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Introduction Totally and Partial Connectivity Indices in Neutrosophic graphs with Application in Behavioral Sciences

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Abstract: Connectivity is one of the most important concepts in graph theory. Since Neutrosophic Graphs are a branch of graphs, connectivity will be very important in this branch as well. In this paper, we will define the connectivity in Neutrosophic graphs using the strength of connectedness between each pair of its vertices. Also in this article, we define two new concepts of Partial connectivity index and totally connectivity index. We present several theorems related to these concepts and prove the theorems.

Keywords: neutrosophic graphs; partial connectivity index; totally connectivity index; m-barbell graph; connected neutrosophic graph

1. Introduction

Neutrosophic graphs are a new branch of graphs that has been very popular among graph theorists in recent decades. Neutrosophic graphs are a generalized form of fuzzy graph theory. One of the features that have been considered in fuzzy graphs is connectivity and types of connectivity indices in fuzzy graphs [7]. The connectivity index is a numerical quantity that can be used to calculate some of the properties of the studied graph in more detail. Many researchers have pointed to different uses of neutrosophic Graphs, such as the use of neutrosophic sets and graphs in medicine [3], social media [4], decision-making problem [9], Economics Theorizing [11] and so on. In this article, after introducing the partial connectivity index and totally connectivity index in neutrosophic graphs, we will point out some applications of it.

In our previous article [8], we also presented the correlation index in neutrosophic graphs and gave an example of its applications. In the following works, we will compare and examine the strengths and weaknesses of each.

2. Preliminaries

In this section, some of the important and basic concepts required are given by mentioning the source.

Definition 1. [4] A single-valued neutrosophic graph on a nonempty V is a pair $G = (N, M)$. Where N is single-valued neutrosophic set in V and M single-valued neutrosophic relation on V such that

$$T_M(uv) \leq \min\{T_N(u), T_N(v)\},$$
$$I_M(uv) \leq \min\{I_N(u), I_N(v)\},$$
$$F_M(uv) \leq \max\{F_N(u), F_N(v)\},$$

For all $u, v \in V$. N is called single-valued neutrosophic vertex set of G and, M is called single-valued neutrosophic edge set of G, respectively.

Definition 2. [4] Let $G = (N, M)$ be the Neutrosophic Graph of G^*. If $H = (N', M')$ is a neutrosophic graph of G^* such that

$$\begin{array}{llll} T'(u) \leq T(u), & I'(u) \geq I(u), & F'(u) \geq F(u), & \forall u \in X, \\ T_{M'}(uv) \leq T_M(uv), & I_{M'}(uv) \geq I_M(uv), & F_{M'}(uv) \geq F_M(uv), & \forall uv \in E, \end{array}$$

Then H is called a Neutrosophic subgraph of the Neutrosophic graph G.

Definition 3. [4] A neutrosophic graph $G = (N, M)$ is called complete if the following conditions are satisfied:

$$T_M(uv) = \min\{T_N(u), T_N(v)\},$$
$$I_M(uv) = \min\{I_N(u), I_N(v)\},$$
$$F_M(uv) = \max\{F_N(u), F_N(v)\},$$

For all $u, v \in V$.

Definition 4. [4] A neutrosophic graph $G_1 = (N_1, M_1)$ of the graph $G_1^* = (V_1, E_1)$ is isomorphic with neutrosophic graph $G_2 = (N_2, M_2)$ of the graph $G_2^* = (V_2, E_2)$ if we have f where $f: V_1 \to V_2$ is a bijection and following relations are satisfied

$$T_{N_1}(u) = T_{N_2}(f(u)), \quad I_{N_1}(u) = I_{N_2}(f(u)), \quad F_{N_1}(u) = F_{N_2}(f(u)),$$

For all $u \in V_1$ and

$$T_{M_1}(uv) = T_{M_2}(f(u)f(v)), \quad I_{M_1}(uv) = I_{M_2}(f(u)f(v)), \quad F_{M_1}(uv) = F_{M_2}(f(u)f(v)),$$

For all $uv \in E_1$.

Definition 5. [4] the m-barbell graph $B_{(m,m)}$ is the simple graph obtained by connecting two copies of a complete graph K_m by abridge.

3. Totally and Partial connectivity index

In this section, which is the main part of the article, we first define the connected neutrosophic graph and connectivity index in the neutrosophic graphs. Note that definitions are provided for a connected neutrosophic graph in some references [5, 6], but the definition we use here will be based on connectivity. After providing some examples, the theorems related to the connectivity index are expressed and proved in neutrosophic graphs.

3.1. Partial connectivity index in neutrosophic graphs

Here we first define the Partial and totally connectivity indices in neutrosophic graphs and provide examples to better understand it. And then in the next part we will present the boundaries for the Partial and totally connectivity indices in neutrosophic graphs.

Definition 6. Let $G = (N, M)$ be the connected Neutrosophic Graph. The **partial connectivity index** of G is defined as

$$PCI_T(G) = \sum_{u,v \in N} T_N(u) T_N(v) CONN_{T_G}(u, v),$$
$$PCI_I(G) = \sum_{u,v \in N} I_N(u) I_N(v) CONN_{I_G}(u, v),$$
$$PCI_F(G) = \sum_{u,v \in N} F_N(u) F_N(v) CONN_{F_G}(u, v),$$

Where $CONN_{T_G}(u,v)$ is the strength of truth, $CONN_{I_G}(u,v)$ the strength of indeterminacy and $CONN_{F_G}(u,v)$ the strength of falsity between two vertices u and v. We have

$$CONN_{T_G}(u,v) = \max\{\min T_M(e) \mid e \in P \text{ and } P \text{ is a path between } u \text{ and } v\},$$
$$CONN_{I_G}(u,v) = \min\{\max I_M(e) \mid e \in P \text{ and } P \text{ is a path between } u \text{ and } v\},$$
$$CONN_{F_G}(u,v) = \min\{\max F_M(e) \mid e \in P \text{ and } P \text{ is a path between } u \text{ and } v\}.$$

Also, the **totally connectivity index** of G is defined as

$$TCI(G) = \frac{4 + 2PCI_T(G) - 2PCI_F(G) - PCI_I(G)}{6}.$$

Definition 7. Let $G = (N, M)$ be the Neutrosophic graph. G called a **connected** neutrosophic graph if for any two vertices $u, v \in N$, $CONN_{T_G}(u,v) > 0$, $CONN_{I_G}(u,v) > 0$, and $CONN_{F_G}(u,v) > 0$.

Example 1. Consider the Neutrosophic graph $G = (N, M)$ with $V = \{a, b, c, d\}$, that shown in figure 1. As can be seen, $(T_N, I_N, F_N)(a) = (0.4, 0.6, 0.5)$, $(T_N, I_N, F_N)(b) = (0.7, 0.5, 0.4)$, $(T_N, I_N, F_N)(c) = (0.7, 0.4, 0.3)$, and $(T_N, I_N, F_N)(d) = (0.5, 0.4, 0.5)$, The edge set contains $(T_M, I_M, F_M)(a,b) = (0.4, 0.5, 0.5)$, $(T_M, I_M, F_M)(b,c) = (0.7, 0.4, 0.4)$, $(T_M, I_M, F_M)(c,d) = (0.5, 0.4, 0.5)$, $(T_M, I_M, F_M)(a,d) = (0.4, 0.4, 0.5)$ and $(T_M, I_M, F_M)(b,d) = (0.3, 0.5, 0.7)$.
By direct calculations, we have

Table 1. The strength of connectedness between each pair of vertices u and v.

	$CONN_{T_G}(u,v)$	$CONN_{I_G}(u,v)$	$CONN_{F_G}(u,v)$
a, b	0.4	0.5	0.5
a, c	0.4	0.4	0.5
a, d	0.4	0.4	0.5
b, c	0.7	0.4	0.4
b, d	0.5	0.4	0.5
c, d	0.5	0.4	0.5

Then the partial connectivity index of G is,

$$\begin{aligned} PCI_T(G) &= \sum_{u,v \in N} T_N(u)T_N(v)CONN_{T_G}(u,v) \\ &= (0.4)(0.7)(0.4) + (0.4)(0.7)(0.4) + (0.4)(0.5)(0.4) + (0.7)(0.7)(0.7) + (0.7)(0.5)(0.5) \\ &\quad + (0.7)(0.5)(0.5) = 0.112 + 0.112 + 0.080 + 0.147 + 0.245 + 0.245 = 0.941, \end{aligned}$$

$$\begin{aligned} PCI_I(G) &= \sum_{u,v \in N} I_N(u)I_N(v)CONN_{I_G}(u,v) \\ &= (0.6)(0.5)(0.5) + (0.6)(0.4)(0.4) + (0.6)(0.4)(0.4) + (0.5)(0.4)(0.4) + (0.5)(0.4)(0.4) \\ &\quad + (0.4)(0.4)(0.4) = 0.180 + 0.096 + 0.096 + 0.080 + 0.080 + 0.064 = 0.596, \end{aligned}$$

$$\begin{aligned} PCI_F(G) &= \sum_{u,v \in N} F_N(u)F_N(v)CONN_{F_G}(u,v) \\ &= (0.5)(0.4)(0.5) + (0.5)(0.3)(0.5) + (0.5)(0.5)(0.5) + (0.4)(0.3)(0.4) + (0.4)(0.5)(0.5) \\ &\quad + (0.3)(0.5)(0.5) = 0.1 + 0.075 + 0.125 + 0.048 + 0.1 + 0.075 = 0.523. \end{aligned}$$

Also by definition 1, we have

$$TCI(G) = \frac{4 + 2PCI_T(G) - 2PCI_F(G) - PCI_I(G)}{6} = \frac{4 + 2(0.941) - 2(0.523) - 0.596}{6} = 0.707.$$

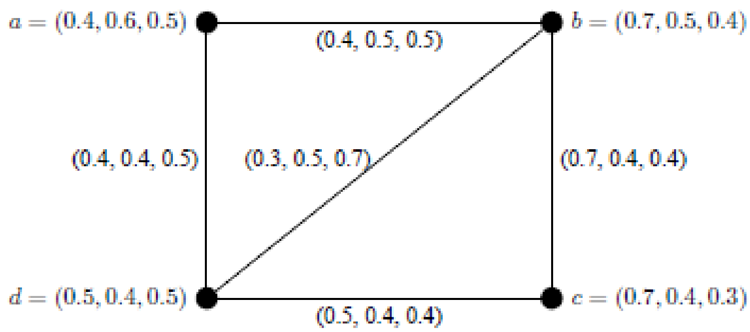

Figure 1. A neutrosophic graph with $V = \{a, b, c, d\}$

Theorem 1. Let $G = (N, M)$ be a connected neutrosophic graph and $H = (N', M')$ is a partial neutrosophic subgraph of G. then

$$PCI_T(H) \leq PCI_T(G),$$
$$PCI_I(H) \geq PCI_I(G),$$
$$PCI_F(H) \geq PCI_F(G),$$

Moreover, we have $TCI(H) \leq TCI(G)$.

Proof. Let $H = (N', M')$ is a partial neutrosophic subgraph of G, and $T_{N'}(u) \leq T_N(u)$ for $u \in V$. Since $T_{M'}(uv) \leq T_M(uv)$ for uv, then $CONN_{TH}(u, v) \leq CONN_{TG}(u, v)$ thus we get

$$PCI_T(H) = \sum_{u,v \in X} T_{N'}(u)T_{N'}(v)CONN_{TH}(u, v) \leq \sum_{u,v \in X} T_N(u)T_N(v)CONN_{TG}(u, v) = PCI_T(G).$$

Using a similar proof, we can show that

$$PCI_I(H) = \sum_{u,v \in X} I_{N'}(u)I_{N'}(v)CONN_{IH}(u, v) \geq \sum_{u,v \in X} I_N(u)I_N(v)CONN_{IG}(u, v) = PCI_I(G),$$

And

$$PCI_F(H) = \sum_{u,v \in X} F_{N'}(u)F_{N'}(v)CONN_{FH}(u, v) \geq \sum_{u,v \in X} F_N(u)F_N(v)CONN_{FG}(u, v) = PCI_F(G).$$

Now, we show that

$$TCI(H) \leq TCI(G).$$

By definition totally connectivity index, and since $PCI_T(H) \leq PCI_T(G), PCI_I(H) \geq PCI_I(G), PCI_F(H) \geq PCI_F(G)$, we have

$$TCI(H) = \frac{4 + 2PCI_T(H) - 2PCI_F(H) - PCI_I(H)}{6} \leq \frac{4 + 2PCI_T(G) - 2PCI_F(H) - PCI_I(H)}{6}$$
$$\leq \frac{4 + 2PCI_T(G) - 2PCI_F(G) - PCI_I(G)}{6} = TCI(G),$$

And, hence $TCI(H) \leq TCI(G)$.
□

Example 2. Consider the neutrosophic graph $G = (N,\ M)$ whit

$$N = \{(a, 0.7, 0.3, 0.4), (b, 0.5, 0.2, 0.3), (c, 0.7, 0.3, 0.6), (d, 0.4, 0.3, 0.5)\},$$

And

$$M = \{(ab, 0.5, 0.2, 0.4), (ac, 0.7, 0.3, 0.6), (bc, 0.5, 0.2, 0.6), (cd, 0.4, 0.3, 0.6)\}.$$

Also, let $H = (N', M')$ be a neutrosophic subgraph of G, whit

$$N' = \{(a, 0.6, 0.3, 0.5), (b, 0.4, 0.2, 0.4), (c, 0.6, 0.3, 0.7), (d, 0.3, 0.3, 0.6)\},$$

And

$$M' = \{(ab, 0.4, 0.2, 0.5), (ac, 0.5, 0.3, 0.7), (bc, 0.4, 0.2, 0.7), (cd, 0.3, 0.3, 0.7)\}.$$

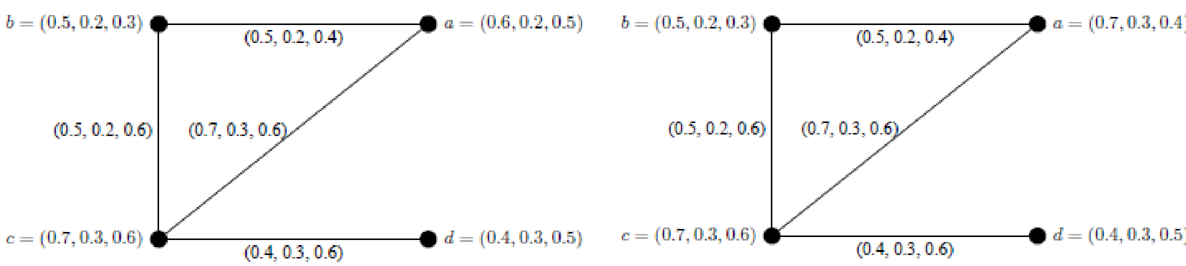

Figure 2. The neutrosophic graph G and the neutrosophic subgraph of G

By direct calculations, we have

$$PCI_T(G) = 0.997, \qquad PCI_I(G) = 0.120, \qquad PCI_F(G) = 0.690,$$

And

$$PCI_T(H) = 0.516, \qquad PCI_I(H) = 0.120, \qquad PCI_F(H) = 1.213.$$

Moreover

$$TCI(G) = \frac{4 + 2PCI_T(G) - 2PCI_F(G) - PCI_I(G)}{6} = \frac{4 + 2(0.997) - 2(0.690) - 0.120}{6} = 0.749.$$

$$TCI(H) = \frac{4 + 2PCI_T(H) - 2PCI_F(H) - PCI_I(H)}{6} = \frac{4 + 2(0.516) - 2(1.213) - 0.120}{6} = 0.622.$$

It is easy to see that $TCI(H) = 0.622 \leq TCI(G) = 0.749$.

Note 1. Note that if $H = (N', M')$ is a partial neutrosophic subgraph of $G = (N, M)$ such that $N' = N \backslash \{v\}$ then $PCI_T(H) < PCI_T(G)$, $PCI_I(H) < PCI_{I|}(G)$, $PCI_F(H) < PCI_F(G)$.

Theorem 2. Let $G_1 = (N_1, M_1)$ be isomorphic with $G_2 = (N_2, M_2)$. Then all of the following equation are established.

$$PCI_T(G_1) = PCI_T(G_2),$$
$$PCI_I(G_1) = PCI_I(G_2),$$
$$PCI_F(G_1) = PCI_F(G_2),$$

Also, we have $TCI(G_1) = TCI(G_2)$.

Proof. Let $G_1 = (N_1, M_1)$ be isomorphic with $G_2 = (N_2, M_2)$, and $f: V_1 \rightarrow V_2$ be the bijection from V_1 to V_2 such that

$$T_{N_1}(u) = T_{N_2}\big(f(u)\big), \qquad I_{N_1}(u) = I_{N_2}\big(f(u)\big), \qquad F_{N_1}(u) = F_{N_2}\big(f(u)\big),$$

For all $u \in V_1$, and

$$T_{M_1}(uv) = T_{M_2}\big(f(u)f(v)\big), \quad I_{M_1}(uv) = I_{M_2}\big(f(u)f(v)\big), \quad F_{M_1}(uv) = F_{M_2}\big(f(u)f(v)\big),$$

For all $uv \in E_1$. Since G_1 isomorphic with G_2, the strength of any strongest path between u and v in G_1 is equal to that between $f(u)$ and $f(v)$ in G_2. Hence

$$CONN_{T_{G_1}}(u, v) = CONN_{T_{G_2}}\big(f(u), f(v)\big), \qquad CONN_{I_{G_1}}(u, v) = CONN_{I_{G_2}}\big(f(u), f(v)\big),$$
$$CONN_{F_{G_1}}(u, v) = CONN_{F_{G_2}}(f(u), f(v)),$$

For $u, v \in N_1^*$. Therefore

$$PCI_T(G_1) = PCI_T(G_2), \qquad PCI_I(G_1) = PCI_I(G_2), \qquad PCI_F(G_1) = PCI_F(G_2),$$

And

$$TCI(G_1) = \frac{4 + 2PCI_T(G_1) - 2PCI_F(G_1) - PCI_I(G_1)}{6} = \frac{4 + 2PCI_T(G_2) - 2PCI_F(G_2) - PCI_I(G_2)}{6} = TCI(G_2).$$

□

Theorem 3. Let $G = (N, M)$ be a complete neutrosophic graph whit $V = \{v_1, v_2, \dots, v_n\}$ such that $t_1 \leq t_2 \leq \cdots \leq t_n$, $i_1 \leq i_2 \leq \cdots \leq i_n$ and $f_1 \geq f_2 \geq \cdots \geq f_n$ where $t_j = T_N\big(v_j\big)$, $i_j = I_N\big(v_j\big)$ and $f_j = F_N\big(v_j\big)$ for $j = 1, 2, \dots, n$. Then

$$PCI_T(G) = \sum_{j=1}^{n-1} t_j^2 \sum_{k=j+1}^{n} t_k,$$
$$PCI_I(G) = \sum_{j=1}^{n-1} i_j^2 \sum_{k=j+1}^{n} i_k,$$
$$PCI_F(G) = \sum_{j=1}^{n-1} f_j^2 \sum_{k=j+1}^{n} f_k.$$

Proof. Suppose v_1 is a vertex with the least Truth-membership value t_1. In a complete neutrosophic graph, $CONN_{TG}(u, v) = T_M(u, v)$ for all $u, v \in V$. Therefore $T_M(v_1 v_k) = t_1$ for $k = 2, 3, \dots, n$ and hence $T_N(v_1)T_N(v_k)CONN_{TG}(v_1, v_k) = t_1^2 t_k$ for $k = 2, 3, \dots, n$. Then for v_1, we have

$$\sum_{k=2}^{n} T_N(v_1)T_N(v_k)CONN_{TG}(v_1, v_k) = \sum_{k=2}^{n} t_1^2 t_k.$$

For v_2, $T_N(v_2)T_N(v_k)CONN_{TG}(v_2, v_k) = t_2^2 t_k$ for $k = 3, 4, \dots, n$.

$$\sum_{k=3}^{n} T_N(v_2)T_N(v_k)CONN_{TG}(v_2, v_k) = \sum_{k=3}^{n} t_2^2 t_k,$$

For v_{n-2}, $T_N(v_{n-2})T_N(v_k)CONN_{TG}(v_{n-2}, v_k) = t_{n-2}^2 t_k$ for $k = n-1, n$.

For v_{n-1}, $T_N(v_{n-1})T_N(v_k)CONN_{TG}(v_{n-1}, v_k) = t_{n-1}^2 t_j$ for $k = n$.

Thus, by summing over v_j, $j = 1, 2, 3, \dots, n-1$, we get

$$PCI_T(G) = \sum_{k=2}^{n} t_1^2 t_k + \sum_{k=3}^{n} t_2^2 t_k + \cdots + \sum_{k=n-1}^{n} t_{n-2}^2 t_k + \sum_{k=n}^{n} t_{n-1}^2 t_k = \sum_{j=1}^{n-1} t_j^2 \sum_{k=j+1}^{n} t_k.$$

Using the same argument, we can prove the other two cases.
□

Theorem 4. Let $G = (N, M)$ be a neutrosophic graph whit $V = \{v_1, v_2, \dots, v_n\}$ such that $G^* = (V, E)$ is a complete bipartite graph and $T_M(uv) = \min\{T_N(u), T_N(v)\}$, $I_M(uv) = \min\{I_N(u), I_N(v)\}$, $F_M(uv) = \max\{F_N(u), F_N(v)\}$ For all $u, v \in V$. Also, $V_1 = \{v_1, v_2, \dots, v_m\}$, and $V_2 = \{v_{m+1}, v_{m+2}, \dots, v_n\}$ whit $t_1 \le t_2 \le \cdots \le t_n$, $i_1 \le i_2 \le \cdots \le i_n$, and $f_1 \ge f_2 \ge \cdots \ge f_n$ where $t_j = T_N(v_j)$, $i_j = I_N(v_j)$ and $f_j = F_N(v_j)$ for $j = 1, 2, \dots, n$. Then

$$PCI_T(G) = \sum_{j=1}^{m} t_j^2 \sum_{k=j+1}^{n} t_k + t_m \sum_{j=m+1}^{n-1} t_j \sum_{k=j+1}^{n} t_k,$$
$$PCI_I(G) = \sum_{j=1}^{m} i_j^2 \sum_{k=j+1}^{n} i_k + i_m \sum_{j=m+1}^{n-1} i_j \sum_{k=j+1}^{n} i_k$$
$$PCI_F(G) = \sum_{j=1}^{m} f_j^2 \sum_{k=j+1}^{n} f_k + f_m \sum_{j=m+1}^{n-1} f_j \sum_{k=j+1}^{n} f_k.$$

Proof. Let $G = (N, M)$ be a neutrosophic graph whit $V = \{v_1, v_2, \dots, v_n\}$ and $G^* = K_{m,n}$, such that $t_1 \le t_2 \le \cdots \le t_n$, $i_1 \le i_2 \le \cdots \le i_n$ and $f_1 \ge f_2 \ge \cdots \ge f_n$.
Here we prove $PCI_F(G)$, states $PCI_T(G)$ and $PCI_I(G)$ are similarly proved.
Using definition, we have

$$PCI_F(G) = \sum_{v_j, v_k \in V} F_N(v_j)F_N(v_k)CONN_{FG}(v_j, v_k).$$

Too, for $v_1, v_k \in V$, we have

$$CONN_{FG}(v_1, v_k) = \min\ \{\max\{f_1\}, \max\{f_1, f_2\}, \dots, \max\{f_1, f_m\}\} = \min\ \{f_1, f_1, \dots, f_1\} = f_1.$$

Accordingly for $v_1, v_k \in V$

$$\sum_{\substack{v_k \ne v_1 \\ v_k \in V}} F_N(v_1)F_N(v_k)CONN_{FG}(v_1, v_k) = f_1 f_1 \sum_{k=2}^{n} f_k.$$

Similarly, for $v_j, v_k \in V$ $j = 2, 3, \dots, m$

$$\sum_{k=j+1} F_N(v_j)F_N(v_k)CONN_{FG}(v_j, v_k) = f_j f_j \sum_{k=j+1}^{n} f_k.$$

On the other hand, we have for $m < j < n$

$$\sum_{k=j+1} F_N(v_j)F_N(v_k)CONN_{FG}(v_j, v_k) = f_m f_j \sum_{k=j+1}^{n} f_k,$$

Then

$$PCI_F(G) = \sum_{v_j,v_k \in V} F_N(v_j)F_N(v_k)CONN_{FG}(v_j,v_k)$$
$$= f_1f_1\sum_{k=2}^{n} f_k + f_2f_2\sum_{k=3}^{n} f_k + \cdots + f_mf_m\sum_{k=m+1}^{n} f_k + f_mf_{m+1}\sum_{k=m+2}^{n} f_k + \cdots + f_mf_{n-1}f_n$$
$$= \sum_{j=1}^{m} f_j^2 \sum_{k=j+1}^{n} f_k + f_m \sum_{j=m+1}^{n-1} f_j \sum_{k=j+1}^{n} f_k.$$

□

Note 2. Clearly, in the above theorem it is enough to have

$$\forall\, v \in V_1\ \forall u \in V_2, \qquad T_N(v) \le T_N(u), \qquad I_N(v) \ge I_N(u), \qquad F_N(v) \ge F_N(u).$$

Then the case will be established. In the following example you can see the correctness of this claim.

Example 3. Consider the neutrosophic graph $G = (N, M)$ whit

$$N = \{(a, 0.2, 0.6, 0.7), (b, 0.4, 0.6, 0.5), (c, 0.7, 0.5, 0.4), (d, 0.5, 0.3, 0.5), (e, 0.6, 0.4, 0.5)\},$$

And

$$M = \{(ac, 0.2, 0.6, 0.7), (ad, 0.2, 0.6, 0.7), (ae, 0.2, 0.6, 0.7),$$
$$(bc, 0.4, 0.6, 0.5), (bd, 0.4, 0.6, 0.5), (be, 0.4, 0.6, 0.5)\}.$$

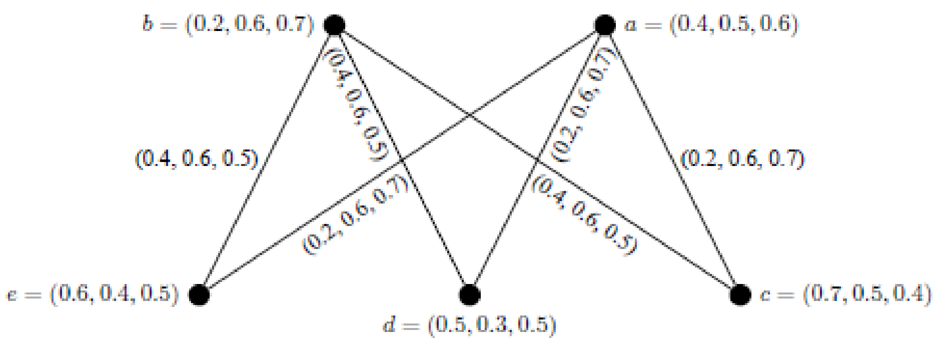

Figure 3. A complete bipartite neutrosophic graph whit $G^* = K_{2,3}$

By direct calculation, we have

$$CONN_{T_G}(a,b) = CONN_{T_G}(a,c) = CONN_{T_G}(a,d) = CONN_{T_G}(a,e) = 0.2 = T_N(a),$$
$$CONN_{T_G}(b,c) = CONN_{T_G}(b,d) = CONN_{T_G}(b,e) = 0.4 = T_N(b),$$
$$CONN_{T_G}(c,d) = CONN_{T_G}(c,e) = 0.4 = T_N(b),$$
$$CONN_{T_G}(d,e) = 0.4 = T_N(b),$$

$$PCI_T(G) = \sum_{u,v \in N} T_N(u)T_N(v)CONN_{T_G}(u,v)$$
$$= (0.2)(0.4)(0.2) + (0.2)(0.7)(0.2) + (0.2)(0.5)(0.2) + (0.2)(0.6)(0.2) + (0.4)(0.4)(0.7) + (0.4)(0.4)(0.5) + (0.4)(0.4)(0.6) + (0.7)(0.5)(0.4) + (0.7)(0.6)(0.4) + (0.5)(0.4)(0.6) = 0.804,$$

Using Theorem 4,

$$PCI_T(G) = \sum_{j=1}^{m} t_j^2 \sum_{k=j+1}^{n} t_k + t_m \sum_{j=m+1}^{n-1} t_j \sum_{k=j+1}^{n} t_k = \sum_{j=1}^{2} t_j^2 \sum_{k=j+1}^{5} t_k + t_m \sum_{3}^{4} t_j \sum_{k=j+1}^{5} t_k$$
$$= (0.2)(0.2)(0.4 + 0.7 + 0.5 + 0.6) + (0.4)(0.4)(0.7 + 0.5 + 0.6) + (0.4)(0.7)(0.5 + 0.6) + (0.4)(0.5)(0.6) = 0.804.$$

As observed, the value of truth- partial connectivity index $PCI_T(G)$ is obtained from both methods equally.

Theorem 5. Let $G = (N, M)$ be a wheel neutrosophic graph whit $V = \{v_1, v_2, \dots, v_n\}$ such that G^* is a wheel graph and for any $uv \in M^*$,

$$T_M(uv) = \min\{T_N(u), T_N(v)\}, \quad I_M(uv) = \min\{I_N(u), I_N(v)\}, \quad F_M(uv) = \max\{F_N(u), F_N(v)\}.$$

If $t_1 \le t_2 \le \cdots \le t_n$, $i_1 \le i_2 \le \cdots \le i_n$ and $f_1 \ge f_2 \ge \cdots \ge f_n$ where $t_j = T_N(v_j)$, $i_j = I_N(v_j)$ and $f_j = F_N(v_j)$ for $j = 1, 2, \dots, n$ and v_1 is the center vertex. Then

$$PCI_T(G) = \sum_{j=1}^{n-1} t_j^2 \sum_{k=j+1}^{n} t_k,$$
$$PCI_I(G) = \sum_{j=1}^{n-1} i_j^2 \sum_{k=j+1}^{n} i_k,$$
$$PCI_F(G) = \sum_{j=1}^{n-1} f_j^2 \sum_{k=j+1}^{n} f_k.$$

Proof. Let $G = (N, M)$ be a wheel neutrosophic graph whit the conditions stated in the theorem. Here we prove $PCI_I(G)$, states $PCI_T(G)$ and $PCI_F(G)$ are similarly proved. Then

Suppose v_1 is the center vertex. Using definition,

$$PCI_I(G) = \sum_{v_j, v_k \in V} I_N(v_j)I_N(v_k)CONN_{IG}(v_j, v_k).$$

Now, for $v_1, v_k \in V$ we have

$$CONN_{IG}(v_1, v_2) = \min\{\max\{i_1\}, \max\{i_1, i_2\}, \max\{i_1, i_2, i_3\}, \dots, \max\{i, i_m\}\} = \min\{i_1, i_2, \dots, i_n\} = i_1,$$

Hence

$$\sum_{k=2}^{n} I_N(v_1)I_N(v_k)CONN_{IG}(v_1, v_k) = i_1 i_1 i_2 + i_1 i_1 i_3 + \cdots + i_1 i_1 i_{n-1} + i_1 i_1 i_n = \sum_{k=2}^{n} i_1^2 i_k.$$

Similarly for $v_j, v_k \in V$ $j = 2, 3, \dots, n-1$

$$CONN_{IG}(v_j, v_k) = \sum_{k=j+1}^{n} I_N(v_j) I_N(v_k) CONN_{IG}(v_j, v_k) = \sum_{k=j+1}^{n} i_j^2 i_k,$$

This shows that

$$PCI_I(G) = \sum_{v_j, v_k \in V} I_N(v_j) I_N(v_k) CONN_{IG}(v_j, v_k) = \sum_{k=2}^{n} i_1^2 i_k + \sum_{k=3}^{n} i_2^2 i_k + \cdots + \sum_{k=j+1}^{n} i_j^2 i_k + \cdots + i_{n-1} i_{n-1} i_n$$
$$= \sum_{j=1}^{n-1} i_j^2 \sum_{k=j+1}^{n} i_k.$$

□

Theorem 6. Let $G = (N, M)$ be a complete neutrosophic graph of $G^* = (V, E)$, and $B_{(m,m)}$ is a m-barbell graph of G. if $t_1 \leq t_2 \leq \cdots \leq t_n$, $i_1 \geq i_2 \geq \cdots \geq i_n$ and $f_1 \geq f_2 \geq \cdots \geq f_n$ where $t_j = T_N(v_j)$, $i_j = I_N(v_j)$ and $f_j = F_N(v_j)$ for $j = 1, 2, \ldots, n$. And uv is a $\mathrm{I} - strong\ edge$ whit $M(uv) = (T_M(uv), I_M(uv), F_M(uv))$, where $T_M(uv) \leq t_1, I_M(uv) \leq i_1, F_M(uv) \geq f_1$, and uv connecting two copies of complete neutrosophic graphs G. Then

$$PCI_T(B_{(m,m)}) = 2\sum_{j=1}^{n-1} t_j^2 \sum_{k=j+1}^{n} t_k + T_M(uv) \sum_{j=1}^{n} t_j \sum_{k=j}^{n} t_k,$$
$$PCI_I(B_{(m,m)}) = 2\sum_{j=1}^{n-1} i_j^2 \sum_{k=j+1}^{n} i_k + I_M(uv) \sum_{j=1}^{n} i_j \sum_{k=j}^{n} i_k,$$
$$PCI_F(B_{(m,m)}) = 2\sum_{j=1}^{n-1} f_j^2 \sum_{k=j+1}^{n} f_k + F_M(uv) \sum_{j=1}^{n} f_j \sum_{k=j}^{n} f_k.$$

Proof. Let $G = (N, M)$ be a wheel neutrosophic graph whit the conditions stated in the theorem. By definition 5, here we have two copies of the complete graph K_m. Also using Theorem 3, for a complete neutrosophic graph

$$PCI_T(G) = \sum_{j=1}^{n-1} t_j^2 \sum_{k=j+1}^{n} t_k,$$
$$PCI_I(G) = \sum_{j=1}^{n-1} i_j^2 \sum_{k=j+1}^{n} i_k,$$
$$PCI_F(G) = \sum_{j=1}^{n-1} f_j^2 \sum_{k=j+1}^{n} f_k.$$

Now it suffices to obtain the connectivity between two vertices from two copies of K_m. Suppose vertex v_j is from one of the two copies of K_m and vertex v_k is from another copy, in which case we have

$$CONN_{TG}(v_j, v_k) = \max\{min\{T_M(uv) \wedge \min\{t_k \mid t_k \in P(v_j\_v_k)\}\} = T_M(uv),$$

Then

$$PCI_T\big(B_{(m,m)}\big) = \sum_{v_j, v_k \in V} I_N(v_j) I_N(v_k) CONN_{IG}(v_j, v_k)$$
$$= \sum_{j=1}^{n-1} t_j^2 \sum_{k=j+1}^{n} t_k + \sum_{j=1}^{n-1} t_j^2 \sum_{k=j+1}^{n} t_k + v_1 v_1 T_M(uv) + v_1 v_2 T_M(uv) + \cdots + v_n v_n T_M(uv)$$
$$= 2 \sum_{j=1}^{n-1} t_j^2 \sum_{k=j+1}^{n} t_k + T_M(uv) \sum_{j=1}^{n} t_j \sum_{k=j}^{n} t_k.$$

The proof will be the same for the other two cases.
□

Example 4. Consider the neutrosophic graph $G = K_4 = (N, M)$ whit

$$N = \{(a, 0.2, 0.6, 0.8), (b, 0.3, 0.5, 0.7), (c, 0.3, 0.4, 0.7), (d, 0.4, 0.4, 0.5)\},$$

And

$$M = \{(ab, 0.2, 0.6, 0.8), (ac, 0.2, 0.6, 0.8), (ad, 0.2, 0.6, 0.8),$$
$$(bc, 0.3, 0.5, 0.7), (bd, 0.3, 0.4, 0.7), (cd, 0.3, 0.4, 0.7)\}.$$

Now suppose that the edge that connects the two complete graphs does not hold true. As shown in figure 4, for example, if we want to go from vertex b in the right graph to vertex a in the left graph, there are paths with different connectivity.

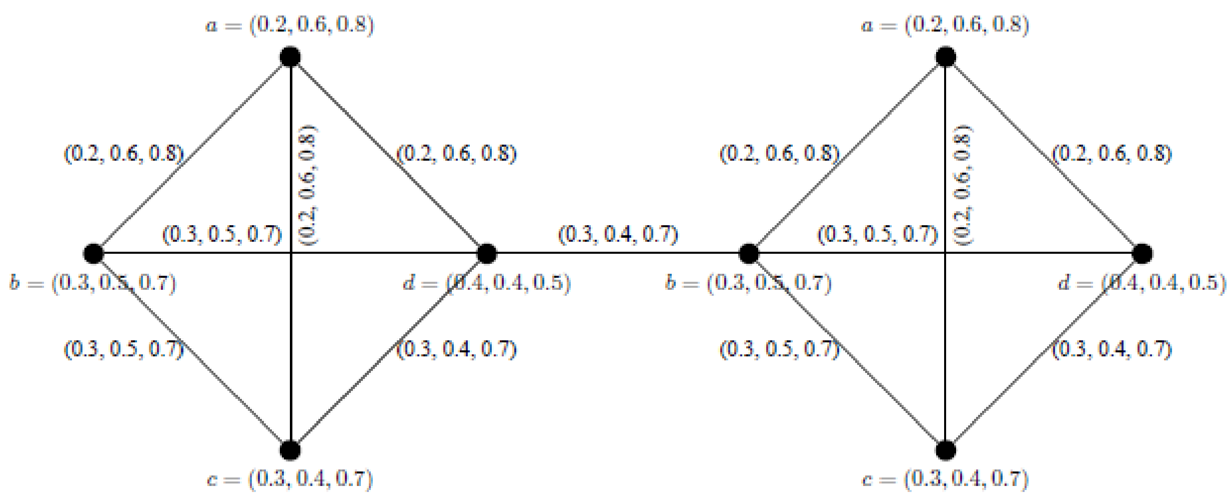

Figure 4. A m-barbell neutrosophic graph whit $G^* = K_4$

3.2. Bounds for connectivity index

In this section, we discuss bunds for partial connectivity index (PCI) and totally connectivity index (TCI). We show that, among all neutrosophic graphs whit a same support, the complete neutrosophic graph will have maximum totally connectivity index.

Theorem 7. Let $G = (N, M)$ be a neutrosophic graph whit $|N| = n$, and $G' = (N', M')$ is the complete neutrosophic graph spanned by the vertex set of G. Then,

$$0 \le PCI_T(G) \le PCI_T(G'),$$

$$0 \le PCI_I(G) \le PCI_I(G'),$$
$$0 \le PCI_F(G) \le PCI_F(G').$$

Also if $I_M(uv) = I_{M'}(uv)$, and $F_M(uv) = F_{M'}(uv)$, for all $uv \in E$ then $0 \le TCI_F(G) \le TCI_F(G')$.

proof. Consider the neutrosophic graph $G = (N, M)$ whit $|N| = n$. If $|E| = 0$ clearly, $PCI_T(G) = PCI_I(G) = PCI_F(G) = TCI(G) = 0$. Let $|E| > 0$ and $G' = (N', M')$ is the complete neutrosophic graph whit $|N'| = n$. Suppose $\left(T_N(u), I_N(u), F_N(u)\right) = (T_{N'}(u), I_{N'}(u), F_{N'}(u))$ for all $u \in X$. Since

$$T_M(uv) \le T_{M'}; \quad I_M(uv) \le I_{M'}(uv); \quad F_M(uv) \le F_{M'}(uv); \quad \forall uv \in E.$$

Therefore, we have $CONN_{TG}(u,v) \le CONN_{TG'}(u,v)$, $CONN_{IG}(u,v) \le CONN_{IG'}(u,v)$ and $CONN_{FG}(u,v) \le CONN_{FG'}(u,v)$. Then

$$0 \le PCI_T(G) = \sum_{u,v \in X} T_N(u)T_N(v)CONN_{TG}(u,v) \le \sum_{u,v \in X} T_{N'}(u)T_{N'}(v)CONN_{TG'}(u,v) = PCI_T(G').$$

Using a similar proof we can show that

$$0 \le PCI_I(G) \le PCI_I(G'), \qquad and \qquad 0 \le PCI_F(G) \le PCI_F(G').$$

Also, according to definition $TCI(G)$, if $I_M(uv) = I_{M'}(uv)$, and $F_M(uv) = F_{M'}(uv)$, for all $uv \in E$, then

$$TCI(G) = \frac{4 + 2PCI_T(G) - 2PCI_F(G) - PCI_I(G)}{6} \le \frac{4 + 2PCI_T(G') - 2PCI_F(G') - PCI_I(G')}{6} = TCI(G').$$

□

Note 3. Note that the above theorem for case $TCI(G) \le TCI(G')$ may not always be true.

4. Applications

Neutrosophic graphs are one of the most practical branches of graph theory. Different applications of it have been studied to date [1-3, 12-20]. Here we will mention another application.

Behavioral sciences, which is one of the branches of humanities, is one of the most extensive sciences in our time. Every day, many theorists in this field create new theories and cause them to expand more and more. So every day they are faced with a lot of new data and information.

Mathematics has always been one of the best tools for modeling and categorizing this data and information. Among these, graphic models are among the most appropriate models that come with the help of behavioral sciences and with proper modeling, provide the conditions for a more accurate analysis of these complex problems. What is very important in behavioral sciences is the existence of a relationship, the relationship between individuals, groups, communities, organizations and institutions, and, so on. Studying and discovering these relationships, categorizing them, and then examining and studying the extent and impact of these relationships on each other is a complex task. Neutrosophic graph models can help with these problems and help answer some of the questions. Questions such as: Which relationship is most effective? Which relationship should end? Which person is more influential in a relationship? And many other questions

Here we are dealing with the relationship between several families. Information related to this problem is data from a real study obtained from a behavioral science study clinic. Of course, given the limitations we had, we have provided a small sample of that data in this article.

In this problem, we studied 5 families that are related. First, each family was studied separately and the behavior of each family member was studied by experts, and then we obtained an average of the behaviors and traits studied in family members. These features were classified into three categories. Good qualities include the ability to communicate, cooperate, be honest, etc; Bad traits include jealousy, misconceptions, lack of anger control, personal aggression, etc; Neutral behaviors include behaviors that do not involve any behavioral actions. The experts then assigned a numerical value to each of these behaviors, which we named T, F, and I, respectively. Experts then studied the relationships between families and the extent of each family's impact on another family and the type of impact of each family. The effect of each family on other families was evaluated using behavioral science criteria. The experts coded these relationships into three categories: good, neutral, and bad, and obtained a numerical quantity for each category based on the coding results.

Here we present a neutrosophic graph model related to 5 families from 137 families surveyed.

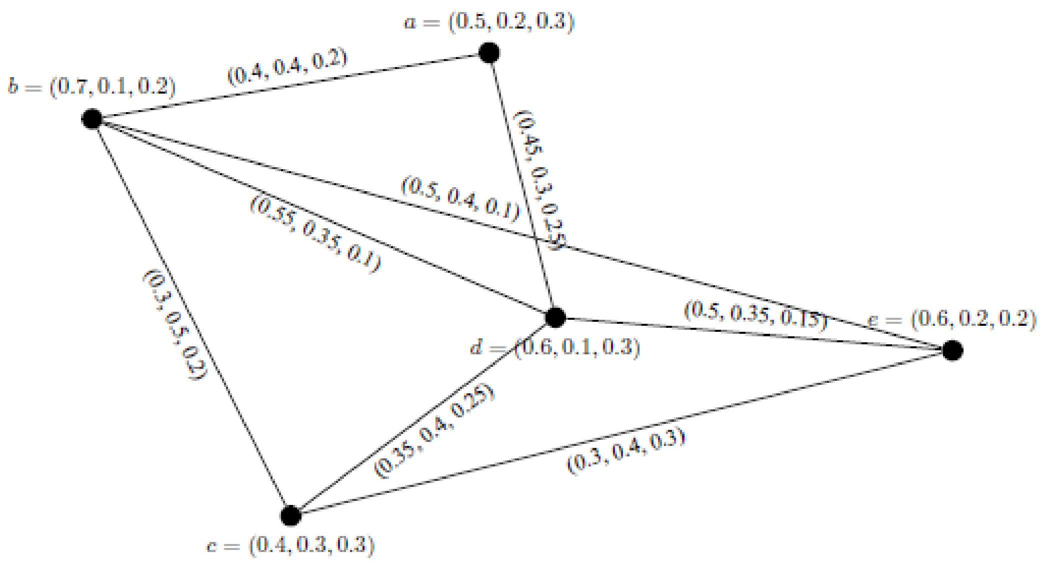

Figure 5. A neutrosophic graph model corresponding to 5 families

By direct calculations

Table 2. The strength of connectedness between each pair of vertices u and v.

	$CONN_{T_G}(u,v)$	$CONN_{I_G}(u,v)$	$CONN_{F_G}(u,v)$
a , b	0.45	0.35	0.2
a , c	0.35	0.4	0.2
a , d	0.45	0.3	0.2
a , e	0.45	0.35	0.2
b , c	0.35	0.4	0.2
b , d	0.55	0.35	0.1
b , e	0.5	0.35	0.1
c , d	0.35	0.4	0.2
c , e	0.35	0.4	0.2
d , e	0.5	0.35	0.1

Then

$$PCI_T(G) = \sum_{u,v \in N} T_N(u)T_N(v)CONN_{T_G}(u,v) = 1.3845,$$
$$PCI_I(G) = \sum_{u,v \in N} I_N(u)I_N(v)CONN_{I_G}(u,v) = 0.519,$$
$$PCI_F(G) = \sum_{u,v \in N} F_N(u)F_N(v)CONN_{F_G}(u,v) = 0.118.$$

Also, we have

$$TCI(G) = \frac{4 + 2PCI_T(G) - 2PCI_F(G) - PCI_I(G)}{6} = \frac{4 + 2(1.3845) - 2(0.118) - 0.519}{6} = 1.002.$$

The connectivity index is used as a numerical index in evaluating the interactions of these five families. Note that the analysis of this problem will be done by behavioral science experts and the results will be presented in detail in another article.

5. Conclusion

Connectivity is one of the major parameters associated with a neutrosophic network and a neutrosophic graph. In this paper, two concepts of partial connectivity index and totally connectivity index were studied. In a neutrosophic graph, according to the parameters of the problem, we can obtain the partial connectivity index and totally connectivity for it. The higher the Truth-partial connectivity index and the lower the Falsity-partial correlation index, the more complete our information is and the more reliable the problem will be.

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