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A Novel Method for Neutrosophic Assignment Problem by using Interval-Valued Trapezoidal Neutrosophic Number

Hamiden Abd El- Wahed Khalifa[1], and Pavan Kumar[2,*]

[1] Operations Research Department, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt, Email: hamiden@cu.edu.eg

[1] Present Address: Mathematics Department, College of Science and Arts, Al- Badaya, Qassim University, Qassim, Saudi Arabia, Email: Ha.Ahmed@qu.edu.sa

[2] Mathematics Department, School of Applied Science & Languages, VIT Bhopal University, Sehore, Madhya Pradesh, INDIA-466114, E-mail: pavankmaths@gmail.com , pavan.kumar@vitbhopal.ac.in

* Correspondence: pavankmaths@gmail.com

Abstract: Assignment problem (AP) is well- studied and important area in optimization. In this research manuscript, an assignment problem in neutrosophic environment, called as neutrosophic assignment problem (NAP), is introduced. The problem is proposed by using the interval-valued trapezoidal neutrosophic numbers in the elements of cost matrix. As per the concept of score function, the interval-valued trapezoidal neutrosophic assignment problem (IVTNAP) is transformed to the corresponding an interval-valued AP. To optimize the objective function in interval form, we use the order relations. These relations are the representations of choices of decision maker. The maximization (or minimization) model with objective function in interval form is changed to multi- objective based on order relations introduced by the decision makers' preference in case of interval profits (or costs). In the last, we solve a numerical example to support the proposed solution methodology.

Keywords: Assignment problem; Interval-valued trapezoidal neutrosophic numbers; Score function; Interval-valued assignment problem; Multi-objective assignment problem; Weighting Tchebycheff program; Decision Making.

Glossary	
AP: Assignment problem.	LP : Linear programming.
DM: Decision makers.	MOLP: Multi-objective linear programming
FN-LPP: Fuzzy neutrosophic LPP.	MOAP: Multi-objective assignment problem
GAMS: General Algebraic Modeling System.	MOOP: Multi-objective optimization problem.
IVN : Interval-valued neutrosophic.	NAP: Neutrosophic assignment problem.

IVTNAP: Interval- valued trapezoidal neutrosophic assignment.	

1. Introduction

In important real-life applications, an AP appears such as production planning, telecommunication, resource scheduling, vehicle routing and distribution, economics, plant location and flexible manufacturing systems, and attracts more and more researchers' attention [10, 13, 37], where it deals with the question how to set n number of people or machines to m number of works in such a way that an optimal assignment can be obtained to minimize the cost (or maximize the profit).

Following these research objectives, the DM has to make an attempt for the optimization of models starting from linear AP to nonlinear AP. In view of this, the linear AP is a special kind of linear programming problem (LPP) where the people or machines are being assigned to various works as one to one rule so that the assignment profit (or cost) is optimized. An optimal assignee for the work is a good description of the AP, where number of rows is equal to the number of columns as explained in Ehrgott et al. [14]. A new approach was developed to study the assignment problem with several objectives, by Bao et al. [4], which was followed with applications to determine the cost-time AP problem as multiple criteria decision making problem by Geetha and Nair [16].

Few decades ago, a large number of authors and policy makers around the world have investigated the basic idea of fuzzy sets. The theory of fuzzy sets was, first, originated by Zadeh [45], which has been intensely applied to study several practical problems, including financial risk management. Then the fuzzy concept is also represented by fuzzy constraints and / or fuzzy quantities. Dubois and Prade [13] suggested the implementation of algebraic operations on crisp numbers to fuzzy numbers with the help of fuzzification method. However, AP representing real-life scenario consists of a set of parameters. The values of these parameters are set by decision makers. DMs required fixing exact values to the parameters that in the conventional approach. In that case, DMs do not precisely estimate the exact value of parameters, therefore the model parameters are generally defined in an uncertain manner. Zimmermann [46] was the first solved LP model having many objectives through suitable membership functions. Bellmann and Zadeh [6] implemented fuzzy set notion to the decision-making problem consisting of imprecision as well as uncertainty.

Sakawa and Yano [39] suggested the idea of fuzzy multiobjective linear programming (MOLP) problems. Hamadameen [18] derived an approach for getting the optimal solution of fuzzy MOLP model considering the coefficients of objective function as triangular fuzzy numbers. The fuzzy MOLP problem was reduced to crisp MOLP with the help of ranking function as explained by Wang [42]. Thereafter, the problem was solved with the help of the fuzzy programming method. Leberling [28] solved vector maximum LP problem using a particular kind of nonlinear membership functions. Bit et al. [7] applied fuzzy methodology for multiple objective transportation model. Belacela and Boulasselb [5] studied a multiple criteria fuzzy AP. Lin and Wen [29] designed an algorithm for the solution of fuzzy AP problem. Kagade and Bajaj [22] discussed interval numbers cost coefficients MOAP problem. Yang and Liu [44] developed a Tabu search method with the help of fuzzy simulation to determine an optimal solution to the fuzzy AP. Moreover, De and Yadav [11] proposed a solution approach to MOAP with the implementation of fuzzy goal programming technique. Mukherjee and Basu [32] solved fuzzy cost AP problem using the ranking method introduced by Yager [43]. Pramanik and Biswas [36] studied multi-objective AP with imprecise costs, time and ineffectiveness. Haddad et al. [17] investigated some generalized AP models in imprecise environment. Emrouznejad et al. An alternative development was suggested for the fuzzy AP with fuzzy profits or fuzzy costs for all possible assignments as explained by Emrouznejad et al. [15]. Kumar and Gupta [26] investigated a methodology to solve fuzzy AP as well as fuzzy travelling salesman problem under various membership functions and ranking index introduced by Yager [43]. Medvedeva and Medvedev [31] applied the concept of the primal and dual for getting the optimal solution to a MOAP. Hamou and Mohamed [19] applied the branch & bound based method to generate the set of each efficient solution to MOAP. Jayalakshmi and Sujatha [21] investigated a novel procedure, referred as optimal flowing method providing the ideal and set of all efficient solutions. Pandian and Anuradha [34] investigated a novel methodology to determine the optimal solution of the problem consisting of zero-point method which was introduced by Pandian and Natarajan [33].

Khalifa and Al- Shabi [23] studied the multi-objective assignment problem with trapezoidal fuzzy numbers. They introduced an interactive approach for solving it and then determined the stability set of the first kind corresponding the solution. Khalifa [25] introduced an approach based on the Weighting Tchebycheff program to solve the multi- objective assignment problem in neutrosophic environment.

The extension of intuitionistic fuzzy set is the neutrosophic set. The neutrosophic set consists of three defining functions. These functions are the membership function, the non-membership function, and the indeterminacy function. All these functions are entirely independent to each other. A new solution approach for the FN-LPP was proposed with real life application by Abdel *et al.* [3]. Kumar et al. [27] investigated a novel solution procedure for the computation of fuzzy pythagorean transportation problem, where they extended the interval basic feasible solution, then existing optimality method to obtain the cost of transportation. Khalifa et al. [24] studied the complex programming problem with neutrosophic concept. They applied the lexicographic order to determine the optimal solution of neutrosophic complex programming. Vidhya et al. [41] studied neutrosophic MOLP problem. Pramanik and Banerjee [35] proposed a goal programming methodology to MOLP problem under neutrosophic numbers. Broumi and Smarandaache [8] introduced some novel operations for interval neutrosophic sets in terms of arithmetic, geometrical, and harmonic means. Rizk-Allah et al. [38] suggested a novel compromise approach for many objective transportation problem, which was further studied by Zimmermann's fuzzy programming approach as well as the neutrosophic set terminology. Abdel- Basset et al. [1] introduced a plithogenic multi- criteria decision- making model based on neutrosophic analytic hierarchy process in order of performance by similarity to the ideal solution of financial performance. Abdel- Basset et al. [2] evaluated a set of measurements for providing sustainable supply chain finance in the gas industry in the uncertain environment. Abdel- Basset et al. [3] proposed an integrated method based on neutrosophic set to evaluate innovation value for smart product- service systems.

In this paper, the assignment problem having interval- valued trapezoidal neutrosophic numbers in all the parameters is introduced. This problem is converted into two objectives assignment problem, then the Weighting Tchebycheff program with the ideal targets are applied for solving it.

The outlay of the proposed research article is organized as follows: In the next Section, we present some sort of preliminaries, which is essential for the present study. Section 3 formulate interval-valued trapezoidal neutrosophic assignment problem. Section 4 proposes solution approach for the determination of preferred solution. A numerical example is solved, in Section 5, to support the efficiency of the solution approach. In the last, some concluding remarks as well as the further research directions are summarized in Section 6.

2. Preliminaries

This section introduces some of basic concepts and results related to fuzzy numbers, neutrosophic set, and their arithmetic operations.

Definition 1. A fuzzy set $\widetilde{P}$ defined on the set of real numbers $\mathbb{R}$ is called fuzzy number when the membership function

$\mu_{\widetilde{P}}(x)\colon \mathbb{R} \to [0,1]$, have the following properties:

1. $\mu_{\widetilde{P}}(x)$ is an upper semi-continuous membership function;
2. $\widetilde{P}$ is convex fuzzy set, i.e., $\mu_{\widetilde{P}}(\delta\, x + (1-\delta)\, y) \geq \min\{\mu_{\widetilde{P}}(x), \mu_{\widetilde{P}}(y)\}$ for all $x, y \in \mathbb{R}; 0 \leq \delta \leq 1$;
3. $\widetilde{P}$ is normal, i.e., $\exists\, x_0 \in \mathbb{R}$ for which $\mu_{\widetilde{P}}(x_0) = 1$;
4. $\text{Supp}\,(\widetilde{P}) = \{x \in \mathbb{R}: \mu_{\widetilde{P}}(x) > 0\}$ is the support of $\widetilde{P}$, and the closure $\text{cl}(\text{Supp}(\widetilde{P}))$ is compact set.

Definition 2. (Ishibuchi and Tanaka [20). An interval on $\mathbb{R}$ is defined as

$A = [a^L, a^R] = \{a: , a^L \leq a \leq a^R, a \in \mathbb{R}\}$, where a^L is left limit and a^R is right limit of A.

Definition 3. (Ishibuchi and Tanaka [20]).The interval is also defined by

$A = \langle a_C, a_W \rangle = \{\mathbf{a}: a_C - a_W \leq \mathbf{a} \leq a_C + a_W, \mathbf{a} \in \mathbb{R}\}$, where $a_C = \frac{1}{2}(a^R + a^L)$ is center and $a_W = \frac{1}{2}(a^R - a^L)$ is width of A.

Definition 4. (Neutrosophic set, Wang et al. [42]). Let X be a nonempty set. Then a neutrosophic set $\overline{P}_N$ of nonempty set X is defined as

$$\overline{P}_N = \left\{\langle x; , T_{\overline{P}_N}, I_{\overline{P}_N}, F_{\overline{P}_N} \rangle : x \in X\right\},$$

where $T_{\overline{P}_N}$, $I_{\overline{P}_N}$, $F_{\overline{P}_N}: X \rightarrow]0_-, 1^+[$ define respectively the degree of membership function, the degree of indeterminacy , and the degree of non-membership of element $x \in X$ to the set $\overline{P}_N$ with the condition:

$$0_- \leq T_{\overline{P}_N} + I_{\overline{P}_N} + F_{\overline{P}_N} \leq 3^+. \quad (1)$$

Definition 5. (Interval-valued neutrosophic set, Broumi and Smarandache [8]). Let X be a nonempty set. Then an interval valued neutrosophic (IVN) set $\overline{P}_N^{IV}$ of X is defined as:

$$\overline{P}_N^{IV} = \left\{\langle x; \left[T^L_{\overline{P}_N}, T^U_{\overline{P}_N}\right], \left[I^L_{\overline{P}_N}, I^U_{\overline{P}_N}\right], \left[F^L_{\overline{P}_N}, F^U_{\overline{P}_N}\right] \rangle : x \in X\right\},$$

where $\left[T^L_{\overline{P}_N}, T^U_{\overline{P}_N}\right], \left[I^L_{\overline{P}_N}, I^U_{\overline{P}_N}\right]$, and $\left[F^L_{\overline{P}_N}, F^U_{\overline{P}_N}\right] \subset [0,1]$ for each $x \in X$.

Definition 6. (Broumi and Smarandache [8]). Let

$$\overline{P}_N^{IV} = \left\{\langle x; \left[T^L_{\overline{P}_N}, T^U_{\overline{P}_N}\right], \left[I^L_{\overline{P}_N}, I^U_{\overline{P}_N}\right], \left[F^L_{\overline{P}_N}, F^U_{\overline{P}_N}\right] \rangle : x \in X\right\} \text{ be IVNS, then}$$

(i) $\overline{P}_N^{IV}$ is empty if $T^L_{\overline{P}_N} = T^U_{\overline{P}_N} = 0, I^L_{\overline{P}_N} = I^U_{\overline{P}_N} = 1, F^L_{\overline{P}_N} = F^U_{\overline{P}_N} = 1$, for all $x \in \overline{P}_N$,

(ii) Let $\underline{0} = \langle x;\ 0,1,1 \rangle$, and $\underline{1} = \langle x;\ 1,0,0 \rangle$.

Definition 7. (Interval-valued trapezoidal neutrosophic number). Let $u_{\tilde{a}}, v_{\tilde{a}}, w_{\tilde{a}} \subset [0,1]$, and $a_1, a_2, a_3, a_4 \in \mathbb{R}$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$. Then an interval-valued trapezoidal fuzzy neutrosophic number,

$$\tilde{a} = \langle (a_1, a_2, a_3, a_4);\ [u^L_{\tilde{a}}, u^U_{\tilde{a}}], [v^L_{\tilde{a}}, v^U_{\tilde{a}}], [w^L_{\tilde{a}}, w^U_{\tilde{a}}] \rangle,$$

whose degrees of membership function, the degrees of indeterminacy, and the degrees of non-membership are

$$\vartheta_{\tilde{a}}(x) = \begin{cases} u_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \leq x \leq a_2, \\ u_{\tilde{a}}, & \text{for } a_2 \leq x \leq a_3, \\ u_{\tilde{a}}\left(\frac{a_4-x}{a_4-a_3}\right), & \text{for } a_3 \leq x \leq a_4, \\ 0, & \text{Otherwise,} \end{cases}$$

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x+v_{\tilde{a}}(x-a_1)}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2, \\ v_{\tilde{a}}, & \text{for } a_2 \leq x \leq a_3, \\ \frac{x-a_3+v_{\tilde{a}}(a_4-x)}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4, \\ 1, & \text{Otherwise,} \end{cases} \quad (2)$$

$$\varphi_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x+w_{\tilde{a}}(x-a_1)}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2, \\ w_{\tilde{a}}, & \text{for } a_2 \leq x \leq a_3, \\ \frac{x-a_3+w_{\tilde{a}}(a_4-x)}{a_4-a_3}, & \text{for } a_3 \leq x \leq a_4, \\ 1, & \text{Otherwise.} \end{cases}$$

Where, $u_{\tilde{a}}$, $v_{\tilde{a}}$, and $w_{\tilde{a}}$ are the upper bound of membership degree, lower bound of indeterminacy degree, and lower bound of non-membership degree, respectively.

Definition 8. (Arithmetic operations). Let $\tilde{a} = \langle(a_1, a_2, a_3, a_4);\ [u_{\tilde{a}}^L, u_{\tilde{a}}^U], [v_{\tilde{a}}^L, v_{\tilde{a}}^U], [w_{\tilde{a}}^L, w_{\tilde{a}}^U]\rangle$, and

$\tilde{b} = \langle(b_1, b_2, b_3, b_4);\ [u_{\tilde{b}}^L, u_{\tilde{b}}^U], [v_{\tilde{b}}^L, v_{\tilde{b}}^U], [w_{\tilde{b}}^L, w_{\tilde{b}}^U]\rangle$ be two IVN numbers. Then,

1. $\tilde{a} \oplus \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4): A, B, C\rangle,$
2. $\tilde{a} \ominus \tilde{b} = \langle(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1): A, B, C\rangle,$
3. $\tilde{a} \odot \tilde{b} = \begin{cases} \langle(a_1b_1, a_2b_2, a_3b_3, a_4b_4): A, B, C\rangle, \text{if } a_4 > 0, b_4 > 0, \\ \langle(a_1b_4, a_2b_3, a_3b_2, a_4b_1): A, B, C\rangle, \text{if } a_4 < 0, b_4 > 0\ , \\ \langle(a_4b_4, a_3b_3, a_2b_2, a_1b_1): A, B, C\rangle, \text{if } a_4 < 0, b_4 < 0. \end{cases}$
4. $\tilde{a} \oslash \tilde{b} = \begin{cases} \langle(a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1): A, B, C\rangle, \text{if } a_4 > 0, b_4 > 0, \\ \langle(a_4/b_4, a_3/b_3, a_2/b_2, a_1/b_1): A, B, C\rangle, \text{if } a_4 < 0, b_4 > 0 \\ \langle(a_4/b_1, a_3/b_2, a_2/b_3, a_1/b_4): A, B, C\rangle, \text{if } a_4 < 0, b_4 < 0. \end{cases}$
5. $k\,\tilde{a} = \begin{cases} \langle(ka_1, k\,a_2, ka_3, ka_4);\ [u_{\tilde{a}}^L, u_{\tilde{a}}^U], [v_{\tilde{a}}^L, v_{\tilde{a}}^U], [w_{\tilde{a}}^L, w_{\tilde{a}}^U]\rangle, \text{if } k > 0 \\ \langle(ka_4, ka_3, ka_2, ka_1);\ [u_{\tilde{a}}^L, u_{\tilde{a}}^U], [v_{\tilde{a}}^L, v_{\tilde{a}}^U], [w_{\tilde{a}}^L, w_{\tilde{a}}^U]\rangle, k < 0. \end{cases}$
6. $\tilde{a}^{-1} = \langle(1/a_4, 1/a_3, 1/a_2, 1/a_1);\ [u_{\tilde{a}}^L, u_{\tilde{a}}^U], [v_{\tilde{a}}^L, v_{\tilde{a}}^U], [w_{\tilde{a}}^L, w_{\tilde{a}}^U]\rangle, \tilde{a} \neq 0.$

Where, $A = [\min(u_{\tilde{a}}^L, u_{\tilde{b}}^L,), \min(u_{\tilde{a}}^U, u_{\tilde{b}}^U)], B = [\max(v_{\tilde{a}}^L, v_{\tilde{b}}^L,), \max(v_{\tilde{a}}^U, v_{\tilde{b}}^U)],$ and

$C = [\max(w_{\tilde{a}}^L, w_{\tilde{b}}^L,), \max(w_{\tilde{a}}^U, w_{\tilde{b}}^U)].$

Definition 9. (Score function, Tharmaraiselvi and Santhi [40]). The score function for the IVN number $\tilde{a} = \langle(a_1, a_2, a_3, a_4);\ [u_{\tilde{a}}^L, u_{\tilde{a}}^U], [v_{\tilde{a}}^L, v_{\tilde{a}}^U], [w_{\tilde{a}}^L, w_{\tilde{a}}^U]\rangle$ is defined as

$$S(\tilde{a}) = \frac{1}{16}(a_1 + a_2 + a_3 + a_4) \times [\vartheta_{\tilde{a}} + (1 - \mu_{\tilde{a}}) + (1 - \varphi_{\tilde{a}})].$$

3. Problem statement and solution concepts

3.1 Assumptions, Index and notation

3.1.1. Assumption

We assume that there are n number of jobs, which must be performed by and n persons, where the costs are based on the specific assignments. Each job must be assigned to exactly one person and each person has to perform exactly one job.

3.1.2. Index

i: Persons.

j: Jobs.

3.1.3. Notation

$(\tilde{c}_{ij})_N^{IV}$: Interval-valued trapezoidal neutrosophic cost of ith person assigned to jth job.

x_{ij}: Number of jth jobs assigned to ith person.

Consider the following interval-valued trapezoidal neutrosophic assignment problem (IVTNAP)

(IVTNAP) Min $\tilde{Z}_N^{IV} = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\tilde{c}_{ij}\right)_N^{IV} x_{ij}$

Subject to

$\sum_{i=1}^{n} x_{ij} = 1, j = 1, 2, \ldots, n$ (only one person would be assigned the jth job)

$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots, n$ (only one job selected by ith person)

$x_{ij} = 0$ or 1.

It obvious that $\left(\tilde{c}_{ij}\right)_N^{IV}$ $(i = j = 1, 2, 3, \ldots, n; 1, 2, 3, \ldots, K)$ are interval-valued trapezoidal neutrosophic numbers.

Based on score function defined in Definition 9, the IVTNAP in converted into the following interval-valued assignment problem (IVAP)

(IVAP) Min $Z^{IV} = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[c_{ij}^{L}, c_{ij}^{U}\right] x_{ij}$

Subject to

$x \in X' = \left\{ \sum_{i=1}^{n} x_{ij} = 1, j = 1, 2, \ldots, n; \sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots, n; x_{ij} = 0 \text{ or } 1\right\}.$

Definition 10. $x \in X'$ is solution of problem IVAP if and only if there is no $\hat{x} \in X'$ satisfies $Z(\hat{x}) \leq_{LR} Z(x)$, or $Z(\hat{x}) <_{CW} Z$.

Or equivalently,

Definition 11. $x \in X'$ is solution of problem IVAP if and only if there is no $\hat{x} \in X'$ satisfies that $Z(\hat{x}) \leq_{RC} Z(x)$.

The solution set of problem IVAP can be obtained as the efficient solution of the following MOAP:

$$\begin{aligned} &\text{Min } (Z^R, Z^C) \\ &\text{Subject to } \quad x \in X'. \end{aligned} \tag{3}$$

Using the Weighting Tchebycheff problem, the Problem (3) is described in the following form

$$\begin{aligned} &\text{Min } \psi \\ &\text{Subject to} \\ &\quad w_1\left[Z^R - \hat{Z}^R\right] \leq \psi, \\ &\quad w_2\left[Z^C - \hat{Z}^C\right] \leq \psi, \\ &\quad x \in X'. \end{aligned} \tag{4}$$

Where $w_1, w_2 \geq 0$; $\hat{Z}^R$, and $\hat{Z}^C$ are defined as the ideal targets.

4. Solution procedure

The steps of the solution procedure to solve the IVTNAP can be summarized as:

Step 1: Formulate the IVTNAP

Step 2: Convert the IVTNAP using the score function (Definition 9) into the IVAP.

Step 3: Estimate the ideal points $\hat{Z}^R$ and $\hat{Z}^C$ for the IVAP from the following relation

$\hat{Z}^R = \text{Min } Z^R,$

Subject to $x \in X'$, and

$$\hat{Z}^C = \text{Min } Z^C,$$

$$\text{Subject to } x \in X'.$$

Step 4: Determine the value of individual maximum and minimum for every objective function subject to given constraints.

Step 5: Compute the weights from the relation

$$w_1 = \frac{\overline{Z}^R - \underline{Z}^R}{(\overline{Z}^R - \underline{Z}^R) + (\overline{Z}^C - \underline{Z}^C)},\ w_2 = \frac{\overline{Z}^C - \underline{Z}^C}{(\overline{Z}^R - \underline{Z}^R) + (\overline{Z}^C - \underline{Z}^C)} \quad (5)$$

Here $\overline{Z}^R$, $\overline{Z}^C$ and $\underline{Z}^R$, $\underline{Z}^C$ are the value of individual maximum and minimum of the Z^R, and Z^C, respectively.

Step 6: Applying the GAMS software to problem (5) to obtain the optimum compromise solution, and hence the fuzzy cost.

Step 7: Stop.

The flowchart of the proposed method is presented in Figure 1, below.

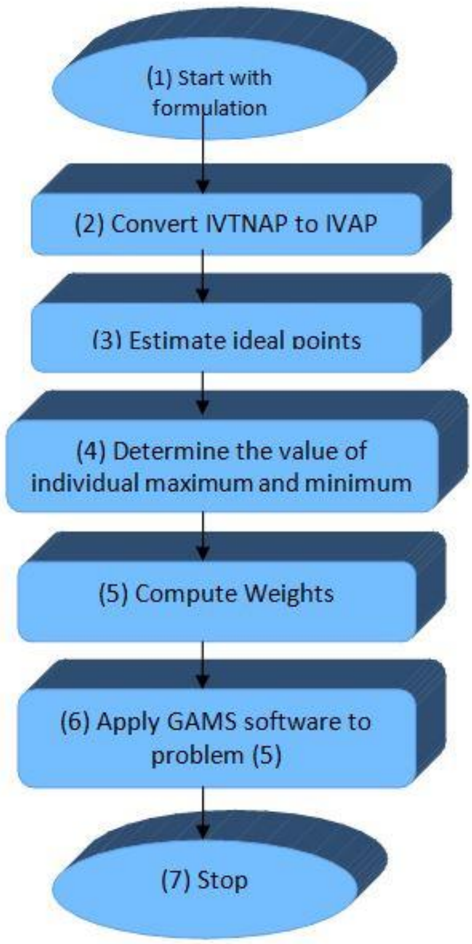

Figure 1: Flowchart of the proposed method

5. Numerical example

Consider the following IVTNAP

$$\text{Min } Z(x)_N^{IV} = \begin{pmatrix} \langle(14,17,21,28);[0.7,0.9],[0.1,0.3],[0.5,0.7]\rangle\, x_{11} \\ \oplus \langle((13,18,20,24));[0.5,0.7],[0.3,0.5],[0.4,0.6]\rangle\, x_{12} \\ \oplus \langle(20,25,30,35);[0.8,1.0],[0.2,0.4],[0.1,0.3]\rangle\, x_{13} \\ \oplus \langle(15,18,23,30);[0.7,1.0],[0.2,0.3],[0.2,0.5]\rangle x_{21} \\ \oplus \langle(6,10,13,15);[0.6,0.8],0.1,0.4],[0.2,0.6]\rangle x_{22} \\ \oplus \langle(15,18,23,30);[0.7,0.9],[\,0.1,0.4],[\,0.3,0.5]\rangle x_{23} \\ \langle(13,18,20,24);[0.3,0.7],[0.1,0.4],[0.3,0.7]\rangle x_{31} \\ \oplus \langle(13,18,20,24);[0.2,0.7],0.2,0.5],[0.3,0.6]\rangle x_{32} \\ \oplus \langle(14,16,21,23);[0.6,0.8],[\,0.3,0.6],[0.2,0.4]\rangle x_{33} \end{pmatrix}$$

Subject to

$$\sum_{i=1}^{3} x_{ij} = 1,\ j = 1,2,3;\ \sum_{j=1}^{3} x_{ij} = 1,\ i = 1,2,3,$$
$$x_{ij} = 0 \text{ or } 1.$$

Step 2:

$$\text{Min } Z(x)^{IV} = \begin{pmatrix} [8.5,11.5]x_{11} + [6.5625,9.375]x_{12} + [14.4375,18.5625]x_{13} \\ [10.2125,13.975]x_{21} + [4.4,6.875]x_{22} + [9.675,13.4375]x_{23} \\ [5.625,10.78125]x_{31} + [5.15625,10.3125]x_{32} + [7.4,10.6375]x_{33} \end{pmatrix}$$

Subject to

$$\sum_{i=1}^{3} x_{ij} = 1,\ j = 1,2,3;\ \sum_{j=1}^{3} x_{ij} = 1,\ i = 1,2,3,$$
$$x_{ij} = 0 \text{ or } 1.$$

Step 4: We determine optimal solution for the following problems individually with respect to the given constraints:

$$\hat{Z}^R = \text{Min } Z^R = \begin{pmatrix} 11.5x_{11} + 9.375x_{12} + 18.5625x_{13} + 13.975x_{21} + 6.875x_{22} \\ 13.4375x_{23} + 10.78125x_{31} + 10.3125x_{32} + 10.6375x_{33} \end{pmatrix}$$

$$\hat{Z}^C = \text{Min } Z^C = \begin{pmatrix} 10x_{11} + 7.96875x_{12} + 16.5x_{13} + 12.09375x_{21} + 5.6375x_{22} \\ 11.55625x_{23} + 8.203125x_{31} + 7.734375x_{32} + 9.01875x_{33} \end{pmatrix}$$

$$\text{Max } Z^R = \begin{pmatrix} 11.5x_{11} + 9.375x_{12} + 18.5625x_{13} + 13.975x_{21} + 6.875x_{22} \\ 13.4375x_{23} + 10.78125x_{31} + 10.3125x_{32} + 10.6375x_{33} \end{pmatrix}$$

$$\text{Max } Z^C = \begin{pmatrix} 10x_{11} + 7.96875x_{12} + 16.5x_{13} + 12.09375x_{21} + 5.6375x_{22} \\ 11.55625x_{23} + 8.203125x_{31} + 7.734375x_{32} + 9.01875x_{33} \end{pmatrix}$$

Subject to

$$\sum_{i=1}^{3} x_{ij} = 1,\ j = 1,2,3;\ \sum_{j=1}^{3} x_{ij} = 1,\ i = 1,2,3,$$
$$X_{ij} = 0 \text{ or } 1.$$

$\hat{Z}^R = \text{Min } Z^R = 29.01$, $\hat{Z}^C = \text{Min } Z^C = 24.66$, $\text{Max } Z^R = 42.85$, $\text{Max } Z^C = 36.33$

Step 5: Calculate the weights

$$w_1 = \frac{13.84}{25.15} = 0.542532, \quad w_2 = \frac{11.67}{25.51} = 0.45747$$

Step6: Determine the optimal solution of the problem:

$$\text{Min } \psi$$

Subject to

$$\begin{pmatrix} 11.5x_{11} + 9.375x_{12} + 18.5625x_{13} + 13.975x_{21} + 6.875x_{22} \\ 13.4375x_{23} + 10.78125x_{31} + 10.3125x_{32} + 10.6375x_{33} - 1.84321\psi \end{pmatrix} \le 29.01,$$

$$\begin{pmatrix} 10x_{11} + 7.96875x_{12} + 16.5x_{13} + 12.09375x_{21} + 5.6375x_{22} \\ 11.55625x_{23} + 8.203125x_{31} + 7.734375x_{32} + 9.01875x_{33} - 2.18595\psi, \end{pmatrix} \le 24.66,$$

$$x \in X'.$$

The optimal compromise solution is $x_{11} = 1, x_{22} = 1, \ x_{33} = 1$,
$x_{12} = x_{13} = \ x_{21} = x_{23} = x_{31} = x_{32} = 0$, and $\psi = 0.0014$.
So, the interval-valued trapezoidal neutrosophic optimum value is
$Z(x)_N^{IV} = \langle(34, 43, 55, 66); [0.6, 0.8], [0.3, 0.6], [0.5, 0.7]\rangle$.
It is evident that the total minimum assigned cost will be greater than 34 and less than 66 . The total minimum assigned cost lies in between 43 and 55, the overall satisfaction lies in between 60% and 80%. Then, for the remaining of total minimum assigned cost, the truthfulness degree is

$$\vartheta_{\tilde{a}}(x) \times 100 = \begin{cases} [0.6, 0.8]\left(\frac{x-34}{43-34}\right), & \text{for } 34 \le x \le 43, \\ [0.6, 0.8], & \text{for } 43 \le x \le 55, \\ [0.6, 0.8]\left(\frac{66-x}{66-55}\right), & \text{for } 55 \le x \le 66, \\ 0, & \text{Otherwise,} \end{cases}$$

Also, the indeterminacy and falsity degrees for the assigned cost are

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{43-x+[0.3,0.6](x-34)}{43-34}, & \text{for } 34 \le x \le 43, \\ v_{\tilde{a}}, & \text{for } 43 \le x \le 55, \\ \frac{x-55+[0.3,0.6](66-x)}{66-55}, & \text{for } 55 \le x \le 66, \\ 1, & \text{Otherwise,} \end{cases}$$

$$\varphi_{\tilde{a}}(x) = \begin{cases} \frac{43-x+[0.5,0.7](x-34)}{43-34}, & \text{for } 34 \le x \le 43, \\ w_{\tilde{a}}, & \text{for } 43 \le x \le 55, \\ \frac{x-55+[0.5,0.7](66-x)}{66-55}, & \text{for } 55 \le x \le 66 \\ 1, & \text{Otherwise.} \end{cases}$$

Thus, the DM concludes that the total interval-valued trapezoidal neutrosophic assigned cost lies in between 34 and 66 with truth, indeterminacy, and falsity degrees lies in between $[0.6, 0.8]$, $[0.3, 0.6]$, and $[0.5, 0.7]$, respectively, and also he is able to schedule the assignment and constraints under budgetary.

6. Concluding remarks and further research directions

The present research article addressed a novel solution methodology to the assignment problem with objective function coefficients characterized by interval-valued trapezoidal neutrosophic numbers. The problem is transformed to the corresponding interval–valued problem, and hence into the multi-objective optimization problem (MOOP). Then, the so obtained MOOP is undertaken for the solution by using the Weighting Tchebycheff problem beside the GAMS software. The advantage of this approach is more flexible than the standard assignment problem, where it allows the DM to choose the targets he is willing.

For further research, one may incorporate this concept in transportation model. Also, one may consider the stochastic nature in assignment problem and develop the same methodology to solve the problem. Additionally, one possible extension might be explored by considering the fuzzy-random, fuzzy-stochastic, etc. In addition, the proposed solution methodology may be applied in different branches (viz. management science, financial management and decision science) where the assignment problems occur in neutrosophic environment.

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Conflicts and Interest

The author declares no conflict of interest.

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