Neutrosophic Sets and Systems
{Special Issue: Impact of Neutrosophy in solving the Latin American's social problems}, Vol. 37, 2020

University of New Mexico

Linking Neutrosophic AHP and Neutrosophic Social Choice Theory for Group Decision Making

Sharon Dinarza Álvarez Gómez[1], Jorge Fernando Goyes García[2], and Bayron Pinda Guanolema[3]

[1] Universidad Regional Autónoma de los Andes (UNIANDES), Km 5 ½ vía a Baños, Ambato, 180166, Ecuador.
E-mail: dirfinanciera@uniandes.edu.ec

[2] Universidad Regional Autónoma de los Andes (UNIANDES), Km 5 ½ vía a Baños Ambato, 180166, Ecuador.
E-mail: admfinanciero@uniandes.edu.ec

[3] Universidad Regional Autónoma de los Andes (UNIANDES), Km 5 ½ vía a Baños Ambato, 180166, Ecuador.
E-mail: dir.contabilidad@uniandes.edu.ec

Abstract. Analytic Hierarchy Process (AHP) is a decision-making technique that has been widely studied and developed by the scientific community. The interest in this tool is because it combines scientific rigor with the simplicity of its application. Additionally, it has been extended to uncertainty frameworks, such as fuzzy and neutrosophic frameworks. This paper aims to define a new method called NAHP+NSC, where the Neutrosophic Analytic Hierarchy Process (NAHP) is combined with the recently introduced Neutrosophic Social Choice (NSC) theory. Neutrosophy incorporates indeterminacy to both the AHP technique and the SC theory, which is an intrinsic condition of any decision-making process. On the other hand, it is possible to count on a group of experts to carry out the NAHP evaluations, where the chosen alternative is the one with the highest votes. Experts are divided into kind of homogeneous sub-groups called Interest Groups (IG), where each IG conjointly evaluates the proposed alternatives, and then tools of NSC are used for choosing the best alternative. The contribution of this new method is that evaluations and results are more accurate when indeterminacy is incorporated.

Keywords: Neutrosophic Analytic Hierarchy Process, neutrosophic social choice theory, neutrosophic preference relations, group decision-making.

1 Introduction

Analytic Hierarchy Process (AHP) is a technique introduced by Thomas L. Saaty for decision-making, [1]. It is a peculiar technique because the result encloses mathematical and psychology rigor. The decision maker starts from a decision tree with different hierarchical levels, where the top level contains a single leaf that represents the goal of the decision, the intermediate levels represent the attributes and sub-attributes necessaries to make the decision, while the level on bottom contains the alternatives to make the decisions.

The decision maker must compare the relative importance among the attributes, the sub-attributes and finally the alternatives are evaluated with respect to the attributes and sub-attributes, in such a way that the relative importance of each alternative is obtained, which are then ordered so that the preferred one is that with the highest index. These measurements are based on a scale introduced by Saaty. In addition, the consistency of each relative comparison is measured.

This technique has been widely studied for its simplicity and applicability in more or less complex decision-making situations. Additionally, the Saaty's scale has been generalized from crisp numbers to fuzzy numbers, to contain the uncertainty of decision-making, [2, 3]. The extension of the method to a Neutrosophic Analytic Hierarchy Process (NAHP) is also introduced; see [4-9], where the pair-wise comparison is performed with triangular neutrosophic numbers. Neutrosophy allows us to incorporate indeterminacy into the method [10] that is the result of lack of knowledge, inconsistencies or contradictions, which is an essential part of any complex decision-making problem[11].

AHP has also hybridized with other techniques such as SWOT or TOPSIS, [5, 7], to enhance its strength as a decision-making tool. However, due to AHP interest, and its development, some new needs have arisen through classical and non-classical AHP, because more than one expert can make the decision. This makes the method more accurate, and the decision is consensual, but on the other hand AHP theory becomes more complex and that yields some additional questions, such as what are the ways to aggregate the elements of the decision tree of experts or what is the way to consider the evaluations of the experts belonging to different interest groups.

B. Srdjevic in [12], defines a method where the classic AHP hybridizes with tools of the Social Choice Theory, [13-17]. He called this hybridization AHP+SC. The theory of social choice deals with making collective decisions based on the preferences of the individuals that make up a society. Considering a set of social alternatives and a society whose individuals have preferences, these preferences are represented by binary relations over the set of alternatives. Keeping in mind that individuals may have different opinions about social alternatives; the Social Choice theory studies the process of aggregation of individual preferences in a social preference. Collective decisions will be made from the social binary relationship that has been obtained by aggregating individual preferences. That is to say, given a set of social alternatives, a social welfare function assigns to each state of opinion a binary relationship; in general, there is infinity of aggregation processes. Specifically, this theory studies voting methods.

Recently, the theory of social choice has been extended to Neutrosophy, [18], with the intention of applying it to decision-making, where the so-called Neutrosophic Social Choice (NSC) theory was introduced. In that paper, Topal et al. define preferences in a neutrosophic environment. In addition, they use a new form of truth representation of neutrosophic theory called Distributed Indeterminacy Form (DIF), as well as an accuracy function H that serves as a de-neutrosophication method.

In this paper authors are inspired by the method proposed in [12] to introduce a new one in the neutrosophic framework. That is, AHP+SC becomes in NAHP+NSC, where NAHP includes indeterminacy within the AHP technique, which binds with the NSC of Topal et al. Specifically, a group of experts is considered, each of which has its own NAHP evaluation. On the other hand, NSC let us to use voting methods to determine the best alternative by consensus. The method in [12] is based on the division of the group of experts into subgroups called Interest Groups (IG) which for simplicity are considered internally homogeneous. Each of the IGs makes an evaluation jointly, therefore there will be as many evaluations as existing IGs. NSC serves to select the best option among IGs' evaluations. Linking NAHP and NSC in a neutrosophic framework let us incorporate indeterminacy in decision-making.

Next, the main concepts and methods on NAHP and NSC are discussed in Section 2 below. In section 3 the method proposed in this paper is introduced and a simulated example is used as case study. The last section contains the conclusions.

2 Preliminary concepts

This section is structured into two subsections. Subsection 2.1 describes the Neutrosophic Analytic Hierarchy process (NAHP) technique. Subsection 2.2 contains the main concepts of the Neutrosophic Social Choice (NSC) theory.

2.1 Neutrosophic Analytic Hierarchy process

In this subsection, we explain basic concepts of Neutrosophy, like neutrosophic set, single-valued neutrosophic set, and the Neutrosophic Analytic Hierarchy Process (NAHP) technique.

Definition 1: ([19-28]) The *Neutrosophic set* N is characterized by three membership functions, which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsehood-membership function F_A, where U is the Universe of Discourse and $\forall x\in U$, $T_A(x), I_A(x), \text{and } F_A(x) \subseteq]^-0, 1^+[$, and $^-0 \le \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Notice that according to the definition, $T_A(x), I_A(x), \text{and } F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ and hence, $T_A(x), I_A(x), \text{and } F_A(x)$ can be subintervals of $[0, 1]$.

Definition 2: ([19-28]) The *Single-Valued Neutrosophic Set* (SVNS) N over U is $A = \{< x;\ T_A(x), I_A(x), F_A(x) >: x\in U\}$, where $T_A: U \to [0, 1], I_A: U \to [0, 1], \text{and } F_A: U \to [0, 1]$, $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

The *Single-Valued Neutrosophic number* (SVNN) is represented by $N = (t, i, f)$, such that $0 \le t, i, f \le 1$ and $0 \le t + i + f \le 3$.

Definition 3: ([19-28]) The *single-valued trapezoidal neutrosophic number*,

$\tilde{a} = \langle (a_1, a_2. a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy and falsehood membership functions are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \alpha_{\tilde{a}}, & a_2 \le x \le a_3 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \quad (2)$$

$$\begin{cases} \frac{\left(a_2 - x + \beta_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{\left(x - a_2 + \beta_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases}$$

$$F_{\tilde{a}}(x) =$$

$$\begin{cases} \frac{\left(a_2 - x + \gamma_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, & a_2 \le x \le a_3 \\ \frac{\left(x - a_2 + \gamma_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_3 \le x \le a_4 \\ 1, & \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0,1]$, $a_1, a_2, a_3, a_4 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3 \le a_4$.

Definition 4: ([19-28]) Given $\tilde{a} = \langle (a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3, b_4); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$ two single-valued trapezoidal neutrosophic numbers and λ any non-null number in the real line. Then, the following operations are defined:

1. Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$
2. Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$
3. Inversion: $\tilde{a}^{-1} = \langle (a_4^{-1}, a_3^{-1}, a_2^{-1}, a_1^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3, a_4 \neq 0$.
4. Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda > 0 \\ \langle (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda < 0 \end{cases}$$

5. Division of two trapezoidal neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_4 > 0 \; and \; b_4 > 0 \\ \langle \left(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_4 < 0 \; and \; b_4 > 0 \\ \langle \left(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_4 < 0 \; and \; b_4 < 0 \end{cases}$$

6. Multiplication of two trapezoidal neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_4 > 0 \; and \; b_4 > 0 \\ \langle (a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_4 < 0 \; and \; b_4 > 0 \\ \langle (a_4 b_4, a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_4 < 0 \; and \; b_4 < 0 \end{cases}$$

Where, $\wedge$ is a t-norm and $\vee$ is a t-conorm.

Definitions 3 and 4 refer to *single-valued triangular neutrosophic number* when the condition $a_2 = a_3$ holds [24].

We can find in [4] the theory of AHP technique in a neutrosophic framework. Thus, we can model the indeterminacy of decision-making from applying neutrosophic AHP or NAHP for short.

Equation 4 contains a generic neutrosophic pair-wise comparison matrix for NAHP.

$$\tilde{A} = \begin{bmatrix} \tilde{1} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{1} \end{bmatrix} \quad (4)$$

Matrix $\tilde{A}$ must satisfy condition $\tilde{a}_{ji} = \tilde{a}_{ij}^{-1}$, based on the inversion operator of Definition 4, according to the scale summarized in Table 1 of triangular neutrosophic numbers.

For converting neutrosophic triangular numbers into crisp numbers, there are two indexes defined in [5], they are the so-called score and accuracy indexes, respectively, see Equations 5 and 6:

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (5)$$

$$A(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (6)$$

Saaty's scale	Definition	Neutrosophic Triangular Scale
1	Equally influential	$\tilde{1} = \langle(1,1,1); 0.50, 0.50, 0.50\rangle$
3	Slightly influential	$\tilde{3} = \langle(2,3,4); 0.30, 0.75, 0.70\rangle$
5	Strongly influential	$\tilde{5} = \langle(4,5,6); 0.80, 0.15, 0.20\rangle$
7	Very strongly influential	$\tilde{7} = \langle(6,7,8); 0.90, 0.10, 0.10\rangle$
9	Absolutely influential	$\tilde{9} = \langle(9,9,9); 1.00, 0.00, 0.00\rangle$
2, 4, 6, 8	Sporadic values between two close scales	$\tilde{2} = \langle(1,2,3); 0.40, 0.65, 0.60\rangle$
		$\tilde{4} = \langle(3,4,5); 0.60, 0.35, 0.40\rangle$
		$\tilde{6} = \langle(5,6,7); 0.70, 0.25, 0.30\rangle$
		$\tilde{8} = \langle(7,8,9); 0.85, 0.10, 0.15\rangle$

Table 1: Saaty's scale translated to a neutrosophic triangular scale.

To get the score and the accuracy degree of $\tilde{a}_{ji}$ the following equations are used:

$$\mathbf{S}(\tilde{a}_{ji}) = 1/\mathbf{S}(\tilde{a}_{ij}) \quad (7)$$

$$\mathbf{A}(\tilde{a}_{ji}) = 1/\mathbf{A}(\tilde{a}_{ij}) \quad (8)$$

With compensation by accuracy degree of each triangular neutrosophic number in the neutrosophic pair-wise comparison matrix, we derive the following deterministic matrix:

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix} \quad (9)$$

Next, we determine the ranking of priorities from the previous matrix as follows:

1. Normalize the column entries by dividing each entry by the sum of the column.
2. Take the total of the row averages.

The *Consistency Index* (CI) is calculated for matrices in formula 9, which is a function depending on λ_{max}, the maximum eigenvalue of the matrix. Saaty establishes that consistency of the evaluations can be determined by equation $CI = \frac{\lambda_{max} - n}{n-1}$, [1],where n is the order of the matrix. Also, the *Consistency Ratio* (CR) is defined by equation CR = CI/RI, where RI is given in Table 2.

Order (n)	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Table 2: RI associated to every order.

If CR≤0.1 we can consider that experts' evaluation is sufficiently consistent and hence we can proceed to use NAHP. We apply this procedure to matrices *A* in Equation 9. Consult [4] for more details on NAHP.

2.2 Neutrosophic Social Choice theory

This subsection summarizes the main concepts of Neutrosophic Social Choice theory developed in [18].

Definition 5: ([18]) Let $a = (T_a, I_a, F_a)$ be a single-valued neutrosophic number with truth value T_a, indeterminacy value I_a, and falsehood value F_a. *Distributed Indeterminacy Form* (DIF) of *a* is defined as $a_{DIF} = (T_a - T_a I_a, 0, F_a - F_a I_a)$.

DIF aims to distribute the indeterminacy result on truth and falsehood, thus, this measures the degree of affectation of the truthiness and falsehood, when indeterminacy varies.

Definition 6: ([18]) Let *a* be a single-valued neutrosophic number. An *accuracy function* H of *a* is:

$$H(a) = \frac{1 + T_a - I_a(1 - T_a) - F_a(1 - I_a)}{2} \quad (10)$$

Where for all a, $H(a) \in [0,1]$. H is an order relation which represents an accuracy score of information of a. If $H(a_1) = H(a_2)$, then $a_1 = a_2$, i.e., they have the same information, whereas, if $H(a_1) < H(a_2)$, then a_2 is larger than a_1.

Let $S = \{s_1, s_2, \ldots, s_n\}$ be a set of alternatives and m be a set of individuals. Each individual declares his or her preferences over S which are represented by an individual neutrosophic preference relation R_k, where $N_{R_k}: S \times S \rightarrow [0,1] \times [0,1] \times [0,1]$ and matrix $R_k = [r_{ij}^k]$, $i, j = 1,2,3,\ldots,n;\ k = 1,2,3,\ldots,m$, where $r_{ij}^k = N_{R_k}(r_i^k, r_j^k)$.

$$R_k = \begin{bmatrix} (0.5, 0.5, 0.5) & r_{12}^k & \cdots & r_{1n}^k \\ r_{21}^k & (0.5, 0.5, 0.5) & \cdots & r_{2n}^k \\ \vdots & \vdots & \cdots & \vdots \\ r_{n1}^k & r_{n2}^k & \cdots & (0.5, 0.5, 0.5) \end{bmatrix}$$

The function H (called *neutrosophic index* or *neutrosophic hesitation function*) assigns each a_{ij} neutrosophic value to a number in $[0, 1]$. Thus, the *neutrosophic index* or *neutrosophic hesitation function* is defined as follows:

$$H(a) = \frac{1 + T(a_{ij}) - I(a_{ij})\left(1 - T(a_{ij})\right) - F(a_{ij})\left(1 - I(a_{ij})\right)}{2} \quad (11)$$

The matrix $R_k^H = [H(r_{ij}^k)]$, $i, j = 1,2,3,\ldots,n;\ k = 1,2,3,\ldots,m$.

$$R_k^H = \begin{bmatrix} H((0.5, 0.5, 0.5)) & H(r_{12}^k) & \cdots & H(r_{1n}^k) \\ H(r_{21}^k) & H((0.5, 0.5, 0.5)) & \cdots & H(r_{2n}^k) \\ \vdots & \vdots & \cdots & \vdots \\ H(r_{n1}^k) & H(r_{n2}^k) & \cdots & H((0.5, 0.5, 0.5)) \end{bmatrix}$$

R_k^H is *quasi-reciprocal* if and only if $H(r_{ij}^k) \leq 1 - H(r_{ji}^k)$. If R_k^H is not quasi-reciprocal, we call k an *irrational individual*.

Other definitions declared in [18] are the following:

$$DIF(R_k) = \begin{bmatrix} (0.5, 0, 0.5) & DIF(r_{12}^k) & \cdots & DIF(r_{1n}^k) \\ DIF(r_{21}^k) & (0.5, 0, 0.5) & \cdots & DIF(r_{2n}^k) \\ \vdots & \vdots & \cdots & \vdots \\ DIF(r_{n1}^k) & DIF(r_{n2}^k) & \cdots & (0.5, 0, 0.5) \end{bmatrix}$$

R_i: preference matrix of the i-th individual,
$DIF(R_i)$: DIF of preference matrix of the i-th individual,
R_i^H: range of preference matrix of the i-th individual under H function,
$r_k^H(ij)$: represents the element at the row i and column j of R_k^H,
$h^k(ij)$: distribution of the kth individual's votes for each pair-wise comparison of alternative's value. It is determined through 0.5 derived from R_i^H,
$[\![h^k]\!]$: the matrix obtained by each element of $h^k(ij)$,
$[\![H_{ij}]\!]$: matrix of the group vote,
A_k: the degree for preference k assigned by the group,
a_{ij}^k: majority determination value for preference k of the group (the element at the row i and column j of $[\![h^k]\!]$,
H_{ij}^k: majority determination value for preference k of the group under H function,

$$h^k(ij) = \begin{cases} 1, \text{if } r_k^H(ij) > 0.5 \\ 0, \quad \text{otherwise} \end{cases}$$

$H_{\pi_{ij}}$: average majority determination value of the group under H function,
H_π: consensus winner determination matrix,
$C(s_i)$: social aggregation function for the alternative (preference) s_i.

Definition 7: ([18])*:* $s_i \in W$ is called a *consensus winner* if and only if $\forall s_j \neq s_i$: $r_{ij} > 0.5$, where $r_{ij} \in H_\pi$.

Definition 8: ([18]) The *social aggregation average function C* is defined to calculate the order of s_i in the group to the extent that individuals are not against option s_i, using the following equation:

$$C(s_i) = \frac{1}{m-1}\sum_{i \neq j} r_{ij} \quad (12)$$

Where $i, j = 1, 2, \cdots, m$.

3 NAHP+NSC method

In this section we introduce the NAHP+NSC method that is proposed in this paper.

First of all we define for any triangular neutrosophic number $\tilde{a}$, the *triangular accuracy function of* $\tilde{a} = \langle(a_1, a_2, a_3, a_4); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}}\rangle$, which is the function **TA** defined as follows:

$$\mathrm{TA}(\tilde{a}) = A\big(\langle(a_1, a_2, a_3); \mathrm{DIF}\big((\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}})\big)\rangle\big) \tag{13}$$

This is the accuracy degree of Equation 6 calculated for the DIF of the neutrosophic number contained in $\tilde{a}$. DIF is included following the idea in [18], where the accuracy function H also calculates the effect of indeterminacy in the truthiness and falsehood.

Let us note that reciprocal or quasi-reciprocal properties in NSC theory are similar to the reciprocal property in NAHP, from the point of view of the decision maker's rationality.

The method consists of the following steps:

1. The goal of the problem is established, and consequently the group of experts is selected. Next, the attributes, sub-attributes and alternatives are specified.
2. The group of experts is divided into M interest sub-groups, let us denote them by $\mathrm{IG} = \{\mathrm{IG}_1, \mathrm{IG}_2, \cdots, \mathrm{IG}_M\}$. We assumed the members of each sub-group form a homogenous decision group.
3. Each expert evaluates his/her own NAHP. However, with respect to every IG_i the equivalent matrices of the members of the sub-group are aggregated using formula 14.

Let $\{\widetilde{A}_{i1}, \widetilde{A}_{i2}, \cdots, \widetilde{A}_{in_i}\}$ be a set of n_i SVTNNs representing the assessment of each member of the i-th sub-group, where $\widetilde{A}_{ij} = \langle(a_{ij}, b_{ij}, c_{ij}); \alpha_{\tilde{a}_{ij}}, \beta_{\tilde{a}_{ij}}, \gamma_{\tilde{a}_{ij}}\rangle$ $(i = 1, 2, \dots, M)$ $(j = 1, 2, \dots, n_i)$, then the *weighted mean of the SVTNNs* is calculated through the following Equation:

$$\widetilde{A}_i = \sum_{j=1}^{n_i} \lambda_{ij}\widetilde{A}_{ij} \tag{14}$$

Where λ_{ij} is the weight of $\widetilde{A}_{ij}$, $\lambda_{ij} \in [0, 1]$ and $\sum_{j=1}^{n_i} \lambda_{ij} = 1$.

Note that λ_{ij} measures the relative importance of the j-th expert in the i-th sub-group.

Each $\widetilde{A}_i$ represents the matrix of pair-wise comparisons of NAHP method in IG_i, to aggregate the matrices of pair-wise comparison of criteria, sub-criteria and alternatives.

$\widetilde{A}_i$s are converted into A_is using Equation 13. This process can be repeated until the results are consistent according to the Consistency Ratio of the NAHP method. In accordance with the NAHP method, we obtain a vector of preference of the alternatives.

Here, the Aggregation of Individual Judgments (AIJ) is used because we are interested in measuring the judgments of the sub-group as a synergistic unit.

Let us denote by $O_i = \{o_{i1}, o_{i2}, \cdots, o_{iN}\}$ the position of each alternative $S_i = \{s_{i1}, s_{i2}, \cdots, s_{iN}\}$, when they are evaluated by the members of the i-th sub-group. For example, $O_1 = \{1,1,3,5,4\}$ means that according to the first subgroup, alternatives 1 and 2 are equally preferred, whereas, the next ones are the third, the fifth, and the fourth alternatives, in that order.

4. For each s_{il} $(l = 1, 2, \dots, N)$, the following triple is formed $V_{il} = (P_{il}, I_{il}, N_{il})$, where $P_{il} = card(\{k \neq l: s_{il} \text{ is strictly preferred over } s_{ik}\})$, $I_{il} = card(\{k \neq l: s_{il} \text{ is equally preferred to } s_{ik}\})$ and $N_{il} = card(\{k \neq l: s_{ik} \text{ is strictly preferred over } s_{il}\})$. See that, $V_{il} \in [0, N-1] \times [0, N-1] \times [0, N-1]$ and $P_{il} + I_{il} + N_{il} = N - 1$.
   Finally, $V_l \in [0, 1] \times [0, 1] \times [0, 1]$, $V_l = (P_l, I_l, N_l)$, aggregates the preference of the l-th alternative for all sub-groups, where $P_l = \frac{\sum_{i=1}^{M} P_{il}}{M(N-1)}$, $I_l = \frac{\sum_{i=1}^{M} I_{il}}{M(N-1)}$, and $N_l = \frac{\sum_{i=1}^{M} N_{il}}{M(N-1)}$.
   Note that this is a neutrosophic voting method.
5. $H(V_l)$ $(l = 1, 2, \cdots, N)$ is calculated, and the alternatives are sorted by order of preference, such that V_{l_1} is preferred over V_{l_2} if and only if $H(V_{l_1}) > H(V_{l_2})$. When, $H(V_{l_1}) = H(V_{l_2})$ we say that "V_{l_1} is equally preferred to V_{l_2}".

Below we illustrate this method with an example.

Example 1: (See [29])

Organizations face the problem of how to invest their resources in the different project alternatives. The correct evaluation and subsequent selection of software development projects provides competitive advantages to organizations. The selection of projects in the field of information technology presents multiple challenges, including the difficulty of evaluating intangible benefits, the existing interdependencies between projects, and the restrictions imposed by organizations.

The goal of this decision making problem is to assess three candidates of Information Technology Projects based on three criteria, namely, cost, project time span and profit. We call alternatives by Project 1, Project 2, and Project 3. The decision tree is depicted in Figure 1.

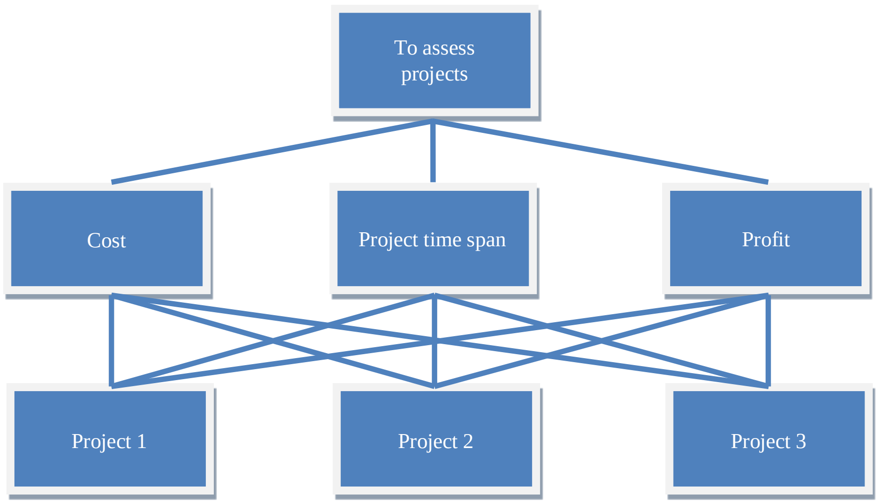

Figure 1: AHP tree of the example. Source [29].

This is a simulated example for illustrating how the NAHP+NSC technique can be applied in a decision-making problem. Suppose a group of 15 experts are selected to make the decision, and three sub-groups are formed, each of them containing 5 members. They are: IG_1 which contains the managers, IG_2 is the sub-group of financial analysts, and IG_3 is the interest group of specialists in information technology.

When the 15 experts give their evaluation according to the NAHP method, matrices of the IG_1 members are aggregated using formula 14, as well as the members of IG_2, and IG_3. Next, the results for each IG are de-neutrosophied using Equation 13, and the NAHP is completed for each interest group. To exemplify these steps, suppose the group assessment of IG_1 is summarized in Table 3 for the pair-wise comparison of the criteria.

	Cost	Project time span	Profit
Cost	$\tilde{1}$	$\tilde{2}$	$\tilde{5}^{-1}$
Project time span	$\tilde{2}^{-1}$	$\tilde{1}$	$\tilde{4}^{-1}$
Profit	$\tilde{5}$	$\tilde{4}$	$\tilde{1}$

Table 3: Group assessment of criteria by the members of IG_1. Source [29].

Table 4 contains the results to calculate TA(·) according to Equation 13.

	Cost	Project time span	Profit
Cost	0.93750	1.7625	0.18713
Project time span	0.56738	0.93750	0.25157
Profit	5.3438	3.9750	0.93750

Table 4: TA(·) of the group assessment of criteria by the members of IG_1

The calculation of consistency is $\lambda_{max} = 3.02075$, then, CI = 0.010375, and CR = 0.019952<0.1, therefore the group decision of IG_1 is consistent.

The weight of every criterion according to the members of IG_1 is the following, 0.19377 for the cost, 0.11788 for the time span, and 0.68835 for the profit.

Tables 5, 6, and 7 summarize the results of pair-wise evaluating projects 1, 2, and 3 with respect to cost, time span and profit criteria, respectively, collectively by members of IG_1. The numbers in parentheses are the crisp values after calculating TA. The rightmost column contains the priority vector of each project.

	Project 1	Project 2	Project3	Priority vector
Project 1	$\tilde{1}$(0.93750)	$\tilde{2}$(1.7625)	$\tilde{5}$(5.3438)	0.496401
Project 2	$\tilde{2}^{-1}$(0.56738)	$\tilde{1}$(0.93750)	$\tilde{5}$(5.3438)	0.422647
Project 3	$\tilde{5}^{-1}$(0.18713)	$\tilde{5}^{-1}$(0.18713)	$\tilde{1}$(0.93750)	0.080952

Table 5: Reciprocal matrix of the projects related to Cost and their priority vector (rightmost column). The parentheses contain TA values of the triangular neutrosophic numbers.

	Project 1	Project 2	Project3	Priority vector
Project 1	$\tilde{1}(0.93750)$	$\tilde{5}^{-1}(0.18713)$	$\tilde{2}^{-1}(0.56738)$	0.13012
Project 2	$\tilde{5}(5.3438)$	$\tilde{1}(0.93750)$	$\tilde{2}(1.7625)$	0.61860
Project 3	$\tilde{2}(1.7625)$	$\tilde{2}^{-1}(0.56738)$	$\tilde{1}(0.93750)$	0.25128

Table 6: Reciprocal matrix of the projects related to Project Time span and their priority vector (rightmost column). The parentheses contain TA values of the triangular neutrosophic numbers.

	Project 1	Project 2	Project3	Priority vector
Project 1	$\tilde{1}(0.93750)$	$\tilde{5}(5.3438)$	$\tilde{2}(1.7625)$	0.61860
Project 2	$\tilde{5}^{-1}(0.18713)$	$\tilde{1}(0.93750)$	$\tilde{2}^{-1}(0.56738)$	0.13012
Project 3	$\tilde{2}^{-1}(0.56738)$	$\tilde{2}(1.7625)$	$\tilde{1}(0.93750)$	0.25128

Table 7: Reciprocal matrix of the projects related to Profit and their priority vector (rightmost column). The parentheses contain TA values of the triangular neutrosophic numbers.

It is easy to check the consistency of the assessments in Tables 5-7.
Table 8 contains the global weights of the three projects by the members of IG_1.

	Costs	Project time span	Profit	Global Weight
Project 1	0.496401	0.13012	0.61860	0.53734
Project 2	0.422647	0.61860	0.13012	0.24438
Project 3	0.080952	0.25128	0.25128	0.21828
Criterion Weight	0.19377	0.11788	0.68835	1.00000

Table 8: Global weight matrix by IG_1.

Therefore, the members of IG_1 sort the projects in the following order, $p_1 \succ p_2 \succ p_3$.

Suppose that also for IG_2 the order of preference is $p_1 \succ p_2 \succ p_3$, whereas, according to the members of IG_3 the order is $p_1 \succ p_2 = p_3$.

The final results of the method are $V_{11} = V_{21} = V_{31} = (2,0,0)$, which means project 1 is preferred over the rest of projects (two of them), there is not any project preferred over project 1 and it is not equally preferred to another project, for the three IG.

$V_{12} = V_{22} = (1,0,1)$ and $V_{32} = (0,1,1)$, that means for the first and second sub-groups, project 2 is preferred over one project and not preferred over the other one, whereas, for the third IG, project 2 is equally preferred to one project and not preferred over the other one. Additionally, $V_{13} = V_{23} = (0,0,2)$, and $V_{33} = (0,1,1)$.

For each alternative we have, $V_1 = \left(\frac{2+2+2}{2(3)}, \frac{0+0+0}{2(3)}, \frac{0+0+0}{2(3)}\right) = (1,0,0)$, $V_2 = \left(\frac{1+1+0}{2(3)}, \frac{0+0+1}{2(3)}, \frac{1+1+1}{2(3)}\right) = \left(\frac{1}{3}, \frac{1}{6}, \frac{1}{2}\right)$, and $V_3 = \left(\frac{0+0+0}{2(3)}, \frac{0+0+1}{2(3)}, \frac{2+2+1}{2(3)}\right) = \left(0, \frac{1}{6}, \frac{5}{6}\right)$. See that here M = N = 3.

Finally, $H(V_1) = 1$, $H(V_2) = 0.40278$, and $H(V_3) = 0.069444$. Then, project 1 is the preferred one.

Conclusion

This paper introduces for the first time a group decision-making method based on neutrosophic analytic hierarchy process associated with elements of the neutrosophic social choice theory, it is called NAHP+NSC. The advantages of this technique are that it incorporates the indeterminacy as part of the decision-making. So, the result is more accurate than methods where indeterminacy is not explicitly considered. The group of experts is divided into interest groups, therefore the result is consistent, and the hybridization with the neutrosophic social choice theory allow decision makers to rigorously select the best option. Briefly, to combine AHP technique with SC theory in a neutrosophic framework is a complete tool for decision-making. An example is used for illustrating the applicability and the advantages of NAHP+NSC. Future works will consider other voting methods, even modelling with both, offsets [30, 31] and voting game theory as in [32].

References

1. Aczél, J. and T.L. Saaty, *Procedures for Synthesizing Ratio Judgments. Journal of Mathematical Psychology*,, 1983. ***27***(1): p. 93-102.
2. Buckley, J.J., *Fuzzy hierarchical analysis. Fuzzy Sets and Systems*,, 1985. ***17***: p. 233-247.
3. Buckley, J.J., T. Feuring, and Y. Hayashi, *Fuzzy hierarchical analysis revisited. European Journal of Operational Research*,, 2001. ***129***: p. 48-64.
4. Abdel-Basset, M., M. Mohamed, Y. Zhou, and I. Hezam, *Multicriteria group decision making based on neutrosophic analytic hierarchy process. Journal of Intelligent & Fuzzy Systems*,, 2017. ***33***(6): p. 4055–4066.
5. Abdel-Basset, M., M. Mohamed, and F. Smarandache, *An Extension of Neutrosophic AHP-SWOT Analysis for Strategic Planning and Decision-Making. Symmetry*,, 2018. ***10***(4): p. 116.
6. Becerra-Arévalo, N.P., M.F. Calles-Carrasco, J.L. Toasa-Espinoza, and M. Velasteguí-Córdova, *Neutrosophic AHP for the prioritization of requirements for a computerized facial recognition system. Neutrosophic Sets and Systems*,, 2020. ***34***: p. 159-168.
7. Gonzalez-Ortega, R., M. Leyva-Vazquez, J.A. Sganderla-Figueiredo, and A. Guijarro-Rodriguez, *Sinos river basin social-environmental prospective assessment of water quality management using fuzzy cognitive maps and neutrosophic AHP-TOPSIS. Neutrosophic Sets and Systems*,, 2018. ***23***: p. 160-171.
8. Tello-Cadena, M.A., E.M. Pinos-Medina, M. Jaramillo-Burgos, and F. Jara-Vaca, *Neutrosophic AHP in the analysis of Business Plan for the company Rioandes bus tours. Neutrosophic Sets and Systems*, , 2020. ***34***: p. 16-23.
9. Zaied, A.-N.H., M. Ismail, and A. Gamal, *An Integrated of Neutrosophic-ANP Technique for Supplier Selection. Neutrosophic Sets and Systems*, , 2019. **27**: p. 237-244.
10. Arulpandy, P., *Reduction of indeterminacy of gray-scale image in bipolar neutrosophic domain.* Neutrosophic Sets and Systems, 2019. **28**(1): p. 2.
11. Ortega, R.G., M.D.O. Rodríguez, M.L. Vázquez, J.E. Ricardo, J.A.S. Figueiredo, and F. Smarandache, *Pestel analysis based on neutrosophic cognitive maps and neutrosophic numbers for the sinos river basin management*. 2019: Infinite Study.
12. Srdjevic, B., *Linking analytic hierarchy process and social choice methods to support group decision-making in water management. Decision Support Systems*,, 2007. ***42*** p. 2261–2273.
13. Kelly, J.S., *Social Choice Theory: An Introduction*. 1988, Berlin: Springer-Verlag.
14. Pennock, D.M., E. Horvitz, and C.L. Giles. *Social choice theory and recommender systems: Analysis of the axiomatic foundations of collaborative filtering*. in *Proceedings of the Seventeenth National Conference on Artificial Intelligence and Twelfth Conference on Innovative Applications of Artificial Intelligence*. 2000. AAAI Press.
15. Faliszewski, P., P. Skowron, A. Slinko, and N. Talmon, *Multiwinner voting: A new challenge for social choice theory. . Trends in computational social choice*,, 2017. ***74***: p. 27-47.
16. Dubois, D. and J.-L. Koning, *Social choice axioms for fuzzy set aggregation.* Fuzzy Set and Systems, 1991. ***43*** p. 257-274.
17. Austen-Smith, D. and J.S. Banks, *Social choice theory, game theory, and positive political theory.* Annual Review of Political Science, 1998. ***1***: p. 259-287.
18. Topal, S., A. Çevik, and F. Smarandache, *A New Group Decision Making Method With Distributed Indeterminacy Form Under Neutrosophic Environment: An Introduction to Neutrosophic Social Choice Theory. IEEE Access*,, 2020. ***8***: p. 42000-42009.
19. Ye, J., *Trapezoidal neutrosophic set and its application to multiple attribute decision-making. Neural Computing and Applications*,, 2015. ***26***(5): p. 1157-1166.
20. Biswas, P., S. Pramanik, and B.C. Giri, *Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets and Systems*,, 2016. ***12***: p. 127-138.
21. Biswas, P., S. Pramanik, and B.C. Giri, *Distance Measure Based MADM Strategy with Interval Trapezoidal Neutrosophic Numbers. Neutrosophic Sets and Systems*,, 2018. ***19***: p. 40-46.
22. Deli, I., *Operators on Single Valued Trapezoidal Neutrosophic Numbers and SVTN-Group Decision Making. Neutrosophic Sets and Systems*,, 2018. ***22***: p. 131-150.
23. Broumi, S., M. Lathamaheswari, R. Tan, D. Nagarajan, T. Mohamed, F. Smarandache, and A. Bakali, *A new distance measure for trapezoidal fuzzy neutrosophic numbers based on the centroids. Neutrosophic Sets and Systems*,, 2020. ***35***: p. 478-502.
24. Abdel-Basset, M., M. Mohamed, A.N. Hussien, and A.K. Sangaiah, *A novel group decision-making model based on triangular neutrosophic numbers. Soft Computing*,, 2018. ***22***(20): p. 6629-6643.

25. Abdel-Aal, S.I., M.M.A. Abd-Ellatif, and M.M. Hassan, *Two Ranking Methods of Single Valued Triangular Neutrosophic Numbers to Rank and Evaluate Information Systems Quality. Neutrosophic Sets and Systems*,, 2018. ***19***: p. 132-141.
26. Mullai, M. and R. Surya, *Neutrosophic Inventory Backorder Problem Using Triangular Neutrosophic Numbers. Neutrosophic Sets and Systems*,, 2020. ***31***: p. 148-155.
27. Saha, A., I. Deli, and S. Broumi, *HESITANT Triangular Neutrosophic Numbers and Their Applications to MADM. Neutrosophic Sets and Systems*, , 2020. **35**: p. 269-298.
28. Deli, I., *Some Operators with IVGSVTrN-Numbers and Their Applications to Multiple Criteria Group Decision Making. Neutrosophic Sets and Systems*, , 2019. ***25***: p. 33-53.
29. Leyva-Vázquez, M., M.A. Quiroz-Martínez, Y. Portilla-Castell, J.R. Hechavarría-Hernández, and E. González-Caballero, *A New Model for the selection of Information Technology Project in a Neutrosophic Environment. Neutrosophic Sets and Systems*,, 2020. ***32***: p. 344-360.
30. Smarandache, F., *Neutrosophic Overset, Neutrosophic Underset, and Neutrosophic Offset:Similarly for Neutrosophic Over-/Under-/Off-Logic, Probability, and Statistics*. 2016, Brussels: Pons Editions.
31. Smarandache, F., M.A. Quiroz-Martínez, J.E. Ricardo, N.B. Hernández, and M.Y.L. Vázquez, *Application of Neutrosophic Offsets for Digital Image Processing*. 2020: Infinite Study.
32. González-Caballero, E., F. Smarandache, and M. Leyva-Vázquez, *On Neutrosophic Offuninorms. Symmetry*,, 2019. ***11***: p. 1136.

Received: April 23, 2020. Accepted: August 25, 2020