decision making process and this is the point of origin of combined plithogenic hypersoft sets. Thus a combined plithogenic hypersoft sets is a plithogenic hypersoft set in which the degree of appurtenance of an element x to the set $\mathcal{M}$, with respect to each attribute value is a combination of either crisp, fuzzy, intuitionistic or neutrosophic.

Combined plithogenic hypersoft sets can be classified into completely combined plithogenic hypersoft sets and partially combined plithogenic hypersoft sets based on the nature and combination of values taken by d_x^0(a). In the above stated example G({low, high, more, maximum}) = { M_1 (1,0.8,0.7,(0.4,0.5)), M_4 (1,0.9,(0.8,0.1,0.1),(0.5,0.6))} is a completely combined plithogenic hypersoft sets as $dx0$(a) takes all possible types of values. Suppose G({low, high, more, maximum}) = { M_1 (0.9,0.8,0.7,(0.4,0.5)), M_4 (0.8,0.9,0.6,(0.5,0.6))} then this combined plithogenic hypersoft set is partial in nature as d_x^0(a) takes only a combination of two types of values. Thus a combined plithogenic hypersoft set which is not complete is partial in its nature.

It is very evident that combined plithogenic hypersoft sets are highly rational in nature and it will certainly play a vital role in receiving the expert's opinion, which is very significant in any multi attribute decision making process. Also the need of such new types of plithogenic hypersoft sets are very essential, because in the manufacturing firms and in business sectors the implementation of certain methods and installation of certain mechanisms and machinery may not be characterized by only crisp or fuzzy values with regard to the degree of appurtenance as the possibility aspect has some extent of participation in it. To handle such situations the combined plithogenic hypersoft sets may lend a helping hand to the decision makers.

3. Application of Combined Plithogenic Hypersoft set in Multi Attribute Decision Making

The previous section presented an elaborate portrayal of combined plithogenic hypersoft set, the significant feature is the realistic representation, but it has certain difficulties in computations as the degree of appurtenance varies for each attribute. To handle such crisis, all the values of d_x^0(a) must be similar in nature, i.e. either all the values must be fuzzy values which is the lower level of fuzzy representation or it must be neutrosophic values, the higher level of fuzzy representation.

A manufacturing sector has decided to enhance its production rate by installing new kinds of machinery. The ultimate aim of incorporating such a change in the production mechanism is quality production and customer satisfaction. The market is flooded with several varieties of well equipped, modern machines and since the manufacturing sector makes huge investment, the decision making process takes place in two phases based on the expert's opinion and advice. In the first phase, ten machines were selected by the manufacturing sector and in the next phase five were selected based on the feedback of the users. The decision making problem begins here, as the company has to purchase only three out of five based on the extent of satisfaction of the attributes by these machines.

Let U = { M_1, M_2, M_3, M_4, M_5, M_6, M_7,M_8, M_9, M_{10}} be the university of discourse and set

T = {M_1, M_3, M_6, M_7, M_9} $\subset$ U.

The attribute system is represented as follows A = { (A_1)Maintenance Cost {Maximum in the initial years of utility(A_1^1), Maximum in the latter years of utility(A_1^2)}, (A_2)Reliability {High with additional expenditure(A_2^1), Moderate with no extra expense(A_2^2)}, (A_3)Flexibility {Single task oriented(A_3^1), Multi task oriented(A_3^2)}, (A_4)Durability {Very high in the beginning years of service(A_4^1), High in the latter years of service(A_4^2), }, (A_5)Profitability {Moderate in the initial years(A_5^1), Maximum in the latter years(A_5^2)}}.

The attributes are quite common, but the attribute values are more realistic as it mirror the actual aspects involved in making decision.

Let the function be: G: $A_1^1 \times A_2^2 \times A_3^2 \times A_4^1 \times A_5^2 \longrightarrow P(U)$. Based on the Expert's opinion, the degree of appurtenance of the elements with respect to the attribute values is represented as follows

G(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2) =

{M1(0.9,(0.7,0.1),0.8,(0.6,0.2),0.5),M3((0.6,0.3),0.5,(0.4,0.1,0.3),0.8,0.7), M6(0.8,0.7,0.6,(0.5,0.2),(0.6,0.1,0.1)),M7((0.7,0.2,0.1),(0.7,0.1),0.9,(0.7,0.2),0.8),M9(1,0.9,0.5,0.8,(0.6,0.1,0.2))}.

The modified lower and higher fuzzy values of the degree of appurtenance of the elements with respect to the attribute values are denoted as G_L(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2) and G_H(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2)

G_L(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2) = {M1(0.9,0.875,0.8,0.75,0.5),M3(0.67,0.5,0.4,0.8,0.7),M6(0.8,0.7,0.6,0.7,0.5), M7(0.67,0.875,0.9,0.78,0.8), M9(1,0.9,0.5,0.8,0.47)}

G_H(A_1^1, A_2^2 , A_3^2 , A_4^1, A_5^2) =

{M1(0.9,0.1,0.1),(0.7,0.2,0.1),(0.8,0.1,0.1),(0.6,0.3,0.2),(0.5,0.2,0.7)),M3((0.6,0.3,0.3), (0.5,0.2,0.7),(0.4,0.1,0.3),(0.8,0.1,0.1),(0.7,0.2,0.1)),M6((0.8,0.1,0.1),(0.7,0.2,0.1),(0.6,0.2,0.3),(0.5,0.3,0.2),(0.6,0.1,0.1)),M7((0.7,0.2,0.1),(0.7,0.1,0.1),(0.9,0.1,0.1),(0.7,0.1,0.2),(0.8,0.1,0.1)),M9((1,0,0),(0.9,0.1,0.1),(0.5,0.2,0.7),(0.8,0.1,0.1),(0.6,0.1,0.2))}

The lower and higher fuzzy values of the degree of appurtenance correspond to single fuzzy value and neutrosophic values. The matrix representation C of the degree of appurtenance of the elements with respect to the attribute values in combined plithogenic hypersoft sets is

	A_1^1	A_2^2	A_3^2	A_4^1	A_5^2
M_1	0.9	(0.7,0.1)	0.8	(0.6,0.2)	0.5
M_3	(0.6,0.3)	0.5	(0.4,0.1,0.3)	0.8	0.7
M_6	0.8	0.7	0.6	(0.5,0.2)	(0.6,0.1,0.1)),
M_7	(0.7,0.2,0.1)	(0.7,0.1)	0.9	(0.7,0.2)	0.8
M_9	1	0.9	0.5	0.8	(0.6,0.1,0.2)

The intuitionistic and neutrosophic values are transformed to the above fuzzy values by the methods of imprecision and Defuzzification [16]

Method I (Imprecision membership): Any neutrosophic fuzzy set $N_A = (T_A, I_A, F_A)$ including neutrosophic fuzzy values are transformed into intuitionistic fuzzy values or vague values as η(A) = (T_A, f_A) where f_A is estimated the formula stated below which is called as Impression membership method.

$$f_A = \begin{cases} F_A + \frac{[1-F_A-I_A][1-F_A]}{[F_A+I_A]} & if\ F_A = 0 \\ F_A + \frac{[1-F_A-I_A][F_A]}{[F_A+I_A]} & if\ 0 < F_A \leq 0.5 \\ F_A + [1-F_A-I_A]\left[0.5 + \frac{F_A-0.5}{F_A+I_A}\right] & if\ 0.5 < F_A \leq 1 \end{cases}$$

Method II (Defuzzification): After Method I (Median membership), intuitionistic (vague),fuzzy values of the form η(A)= (T_A, f_A) are transformed into fuzzy set including fuzzy values as $<\Delta(A)> = <\frac{T_A}{[T_A+f_A]}>$.

The matrix representation C_L of the lower fuzzy values of the degree of appurtenance of the elements with respect to the attribute values in combined plithogenic hypersoft sets is

	A_1^1	A_2^2	A_3^2	A_4^1	A_5^2
M_1	0.9	0.875	0.8	0.75	0.5
M_3	0.67	0.5	0.4	0.8	0.7
M_6	0.8	0.7	0.6	0.7	0.5
M_7	0.67	0.875	0.9	0.78	0.8
M_9	1	0.9	0.5	0.8	0.47

By using the procedure of ranking as discussed by Shazia Rana et. al [15] the machines are ranked by considering the values of C_L.

The frequency matrix F_L representing the ranking of the machines is

	R_1	R_2	R_3	R_4	R_5
M_1	1	2	0	0	0
M_3	0	0	0	1	2
M_6	0	1	0	2	0
M_7	2	0	1	0	0
M_9	1	1	1	0	0

The percentage measure of authenticity of ranking is presented below in Table 3.1

Table 3.1

R_1	M_7	50%
R_2	M_1	50%
R_3	M_9	50%
R_4	M_6	67%
R_5	M_3	100%

The matrix representation C_H of higher fuzzy values (neutrosophic representations) of the degree of appurtenance of the elements with respect to the attribute values in combined plithogenic hypersoft sets is

	A_1^1	A_2^2	A_3^2	A_4^1	A_5^2
M_1	(0.9,0.1,0.1)	(0.7,0.2,0.1)	(0.8,0.1,0.1)	(0.6,0.3,0.2)	(0.5,0.2,0.7)
M_3	(0.6,0.3,0.3)	(0.5,0.2,0.7)	(0.4,0.1,0.3)	(0.8,0.1,0.1)	(0.7,0.2,0.1)
M_6	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.6,0.2,0.3)	(0.5,0.3,0.2)	(0.6,0.1,0.1)
M_7	(0.7,0.2,0.1)	(0.7,0.1,0.1)	(0.9,0.1,0.1)	(0.7,0.1,0.2)	(0.8,0.1,0.1)
M_9	(1,0,0)	(0.9,0.1,0.1)	(0.5,0.2,0.7)	(0.8,0.1,0.1)	(0.6,0.1,0.2)

To make the ranking of the machines based on the higher values in C_H the score values K(A) of the single valued neutrosophic representations [say A = (a,b,c)] are determined by $K(A) = \frac{1+a-2b-c}{2}$ [17]

	A_1^1	A_2^2	A_3^2	A_4^1	A_5^2
M_1	0.8	0.6	0.75	0.4	0.2
M_3	0.35	0.2	0.45	0.75	0.6
M_6	0.75	0.6	0.45	0.35	0.65
M_7	0.6	0.7	0.8	0.65	0.75
M_9	1	0.8	0.2	0.75	0.6

The frequency matrix F_H representing the ranking of machines is

	R_1	R_2	R_3	R_4	R_5
M_1	1	0	1	1	0
M_3	0	0	1	1	1
M_6	0	1	1	1	0
M_7	3	0	0	0	0
M_9	1	1	1	0	0

The percentage measure of authenticity of ranking is presented below in Table 3.2

Table 3.2

R_1	M_7	60%
R_2	M_9	50%
R_3	M_6	25%
R_4	M_1	33%
R_5	M_3	100%

4. Discussion

The combined plithogenic hypersoft set representations are so deliberate in nature. The resultant of computations in making decisions in two ways is represented in Table 3.1 and 3.2. The machines M_7 and M_3 occupy first and fifth rank respectively in both kinds of representation of degree of appurtenance. Also by making inferences from the table values M_1, M_3 and M_6 can be ranked in second ,third and fourth positions respectively. It is very evident that the transformation of combined attribute values to lower order fuzzy values yields best results in ranking the machines, but still the higher order values will also yield optimum results based on the selection of the score functions. The methods of converting combined attribute value to the values of similar fashion have to be constituted in the upcoming research works to attain feasible solutions to the decision making problems.

5. Conclusions

This research work encompasses the discussion of the new concept of combined plithogenic hypersoft set and its application in multi attribute decision making. Besides these types of appurtenance degrees, others can be used under the plithogenic umbrella, such as: Pythagorean, picture fuzzy, spherical fuzzy, spherical neutrosophic, etc. and even the most general one, refined neutrosophic type of appurtenance degree. The combined plithogenic hypersoft set can be extended to interval-valued combined plithogenic hypersoft sets and it can be converted to simple fuzzy values using score functions. The matrix representations of degree of appurtenance in combined plithogenic hypersoft set has induced the author to extend the proposed theoretical conceptualization to plithogenic concentric hypergraphs, most probably the scope and future research work.

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