



Application of Single Valued Trapezoidal Neutrosophic Numbers in Transportation Problem

Rajesh Kumar Saini*¹ Atul Sangal² Manisha³

^{1,3}Deptt. of Mathematical Sciences and Computer Applications, Bundelkhand University, Jhansi, India,
prof.rksaini@bujhansi.ac.in*

²Deptt. of Management Studies, Sharda University, Greater Noida, Uttar Pradesh, India
atul.sangal@sharda.ac.in

* Correspondence: prof.rksaini@bujhansi.ac.in, Mob. 9412322576(091)

Abstract: In the present paper, we introduced the concept of single valued trapezoidal neutrosophic number, which is generalization of single valued neutrosophic number. A generalization of crisp, fuzzy and intuitionistic fuzzy sets represents as neutrosophic sets, which have uncertainty, inconsistent, and incompleteness information in real world problem. De-neutrosophication is a process to convert neutrosophic number into a crisp number for practical applications. For unbalanced neutrosophic transportation problem, we also use here minimum row column method and set a comparison among crisp and neutrosophic optimal solutions. Here we use two models of transportation problems to understand the applications in neutrosophic environment.

Keywords: Fuzzy Number, Single valued trapezoidal neutrosophic number, De-neutrosophication, neutrosophic transportation problem.

1. Introduction

In present scenario the classical theory of mathematics can't be handling the different kind of uncertainties, vagueness or impreciseness of mathematical problems. Many researchers around the world define many approaches to understand or define it. In 1965, Zadeh [37] first time introduce the mathematical formulation of a fuzzy set (FS) as a set with its membership function or membership grade. Sometimes the membership function in FS was not suitable one to describe the ambiguity of a problem.

After development of FS theory in various fields of uncertainty, Atanassov [1] in 1986, believe about the belongingness and non-belongingness in fuzzy set and present it's extension as intuitionistic fuzzy set (IFS) theory, which included the degree of membership and degree of non-membership function of each element in the set. More development of IFS theory in decision problems plays key role in recent scenario [17, 20]. In real life decision making problems, the theory of FS and IFS is much applicable, IFS approach in the solution of transportation problems used by many researchers [15, 22, 23].

The basic theme of a transportation problem is to find a direct connection between source and destination in minimum time with minimum cost. Hitchcock [12] was first, who originally developed the basic results of transportation problem by simplex method, which was recognized as special mathematical module. Since in early stage the transportation parameters like transportation cost, demand and supply were on the crisp values. In present time the real life transportation problems have uncertain, uncontrolled factors as the transportation cost, supply and demand are in fuzzy values.

In that period many research problems related to fuzzy transportation problem (FTP) were solved, in which some are partial fuzzy and some are fully fuzzy. A FTP in which cost demand and supply are

as fuzzy number is called fully FTP while in case of either cost, demand or supply are in fuzzy number, then it is FTP see [24, 7]. In a fuzzy solid transportation problem the parameters are trapezoidal fuzzy number (TrFN), introduced by Jim'enez and Verdegay [16] in 1999. For more research work about FTP, see [18, 19, 22, 25].

In current scenario, due to uncertainty, unawareness, vagueness, ambiguity in the constraints or some poor handling of data, the indeterminacy exists in transportation problems. The IFS theory can handle the problems of incomplete information but not the indeterminate and inconsistent information exists in transportation modal.

The problems with inconsistent information or indeterminate cannot be handled by any evocation of fuzzy set, so to overpower of such problems, Smarandache [27] introduced the neutrosophic set (NS) in 1988, which was an extension of classical set, FS and IFS. The well applicable fundamentals of NS, to represent the indeterminacy and inconsistent information are truth-membership degree, indeterminacy membership degree, and falsity-membership degree. The NS becomes the IFS, if indeterminacy membership degree $I(x)$ of NS is equal to hesitancy degree $h(x)$ of IFS. For practical applications and some technical references in NS, Wang et. al. [31] in 2010 introduced the idea of single valued neutrosophic set (SVNS). The notion of SVNS is more suitable and effective in solving many real life problems of decision making and supply chain management. For more applications of FS, IFS and NS in some different fields see [1- 10, 14, 21, 29- 32, 34, 36].

Since the study of transportation models with optimal and effective cost play a key role in every real life situation. Many researchers formulated efficient mathematical models in various uncertain environments. For practical application, two models of neutrosophic transportation problem (NTP) with all entries such as cost, demand, and supply are as single valued trapezoidal neutrosophic number (SVTrNN). Here we also use minimum row column method (MRCM) for balance the unbalance crisp transportation problem (CTP) and NTP with some existing method.

The main features of the paper are obtaining the optimal solutions of CTP and NTP after balancing with different methods and to compare the results. The paper is well organized in seven sections. In section first, introduction of the present paper with some earlier research are given. In second section, the basics concepts of FS, IFS and NS are discussed and reviewed. In third section, introduce the de-neutrosophication as score function to convert neutrosophic values into crisp values. Section fourth composes the classification and mathematical formulation of CTP & NTP of type-2 & 3. In fifth section, we introduce the procedures for solutions of CTP & NTP. In section six, seven and eight, we introduce two models of transportation with their solutions in different tables, their comparison, result and discussion. The conclusion and future aspects of research work exhibit in last section of the paper.

2. Preliminaries

2.1. Some basic definitions and examples

Definition 2.1.1. (FS [37]): A FS \tilde{A} of a non empty set X is defined as $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x) \rangle / x \in X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function such that $\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]$.

Definition 2.1.2. (FN): A convex, normalized fuzzy set \tilde{A} is called fuzzy number on the universal set of real numbers \mathbb{R} , if the membership function $\mu_{\tilde{A}}$ of \tilde{A} has the following belongingness:

- (i) $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ is continuous
- (ii) $\mu_{\tilde{A}}(x) = 0$, for all $x \in (-\infty, a] \cup [d, \infty)$
- (iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$

(iv) $\mu_{\tilde{A}}(x) = 1$, for all $x \in [b, c]$, where $a \leq b \leq c \leq d$.

Definition 2.1.3. (TrFN[19]): A trapezoidal fuzzy number (TrFN) denoted as $\tilde{A} = (a, b, c, d)$, with its membership function $\mu_{\tilde{A}}(x)$, on \mathbf{R} , is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & \text{for } a \leq x < b \\ 1, & \text{for } b \leq x < c \\ (d-x)/(d-c), & \text{for } c < x \leq d \\ 0, & \text{otherwise} \end{cases}$$

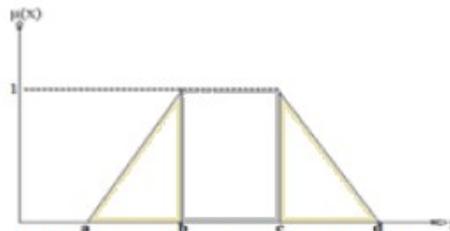


Figure1: Trapezoidal Fuzzy Number

If $b = c$ in TrFN $\tilde{A} = (a, b, c, d)$, then it becomes TFN $\tilde{A} = (a, b, d)$.

Definition 2.1.4. An IFS in a non-empty set X is denoted by \tilde{A}^I and defined as $\tilde{A}^I = \{ \langle x, \mu_{\tilde{A}^I}, \nu_{\tilde{A}^I} \rangle : x \in X \}$, where $\mu_{\tilde{A}^I}, \nu_{\tilde{A}^I} : X \rightarrow [0, 1]$, such that $0 \leq \mu_{\tilde{A}^I}, \nu_{\tilde{A}^I} \leq 1, \forall x \in X$. The degree of membership $\mu_{\tilde{A}^I}$ and degree of non-membership $\nu_{\tilde{A}^I}$ are functions from X to $[0, 1]$ in \tilde{A}^I . The degree of hesitation is defined as $h(x) = 1 - \mu_{\tilde{A}^I} - \nu_{\tilde{A}^I} \leq 1, \forall x \in X$ in \tilde{A}^I .

Definition 2.1.5. (ITrFN [20]): An Intuitionistic trapezoidal fuzzy number (ITrFN) is denoted by $\tilde{A}^I = (a_1, a_2, a_3, a_4)(a'_1, a'_2, a'_3, a'_4)$ where $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq a'_4$ with membership function $\mu_{\tilde{A}^I}$ and non-membership function $\nu_{\tilde{A}^I}$ defined by

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0, & \text{for } x < a_1, \\ \frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2, \\ 1, & \text{for } a_2 \leq x \leq a_3, \\ \frac{a_4-x}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4, \\ 0, & \text{for } x > a_4. \end{cases} \quad \nu_{\tilde{A}^I}(x) = \begin{cases} 1, & \text{for } x < a'_1, \\ \frac{x-a'_1}{a_2-a'_1}, & \text{for } a'_1 \leq x \leq a_2, \\ 0, & \text{for } a_2 \leq x \leq a_3, \\ \frac{a'_4-x}{a'_4-a_3}, & \text{for } a_3 \leq x \leq a'_4, \\ 1, & \text{for } x > a'_4. \end{cases}$$

If $a_2 = a_3$ then ITrFN becomes ITFN denoted as $\tilde{A}^I = (a_1, a_2, a_3)(a'_1, a'_2, a'_3)$ where $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$.

Definition 2.1.6. ([4]): Let x be a generic element of a non empty set X . A neutrosophic number \tilde{A}^N in X is defined as $\tilde{A}^N = \{ \langle x, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x) \rangle / x \in X \}$, $\forall T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x)$ and $F_{\tilde{A}^N}(x) \in]-0, 1+[$ where $T_{\tilde{A}^N} : X \rightarrow]0^-, 1^+[$, $I_{\tilde{A}^N} : X \rightarrow]0^-, 1^+[$ and $F_{\tilde{A}^N} : X \rightarrow]0^-, 1^+[$ are functions of truth-membership, indeterminacy membership and falsity-membership in \tilde{A}^N respectively also there is no restrictions on the sum of $T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x)$ and $F_{\tilde{A}^N}(x)$ so that $-0 \leq T_{\tilde{A}^N}(x) + I_{\tilde{A}^N}(x) + F_{\tilde{A}^N}(x) \leq 3^+$.

For the practical applications it is difficult to apply directly NS theory, hence the notion of SVNS as well as single valued neutrosophic numbers [SVNN] introduced by Deli I., S, uba Y[8] in 2014 .

Definition 2.1.7. (SVNS [8]): Let x be the generic point of a non-empty space X . A SVNS is denoted and defined as $\tilde{A}^N = \{\langle x, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x) \rangle / x \in X\}$, where for each point x in X , $T_{\tilde{A}^N}(x)$, called truth membership $I_{\tilde{A}^N}(x)$ called indeterminacy membership and $F_{\tilde{A}^N}(x)$ called falsity membership function in $[0, 1]$ and $0 \leq T_{\tilde{A}^N}(x) + I_{\tilde{A}^N}(x) + F_{\tilde{A}^N}(x) \leq 3$.

For continuous SVNS \tilde{A}^N can be written as

$$\tilde{A}^N = \int_{\tilde{A}^N} \langle T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x) \rangle / x, \quad x \in X$$

When X is discrete, a SVNS \tilde{A}^N can be written as

$$\tilde{A}^N = \sum_{i=1}^n \langle T_{\tilde{A}^N}(x_i), I_{\tilde{A}^N}(x_i), F_{\tilde{A}^N}(x_i) \rangle / x_i, \quad x_i \in X$$

Example 2.1.1. Let X be a space with capability x_1 , trustworthiness x_2 and price x_3 in $[0,1]$. If expert wants “degree of good services”, “degree of indeterminacy” and degree of poor services”, then a SVNS \tilde{A}^N of X is defined as

$$\tilde{A}^N = \langle 0.7, 0.1, 0.3 \rangle / x_1 + \langle 0.4, 0.2, 0.7 \rangle / x_2 + \langle 0.5, 0.1, 0.6 \rangle / x_3.$$

Definition 2.1.8. An (α, β, γ) -cut set of SVNS \tilde{A}^N , a crisp subset of \mathbf{R} is defined by

$$\tilde{A}_{\alpha, \beta, \gamma}^N = \{x : T_{\tilde{A}^N}(x) \geq \alpha, I_{\tilde{A}^N}(x) \leq \beta, F_{\tilde{A}^N}(x) \leq \gamma\}$$

where $0 \leq \alpha \leq 1, 0 \leq \beta \leq 1, 0 \leq \gamma \leq 1$ and $0 \leq \alpha + \beta + \gamma \leq 3$.

Definition 2.1.9. A SVNS $\tilde{A}^N = \{\langle x, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x) \rangle : x \in X\}$ is called neut-normal, if there exist at least three points $x_1, x_2, x_3 \in X$ such that $T_{\tilde{A}^N}(x_1) = 1, I_{\tilde{A}^N}(x_2) = 1, F_{\tilde{A}^N}(x_3) = 1$.

Definition 2.1.10. A SVNS $\tilde{A}^N = \{\langle x, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x) \rangle : x \in X\}$ is called neut-convex set on the real line; if the following conditions are satisfied $\forall x_1, x_2, x_3 \in \mathbf{R}$ and $\lambda \in [0, 1]$

- (i) $T_{\tilde{A}^N}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(T_{\tilde{A}^N}(x_1), T_{\tilde{A}^N}(x_2))$
- (ii) $I_{\tilde{A}^N}(\lambda x_1 + (1 - \lambda)x_2) \leq \max(I_{\tilde{A}^N}(x_1), I_{\tilde{A}^N}(x_2))$
- (iii) $F_{\tilde{A}^N}(\lambda x_1 + (1 - \lambda)x_2) \leq \max(F_{\tilde{A}^N}(x_1), F_{\tilde{A}^N}(x_2))$

Definition 2.1.11. (SVTrNN [8]): A single valued trapezoidal neutrosophic number (SVTrNN) $\tilde{a}^N = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, v_{\tilde{a}} \rangle$ is a special NS on the real line \mathbf{R} , whose truth-membership $T_{\tilde{a}^N}(x)$, indeterminacy-membership $I_{\tilde{a}^N}(x)$, and a falsity-membership $F_{\tilde{a}^N}(x)$ are given as follows:

$$T_{\tilde{a}^N}(x) = \begin{cases} \frac{((x-a_1)w_{\tilde{a}^N})}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2, \\ w_{\tilde{a}^N}, & \text{for } a_2 \leq x \leq a_3, \\ \frac{(a_4-x)w_{\tilde{a}^N}}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4, \\ 0, & \text{for } x > a_4 \text{ and } x < a_1 \end{cases}$$

$$I_{\tilde{a}^N}(x) = \begin{cases} \frac{a_2-x+(x-a_1)u_{\tilde{a}^N}}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2, \\ u_{\tilde{a}^N}, & \text{for } a_2 \leq x \leq a_3, \\ \frac{x-a_3+(a_4-x)u_{\tilde{a}^N}}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4, \\ 1, & \text{for } x > a_4 \text{ and } x < a_1 \end{cases}$$

$$F_{\tilde{a}^N}(x) = \begin{cases} \frac{a_2-x+(x-a_1)v_{\tilde{a}^N}}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2, \\ v_{\tilde{a}^N}, & \text{for } a_2 \leq x \leq a_3, \\ \frac{x-a_3+(a_4-x)v_{\tilde{a}^N}}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4, \\ 1, & \text{for } x > a_4 \text{ and } x < a_1 \end{cases}$$

where $w_{\tilde{a}}$, $u_{\tilde{a}}$, and $v_{\tilde{a}}$ denotes the maximum truth-membership degree, minimum-indeterminacy membership degree and minimum falsity-membership degree in $[0,1]$ respectively and $a_1, a_2, a_3, a_4 \in \mathbf{R}$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$. When $a_1 > 0, \tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, v_{\tilde{a}} \rangle$ is called positive SVTrNN, denoted by $\tilde{a} > 0$, and if $a_4 \leq 0$, then $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, v_{\tilde{a}} \rangle$ becomes a negative SVTrNN, denoted by $\tilde{a} < 0$.

If $0 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq 1$, $w_{\tilde{a}}, u_{\tilde{a}}, v_{\tilde{a}} \in [0,1]$, then \tilde{a} called a normalized SVTrNN. When $I_{\tilde{a}^N} = 1 - T_{\tilde{a}^N} - F_{\tilde{a}^N}$, then SVTrNN reduces as TIFN. If $a_2 = a_3$, then SVTrNN is reduces single valued triangular neutrosophic number (SVTNN), denoted as $\tilde{a} = \langle (a_1, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}}, v_{\tilde{a}} \rangle$.

Definition 2.1.12. A single valued trapezoidal neutrosophic number (SVTrNN) with twelve components is defined and denoted as:

$$\tilde{A}^N = \langle (p_1, p_2, p_3, p_4); (q_1, q_2, q_3, q_4); (r_1, r_2, r_3, r_4); w_{\tilde{A}^N}, u_{\tilde{A}^N}, v_{\tilde{A}^N} \rangle$$

where $r_1 \leq q_1 \leq p_1 \leq r_2 \leq q_2 \leq p_2 \leq r_3 \leq q_3 \leq p_3 \leq r_4 \leq q_4 \leq p_4$ in which the quantity of the truth membership, indeterminacy membership and falsity membership are not dependent and is defined as follows:

$$T_{\tilde{A}^N}(x) = \begin{cases} \frac{(x-p_1)w_{\tilde{A}^N}}{p_2-p_1}, & \text{for } p_1 \leq x \leq p_2, \\ w_{\tilde{A}^N}, & \text{for } p_2 \leq x \leq p_3, \\ \frac{(p_4-x)w_{\tilde{A}^N}}{p_4-p_3}, & \text{for } p_3 \leq x \leq p_4, \\ 0, & \text{for } x > p_4 \text{ and } x < p_1 \end{cases}$$

$$I_{\tilde{A}^N}(x) = \begin{cases} \frac{q_2 - x + (x - q_1)u_{\tilde{A}^N}}{q_2 - q_1}, & \text{for } q_1 \leq x \leq q_2, \\ u_{\tilde{A}^N}, & \text{for } q_2 \leq x \leq q_3, \\ \frac{x - q_3 + (q_4 - x)u_{\tilde{A}^N}}{q_4 - q_3}, & \text{for } q_3 \leq x \leq q_4, \\ 1, & \text{for } x > q_4 \text{ and } x < q_1 \end{cases}$$

$$F_{\tilde{A}^N}(x) = \begin{cases} \frac{r_2 - x + (x - r_1)v_{\tilde{A}^N}}{r_2 - r_1}, & \text{for } r_1 \leq x \leq r_2, \\ v_{\tilde{A}^N}, & \text{for } r_2 \leq x \leq r_3, \\ \frac{x - r_3 + (r_4 - x)v_{\tilde{A}^N}}{r_4 - r_3}, & \text{for } r_3 \leq x \leq r_4, \\ 1, & \text{for } x > r_4 \text{ and } x < r_1 \end{cases}$$

where $0 \leq T_{\tilde{A}^N}(x) + I_{\tilde{A}^N}(x) + F_{\tilde{A}^N}(x) \leq 3, x \in \tilde{A}^N$.

Definition 2.1.13. The parametric form \tilde{A}^N of SVTrNN for some $0 \leq \alpha \leq 1, 0 \leq \beta \leq 1, 0 \leq \gamma \leq 1$ and $0 \leq \alpha + \beta + \gamma \leq 3$ is defined as $(\tilde{A}^N)_{\alpha, \beta, \gamma} = [T'_{\tilde{A}^N}(\alpha), T''_{\tilde{A}^N}(\alpha), I'_{\tilde{A}^N}(\beta), I''_{\tilde{A}^N}(\beta), F'_{\tilde{A}^N}(\gamma), F''_{\tilde{A}^N}(\gamma)]$,

where

$$T'_{\tilde{A}^N}(\alpha) = p_1 + \frac{\alpha}{w_{\tilde{A}^N}}(p_2 - p_1), \quad T''_{\tilde{A}^N}(\alpha) = p_4 - \frac{\alpha}{w_{\tilde{A}^N}}(p_4 - p_3)$$

$$I'_{\tilde{A}^N}(\beta) = \frac{q_1 u_{\tilde{A}^N} - q_2 + \beta(q_2 - q_1)}{u_{\tilde{A}^N} - 1}, \quad I''_{\tilde{A}^N}(\beta) = \frac{q_3 - q_4 u_{\tilde{A}^N} + \beta(q_4 - q_3)}{1 - u_{\tilde{A}^N}}$$

$$F'_{\tilde{A}^N}(\gamma) = \frac{r_1 v_{\tilde{A}^N} - r_2 + \gamma(r_2 - r_1)}{v_{\tilde{A}^N} - 1}, \quad F''_{\tilde{A}^N}(\gamma) = \frac{r_3 - r_4 v_{\tilde{A}^N} + \gamma(r_4 - r_3)}{1 - v_{\tilde{A}^N}}$$

Example 2.1.2. let us take $\tilde{A}^N = \langle (7, 12, 16, 22), (6, 11, 15, 20), (5, 10, 14, 19); 0.4, 0.6, 0.6 \rangle$. The parametric representation is

$$T'_{0.4}(\alpha) = 7 + 12.5\alpha, \quad T''_{0.4}(\alpha) = 22 - 15\alpha,$$

$$I'_{0.6}(\beta) = 18.5 - 12.5\beta, \quad I''_{0.6}(\beta) = 7.5 + 12.5\beta,$$

$$F'_{0.6}(\gamma) = 17.5 - 12.5\gamma, \quad F''_{0.6}(\gamma) = 6.5 + 12.5\gamma$$

For different values of α, β, γ the degree of truthfulness, degree of indeterminacy and degree of falsity shown in table 1 and their graphical representation in figure 2:

Table 1

α, β, γ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$T'_{\tilde{A}^N}(\alpha)$	7.00	8.25	9.50	10.75	12.00	13.25	14.50	15.75	17.00	18.25	19.50
$T''_{\tilde{A}^N}(\alpha)$	22.00	20.50	19.5	17.50	16.00	14.50	13.00	11.50	10.00	8.50	7.00
$I'_{\tilde{A}^N}(\beta)$	18.5	17.25	16.00	14.75	13.50	12.25	11.00	9.75	8.50	7.25	6
$I''_{\tilde{A}^N}(\beta)$	7.50	8.75	10.00	11.25	12.50	13.75	15.00	16.25	17.50	18.75	20.00
$F'_{\tilde{A}^N}(\gamma)$	17.5	16.25	15.00	13.75	12.50	11.25	10.00	8.75	7.50	6.25	5.00
$F''_{\tilde{A}^N}(\gamma)$	6.50	7.75	9.00	10.25	11.50	12.75	14.00	15.25	16.50	17.75	19.50

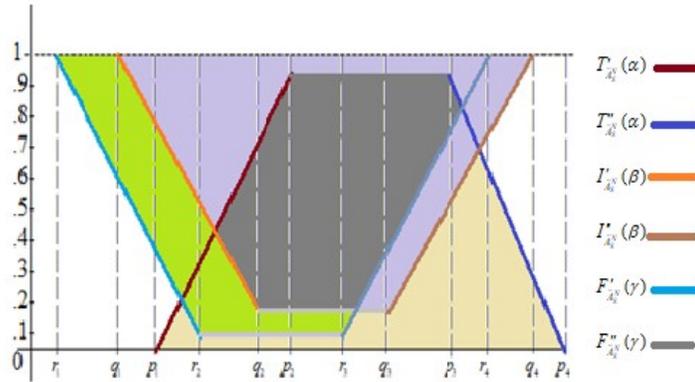


Figure 2: Graphical representation of

$$\tilde{A}^N = \langle (p_1, p_2, p_3, p_4); (q_1, q_2, q_3, q_4); (r_1, r_2, r_3, r_4); w_{\tilde{A}^N}, u_{\tilde{A}^N}, v_{\tilde{A}^N} \rangle$$

where $r_1 \leq q_1 \leq p_1 \leq r_2 \leq q_2 \leq p_2 \leq r_3 \leq q_3 \leq p_3 \leq r_4 \leq q_4 \leq p_4$

2.2. Operational Laws on SVTrNN

Definition 2.2. 1. If \tilde{A}^N and \tilde{B}^N are two SVTrNN with twelve components having truth-membership $T'_{\tilde{A}^N}(x), T''_{\tilde{B}^N}(x)$, indeterminacy-membership $I'_{\tilde{A}^N}(x), I''_{\tilde{B}^N}(x)$ and falsity-membership $F'_{\tilde{A}^N}(x), F''_{\tilde{B}^N}(x)$ respectively and three real numbers in [0.1], such as

$$\tilde{A}^N = \langle (p'_1, p'_2, p'_3, p'_4); (q'_1, q'_2, q'_3, q'_4); (r'_1, r'_2, r'_3, r'_4); w'_{\tilde{A}^N}, u'_{\tilde{A}^N}, v'_{\tilde{A}^N} \rangle$$

$$\tilde{B}^N = \langle (p''_1, p''_2, p''_3, p''_4); (q''_1, q''_2, q''_3, q''_4); (r''_1, r''_2, r''_3, r''_4); w''_{\tilde{B}^N}, u''_{\tilde{B}^N}, v''_{\tilde{B}^N} \rangle$$

Addition of SVTrNN:

$$\tilde{C}^N = \tilde{A}^N + \tilde{B}^N = \langle (p'_1 + p''_1, p'_2 + p''_2, p'_3 + p''_3, p'_4 + p''_4); (q'_1 + q''_1, q'_2 + q''_2, q'_3 + q''_3, q'_4 + q''_4); (r'_1 + r''_1, r'_2 + r''_2, r'_3 + r''_3, r'_4 + r''_4); w'_{\tilde{A}^N} \wedge w''_{\tilde{B}^N}, u'_{\tilde{A}^N} \vee u''_{\tilde{B}^N}, v'_{\tilde{A}^N} \vee v''_{\tilde{B}^N} \rangle$$

Negative of SVTrNN: If $\tilde{A}^N = \langle (p_1, p_2, p_3, p_4); (q_1, q_2, q_3, q_4); (r_1, r_2, r_3, r_4); w_{\tilde{A}^N}, u_{\tilde{A}^N}, v_{\tilde{A}^N} \rangle$, then

$$-\tilde{A}^N = \langle (-p_4, -p_3, -p_2, -p_1); (-q_4, -q_3, -q_2, -q_1); (-r_4, -r_3, -r_2, -r_1); w_{\tilde{A}^N}, u_{\tilde{A}^N}, v_{\tilde{A}^N} \rangle$$

Subtraction of SVTrNN:

$$\tilde{A}^N - \tilde{B}^N = \langle (p'_1 - p''_4, p'_2 - p''_3, p'_3 - p''_2, p'_4 - p''_1); (q'_1 - q''_4, q'_2 - q''_3, q'_3 - q''_2, q'_4 - q''_1); (r'_1 - r''_4, r'_2 - r''_3, r'_3 - r''_2, r'_4 - r''_1); w'_{\tilde{A}^N} \wedge w''_{\tilde{B}^N}, u'_{\tilde{A}^N} \vee u''_{\tilde{B}^N}, v'_{\tilde{A}^N} \vee v''_{\tilde{B}^N} \rangle$$

Multiplication of SVTrNN:

$$\tilde{A}^N \cdot \tilde{B}^N = \begin{cases} \langle (p'_1 \cdot p''_1, p'_2 \cdot p''_2, p'_3 \cdot p''_3, p'_4 \cdot p''_4); (q'_1 \cdot q''_1, q'_2 \cdot q''_2, q'_3 \cdot q''_3, q'_4 \cdot q''_4); (r'_1 \cdot r''_1, r'_2 \cdot r''_2, r'_3 \cdot r''_3, r'_4 \cdot r''_4); w'_{\tilde{A}^N} \wedge w''_{\tilde{B}^N}, u'_{\tilde{A}^N} \vee u''_{\tilde{B}^N}, v'_{\tilde{A}^N} \vee v''_{\tilde{B}^N} \rangle \\ \text{if } p'_4 > 0, p''_4 > 0; q'_4 > 0, q''_4 > 0; r'_4 > 0, r''_4 > 0 \\ \langle (p'_1 \cdot p''_4, p'_2 \cdot p''_3, p'_3 \cdot p''_2, p'_4 \cdot p''_1); (q'_1 \cdot q''_4, q'_2 \cdot q''_3, q'_3 \cdot q''_2, q'_4 \cdot q''_1); (r'_1 \cdot r''_4, r'_2 \cdot r''_3, r'_3 \cdot r''_2, r'_4 \cdot r''_1); w'_{\tilde{A}^N} \wedge w''_{\tilde{B}^N}, u'_{\tilde{A}^N} \vee u''_{\tilde{B}^N}, v'_{\tilde{A}^N} \vee v''_{\tilde{B}^N} \rangle \\ \text{if } p'_4 < 0, p''_4 > 0; q'_4 < 0, q''_4 > 0; r'_4 < 0, r''_4 > 0 \\ \langle (p'_4 \cdot p''_4, p'_3 \cdot p''_3, p'_2 \cdot p''_2, p'_1 \cdot p''_1); (q'_4 \cdot q''_4, q'_3 \cdot q''_3, q'_2 \cdot q''_2, q'_1 \cdot q''_1); (r'_4 \cdot r''_4, r'_3 \cdot r''_3, r'_2 \cdot r''_2, r'_1 \cdot r''_1); w'_{\tilde{A}^N} \wedge w''_{\tilde{B}^N}, u'_{\tilde{A}^N} \vee u''_{\tilde{B}^N}, v'_{\tilde{A}^N} \vee v''_{\tilde{B}^N} \rangle \\ \text{if } p'_4 < 0, p''_4 < 0; q'_4 < 0, q''_4 < 0; r'_4 < 0, r''_4 < 0 \end{cases}$$

Scalar multiplication of SVTrNN:

$$k\tilde{A}^N = \begin{cases} \langle (kp_1, kp_2, kp_3, kp_4); (kq_1, kq_2, kq_3, kq_4); (kr_1, kr_2, kr_3, kr_4); w_{\tilde{A}^N}, u_{\tilde{A}^N}, v_{\tilde{A}^N} \rangle, & \text{if } k > 0, \\ \langle (kp_4, kp_3, kp_2, kp_1); (kq_4, kq_3, kq_2, kq_1); (kr_4, kr_3, kr_2, kr_1); w_{\tilde{A}^N}, u_{\tilde{A}^N}, v_{\tilde{A}^N} \rangle, & \text{if } k < 0. \end{cases}$$

Inverse of SVTrNN:

$$(\tilde{A}^N)^{-1} = \frac{1}{\tilde{A}^N} = \begin{cases} \langle (\frac{1}{p_4}, \frac{1}{p_3}, \frac{1}{p_2}, \frac{1}{p_1}); (\frac{1}{q_4}, \frac{1}{q_3}, \frac{1}{q_2}, \frac{1}{q_1}); (\frac{1}{r_4}, \frac{1}{r_3}, \frac{1}{r_2}, \frac{1}{r_1}); w_{\tilde{A}^N}, u_{\tilde{A}^N}, v_{\tilde{A}^N} \rangle, & \text{if } p_s > 0, q_s > 0, r_s > 0, \\ \langle (\frac{1}{p_1}, \frac{1}{p_2}, \frac{1}{p_3}, \frac{1}{p_4}); (\frac{1}{q_1}, \frac{1}{q_2}, \frac{1}{q_3}, \frac{1}{q_4}); (\frac{1}{r_1}, \frac{1}{r_2}, \frac{1}{r_3}, \frac{1}{r_4}); w_{\tilde{A}^N}, u_{\tilde{A}^N}, v_{\tilde{A}^N} \rangle, & \text{if } p_s < 0, q_s < 0, r_s < 0. \end{cases}$$

Division of SVTrNN:

$$\frac{\tilde{A}^N}{\tilde{B}^N} = \begin{cases} \langle (\frac{p'_1}{p''_4}, \frac{p'_2}{p''_3}, \frac{p'_3}{p''_2}, \frac{p'_4}{p''_1}); (\frac{q'_1}{q''_4}, \frac{q'_2}{q''_3}, \frac{q'_3}{q''_2}, \frac{q'_4}{q''_1}); (\frac{r'_1}{r''_4}, \frac{r'_2}{r''_3}, \frac{r'_3}{r''_2}, \frac{r'_4}{r''_1}); w'_{\tilde{A}^N} \wedge w''_{\tilde{B}^N}, u'_{\tilde{A}^N} \vee u''_{\tilde{B}^N}, v'_{\tilde{A}^N} \vee v''_{\tilde{B}^N} \rangle & \text{if } p'_4 > 0, p''_4 > 0; q'_4 > 0, q''_4 > 0; r'_4 > 0, r''_4 > 0 \\ \langle (\frac{p'_4}{p''_1}, \frac{p'_3}{p''_2}, \frac{p'_2}{p''_3}, \frac{p'_1}{p''_4}); (\frac{q'_4}{q''_1}, \frac{q'_3}{q''_2}, \frac{q'_2}{q''_3}, \frac{q'_1}{q''_4}); (\frac{r'_4}{r''_1}, \frac{r'_3}{r''_2}, \frac{r'_2}{r''_3}, \frac{r'_1}{r''_4}); w'_{\tilde{A}^N} \wedge w''_{\tilde{B}^N}, u'_{\tilde{A}^N} \vee u''_{\tilde{B}^N}, v'_{\tilde{A}^N} \vee v''_{\tilde{B}^N} \rangle & \text{if } p'_4 < 0, p''_4 > 0; q'_4 < 0, q''_4 > 0; r'_4 < 0, r''_4 > 0 \\ \langle (\frac{p'_4}{p''_1}, \frac{p'_3}{p''_2}, \frac{p'_2}{p''_3}, \frac{p'_1}{p''_4}); (\frac{q'_4}{q''_1}, \frac{q'_3}{q''_2}, \frac{q'_2}{q''_3}, \frac{q'_1}{q''_4}); (\frac{r'_4}{r''_1}, \frac{r'_3}{r''_2}, \frac{r'_2}{r''_3}, \frac{r'_1}{r''_4}); w'_{\tilde{A}^N} \wedge w''_{\tilde{B}^N}, u'_{\tilde{A}^N} \vee u''_{\tilde{B}^N}, v'_{\tilde{A}^N} \vee v''_{\tilde{B}^N} \rangle & \text{if } p'_4 < 0, p''_4 < 0; q'_4 < 0, q''_4 < 0; r'_4 < 0, r''_4 < 0 \end{cases}$$

Example 2.2.1. let $\tilde{A}^N = \langle (7, 11, 16, 21), (6, 10, 15, 20), (5, 9, 14, 19); 0.4, 0.6, 0.6 \rangle$ and

$\tilde{B}^N = \langle (6, 11, 13, 20), (5, 10, 12, 18), (3, 8, 11, 16); 0.3, 0.6, 0.7 \rangle$ be two SVTrNN, then

$$\tilde{A}^N + \tilde{B}^N = \langle (13, 22, 29, 41), (11, 20, 27, 38), (8, 17, 25, 35); 0.4, 0.6, 0.6 \rangle$$

$$\tilde{A}^N - \tilde{B}^N = \langle (-13, -2, 5, 15), (-12, -2, 5, 15), (-11, -2, 6, 16); 0.4, 0.6, 0.6 \rangle$$

$$\tilde{A}^N \cdot \tilde{B}^N = \langle (42, 121, 208, 420), (30, 100, 180, 360), (15, 72, 154, 304); 0.4, 0.6, 0.6 \rangle$$

$$\tilde{A}^N / \tilde{B}^N = \langle (0.35, 0.85, 1.45, 3.50), (0.33, 0.83, 1.50, 4.00), (0.31, 0.81, 1.75, 6.33); 0.4, 0.6, 0.6 \rangle$$

$$5\tilde{A}^N = \langle (35, 55, 80, 105), (30, 50, 75, 100), (25, 45, 70, 95); 0.4, 0.6, 0.6 \rangle$$

3. De-Neutrosophication by using score function

We use the score and accuracy functions of a SVTrNN, is defined by an expert [31] to compare any two SVTrNN. So that the score function is defined as

$$S(\tilde{A}^N) = \left(\frac{p_1 + p_2 + p_3 + p_4 - q_1 - q_2 - q_3 - q_4 - r_1 - r_2 - r_3 - r_4}{12} \right) \times (2 + w_{\tilde{A}^N} - u_{\tilde{A}^N} - v_{\tilde{A}^N})$$

and accuracy function is

$$A(\tilde{A}^N) = \left(\frac{p_1 + p_2 + p_3 + p_4 - q_1 - q_2 - q_3 - q_4}{4} \right) \times (2 + w_{\tilde{A}^N} - u_{\tilde{A}^N} + v_{\tilde{A}^N})$$

Example 3.1. Let $\tilde{A}^N = \langle (7, 11, 16, 21), (6, 10, 15, 20), (5, 10, 14, 19); 0.4, 0.6, 0.6 \rangle$ then $S(\tilde{A}^N) = -4.4$ and $A(\tilde{A}^N) = -0.7$

Definition 3.1. (Comparison of SVTrNN). Let \tilde{A}^N and \tilde{B}^N be any two SVTrNN, then one has the following:

- (a) $S(\tilde{A}^N) < S(\tilde{B}^N) \Rightarrow \tilde{A}^N < \tilde{B}^N$
- (b) If $S(\tilde{A}_s^N) = S(\tilde{B}_s^N)$ and if
 - (i) $A(\tilde{A}^N) < A(\tilde{B}^N)$ then $\tilde{A}^N < \tilde{B}^N$
 - (ii) $A(\tilde{A}^N) > A(\tilde{B}^N)$ then $\tilde{A}^N > \tilde{B}^N$
 - (iii) $A(\tilde{A}^N) = A(\tilde{B}^N)$ then $\tilde{A}^N = \tilde{B}^N$

Example 3.2. Let $\tilde{A}^N = \langle (6, 10, 16, 20), (5, 9, 14, 19), (3, 8, 12, 18); 0.3, 0.6, 0.7 \rangle$ and

$$\tilde{B}^N = \langle (7, 11, 16, 21), (6, 15, 14, 20), (5, 10, 14, 19); 0.3, 0.6, 0.7 \rangle$$

$$\tilde{C}^N = \langle (8, 11, 16, 22), (6, 15, 14, 21), (5, 10, 14, 20); 0.3, 0.6, 0.7 \rangle$$

$$S(\tilde{A}^N) = -3.00, A(\tilde{A}^N) = 1.25, S(\tilde{B}^N) = -0.4, A(\tilde{B}^N) = 0.0, \text{ and } S(\tilde{C}^N) = -0.4, A(\tilde{C}^N) = 0.25,$$

which implies that if $S(\tilde{A}^N) < S(\tilde{B}^N)$ then $\tilde{A}^N < \tilde{B}^N$

$$\text{Also } S(\tilde{B}^N) = S(\tilde{C}^N) \text{ and } A(\tilde{B}^N) < A(\tilde{C}^N) \text{ then } \tilde{B}^N < \tilde{C}^N.$$

4. Neutrosophic Transportation Problem (NTP) and its Mathematical formulation

4.1. Classification of NTP

Definition 4.1.1. In a TP, if atleast one parameter such as cost, demand or supply is in form of neutrosophic numbers, the TP is termed as NTP.

Definition 4.1.2. A NTP having neutrosophic availabilities and neutrosophic demand but crisp cost, is classified as NTP of type-1.

Definition 4.1.3. The NTP having crisp availabilities and crisp demand but neutrosophic cost, is classified as NTP of type-2.

Definition 4.1.4. If all the specifications of TP such as cost, demand and availabilities are combination of crisp, triangular or trapezoidal neutrosophic numbers, then NTP classified as NTP of type-3.

Definition 4.1.5. If all the specifications of TP must be in neutrosophic numbers form, then TP is said to be NTP of type-4 or fully NTP.

4.2. Mathematical Formulation of NTP

The TP is very important for transporting goods from one source to another destination. In TP if ambiguity occurs in cost, demand or supply then it is more difficult to solve it. To handle this type of impreciseness in cost to transferred product from i^{th} sources to j^{th} destination or uncertainty in supply and demand the decision maker introduce NTP of SVTrNN.

Here we consider two models in which the decision maker is unsettled about the specific values i.e. cost from i^{th} sources to j^{th} destination and also certainty or uncertainty in supply or demand of the product, so that a new type of TP is introduced namely NTP with parameters like cost, demand and supply as SVTrNN. The NTP with assumptions and constraints is defined as the number of unites x_{ij}^N

and the neutrosophic cost \tilde{c}_{ij}^N are transported from i^{th} sources to j^{th} destination. For balance NTP

$$\sum_{i=0}^m a_i^N = \sum_{j=0}^n b_j^N \text{ i.e. total supply is equal to total demand.}$$

For the formulation of NTP the following assumptions and constraints are required:

- m Total number of source point
- n Total number of destination point
- i Table of source (for all m)
- j Table of destination (for all n)
- \tilde{x}_{ij}^N Number of transported neutrosophic unites from i^{th} source to j^{th} destination
- \tilde{c}_{ij}^N Neutrosophic cost of one unit transported from i^{th} source to j^{th} destination
- \tilde{a}_i^N Available neutrosophic supply quantity from i^{th} source
- \tilde{b}_j^N Required neutrosophic demand quantity to j^{th} destination
- c_{ij} Crisp cost of one unit quantity
- x_{ij} Number of transported crisp unites from i^{th} source to j^{th} destination
- a_i Available crisp supply quantity from i^{th} source
- b_j Required crisp demand quantity to j^{th} destination

Modal I

In NTP the objective is to minimize the cost of transported product from source to destination. The mathematical formulation of NTP with uncertain transported units and transportation cost, demand and supply is:

$$\begin{aligned} \text{Minimum } \tilde{Z}^N &= \sum_{i=0}^m \sum_{j=0}^n \tilde{x}_{ij}^N \tilde{c}_{ij}^N \\ \text{Subject to } \sum_{j=0}^n \tilde{x}_{ij}^N &= \tilde{a}_i^N, \forall i(\text{sources}) = 1, 2, 3, \dots, m, \\ \sum_{i=0}^m \tilde{x}_{ij}^N &= \tilde{b}_j^N, \forall j(\text{destination}) = 1, 2, 3, \dots, n, \\ \tilde{x}_{ij}^N &\geq 0, \forall i = 1, 2, 3, \dots, m, j = 1, 2, 3, \dots, n. \end{aligned} \tag{P1}$$

Modal II

The mathematical formulation of NTP with uncertain transported units and transportation cost but curtained about demand and supply is termed a NTP of type-2 is:

$$\begin{aligned} \text{Minimum } \tilde{Z}^N &= \sum_{i=0}^m \sum_{j=0}^n x_{ij} \tilde{c}_{ij}^N \\ \text{Subject to } \sum_{j=0}^n x_{ij} &= a_i, \forall i(\text{sources}) = 1, 2, 3, \dots, m, \\ \sum_{i=0}^m x_{ij} &= b_j, \forall j(\text{destination}) = 1, 2, 3, \dots, n, \\ x_{ij} &\geq 0, \forall i = 1, 2, 3, \dots, m, j = 1, 2, 3, \dots, n. \end{aligned} \tag{P2}$$

5. Procedure for Proposed Algorithms for solution of CTP and NTP

5.1. Basic Assumptions of the Proposed Algorithms

The total transportation cost does not depends on the mode of transportation and distance, also the framework of the problem will be denoted by either crisp or SVTrNN.

$$\text{If } \sum_{i=0}^m \tilde{a}_i^N < \text{ or } > \sum_{j=0}^n \tilde{b}_j^N, \forall i, j, \text{ then first one can make sure to balance the TP as } \sum_{i=0}^m \tilde{a}_i^N = \sum_{j=0}^n \tilde{b}_j^N, \forall i, j,$$

5.2. Steps for solution of CTP after balancing by existing method

- Step5.2.1.** To change the each neutrosophic cost \tilde{c}_{ij}^N , neutrosophic supply \tilde{a}_i^N and neutrosophic demand \tilde{b}_j^N of NTP in cost matrix to crisp values, we use here score function method i.e. we convert these by using $S(\tilde{A}^N)$.
- Step5.2.2.** For balance TP, verify that the sum of demands is equal to the sum of supply i.e. $\sum_{i=0}^m \tilde{a}_i = \sum_{j=0}^n \tilde{b}_j, \forall i, j$. If $\sum_{i=0}^m \tilde{a}_i < \text{ or } > \sum_{j=0}^n \tilde{b}_j, \forall i, j$, then first one can make sure to balance the TP by adding a row or column with zero entries in cost matrix [30].
- Step5.2.3.** After conversion of NTP into TP, choose the minimum entry in each row and subtract it to all other entries in that row. The same way is applicable in each column to find minimum one zero in each row and each column in TP matrix. For better (see table 4 and table 6).
- Step5.2.4.** Verify that the sum of demands is greater than the supply in each row and the sum of supplies are greater than the demand in each column, if ok go on step 5.2.6, otherwise go on step 5.2.5.
- Step5.2.5.** Draw the horizontal and vertical lines that cover all the zeros and equal to minimum number of order of matrix or reduced table. Now if number of lines is less than to the minimum number, revise table by choose the least element which is not under horizontal or vertical line and add it to the entry at the cross point of the lines. Again go to step 5.2.3 to check the condition.
- Step5.2.6.** To allot the maximum possible units of supply or demand in the cost cell, choose a cell of maximum cost in the reduced cost matrix. If the maximum cost exists more than one place, choose any one cell of maximum supply or demand.
- Step5.2.7.** If none cell occur for the maximum cost then go for next maximum. If such cell does not occur for any value, then choose any cell at random, whose reduced cost is zero.
- Step5.2.8.** From the reduced table omit the row which are fully exhausted or column which are fully satisfied, then repeats steps and again. Repeat the procedure until all the demand units and all the supply units are fully received respectively.

The procedure for the solution of NTP by using existing method is same as steps in 5.2, while the cost, demand, supply and solution vales are in SVTrNN.

5.3. Steps for solution of CTP after balancing by MRCM

For balance the unbalance CTP, we use MRCM which is generalization of method in [27]. We use the following steps for solution of CTP by MRCM:

Step5.3.1. Convert neutrosophic cost \tilde{c}_{ij}^N , neutrosophic supply \tilde{a}_i^N and neutrosophic demand \tilde{b}_j^N of NTP in cost matrix to crisp values by using score function $S(\tilde{A}^N)$ i.e NTP convert into CTP.

Step5.3.2 If CTP is unbalance then to make it balance one by applying the steps of MRCM that is if sum of supply is less than the sum of demand, then add a row of minimum costs in each row with a supply equal to sum of supplies and add a column of minimum costs in each column with demand equal to the difference value from sum of all supplies differ to sum of demand. The same is applicable when sum of demand is less than the sum of supply. i.e.

$$\tilde{a}_{m+1} = \sum_{i=0}^m \tilde{a}_i \quad \text{and} \quad \tilde{b}_{n+1} = \sum_{j=0}^n \tilde{a}_j \oplus \text{excess supply,}$$

or

$$\tilde{b}_{n+1} = \sum_{j=0}^n \tilde{b}_j \quad \text{and} \quad \tilde{a}_{m+1} = \sum_{i=1}^m \tilde{b}_i \oplus \text{excess demand.}$$

The unit transportation costs are taken as follows:

$$\tilde{c}_{i(n+1)} = \min_{1 \leq j \leq n} \tilde{c}_{ij}, \quad 1 \leq i \leq m, \quad \tilde{c}_{(m+1)j} = \min_{1 \leq i \leq m} \tilde{c}_{ij}, \quad 1 \leq j \leq n,$$

$$\tilde{c}_{ij} = \tilde{c}_{ji}, \quad 1 \leq i \leq m, \quad 1 \leq j \leq n, \quad \text{and} \quad \tilde{c}_{(m+1)(n+1)} = 0.$$

Step5.3.3 Obtain optimal solution of converted CTP after balance it by existing method using excel solver. Let the crisp optimal solution be \tilde{x}_{ij} , $1 \leq i \leq m+1$, $1 \leq j \leq n+1$.

Step5.3.4 By assuming $\tilde{\omega}_{m+1}^i = 0$ and using the relation $\tilde{\omega}_i^i \oplus \tilde{v}_j^j = \tilde{\sigma}_{ij}^i$ for basic variables, find the values of all the dual variables $\tilde{\omega}_i^i$, $1 \leq i \leq m$ and \tilde{v}_j^j , $1 \leq j \leq n+1$,

Step5.3.5. According to MRCM, $\tilde{\omega}_i^i = \tilde{\omega}_i$ and $\tilde{v}_j^j = \tilde{v}_j$ for $1 \leq i \leq m, 1 \leq j \leq n$, obtain only central rank zero duals. After that in terms of original supply S_i and demand M_j find the neutrosophic optimal solution of the problem.

5.4. Steps for solution of NTP after balancing by MRCM

Step5.4.1. Convert neutrosophic cost \tilde{c}_{ij}^N , neutrosophic supply \tilde{a}_i^N and neutrosophic demand \tilde{b}_j^N of NTP in cost matrix to crisp values by using score function $S(\tilde{A}^N)$ to check either it is balance or unbalance.

Step5.4.2 If NTP is unbalance than same procedure as in 5.3 is applicable. i.e.

$$\tilde{a}_{m+1}^N = \sum_{i=0}^m \tilde{a}_i^N \quad \text{and} \quad \tilde{b}_{n+1}^N = \sum_{j=0}^n \tilde{a}_j^N \oplus \text{excess supply,}$$

or

$$\tilde{b}_{n+1}^N = \sum_{j=0}^n \tilde{b}_j^N \quad \text{and} \quad \tilde{a}_{m+1}^N = \sum_{i=1}^m \tilde{b}_i^N \oplus \text{excess demand.}$$

The unit transportation costs are taken as follows:

$$\tilde{c}_{i(n+1)}^N = \min_{1 \leq j \leq n} \tilde{c}_{ij}^N, \quad 1 \leq i \leq m, \quad \tilde{c}_{(m+1)j}^N = \min_{1 \leq i \leq m} \tilde{c}_{ij}^N, \quad 1 \leq j \leq n,$$

$$\tilde{c}_{ij}^N = \tilde{c}_{ji}^N, \quad 1 \leq i \leq m, \quad 1 \leq j \leq n, \quad \text{and} \quad \tilde{c}_{(m+1)(n+1)}^N = 0.$$

Step5.4.3 Obtain optimal solution of NTP by excel solver. Let the neutrosophic optimal solution obtained be \tilde{x}_{ij}^N , $1 \leq i \leq m+1$, $1 \leq j \leq n+1$.

Step5.4.4 By assuming $\tilde{\omega}_{m+1}^N = 0$ and using the relation $\tilde{\omega}_i^N \oplus \tilde{v}_j^N = \tilde{\sigma}_{ij}^N$ for basic variables, find the values of all the dual variables $\tilde{\omega}_i^N$, $1 \leq i \leq m$ and \tilde{v}_j^N , $1 \leq j \leq n+1$,

Step5.4.5. According to MRCM, $\tilde{\omega}_i^N = \tilde{\omega}_i^N$ and $\tilde{v}_j^N = \tilde{v}_j^N$ for $1 \leq i \leq m, 1 \leq j \leq n$, obtain only central rank zero duals.

6. Numerical Example

6.1. Modal I (NTP of type-3)

Let us consider a NTP with three sources say S_1, S_2, S_3 in which wheat are initially stored and ready to transport in three flour mills namely M_1, M_2, M_3 with unit transportation cost, demand and supply are as SVTrNN. The input data of SVTrNN -TP given in table 2 as follows:

Table 2

	M_1	M_2	M_3	<i>Supply</i>
S_1	$\left\langle \left(\begin{matrix} (3,5,7.5,11) \\ (2,4,7,10) \\ (1,3,5,6,9) \end{matrix} \right) \right\rangle_{0.2;0.4;0.8}$	$\left\langle \left(\begin{matrix} (2,4.5,10,15) \\ (0.5,3.5,8,14) \\ (0,2.5,6,12) \end{matrix} \right) \right\rangle_{0.3;0.5;0.8}$	$\left\langle \left(\begin{matrix} (1,5,9,14.5) \\ (-3,3.5,8,12) \\ (-4,2,7,11) \end{matrix} \right) \right\rangle_{0.2;0.5;0.7}$	$\left\langle \left(\begin{matrix} (9,17,26,36) \\ (6,14,23,33) \\ (3,11,20,30) \end{matrix} \right) \right\rangle_{0.4;0.7;0.7}$
S_2	$\left\langle \left(\begin{matrix} (1,7,11.5,16) \\ (-1,5,10,14) \\ (-3,3,8.5,12) \end{matrix} \right) \right\rangle_{0.4;0.5;0.7}$	$\left\langle \left(\begin{matrix} (1,6,9.5,12) \\ (-1,4,8.5,11) \\ (-2,2,8,10) \end{matrix} \right) \right\rangle_{0.5;0.7;0.8}$	$\left\langle \left(\begin{matrix} (-1,4,8,15) \\ (-2,3,6,12) \\ (-3,2,5,11) \end{matrix} \right) \right\rangle_{0.3;0.6;0.6}$	$\left\langle \left(\begin{matrix} (7,17,25,31) \\ (3,12,22.5,29) \\ (1,10,19.5,27) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7}$
S_3	$\left\langle \left(\begin{matrix} (3,6,9,12) \\ (2,5,8,11) \\ (1,4,7,10) \end{matrix} \right) \right\rangle_{0.2;0.4;0.8}$	$\left\langle \left(\begin{matrix} (-1,3.5,9,12) \\ (-2,2.5,7,11) \\ (-4,1,5,10) \end{matrix} \right) \right\rangle_{0.2;0.4;0.8}$	$\left\langle \left(\begin{matrix} (0,5,8,14) \\ (-2,3,7,12) \\ (-4,1,6,10) \end{matrix} \right) \right\rangle_{0.2;0.6;0.6}$	$\left\langle \left(\begin{matrix} (9,16,22,31) \\ (5,14,20,27) \\ (1,12,18,23) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7}$
<i>Demand</i>	$\left\langle \left(\begin{matrix} (12,21,30,37) \\ (9,19,28,34) \\ (6,16,25,33) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7}$	$\left\langle \left(\begin{matrix} (10,16,22,27) \\ (5,14,20,25) \\ (0,12,18,23) \end{matrix} \right) \right\rangle_{0.4;0.7;0.7}$	$\left\langle \left(\begin{matrix} (7,12,19,27) \\ (4,11,18,24) \\ (1,9,15,21) \end{matrix} \right) \right\rangle_{0.2;0.6;0.6}$	

6.2. Neutrosophic optimal solution with score function method

One can use score function to convert SVTrNN cost, demand and supply to obtain the crisp numbers in TP of table 2 as follows:

$$S(\tilde{A}^N) = \left(\frac{p_1 + p_2 + p_3 + p_4 - q_1 - q_2 - q_3 - q_4 - r_1 - r_2 - r_3 - r_4}{12} \right) \times (2 + w_{\tilde{A}^N} - u_{\tilde{A}^N} - v_{\tilde{A}^N})$$

Here $S(\tilde{c}_{11}^N) = \left\langle \left(\begin{matrix} (3,5,7.5,11) \\ (2,4,7,10) \\ (1,3,5,6,9) \end{matrix} \right) \right\rangle_{0.2;0.4;0.8} = \left(\frac{3+5+7.5+11-2-4-7-10-1-3.5-6-9}{12} \right) \times (0.2+0.6+0.2) = -1.33$

$S(\tilde{c}_{12}^N) = -1.25, S(\tilde{c}_{13}^N) = -0.58, S(\tilde{c}_{21}^N) = -1.08, S(\tilde{c}_{22}^N) = -1.00, S(\tilde{c}_{23}^N) = -0.67, S(\tilde{c}_{31}^N) = -1.50,$
 $S(\tilde{c}_{32}^N) = -0.50, S(\tilde{c}_{33}^N) = -0.50, S(\tilde{a}_1^N) = -4.33, S(\tilde{a}_2^N) = -3.67, S(\tilde{a}_3^N) = -3.50, S(\tilde{b}_1^N) = -5.83,$
 $S(\tilde{b}_2^N) = -3.50, S(\tilde{b}_3^N) = -3.17.$

After converting cost, demand and supply of NTP from SVTrNN to the crisp numbers by using score function method, the unbalance CTP cost matrix is given in table 3:

Table 3

	M ₁	M ₂	M ₃	Supply
S ₁	-1.33	-1.25	-0.58	-4.33
S ₂	-1.08	-1.00	-0.67	-3.67
S ₃	-1.50	-0.50	-0.50	-3.50
Demand	-5.83	-3.50	-3.17	

By using the steps in 5.2, the optimal crisp solution of CTP and their allotment of demand and supply in cost matrix shown in table 4:

Table 4

	M ₁	M ₂	M ₃	Supply
S ₁	-1.33(-2.16)	-1.25(-2.17)	-0.58	-4.33
S ₂	-1.08(-3.67)	-1.00	-0.67	-3.67
S ₃	-1.50	-0.50(-0.33)	-0.50(-3.17)	-3.50
S ₄	0	0(-1.00)	0	-1.00
Demand	-5.83	-3.50	-3.17	

The complete solution of CTP is x₁₁ = -2.16, x₁₂ = -2.17, x₂₁ = -3.67, x₃₂ = -0.33, x₃₃ = -3.17, x₄₂ = -1.00, and $\tilde{Z} = 11.30$. The corresponding optimal solution of NTP with allotment of SVTrNN is shown in table 5 as follows:

Table 5

	M ₁	M ₂	M ₃	Supply
S ₁	$\left\langle \left(\begin{matrix} (-19,-4,13,30) \\ (-20,-3.5,16,31) \\ (-21,-3.5,15,32) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7}$	$\left\langle \left(\begin{matrix} (-21,4,30,55) \\ (-25,-2,26.5,53) \\ (-29,-4,23.5,51) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7}$	-	-
S ₂	$\left\langle \left(\begin{matrix} (7,17,25,31) \\ (3,12,22.5,29) \\ (1,10,19.5,27) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7}$	-	-	-
S ₃	-	$\left\langle \left(\begin{matrix} (-18,-3,10,24) \\ (-19,-4,9,23) \\ (-20,-3,9,22) \end{matrix} \right) \right\rangle_{0.2;0.6;0.6}$	$\left\langle \left(\begin{matrix} (7,12,19,27) \\ (4,11,18,24) \\ (1,9,15,21) \end{matrix} \right) \right\rangle_{0.2;0.6;0.6}$	-
S ₄	-	$\left\langle \left(\begin{matrix} (-69,-24,21,66) \\ (-71,-21.5,26,69) \\ (-73,-20.5,25,72) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7}$	-	-
Demand	-	-	-	-

$$\tilde{Z}^N = \left\langle \left(\begin{matrix} (3,5,7,5,11) \\ (2,4,7,10) \\ (1,3,5,6,9) \end{matrix} \right) \right\rangle_{0.2;0.4;0.8} \left\langle \left(\begin{matrix} (-19,-4,13,30) \\ (-20,-3.5,16,31) \\ (-21,-3.5,15,32) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7} + \left\langle \left(\begin{matrix} (2,4,5,10,15) \\ (0,5,3,5,8,14) \\ (0,2,5,6,12) \end{matrix} \right) \right\rangle_{0.3;0.5;0.8} \left\langle \left(\begin{matrix} (-21,4,30,55) \\ (-25,-2,26.5,53) \\ (-29,-4,23.5,51) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7}$$

i.e.

$$+ \left\langle \left(\begin{matrix} (1,7,11,5,16) \\ (-1,5,10,14) \\ (-3,3,8.5,12) \end{matrix} \right) \right\rangle_{0.4;0.5;0.7} \left\langle \left(\begin{matrix} (7,17,25,31) \\ (3,12,22.5,29) \\ (1,10,19.5,27) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7} + \left\langle \left(\begin{matrix} (-1,3,5,9,12) \\ (-2,2,5,7,11) \\ (-4,1,5,10) \end{matrix} \right) \right\rangle_{0.2;0.4;0.8} \left\langle \left(\begin{matrix} (-18,-3,10,24) \\ (-19,-4,9,23) \\ (-20,-3,9,22) \end{matrix} \right) \right\rangle_{0.2;0.6;0.6}$$

$$+ \left\langle \left(\begin{matrix} (0,5,8,14) \\ (-2,3,7,12) \\ (-4,1,6,10) \end{matrix} \right) \right\rangle_{0.2;0.6;0.6} \left\langle \left(\begin{matrix} (7,12,19,27) \\ (4,11,18,24) \\ (1,9,15,21) \end{matrix} \right) \right\rangle_{0.2;0.6;0.6} + \left\langle \left(\begin{matrix} (0,0,0,0) \\ (0,0,0,0) \\ (0,0,0,0) \end{matrix} \right) \right\rangle_{0.2;0.6;0.6} \left\langle \left(\begin{matrix} (-69,-24,21,66) \\ (-71,-21.5,26,69) \\ (-73,-20.5,25,72) \end{matrix} \right) \right\rangle_{0.3;0.6;0.7}$$

$$\tilde{Z}^N = \left\langle \left(\begin{matrix} (-74,187.5,927,2317) \\ (-25.5,62,738,1999) \\ (52,13.75,531.75,1654) \end{matrix} \right) \begin{matrix} 0.4 \\ 0.4 \\ 0.6 \end{matrix} \right\rangle = -194.54$$

Now for application of MRCM, we use steps in 5.3 to balance the unbalance CTP of table 2 as follows in table 6:

Table 6

	M ₁	M ₂	M ₃	M ₄	Supply
S ₁	$\left\langle \left(\begin{matrix} (3,5,7,5,11) \\ (2,4,7,10) \\ (1,3,5,6,9) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.4 \\ 0.8 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (2,4,5,10,15) \\ (0,5,3,5,8,14) \\ (0,2,5,6,12) \end{matrix} \right) \begin{matrix} 0.3 \\ 0.5 \\ 0.8 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (1,5,9,14,5) \\ (-3,3,5,8,12) \\ (-4,2,7,11) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.5 \\ 0.7 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (1,5,9,14,5) \\ (-3,3,5,8,12) \\ (-4,2,7,11) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.5 \\ 0.7 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (9,17,26,36) \\ (6,14,23,33) \\ (3,11,20,30) \end{matrix} \right) \begin{matrix} 0.4 \\ 0.7 \\ 0.7 \end{matrix} \right\rangle$
S ₂	$\left\langle \left(\begin{matrix} (1,7,11,5,16) \\ (-1,5,10,14) \\ (-3,3,8,5,12) \end{matrix} \right) \begin{matrix} 0.4 \\ 0.5 \\ 0.7 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (1,6,9,5,12) \\ (-1,4,8,5,11) \\ (-2,2,8,10) \end{matrix} \right) \begin{matrix} 0.5 \\ 0.7 \\ 0.8 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (-1,4,8,15) \\ (-2,3,6,12) \\ (-3,2,5,11) \end{matrix} \right) \begin{matrix} 0.3 \\ 0.6 \\ 0.6 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (-1,4,8,15) \\ (-2,3,6,12) \\ (-3,2,5,11) \end{matrix} \right) \begin{matrix} 0.3 \\ 0.6 \\ 0.6 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (7,17,25,31) \\ (3,12,22,5,29) \\ (1,10,19,5,27) \end{matrix} \right) \begin{matrix} 0.3 \\ 0.6 \\ 0.7 \end{matrix} \right\rangle$
S ₃	$\left\langle \left(\begin{matrix} (3,6,9,12) \\ (2,5,8,11) \\ (1,4,7,10) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.4 \\ 0.8 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (-1,3,5,9,12) \\ (-2,2,5,7,11) \\ (-4,1,5,10) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.4 \\ 0.8 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (0,5,8,14) \\ (-2,3,7,12) \\ (-4,1,6,10) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.6 \\ 0.6 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (0,5,8,14) \\ (-2,3,7,12) \\ (-4,1,6,10) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.6 \\ 0.6 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (9,16,22,31) \\ (5,14,20,27) \\ (1,12,18,23) \end{matrix} \right) \begin{matrix} 0.3 \\ 0.6 \\ 0.7 \end{matrix} \right\rangle$
S ₄	$\left\langle \left(\begin{matrix} (1,7,11,5,16) \\ (-1,5,10,14) \\ (-3,3,8,5,12) \end{matrix} \right) \begin{matrix} 0.4 \\ 0.5 \\ 0.7 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (-1,3,5,9,12) \\ (-2,2,5,7,11) \\ (-4,1,5,10) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.4 \\ 0.8 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (0,5,8,14) \\ (-2,3,7,12) \\ (-4,1,6,10) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.6 \\ 0.6 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (0,0,0,0) \\ (0,0,0,0) \\ (0,0,0,0) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.6 \\ 0.6 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (25,50,73,98) \\ (14,40,65,5,89) \\ (5,33,57,5,80) \end{matrix} \right) \begin{matrix} 0.3 \\ 0.6 \\ 0.7 \end{matrix} \right\rangle$
Demand	$\left\langle \left(\begin{matrix} (12,21,30,37) \\ (9,19,28,34) \\ (6,16,25,33) \end{matrix} \right) \begin{matrix} 0.3 \\ 0.6 \\ 0.7 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (10,16,22,27) \\ (5,14,20,25) \\ (0,12,18,23) \end{matrix} \right) \begin{matrix} 0.4 \\ 0.7 \\ 0.7 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (7,12,19,27) \\ (4,11,18,24) \\ (1,9,15,21) \end{matrix} \right) \begin{matrix} 0.2 \\ 0.6 \\ 0.6 \end{matrix} \right\rangle$	$\left\langle \left(\begin{matrix} (-41,29,97,167) \\ (-55,14,87,160) \\ (-67,8,78,153) \end{matrix} \right) \begin{matrix} 0.3 \\ 0.6 \\ 0.7 \end{matrix} \right\rangle$	

After converting cost, demand and supply of NTP in table 6 from SVTrNN to the crisp numbers by using score function method, the balance CTP cost matrix is given in table 7:

Table 7

	M ₁	M ₂	M ₃	M ₄	Supply
S ₁	-1.33	-1.25	-0.58	-0.58	-4.33
S ₂	-1.08	-1.00	-0.67	-0.67	-3.67
S ₃	-1.50	-0.50	-0.50	-0.50	-3.50
S ₄	-1.08	-0.50	-0.50	0	-11.50
Demand	-5.83	-3.50	-3.17	-10.50	

The complete allotment of demand and supply in cost matrix of CTP shown in table 8:

Table 8

	M ₁	M ₂	M ₃	M ₄	Supply
S ₁	-1.33(-1.16)	-1.25	-0.58(-3.17)	-0.58	-4.33
S ₂	-1.08(-3.67)	-1.00	-0.67	-0.67	-3.67
S ₃	-1.50	-0.50(-3.50)	-0.50	-0.50	-3.50
S ₄	-1.08(-1.00)	-0.50	-0.50	0(-10.50)	-11.50
Demand	-5.83	-3.50	-3.17	-10.50	

The optimal crisp solution and minimum cost of balance CTP of table 8 is $x_{11} = -1.16$, $x_{13} = -3.17$, $x_{21} = -3.67$, $x_{32} = -3.50$, $x_{41} = -1.00$, $x_{44} = -10.50$ and $Z = 10.18$.

Similarly after balance the unbalance NTP by MRCM, the corresponding optimal solution of balance NTP with allotment of SVTrNN is shown in table 9 as follows:

Table 9

	M_1	M_2	M_3	M_4	Supply
S_1	$\left\langle \begin{matrix} (-18,-2,14,29) \\ (-18,-4,12,29) \\ (-18,-4,11,29) \end{matrix} \right\rangle_{0.4,0.6,0.6}$	-	$\left\langle \begin{matrix} (7,12,19,27) \\ (4,11,18,24) \\ (1,9,15,21) \end{matrix} \right\rangle_{0.2,0.6,0.6}$	-	-
S_2	$\left\langle \begin{matrix} (7,17,25,31) \\ (3,12,22.5,29) \\ (1,10,19.5,27) \end{matrix} \right\rangle_{0.3,0.6,0.7}$	-	-	-	-
S_3	-	$\left\langle \begin{matrix} (9,16,22,31) \\ (5,14,20,27) \\ (1,12,18,23) \end{matrix} \right\rangle_{0.3,0.6,0.7}$	-	-	-
S_4	$\left\langle \begin{matrix} (-142,-47,44,139) \\ (-146,-47,51.5,144) \\ (-148,-45,49.5,147) \end{matrix} \right\rangle_{0.3,0.6,0.7}$	-	-	$\left\langle \begin{matrix} (-41,29,97,167) \\ (-55,14,87,160) \\ (-67,8,78,153) \end{matrix} \right\rangle_{0.3,0.6,0.7}$	-
Demand		-	-	-	-

$$\begin{aligned} \tilde{Z}^N = & \left\langle \begin{matrix} (3,5,7.5,11) \\ (2,4,7,10) \\ (1,3.5,6,9) \end{matrix} \right\rangle_{0.2,0.4,0.8} \cdot \left\langle \begin{matrix} (-18,-2,14,29) \\ (-18,-4,12,29) \\ (-18,-4,11,29) \end{matrix} \right\rangle_{0.4,0.6,0.6} + \left\langle \begin{matrix} (1,5,9,14.5) \\ (-3,3.5,8,12) \\ (-4,2,7,11) \end{matrix} \right\rangle_{0.2,0.5,0.7} \cdot \left\langle \begin{matrix} (7,12,19,27) \\ (4,11,18,24) \\ (1,9,15,21) \end{matrix} \right\rangle_{0.2,0.6,0.6} + \\ & + \left\langle \begin{matrix} (1,7,11.5,16) \\ (-1,5,10,14) \\ (-3,3,8.5,12) \end{matrix} \right\rangle_{0.4,0.5,0.7} \cdot \left\langle \begin{matrix} (7,17,25,31) \\ (3,12,22.5,29) \\ (1,10,19.5,27) \end{matrix} \right\rangle_{0.3,0.6,0.7} + \left\langle \begin{matrix} (-1,3.5,9,12) \\ (-2,2.5,7,11) \\ (-4,1.5,10) \end{matrix} \right\rangle_{0.2,0.4,0.8} \cdot \left\langle \begin{matrix} (9,16,22,31) \\ (5,14,20,27) \\ (1,12,18,23) \end{matrix} \right\rangle_{0.3,0.6,0.7} + \\ & + \left\langle \begin{matrix} (1,7,11.5,16) \\ (-1,5,10,14) \\ (-3,3,8.5,12) \end{matrix} \right\rangle_{0.4,0.5,0.7} \cdot \left\langle \begin{matrix} (-142,-47,44,139) \\ (-146,-47,51.5,144) \\ (-148,-45,49.5,147) \end{matrix} \right\rangle_{0.3,0.6,0.7} + \left\langle \begin{matrix} (0,0,0,0) \\ (0,0,0,0) \\ (0,0,0,0) \end{matrix} \right\rangle_{0.2,0.6,0.6} \cdot \left\langle \begin{matrix} (-41,29,97,167) \\ (-55,14,87,160) \\ (-67,8,78,153) \end{matrix} \right\rangle_{0.3,0.6,0.7} \end{aligned}$$

this implies $\tilde{Z}^N = \left\langle \begin{matrix} (-191,-104,1267.5,3802.5) \\ (85,-117.5,1108,3297) \\ (415,-89,847.5,2810) \end{matrix} \right\rangle_{0.4,0.6} = -417.77$

6.3. Model II (NTP of type-2)

For solution of NTP of type-2 i.e. a problem in which costs are in SVTrNN while demand and supply are given in crisp numbers. Here we are taking the problem in table 2 in which costs are in SVTrNN while demand and supply are as crisp numbers given as follows in table 10:

Table 10

	M_1	M_2	M_3	<i>Supply</i>
S_1	$\left\langle \begin{pmatrix} (3,5,7.5,11) \\ (2,4,7,10) \\ (1,3.5,6,9) \end{pmatrix} \right\rangle_{0.2;0.4;0.8}$	$\left\langle \begin{pmatrix} (2,4.5,10,15) \\ (0.5,3.5,8,14) \\ (0,2.5,6,12) \end{pmatrix} \right\rangle_{0.3;0.5;0.8}$	$\left\langle \begin{pmatrix} (1,5,9,14.5) \\ (-3,3.5,8,12) \\ (-4,2,7,11) \end{pmatrix} \right\rangle_{0.2;0.5;0.7}$	-4.33
S_2	$\left\langle \begin{pmatrix} (1,7,11.5,16) \\ (-1,5,10,14) \\ (-3,3,8.5,12) \end{pmatrix} \right\rangle_{0.4;0.5;0.7}$	$\left\langle \begin{pmatrix} (1,6,9,5,12) \\ (-1,4,8.5,11) \\ (-2,2,8,10) \end{pmatrix} \right\rangle_{0.5;0.7;0.8}$	$\left\langle \begin{pmatrix} (-1,4,8,15) \\ (-2,3,6,12) \\ (-3,2,5,11) \end{pmatrix} \right\rangle_{0.3;0.6;0.6}$	-3.67
S_3	$\left\langle \begin{pmatrix} (3,6,9,12) \\ (2,5,8,11) \\ (1,4,7,10) \end{pmatrix} \right\rangle_{0.2;0.4;0.8}$	$\left\langle \begin{pmatrix} (-1,3.5,9,12) \\ (-2,2.5,7,11) \\ (-4,1,5,10) \end{pmatrix} \right\rangle_{0.2;0.4;0.8}$	$\left\langle \begin{pmatrix} (0,5,8,14) \\ (-2,3,7,12) \\ (-4,1,6,10) \end{pmatrix} \right\rangle_{0.2;0.6;0.6}$	-3.50
<i>Demand</i>	-5.83	-3.50	-3.17	

The optimal crisp solution of NTP type-2 is shown in table 11 as follows:

Table 11

	M_1	M_2	M_3	<i>Supply</i>
S_1	-2.16	-2.17	-	-4.33
S_2	-3.67	-	-	-3.67
S_3	-	-0.33	-3.17	-3.50
S_4	-	-1.00	-	-3.50
<i>Demand</i>	-5.83	-3.50	-3.17	

The corresponding neutrosophic solution of NTP type-2 is:

$$\begin{aligned} \tilde{Z}_{t_2}^N = & -2.16 \times \left\langle \begin{pmatrix} (3,5,7.5,11) \\ (2,4,7,10) \\ (1,3.5,6,9) \end{pmatrix} \right\rangle_{0.2;0.4;0.8} - 2.17 \times \left\langle \begin{pmatrix} (2,4.5,10,15) \\ (0.5,3.5,8,14) \\ (0,2.5,6,12) \end{pmatrix} \right\rangle_{0.3;0.5;0.8} - 3.67 \times \left\langle \begin{pmatrix} (1,7,11.5,16) \\ (-1,5,10,14) \\ (-3,3,8.5,12) \end{pmatrix} \right\rangle_{0.4;0.5;0.7} \\ & - 0.33 \times \left\langle \begin{pmatrix} (-1,3.5,9,12) \\ (-2,2.5,7,11) \\ (-4,1,5,10) \end{pmatrix} \right\rangle_{0.2;0.4;0.8} - 3.17 \times \left\langle \begin{pmatrix} (0,5,8,14) \\ (-2,3,7,12) \\ (-4,1,6,10) \end{pmatrix} \right\rangle_{0.2;0.6;0.6} = \left\langle \begin{pmatrix} (-14.16, -63.27, -108.44, -163.37) \\ (5.26, -44.93, -93.68, -145.07) \\ (22.85, -27.5, -77.87, -124.52) \end{pmatrix} \right\rangle_{0.2;0.6;0.6} = 15.89 \end{aligned}$$

Similarly after balance the unbalance NTP of type-2 by MRCM, the corresponding optimal neutrosophic solution of balance NTP of type-2 with allotment is as follows:

$$\begin{aligned} \tilde{Z}_{t_2}^N = & -1.16 \times \left\langle \begin{pmatrix} (3,5,7.5,11) \\ (2,4,7,10) \\ (1,3.5,6,9) \end{pmatrix} \right\rangle_{0.2;0.4;0.8} - 3.67 \times \left\langle \begin{pmatrix} (1,5,9,14.5) \\ (-3,3.5,8,12) \\ (-4,2,7,11) \end{pmatrix} \right\rangle_{0.2;0.5;0.7} - 3.17 \times \left\langle \begin{pmatrix} (1,7,11.5,16) \\ (-1,5,10,14) \\ (-3,3,8.5,12) \end{pmatrix} \right\rangle_{0.4;0.5;0.7} - 3.50 \times \left\langle \begin{pmatrix} (-1,3.5,9,12) \\ (-2,2.5,7,11) \\ (-4,1,5,10) \end{pmatrix} \right\rangle_{0.2;0.4;0.8} \\ & - 1.00 \times \left\langle \begin{pmatrix} (1,7,11.5,16) \\ (-1,5,10,14) \\ (-3,3,8.5,12) \end{pmatrix} \right\rangle_{0.4;0.5;0.7} - 10.50 \times \left\langle \begin{pmatrix} (0,0,0,0) \\ (0,0,0,0) \\ (0,0,0,0) \end{pmatrix} \right\rangle_{0.2;0.6;0.6} = \left\langle \begin{pmatrix} 7.82, 65.59, 121.85, 174.70 \\ -19.86, 47.09, 103.68, 152.52 \\ -40.03, 27.41, 85.60, 135.85 \end{pmatrix} \right\rangle_{0.2;0.6;0.6} = 14.35 \end{aligned}$$

7. Comparative Study

Real life application of single valued trapezoidal neutrosophic numbers in transportation problem have been solved by some existing and proposed MRCM methods. In present paper, the minimum cost obtained through proposed method with some existing method discussed in [30] have been summarized in table 12. From the table it is clear that minimum cost obtained by using MRCM is better than to the existing method in both either crisp or in neutrosophic environment. Figure 3 shows the graphical representation of the minimum crisp or neutrosophic cost degree of satisfaction by different approaches.

Comparison				
Model	Balance by existing method		Balance by MRCM	
	Model I	Crisp cost of CTP $Z = 11.30$	The neutrosophic cost of NTP $\tilde{Z}^N = \left\langle \left(\begin{matrix} (-74,187.5,927,2317) \\ (-25.5,62,738,1999) \\ (52,13.75,531.75,1654) \end{matrix} \right) \begin{matrix} 0.4 \\ ;0.4 \\ 0.6 \end{matrix} \right\rangle$ corresponding Crisp cost of NTP $\tilde{Z}^N = -194.54$	Crisp cost of CTP $Z = 10.18$
Model II	The neutrosophic cost of NTP $\tilde{Z}_{t_2}^N = \left\langle \left(\begin{matrix} (-14.16,-63.27,-108.44,-163.37) \\ (5.26,-44.93,-93.68,-145.07) \\ (22.85,-27.5,-77.87,-124.52) \end{matrix} \right) \begin{matrix} 0.2 \\ ;0.6 \\ 0.6 \end{matrix} \right\rangle$		The neutrosophic cost of NTP $\tilde{Z}_{t_2}^N = \left\langle \left(\begin{matrix} (7.82,65.59,121.85,174.70) \\ (-19.86,47.09,103.68,152.52) \\ (-40.03,27.41,85.60,135.85) \end{matrix} \right) \begin{matrix} 0.2 \\ ;0.6 \\ 0.6 \end{matrix} \right\rangle$	
	corresponding Crisp cost of NTP $\tilde{Z}_{t_2}^N = 15.89$		corresponding Crisp cost of NTP $\tilde{Z}_{t_2}^N = 14.35$	

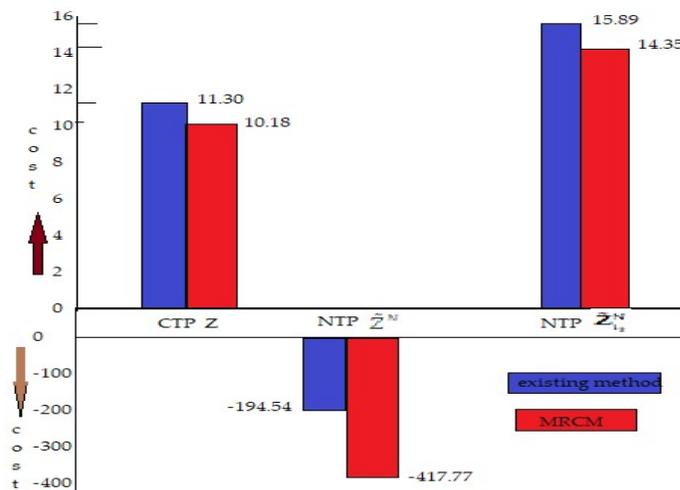


Figure 3: Comparison of results with proposed MRCM and existing method

8. Result and discussion

In this present study the optimal transportation crisp cost and optimal transportation neutrosophic cost of unbalanced NTP using MRCM is minimum than the existing method in [30]. It is also verified that in de-neutrosophication, the crisp values before and after conversion from neutrosophic to crisp and crisp to neutrosophic are different in score function method.

For the real life applications one can find the degree of result. The best of minimum neutrosophic cost of unbalanced NTP is $\tilde{z}^N = \left\langle \left(\begin{matrix} (-191, -104, 1267.5, 3802.5) \\ (85, -117.5, 1108, 3297) \\ (415, -89, 847.5, 2810) \end{matrix} \right); 0.4 \right\rangle$ i.e. total minimum transportation cost

lies between -191 to 3802.5 for level of truthfulness or acceptance, 85 to 3297 for level of indeterminacy and 415 to 2810 for level of falsity. The degree of truthfulness or acceptance, degree of indeterminacy and degree of falsity is defined as $w_{T_{\tilde{z}^N}}(x) \times 100$, $u_{I_{\tilde{z}^N}}(x) \times 100$ and $v_{F_{\tilde{z}^N}}(x) \times 100$ respectively, where x denotes the total cost and

$$w_{T_{\tilde{z}^N}}(x) = \begin{cases} \frac{0.4(x + 191)}{191 - 104}, & \text{for } -191 \leq x \leq -104, \\ 0.4, & \text{for } -104 \leq x \leq 1267.5, \\ \frac{0.4(3802.5 - x)}{3802.5 - 1267.5}, & \text{for } 1267.5 \leq x \leq 3802.5, \\ 0, & \text{for otherwise.} \end{cases}$$

$$u_{I_{\tilde{z}^N}}(x) = \begin{cases} \frac{(-117.5 - x) + 0.4(x - 85)}{62 + 25.5}, & \text{for } -117.5 \leq x \leq 85, \\ 0.4, & \text{for } -117.5 \leq x \leq 1108, \\ \frac{(x - 1108) + 0.4(3297 - x)}{3297 - 1108}, & \text{for } 1108 \leq x \leq 3297, \\ 0, & \text{for otherwise.} \end{cases}$$

$$v_{F_{\tilde{z}^N}}(x) = \begin{cases} \frac{(-89 - x) + 0.6(x - 415)}{13.75 - 52}, & \text{for } -89 \leq x \leq 415, \\ 0.6, & \text{for } -89 \leq x \leq 847, \\ \frac{(x - 847) + 0.6(2810 - x)}{2810 - 847}, & \text{for } 847 \leq x \leq 2810, \\ 0, & \text{for otherwise.} \end{cases}$$

$x \rightarrow$ Degree \downarrow	-100	0	500	1000	2000	3000
$w_{T_{\tilde{z}^N}} \times 100$	40.0	40.0	40.0	40.0	30.0	12.6
$u_{I_{\tilde{z}^N}} \times 100$	40.0	40.0	40.0	40.0	64.4	91.8
$v_{F_{\tilde{z}^N}} \times 100$	60.0	60.0	60.0	63.1	83.4	0

With the help of degree of truthfulness, degree of indeterminacy and degree of falsity, we can conclude the total neutrosophic cost from the range of -191 to 3802.5 for truthfulness, 85 to 3297 for indeterminacy and 415 to 2810 for falsity to scheduled the transportation and budget allocation.

9. Conclusions

In recent scenario the applied mathematical modeling with uncertainty or vagueness is necessity of the society. Nowadays the concept of neutrosophic number is very popular to handle such type of problems. In this research paper, we study of unbalance NTP and introduced a new approach MRCM for optimal solution with the concept of single valued trapezoidal neutrosophic number of twelve components from different viewpoints. Also the optimal neutrosophic solution and minimum cost obtained by using MRCM is better than by using some existing methods. The proposed method provides the more practical structure and considers the various characteristics of transportation problems in neutrosophic environment. In future the proposed MRCM is applied to the unbalance multi-attribute transportation problem, assignment problems and multilevel programming problem in neutrosophic environment. The present research will be a mile stone for transportation problems with generalization of the pick value of truth, indeterminacy and falsity functions by considering, which are very important for uncertainty theory.

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