



On Neutrosophic Generalized Alpha Generalized Continuity

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Abstract: This article demonstrates a further class of neutrosophic closed sets named neutrosophic generalized α g-closed sets and discuss their essential characteristics in neutrosophic topological spaces. Moreover, we submit neutrosophic generalized α g-continuous functions with their elegant features.

Keywords: neutrosophic generalized α g-closed sets, neutrosophic generalized α g-continuous functions, and neutrosophic generalized α g-irresolute functions.

1. Introduction

Smarandache [1,2] originally handed the theory of "neutrosophic set". Recently, Abdel-Basset et al. discussed a novel neutrosophic approach [3-8] in several fields, for a few names, information and communication technology. Salama et al. [9] gave the clue of neutrosophic topological space (or simply *NTS*). Arokiarani et al. [10] added the view of neutrosophic α -open subsets of neutrosophic topological spaces. Imran et al. [11] presented the idea of neutrosophic semi- α -open sets in neutrosophic topological spaces. Dhavaseelan et al. [12] presented the idea of neutrosophic α^m -continuity. Our aim is to introduce a new idea of neutrosophic generalized α g-closed sets and examine their vital merits in neutrosophic topological spaces. Additionally, we propose neutrosophic generalized α g-closed sets and emphasizing some of their primary characteristics.

2. Preliminaries

Everywhere of these following sections, we assume that *NTSs* $(\mathcal{U}, \xi), (\mathcal{V}, \varrho)$ and (\mathcal{W}, μ) are briefly denoted as \mathcal{U}, \mathcal{V} , and \mathcal{W} , respectively. Let \mathcal{C} be a neutrosophic set in \mathcal{U} , and we are easily symbolized it by *NS*, then the complement of \mathcal{C} is basically given by $\overline{\mathcal{C}}$. If \mathcal{C} is a neutrosophic open set in \mathcal{U} and shortly indicated by Ne-OS. Then, $\overline{\mathcal{C}}$ is termed a neutrosophic closed set in \mathcal{U} and simply referred by Ne-CS. The neutrosophic closure and the neutrosophic interior of \mathcal{C} are merely signified by Ne-*cl*(\mathcal{C}) and Ne-*int*(\mathcal{C}), correspondingly.

Definition 2.1 [10]: A *NS* C in a *NTS* U is named a neutrosophic α -open set and simply written as Ne- α OS if $C \subseteq$ Ne-*int*(Ne-*cl*(Ne-*int*(C))). Besides, if Ne-*cl*(Ne-*int*(Ne-*cl*(C))) $\subseteq C$, then C is called a neutrosophic α -closed set, and we are shortly given it as Ne- α CS. The collection of all such these

Ne- α OSs (correspondently, Ne- α CSs) in \mathcal{U} is referred to Ne- α O(\mathcal{U}) (correspondently, Ne- α C(\mathcal{U})). The intersection of all Ne- α CSs that contain \mathcal{C} is called the neutrosophic α -closure of \mathcal{C} in \mathcal{U} and represented by Ne- α *cl*(\mathcal{C}).

Definition 2.2 [13]: A *NS* C in *NTS* U is so-called a neutrosophic generalized closed set and denoted by Ne-gCS if for any Ne-OS \mathcal{M} in U such that $C \subseteq \mathcal{M}$, then Ne-*cl*(C) $\subseteq \mathcal{M}$. Moreover, its complement is named a neutrosophic generalized open set and referred to Ne-gOS.

Definition 2.3 [14]: A *NS* C in *NTS* U is so-called a neutrosophic α g-closed set and indicated by Ne- α gCS if for any Ne-OS \mathcal{M} in \mathcal{U} such that $C \subseteq \mathcal{M}$, then Ne- $\alpha cl(C) \subseteq \mathcal{M}$. Furthermore, its complement is named a neutrosophic α g-open set and symbolized by Ne- α gOS.

Definition 2.4 [15]: A *NS* C in *NTS* U is so-called a neutrosophic g α -closed set and signified by Ne-g α CS if far any Ne- α OS \mathcal{M} in U such that $C \subseteq \mathcal{M}$, then Ne- $\alpha cl(C) \subseteq \mathcal{M}$. Besides, its complement is named a neutrosophic g α -open set and briefly written as Ne-g α OS.

Theorem 2.5 [10,13-15]: For any *NTS U*, the next declarations valid and but not vice versa:
(i) for all Ne-OSs (correspondingly, Ne-CSs) are Ne-αOSs (correspondingly, Ne-αCSs).
(ii) for all Ne-OSs (correspondingly, Ne-CSs) are Ne-gOSs (correspondingly, Ne-gCSs).
(iii) for all Ne-gOSs (correspondingly, Ne-gCSs) are Ne-αgOSs (correspondingly, Ne-αgCSs).
(iv) for all Ne-αOS (correspondingly, Ne-αCSs) are Ne-gαOSs (correspondingly, Ne-gαCSs).
(v) for all Ne-gαOSs (correspondingly, Ne-gαCSs) are Ne-αgOSs (correspondingly, Ne-gαCSs).

Definition 2.6: Let (\mathcal{U}, ξ) and (\mathcal{V}, ϱ) be NTSs and $\eta: (\mathcal{U}, \xi) \to (\mathcal{V}, \varrho)$ be a mapping, we have (i) if for each Ne-OS (correspondingly, Ne-CS) \mathcal{K} in \mathcal{V} , $\eta^{-1}(\mathcal{K})$ is a Ne-OS (correspondingly, Ne-CS) in \mathcal{U} , then η is known as neutrosophic continuous and denoted by Ne-continuous. [16] (ii) if for each Ne-OS (correspondingly, Ne-CS) \mathcal{K} in \mathcal{V} , $\eta^{-1}(\mathcal{K})$ is a Ne- α OS (correspondingly, Ne- α CS) in \mathcal{U} , then η is known as neutrosophic α -continuous and referred to Ne- α -continuous. [10] (iii) if for each Ne-OS (correspondingly, Ne-CS) \mathcal{K} in \mathcal{V} , $\eta^{-1}(\mathcal{K})$ is a Ne-gOS (correspondingly, Ne- α CS) in \mathcal{U} , then η is known as neutrosophic α -continuous and signified by Ne-g-continuous. [17]

Remark 2.7 [17,10]: Let $\eta: (\mathcal{U}, \xi) \to (\mathcal{V}, \varrho)$ be a map, the next declarations valid and but not vice versa:

(i) For all Ne-continuous functions are Ne- α -continuous.

(ii) For all Ne-continuous functions are Ne-g-continuous.

3. Neutrosophic Generalized ag-Closed Sets

The neutrosophic generalized α g-closed sets and their features are studied and discussed in this part of the paper.

Definition 3.1: Let C be a *NS* in *NTS* U, then it called a neutrosophic generalized α g-closed set and denoted by Ne-g α gCS if for any Ne- α gOS \mathcal{M} in \mathcal{U} such that $C \subseteq \mathcal{M}$, then Ne- $cl(C) \subseteq \mathcal{M}$.We indicated the collection of all Ne-g α gCSs in *NTS* U by Ne-g α gC(U).

Definition 3.2: Let C be a *NS* in *TS* U, then its neutrosophic gag-closure is the intersection of each Ne-gagCS in U including C, and we are shortly written it as Ne-gag*cl*(C). In other words, Ne-gag*cl*(C) = $\bigcap \{\mathcal{D} : C \subseteq D, D \text{ is a Ne-gagCS} \}$.

Theorem 3.3: The subsequent declarations are valid in any *TS U*:

(i) for all Ne-CSs are Ne-g α gCSs.

(ii) for all Ne-gagCSs are Ne-gCSs.

(iii) for all Ne-gαgCSs are Ne-αgCSs.

(iv) for all Ne-gαgCSs are Ne-gαCSs.

Proof:

(i) Suppose a Ne-CS C is in *TS* U. For any Ne- α gOS M, including C, we have $M \supseteq C = \text{Ne-}cl(C)$. Therefore, C stands a Ne-g α gCS.

(ii) Suppose Ne-gagCS C is in *TS* U. For any Ne-OS \mathcal{M} , including C, we have theorem (2.5) states that \mathcal{M} stands a Ne-agOS in U. Because a Ne-gagCS C satisfying this fact Ne- $cl(C) \subseteq \mathcal{M}$. As a result, C considers a Ne-gCS.

(iii) Assume Ne-gagCS C is in *TS* U. For any Ne-OS \mathcal{M} , including C, we have theorem (2.5) states that \mathcal{M} remains a Ne- α gOS in U. Subsequently, Ne-g α gCS C satisfying this statement Ne- $\alpha cl(C) \subseteq$ Ne- $cl(C) \subseteq \mathcal{M}$. Therefore, C becomes a Ne- α gCS.

(iv) Assume Ne-gagCS C is in *TS* U. For any Ne- α OS \mathcal{M} including C, we have theorem (2.5) states that \mathcal{M} remains a Ne- α gOS in U. Subsequently, Ne-g α gCS C satisfying this statement Ne- $\alpha cl(C) \subseteq$ Ne- $cl(C) \subseteq \mathcal{M}$. Therefore, C considers a Ne-g α CS.

The opposite conditions for this previous theorem do not look accurate by the next obvious examples.

Example 3.4: Suppose $\mathcal{U} = \{p, q\}$ and let $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$, such that we have the sets $\mathcal{A} = \langle u, (0.6, 0.7), (0.1, 0.1), (0.4, 0.2) \rangle$ and $\mathcal{B} = \langle u, (0.1, 0.2), (0.1, 0.1), (0.8, 0.8) \rangle$, so that (\mathcal{U}, ξ) is a *NTS*. However, the *NS* $\mathcal{C} = \langle u, (0.2, 0.2), (0.1, 0.1), (0.6, 0.7) \rangle$ is a Ne-gagCS but not a Ne-CS.

Example 3.5: Suppose $\mathcal{U} = \{p, q, r\}$ and let $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$, where such that we have the sets $\mathcal{A} = \langle u, (0.5, 0.5, 0.4), (0.7, 0.5, 0.5), (0.4, 0.5, 0.5) \rangle$ and $\mathcal{B} = \langle u, (0.3, 0.4, 0.4), (0.4, 0.5, 0.5), (0.3, 0.4, 0.6) \rangle$, so that (\mathcal{U}, ξ) is a *NTS*. However, the *NS* $\mathcal{C} = \langle u, (0.4, 0.6, 0.5), (0.4, 0.3, 0.5), (0.5, 0.6, 0.4) \rangle$ is a Ne-gCS but not a Ne-gagCS.

Example 3.6: Suppose $\mathcal{U} = \{p, q\}$ and let $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$, where such that we have the sets $\mathcal{A} = \langle u, (0.5, 0.6), (0.3, 0.2), (0.4, 0.1) \rangle$ and $\mathcal{B} = \langle u, (0.4, 0.4), (0.4, 0.3), (0.5, 0.4) \rangle$, so that (\mathcal{U}, ξ) is a *NTS*.

However, the *NS* $C = \langle u, (0.5, 0.4), (0.4, 0.4), (0.4, 0.5) \rangle$ is a Ne- α gCS and hence Ne-g α CS but not a Ne-g α gCS.

Definition 3.7: Let C be any *NS* in *TS* U, then it is called a neutrosophic generalized α g-open set and referred to by Ne-g α gOS iff the set U - C is a Ne-g α gCS. The collection of the whole Ne-g α gOSs in *NTS* U indicated by Ne-g α gO(U).

Definition 3.8: The union of the whole Ne-g α gOSs in *NTS* \mathcal{U} included in *NS* \mathcal{C} is termed neutrosophic g α g-interior of \mathcal{C} and symbolized by Ne-g α gint(\mathcal{C}). In symbolic form, we have this thing Ne-g α gint(\mathcal{C}) = U{ $\mathcal{D}: \mathcal{C} \supseteq \mathcal{D}, \mathcal{D}$ is a Ne-g α gOS}.

Proposition 3.9: For any *NS* \mathcal{M} in *TS* \mathcal{U} , the subsequent features stand:

(i) Ne-gagint(\mathcal{M}) = \mathcal{M} iff \mathcal{M} is a Ne-gagOS.

(ii) Ne-gag $cl(\mathcal{M}) = \mathcal{M}$ iff \mathcal{M} is a Ne-gagCS.

(iii) Ne-gagint(\mathcal{M}) is the biggest Ne-gagOS included in \mathcal{M} .

(iv) Ne-gag $cl(\mathcal{M})$ is the littlest Ne-gagCS, including \mathcal{M} .

Proof: the features (i-iv) are understandable.

Proposition 3.10: For any *NS* \mathcal{M} in *TS* \mathcal{U} , the subsequent features stand:

(i) Ne-gagint($\overline{\mathcal{M}}$) = $\overline{(Ne - gagcl(\mathcal{M}))}$, (ii) Ne-gagcl($\overline{\mathcal{M}}$) = $\overline{(Ne - gagint(\mathcal{M}))}$. **Proof:**

(i) The proof will be evident by symbolic definition, Ne-gag $cl(\mathcal{M}) = \bigcap \{\mathcal{D}: \mathcal{M} \subseteq \mathcal{D}, \mathcal{D} \text{ is a Ne-gagCS} \}$

 $\overline{(\text{Ne} - g\alpha gcl(\mathcal{M}))} = \bigcap \{\overline{D} : \overline{\mathcal{M}} \subseteq \overline{D}, \overline{D} \text{ is a Ne-} g\alpha gCS} \\ = \bigcup \{\overline{D} : \overline{\mathcal{M}} \subseteq \overline{D}, \overline{D} \text{ is a Ne-} g\alpha gCS} \\ = \bigcup \{\mathcal{N} : \mathcal{M} \supseteq \mathcal{N}, \mathcal{N} \text{ is a Ne-} g\alpha gOS} \\ = \operatorname{Ne-} g\alpha gint(\overline{\mathcal{M}}).$

(ii) This feature has undeniable proof analogous to feature (i).

Theorem 3.11: For any Ne-OS C in *TS* U, then this set is a Ne-gagOS. **Proof:** Suppose Ne-OS C in *TS* U, so we obtain that \overline{C} is a Ne-CS. Therefore, \overline{C} is a Ne-gagCS via the previous theorem (3.3), part (i). Consequently, C is a Ne-gagOS.

Theorem 3.12: For any Ne-gagOS C in *TS* U, then this set is a Ne-gOS. **Proof:** Suppose Ne-gagOS C in *TS* U, so we obtain that \overline{C} is a Ne-gagCS. Therefore, \overline{C} is a Ne-gCS via the previous theorem (3.3), part (ii). Consequently, C is a Ne-gOS.

Lemma 3.13: For any Ne-g α gOS C in *TS* U, then this set is Ne- α gOS (correspondingly, Ne-g α OS). **Proof:** The proof of this lemma is similar to one of the previous theorem.

Proposition 3.14: For any two Ne-gagCSs C and D in *TS* U, then the set CUD is a Ne-gagCS.

Proof: Suppose any two Ne-gagCSs C and D in *NTS* U and M is a Ne-agOS, including C and D. In other words, we have $C \cup D \subseteq M$. So, $C, D \subseteq M$. Because C and D are Ne-gagCSs, we get that Ne-*cl*(C), Ne-*cl*(D) $\subseteq M$. Therefore, Ne-*cl*($C \cup D$) = Ne-*cl*(C)UNe-*cl*(D) $\subseteq M$. Then Ne-*cl*($C \cup D$) $\subseteq M$. Thus, $C \cup D$ stands a Ne-gagCS.

Proposition 3.15: For any two Ne-gagOSs C and D in *TS* U, then the set $C \cap D$ is a Ne-gagOS. **Proof:** Suppose any two Ne-gagOSs C and D in *TS* U. Subsequently, we have that \overline{C} and \overline{D} are Ne-gagCSs. So, we reach to this fact $\overline{C} \cup \overline{D}$ is a Ne-gagCS by proposition (3.14). Because $\overline{C} \cup \overline{D} = \overline{(C \cap D)}$, we obtain to our final result $C \cap D$ is a Ne-gagOS.

Proposition 3.16: Let Ne-gagCS C be in *TS* U, then Ne-*cl*(C) – C does not include non-empty Ne-CS in U.

Proof: Assume we have Ne-g α gCS C and Ne-CS F in *NTS* U so as $F \subseteq$ Ne-cl(C) - C. Because C stands a Ne-g α gCS, this gives us the fact Ne- $cl(C) \subseteq \overline{F}$. The latter means $F \subseteq \overline{Ne} - cl(\overline{C})$. Subsequently, we arrive to $F \subseteq Ne-cl(C)\cap(\overline{Ne} - cl(\overline{C})) = 0_N$. Therefore, $F = 0_N$ and so, we reach to our final result Ne-cl(C) - C does not include non-empty Ne-CS.

Proposition 3.17: Let Ne-gagCS C be in *NTS* U iff Ne-cl(C) - C does not include non-empty Ne-agCS in U.

Proof: Assume we have Ne-gagCS C and Ne-agCS G in *NTS* U so as $G \subseteq \text{Ne-}cl(C) - C$. Because C considers a Ne-gagCS, this gives us the fact $\text{Ne-}cl(C) \subseteq \overline{G}$. The latter means $G \subseteq \overline{\text{Ne} - cl(C)}$. Subsequently, we arrive to $G \subseteq \text{Ne-}cl(C) \cap (\overline{\text{Ne} - cl(C)}) = 0_N$. Therefore, G is empty.

On The Other Hand, let us assume that Ne- $cl(\mathcal{C}) - \mathcal{C}$ does not include non-empty Ne- α gCS in \mathcal{U} . Suppose \mathcal{M} is Ne- α gOS so as $\mathcal{C} \subseteq \mathcal{M}$. If we have this truth Ne- $cl(\mathcal{C}) \subseteq \mathcal{M}$ but then we get this fact Ne- $cl(\mathcal{C}) \cap (\overline{\mathcal{M}})$ is non-empty. Meanwhile, we know that Ne- $cl(\mathcal{C})$ is Ne-CS and at the same time, we have $\overline{\mathcal{M}}$ is Ne- α gCS, so Ne- $cl(\mathcal{C}) \cap (\overline{\mathcal{M}})$ is non-empty Ne- α gCS included Ne- $cl(\mathcal{C}) - \mathcal{C}$. This leads us to a contradiction. Consequently Ne- $cl(\mathcal{C}) \not\subseteq \mathcal{M}$. Therefore, \mathcal{C} considers a Ne- α gCS.

Theorem 3.18: Let Ne- α gOS and Ne- α gCS C be in *TS* U, then C considers a Ne-CS in U. **Proof:** Assume we have Ne- α gOS and Ne- α gCS C is in *TS* U, so we get that Ne- $cl(C) \subseteq C$ and subsequently, we reach to $C \subseteq Ne-cl(C)$. Consequently, Ne-cl(C) = C. Therefore, C stands a Ne-CS.

Theorem 3.19: Let Ne-gagCS C be in *NTS* U so as $C \subseteq D \subseteq \text{Ne-cl}(C)$, but then again D considers a Ne-gagCS in U.

Proof: Assume we have Ne-gagCS C and Ne-agOS \mathcal{M} are in *NTS* \mathcal{U} so as $\mathcal{D} \subseteq \mathcal{M}$. Later, $C \subseteq \mathcal{M}$. Subsequently, C stands a Ne-gagCS; this fact pursues Ne- $cl(C) \subseteq \mathcal{M}$. So, $\mathcal{D} \subseteq \text{Ne-}cl(C)$ infers Ne- $cl(\mathcal{D}) \subseteq \text{Ne-}cl(\text{Ne-}cl(C)) = \text{Ne-}cl(C)$. Consequently, Ne- $cl(\mathcal{D}) \subseteq \mathcal{M}$. Therefore, \mathcal{D} exists a Ne-gagCS.

Theorem 3.20: Let Ne-gagOS C be in *NTS* U so as Ne-*int*(C) $\subseteq D \subseteq C$, but then again D considers a Ne-gagOS in U.

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Proof: Assume we have Ne-g α gOS C is in *NTS* U so as Ne-*int*(C) $\subseteq D \subseteq C$. After that, U - C stands a Ne-g α gCS as well as $\overline{C} \subseteq \overline{D} \subseteq \text{Ne-}cl(\overline{C})$. But then again, we depend on theorem (3.19) to get U - D is a Ne-g α gCS. Therefore, D exists a Ne-g α gOS.

Theorem 3.21: A *NS* C is Ne-gagOS iff $\mathcal{P} \subseteq \text{Ne-}int(\mathcal{C})$ so as $\mathcal{P} \subseteq C$ and \mathcal{P} considers a Ne-gagCS. **Proof:** Assume we have that Ne-gagCS \mathcal{P} satisfying $\mathcal{P} \subseteq C$ and $\mathcal{P} \subseteq \text{Ne-}int(\mathcal{C})$. Afterward, $\overline{\mathcal{C}} \subseteq \overline{\mathcal{P}}$ and we have by lemma (3.13), $\overline{\mathcal{P}}$ remains a Ne-agOS. Accordingly, Ne- $cl(\overline{\mathcal{C}}) = \overline{\text{Ne} - int(\mathcal{C})} \subseteq \overline{\mathcal{P}}$. Subsequently, $\overline{\mathcal{C}}$ stands a Ne-gagCS. Therefore, \mathcal{C} stands a Ne-gagOS.

On the contrary, we assume Ne-gagOS C and Ne-gagCS \mathcal{P} is so as $\mathcal{P} \subseteq C$. Subsequently, $\overline{C} \subseteq \overline{\mathcal{P}}$. While \overline{C} exists a Ne-gagCS and $\overline{\mathcal{P}}$ remains a Ne-agOS, we reach to that Ne- $cl(\overline{C}) \subseteq \overline{\mathcal{P}}$. Therefore, $\mathcal{P} \subseteq \text{Ne-}int(C)$.





4. Neutrosophic Generalized ag-Continuous Functions

In this part of this paper, the neutrosophic generalized α g-continuous functions are performed and examined their fundamental features.

Definition 4.1: Let $\eta: (\mathcal{U}, \xi) \to (\mathcal{V}, \varrho)$ be a map so as \mathcal{U} and \mathcal{V} are *NTSs*, then: (i) η is named a neutrosophic αg -continuous and signified by Ne- αg -continuous if for every Ne-OS (correspondingly, Ne-CS) \mathcal{K} in \mathcal{V} , $\eta^{-1}(\mathcal{K})$ is a Ne- αg OS (correspondingly, Ne- αg CS) in \mathcal{U} . (ii) η is named a neutrosophic g α -continuous and signified by Ne- $g\alpha$ -continuous if for every Ne-OS (correspondingly, Ne-CS) \mathcal{K} in \mathcal{V} , $\eta^{-1}(\mathcal{K})$ is a Ne- $g\alpha$ OS (correspondingly, Ne- $g\alpha$ CS) in \mathcal{U} .

Theorem 4.2: Let η be a function on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . So, we have the following:

(i) all Ne-g-continuous functions are Ne- $\alpha g\text{-continuous}.$

(ii) all Ne- α -continuous functions are Ne-g α -continuous.

(iii) all Ne-ga-continuous functions are Ne-ag-continuous.

Proof:

(i) Let Ne-CS \mathcal{K} be in *NTS* \mathcal{V} and Ne-g-continuous function η defined on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . By definition of Ne-g-continuous, $\eta^{-1}(\mathcal{K})$ remains a Ne-gCS in \mathcal{U} . So, we have $\eta^{-1}(\mathcal{K})$ is a Ne- α gCS in \mathcal{U} because of theorem (2.5) part (iii). As a result, η stands a Ne- α g-continuous.

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(ii) Let Ne-CS \mathcal{K} be in *NTS* \mathcal{V} and Ne- α -continuous function η defined on *NTS* \mathcal{U} and valued in *NTS* \mathcal{V} . By definition of Ne- α -continuous, $\eta^{-1}(\mathcal{K})$ remains a Ne- α CS in \mathcal{U} . So, we have $\eta^{-1}(\mathcal{K})$ is a Ne- α CS in \mathcal{U} because of theorem (2.5) part (iv). As a result, η stands a Ne- α -continuous.

(iii) Let Ne-CS \mathcal{K} be in *NTS* \mathcal{V} and Ne-g α -continuous function η defined on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . So, we have $\eta^{-1}(\mathcal{K})$ is a Ne-g α CS and then $\eta^{-1}(\mathcal{K})$ is a Ne- α gCS in \mathcal{U} because of theorem (2.5) part (v). Therefore, η stands a Ne- α g-continuous.

The reverse of the beyond proposition does not become valid as shown in the next examples.

Example 4.3: (i) Assume $\mathcal{U} = \{p, q\}$ and $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$ and $\varrho = \{0_N, \mathcal{B}, \mathcal{C}, 1_N\}$, where $\mathcal{A} = \langle u, (0.6, 0.7), (0.4, 0.3), (0.5, 0.2) \rangle$, $\mathcal{B} = \langle u, (0.5, 0.5), (0.5, 0.4), (0.6, 0.5) \rangle$ and $\mathcal{C} = \langle u, (0.5, 0.5), (0.6, 0.4), (0.7, 0.5) \rangle$ are the neutrosophic sets, then (\mathcal{U}, ξ) and (\mathcal{U}, ϱ) are NTSs. Define $\eta: (\mathcal{U}, \xi) \rightarrow (\mathcal{U}, \varrho)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then η is Ne- α g- continuous. But $\overline{\mathcal{C}} = \langle u, (0.7, 0.5), (0.6, 0.4), (0.5, 0.5) \rangle$ is a Ne-CS in $(\mathcal{U}, \varrho), \eta^{-1}(\overline{\mathcal{C}})$ is not a Ne-gCS in (\mathcal{U}, ξ) . Thus η is not a Ne-g-continuous.

(ii) Let $\mathcal{U} = \{p,q\}$ and let $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$ and $\varrho = \{0_N, \mathcal{B}, \mathcal{C}, 1_N\}$, where $\mathcal{A} = \langle u, (0.6, 0.7), (0.4, 0.3), (0.5, 0.2) \rangle$, $\mathcal{B} = \langle u, (0.5, 0.5), (0.5, 0.4), (0.6, 0.5) \rangle$ and $\mathcal{C} = \langle u, (0.5, 0.5), (0.5, 0.5), (0.4, 0.5) \rangle$ are the neutrosophic sets, then (\mathcal{U}, ξ) and (\mathcal{U}, ϱ) are NTSs. Define $\eta : (\mathcal{U}, \xi) \rightarrow (\mathcal{U}, \varrho)$ as a $\eta(p) = p$ and $\eta(q) = q$. Then η is Ne-g α - continuous. But $\overline{\mathcal{C}} = \langle u, (0.4, 0.5), (0.5, 0.5), (0.5, 0.5) \rangle$ is a Ne-CS in $(\mathcal{U}, \varrho), \eta^{-1}(\overline{\mathcal{C}})$ is not a Ne- α CS in (\mathcal{U}, ξ) . Thus η is not a Ne- α -continuous.

(iii) Let $\mathcal{U} = \{p,q\}$ and let $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$ and $\varrho = \{0_N, \mathcal{B}, \mathcal{C}, 1_N\}$, where $\mathcal{A} = \langle u, (0.6, 0.7), (0.4, 0.3), (0.5, 0.2) \rangle$, $\mathcal{B} = \langle u, (0.5, 0.5), (0.5, 0.4), (0.6, 0.5) \rangle$ and $\mathcal{C} = \langle u, (0.5, 0.5), (0.6, 0.4), (0.7, 0.5) \rangle$ are the neutrosophic sets, then (\mathcal{U}, ξ) and (\mathcal{U}, ϱ) are NTSs. Define $\eta : (\mathcal{U}, \xi) \rightarrow (\mathcal{U}, \varrho)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then η is Ne- α g- continuous. But $\overline{\mathcal{C}} = \langle u, (0.5, 0.5), (0.5, 0.5), (0.6, 0.4) \rangle$ is a Ne-CS in $(\mathcal{U}, \varrho), \eta^{-1}(\overline{\mathcal{C}})$ is not a Ne- $g\alpha$ CS in (\mathcal{U}, ξ) . Thus η is not a Ne- $g\alpha$ -continuous.

Definition 4.4: Let η be a function on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . Then, we named η as neutrosophic generalized α g-continuous and shortly wrote it as Ne-g α g-continuous if for each Ne-CS \mathcal{K} in \mathcal{V} , $\eta^{-1}(\mathcal{K})$ is a Ne-g α gCS in \mathcal{U} .

Theorem 4.5: Let η be a function on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . Afterward, η remains a Ne-gag-continuous function iff for each Ne-OS \mathcal{K} in \mathcal{V} , $\eta^{-1}(\mathcal{K})$ is a Ne-gagOS in \mathcal{U} . **Proof:** Let Ne-OS \mathcal{K} and Ne-CS $\overline{\mathcal{K}}$ are in \mathcal{V} . Therefore, $\eta^{-1}(\overline{\mathcal{K}}) = (\overline{\eta^{-1}(\mathcal{K})})$ remains a Ne-gagCS

in \mathcal{U} . Consequently, $\eta^{-1}(\mathcal{K})$ exists a Ne-gagOS in \mathcal{U} . The reverse proof is evident.

Proposition 4.6: For all Ne-gαg-continuous functions are Ne-αg-continuous.

Proof: Let Ne-CS \mathcal{K} be in *NTS* \mathcal{V} and Ne-g α g-continuous function η defined on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . By definition of Ne-g α g-continuous function, $\eta^{-1}(\mathcal{K})$ stands a Ne-g α gCS in \mathcal{U} . So, we have $\eta^{-1}(\mathcal{K})$ remains a Ne- α gCS in \mathcal{U} because of theorem (3.3) part (iii). As a result, η exists a Ne- α g-continuous.

Proposition 4.7: For all Ne-gαg-continuous functions are Ne-gα-continuous.

Proof: Let Ne-CS \mathcal{K} be in *NTS* \mathcal{V} and Ne-g α g-continuous function η defined on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . By definition of Ne-g α g-continuous function, $\eta^{-1}(\mathcal{K})$ stands a Ne-g α gCS in \mathcal{U} . So, we have $\eta^{-1}(\mathcal{K})$ remains a Ne-g α CS in \mathcal{U} because of theorem (3.3) part (iv). As a result, η exists a Ne-g α -continuous.

The reverse of the beyond proposition does not become valid as shown in the next examples.

Example 4.8: Let $\mathcal{U} = \{p,q\}$ and let $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$ and $\varrho = \{0_N, \mathcal{C}, 1_N\}$, where $\mathcal{A} = \langle u, (0.5, 0.6), (0.3, 0.2), (0.4, 0.1) \rangle$, $\mathcal{B} = \langle u, (0.4, 0.4), (0.4, 0.3), (0.5, 0.4) \rangle$ and $\mathcal{C} = \langle u, (0.5, 0.4), (0.4, 0.4), (0.4, 0.5) \rangle$ are the neutrosophic sets, then (\mathcal{U}, ξ) and (\mathcal{U}, ϱ) are NTSs. Define $\eta : (\mathcal{U}, \xi) \rightarrow (\mathcal{U}, \varrho)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then η is Ne- α g- continuous. But $\mathcal{C} = \langle u, (0.4, 0.5), (0.4, 0.4), (0.5, 0.4) \rangle$ is a Ne-CS in $(\mathcal{U}, \varrho), \eta^{-1}(\overline{\mathcal{C}})$ is a Ne- α gCS but not a Ne-g α gCS in (\mathcal{U}, ξ) . Thus η is not a Ne-g α g-continuous.

Example 4.9: Let $\mathcal{U} = \{p,q\}$ and let $\xi = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$ and $\varrho = \{0_N, \mathcal{C}, 1_N\}$, where $\mathcal{A} = \langle u, (0.5, 0.6), (0.3, 0.2), (0.4, 0.1) \rangle$, $\mathcal{B} = \langle u, (0.4, 0.4), (0.4, 0.3), (0.5, 0.4) \rangle$ and $\mathcal{C} = \langle u, (0.5, 0.4), (0.4, 0.4), (0.4, 0.5) \rangle$ are the neutrosophic sets, then (\mathcal{U}, ξ) and (\mathcal{U}, ϱ) are NTSs. Define $\eta : (\mathcal{U}, \xi) \rightarrow (\mathcal{U}, \varrho)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then η is Ne-g α - continuous. But $\mathcal{C} = \langle u, (0.4, 0.5), (0.4, 0.4), (0.5, 0.4) \rangle$ is a Ne-CS in $(\mathcal{U}, \varrho), \eta^{-1}(\overline{\mathcal{C}})$ is a Ne-g α CS but not a Ne-g α gCS in (\mathcal{U}, ξ) . Thus η is not a Ne-g α g-continuous.

Definition 4.10: Let η be a function on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . Then, we named η as neutrosophic generalized αg -irresolute and shortly wrote it as Ne-g αg -irresolute if for each Ne-g αg CS \mathcal{K} in \mathcal{V} , $\eta^{-1}(\mathcal{K})$ is a Ne-g αg CS in \mathcal{U} .

Theorem 4.11: Let η be a function on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . Afterward, η remains a Ne-gag-irresolute function iff for each Ne-gagOS \mathcal{K} in \mathcal{V} , $\eta^{-1}(\mathcal{K})$ is a Ne-gagOS in \mathcal{U} . **Proof:** Let Ne-gagOS \mathcal{K} and Ne-gagCS $\overline{\mathcal{K}}$ are in \mathcal{V} . Therefore, $\eta^{-1}(\overline{\mathcal{K}}) = (\overline{\eta^{-1}(\mathcal{K})})$ remains a

Ne-gagCS in \mathcal{U} . Consequently, $\eta^{-1}(\mathcal{K})$ exists a Ne-gagOS in \mathcal{U} . The reverse proof is evident.

Proposition 4.12: For all Ne-gαg-irresolute functions are Ne-gαg-continuous.

Proof: Let Ne-CS \mathcal{K} be in *NTS* \mathcal{V} and Ne-g α g-irresolute function η defined on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} . So, we have \mathcal{K} stands a Ne-g α gCS in \mathcal{V} by theorem (3.3) part (i). By definition of Ne-g α g-irresolute function, $\eta^{-1}(\mathcal{K})$ stands a Ne-g α gCS in \mathcal{U} . As a result, η exists a Ne-g α g-continuous.

The subsequent example explains that the inverse of the overhead proposition does not work.

Example 4.13: Suppose $\mathcal{U} = \{p, q\}$ and let $\xi = \{0_N, \mathcal{B}, 1_N\}$ and $\varrho = \{0_N, \mathcal{A}, \mathcal{B}, 1_N\}$, where $\mathcal{A} = \langle u, (0.6, 0.7), (0.4, 0.3), (0.5, 0.2) \rangle$ and $\mathcal{B} = \langle u, (0.5, 0.5), (0.5, 0.4), (0.6, 0.5) \rangle$ are the neutrosophic sets, then (\mathcal{U}, ξ) and (\mathcal{U}, ϱ) are NTSs. Define $\eta : (\mathcal{U}, \xi) \to (\mathcal{U}, \varrho)$ as a $\eta(p) = q$ and $\eta(q) = p$. Then η is Ne-gag-continuous. But $\mathcal{C} = \langle u, (0.5, 0.5), (0.6, 0.4), (0.5, 0.7) \rangle$ is a Ne-gagCS in $(\mathcal{U}, \varrho), \eta^{-1}(\mathcal{C})$ is not a Ne-gagCS in (\mathcal{U}, ξ) . Thus η is not a Ne-gag-irresolute.

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Definition 4.14: We called a *NTS* \mathcal{U} with a neutrosophic $T_{\frac{1}{2}}$ -space if for each Ne-gCS in \mathcal{U} is a Ne-CS and we denoted it by Ne- $T_{\frac{1}{2}}$ -space.

Definition 4.15: We called a *NTS* \mathcal{U} with a neutrosophic $T_{g\alpha g}$ -space if for each Ne-g α gCS in \mathcal{U} is a Ne-CS and we denoted by Ne- $T_{g\alpha g}$ -space.

Proposition 4.16: Every Ne- $T_{\frac{1}{2}}$ -space stands a Ne- $T_{g\alpha g}$ -space.

Proof: Let C be a Ne-g α gCS in Ne- $T_{\frac{1}{2}}$ -space U. By theorem (3.3) part (ii), we obtain C is a Ne-gCS. By definition of Ne- $T_{\frac{1}{2}}$ -space, we reach to that C is a Ne-CS in U. Therefore, U endures a Ne- $T_{g\alpha g}$ -space.

Theorem 4.17: Let η_1 be a Ne-gag-continuous function on *NTS* \mathcal{U} and valued in *NTS* \mathcal{V} and let η_2 be a Ne-g-continuous function on *NTS* \mathcal{V} and valued in *TS* \mathcal{W} . If \mathcal{V} is a Ne- $T_{\frac{1}{2}}$ -space, then $\eta_2 \circ \eta_1$ is a Ne-gag-continuous function.

Proof: Assume Ne-CS \mathcal{K} is in \mathcal{W} . Meanwhile, we have a Ne-g-continuous function η_2 defined on a

Ne- $T_{\frac{1}{2}}$ -space \mathcal{V} , then $\eta_2^{-1}(\mathcal{K})$ stands a Ne-CS in \mathcal{V} . Subsequently, we also see a Ne-g α g-continuous

function η_1 defined on \mathcal{U} , then $\eta_1^{-1}(\eta_2^{-1}(\mathcal{K}))$ stands a Ne-g α gCS in \mathcal{U} . Therefore, $\eta_2 \circ \eta_1$ stands a Ne-g α g-continuous.

Theorem 4.18: Let η be a function on *NTS* \mathcal{U} and valued in *TS* \mathcal{V} , we have the following results:

(i) If *NTS* \mathcal{U} stands a Ne-T₁-space then the function η becomes a Ne-g-continuous iff it considers a

a Ne-gαg-continuous.

(ii) If *NTS* \mathcal{U} stands a Ne-T_{gag}-space then the function η becomes a Ne-continuous iff it considers a Ne-gag-continuous.

Proof:

(i) Let Ne-CS \mathcal{K} be in \mathcal{V} and η be a Ne-g-continuous function. By definition of Ne-g-continuous, $\eta^{-1}(\mathcal{K})$ is a Ne-gCS in \mathcal{U} . Besides, the definition of Ne- $T_{\frac{1}{2}}$ -space states $\eta^{-1}(\mathcal{K})$ is a Ne-CS. So,

 $\eta^{-1}(\mathcal{K})$ is a Ne-gagCS in \mathcal{U} by theorem (3.3) part (i). Therefore, η is a Ne-gag-continuous.

On the contrary, let Ne-CS \mathcal{K} be in \mathcal{V} and let η be a Ne-g α g-continuous. By definition of Ne-g α g-continuous, $\eta^{-1}(\mathcal{K})$ is a Ne-g α gCS in \mathcal{U} . Besides, we have $\eta^{-1}(\mathcal{K})$ is a Ne-gCS in \mathcal{U} by theorem (3.3) part (ii). Therefore, η is a Ne-g-continuous.

(ii) Let Ne-CS \mathcal{K} be in \mathcal{V} and let η be a Ne-continuous. By definition of Ne-continuous, $\eta^{-1}(\mathcal{K})$ is a Ne-CS in \mathcal{U} . So, we have $\eta^{-1}(\mathcal{K})$ is a Ne-gagCS in \mathcal{U} by theorem (3.3) part (i). Therefore, η is a Ne-gag-continuous.

On the contrary, let Ne-CS \mathcal{K} be in \mathcal{V} and let η be a Ne-gag-continuous. Besides, we have $\eta^{-1}(\mathcal{K})$ is a Ne-gagCS in \mathcal{U} . Furthermore, the definition of Ne-T_{gag}-space gives $\eta^{-1}(\mathcal{K})$ is a Ne-CS in \mathcal{U} . Therefore, η is a Ne-continuous.

Remark 4.19: The subsequent illustration indicates the relative among the various kinds of Ne-continuous functions:



5. Conclusion

The class of Ne-gagCS described employing Ne- α gCS structures a neutrosophic topology and deceptions between the classes of Ne-CS and Ne-gCS. We as well illustration Ne-gag-continuous functions by applying Ne-gagCS. The Ne-gagCS know how to be developed to establish another neutrosophic homeomorphism.

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