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Generalized Aggregate Operators on Neutrosophic Hypersoft Set

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Abstract: Multi-criteria decision making (MCDM) is concerned about coordinating as well as looking after selection as well as planning problems which included multi-criteria. The neutrosophic soft set cannot handle the environment which involved more than one attribute. To overcome those hurdles neutrosophic hypersoft set (NHSS) is defined. In this paper, we proposed the generalized aggregate operators on NHSS such as extended union, extended intersection, OR-operation, AND-operation, etc. with their properties. Finally, the necessity and possibility operations on NHSS with suitable examples and properties are presented in the following research.

Keywords: Soft set; Neutrosophic Set; Neutrosophic soft set; Hypersoft set; Neutrosophic hypersoft set.

1. Introduction

Zadeh developed the notion of fuzzy sets [1] to solve those problems which contain uncertainty and vagueness. It is observed that in some cases circumstances cannot be handled by fuzzy sets, to overcome such types of situations Turksen [2] gave the idea of interval-valued fuzzy set. In some cases, we must deliberate membership unbiassed as the non- membership values for the suitable representation of an object in uncertain and indeterminate conditions that could not be handled by fuzzy sets nor interval-valued fuzzy sets. To overcome these difficulties Atanassov presented the notion of Intuitionistic fuzzy sets in [3]. The theory which was presented by Atanassov only deals the insufficient data considering both the membership and non-membership values, but the intuitionistic fuzzy set theory cannot handle the incompatible and imprecise information. To deal with such incompatible and imprecise data the idea of the neutrosophic set (NS) was developed by Smarandache [4].

A general mathematical tool was proposed by Molodtsov [5] to deal with indeterminate, fuzzy, and not clearly defined substances known as a soft set (SS). Maji et al. [6] extended the work on SS and defined some operations and their properties. In [7], they also used the SS theory for decision making. Ali et al. [8] revised the Maji approach to SS and developed some new operations with their properties. De Morgan's Law on SS theory was proved in [9] by using different operators. Cagman and Enginoglu [10] developed the concept of soft matrices with operations and discussed their properties, they also introduced a decision-making method to resolve those problems which contain uncertainty. In [11], they revised the operations proposed by Molodtsov's SS. In [12], the author's proposed some new operations on soft matrices such as soft difference product, soft restricted difference product, soft extended difference product, and soft weak-extended difference product with their properties.

Maji [13] offered the idea of a neutrosophic soft set (NSS) with necessary operations and properties. The idea of the possibility NSS was developed by Karaaslan [14] and introduced a possibility of neutrosophic soft decision-making method to solve those problems which contain uncertainty based on And-product. Broumi [15] developed the generalized NSS with some operations and properties and used the proposed concept for decision making. To solve MCDM problems with single-valued Neutrosophic numbers presented by Deli and Subas in [16], they constructed the concept of cut sets of single-valued Neutrosophic numbers. On the base of the correlation of intuitionistic fuzzy sets, the term correlation coefficient of SVNSs [17] was introduced. In [18], the idea of simplified NSs introduced with some operational laws and aggregation operators such as real-life Neutrosophic weighted arithmetic average operator and weighted geometric average operator. They constructed an MCDM method on the base of proposed aggregation operators.

Smarandache [19] generalized the SS to hypersoft set (HSS) by converting the function to a multi-attribute function to deal with uncertainty. Saqlain et al. [20] developed the generalization of TOPSIS for the NHSS, by using the accuracy function they transformed the fuzzy neutrosophic numbers to crisp form. In [21],s the author's proposed the fuzzy plithogenic hypersoft set in matrix form with some basic operations and properties. Martin and Smarandache developed the plithogenic hypersoft set by combining the plithogenic sets and hypersoft set in [22]. Saqlain et al. [23] proposed the aggregate operators and similarity measure [24] on NHSS. In [25], Abdel basset et al. applied TODIM and TOPSIS methods based on the best-worst method to increase the accuracy of evaluation under uncertainty according to the neutrosophic set. They also used the plithogenic set theory to solve the uncertain information and evaluate the financial performance of manufacturing industries, they used the AHP method to find the weight vector of the financial ratios to achieve this goal after that they used the VIKOR and TOPSIS methods to utilized the companies ranking in [26].

In the following paragraph, we explain some positive impacts of this research. The main focus of this study is too generalized the aggregate operators of the neutrosophic hypersoft set. We will use the proposed aggregate operators to solve multi-criteria decision-making problems after developing distance-based similarity measures. Saqlain et al. [23], developed the aggregate operators on NHSS but in some cases, we face some limitations such as in union and intersection. To overcome these limitations we develop the generalized version of aggregate operators on NHSS.

The following research is organized as follows: In section 2, we recall some basic definitions used in the following research such as SS, NS, NSS, HSS, and NHSS. We develop the generalized aggregate operators on NHSS such as extended union, extended intersection, And-operation, etc. in section 3 with properties. In section 4, the necessity and possibility of operations are presented with examples and properties.

2. Preliminaries

In this section, we recall some basic definitions such as SS, NSS, and NHSS which use in the following sequel.

Definition 2.1 [5] **Soft Set**

The soft set is a pair (F, Λ) over Ű if and only if F: Λ → P(Ű) is a mapping. That is the parameterized family of subsets of Ű known as a SS.

Definition 2.2 [4] **Neutrosophic Set**

Let Ű be a universe and Λ be an NS on Ű is defined as Λ = $\{< u, T_{\Lambda}(u), I_{\Lambda}(u), F_{\Lambda}(u) >: u \in$ Ű$\}$, where T, I, F: Ű $\rightarrow\ ]0^{-},\ 1^{+}[$ and $0^{-} \leq T_{\Lambda}(u) + I_{\Lambda}(u) + F_{\Lambda}(u) \leq 3^{+}$.

Definition 2.3 [13] **Neutrosophic Soft Set**

Let Ű and Ḝ are universal set and set of attributes respectively. Let P(Ű) be the set of Neutrosophic values of Ű and Λ ⊆ Ḝ. A pair (F, Λ) is called an NSS over Ű and its mapping is given as

F: Λ → P(Ű)

Definition 2.4 [19] **Hypersoft Set**

Let Ű be a universal set and P(Ű) be a power set of Ű and for $n \geq 1$, there are n distinct attributes such as $k_1,\ k_2,\ k_3, \ldots,\ k_n$ and $K_1,\ K_2,\ K_3, \ldots,\ K_n$ are sets for corresponding values attributes respectively with following conditions such as $K_i \cap K_j = \emptyset$ $(i \neq j)$ and $i, j \in \{1,2,3 \ldots n\}$. Then the pair (F, $K_1 \times K_2 \times K_3 \times \ldots \times K_n$) is said to be Hypersoft set over Ű where F is a mapping from $K_1 \times K_2 \times K_3 \times \ldots \times K_n$ to P(Ű).

Definition 2.5 [22] **Neutrosophic Hypersoft Set (NHSS)**

Let Ű be a universal set and P(Ű) be a power set of Ű and for $n \geq 1$, there are n distinct attributes such as $k_1,\ k_2,\ k_3, \ldots,\ k_n$ and $K_1,\ K_2,\ K_3, \ldots,\ K_n$ are sets for corresponding values attributes respectively with following conditions such as $K_i \cap K_j = \emptyset\ (i \neq j)$ and $i, j \in \{1,2,3 \ldots n\}$. Then the pair (F, Λ) is said to be NHSS over Ű if there exists a relation $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \Lambda$. F is a mapping from $K_1 \times K_2 \times K_3 \times \ldots \times K_n$ to P(Ű) and F($K_1 \times K_2 \times K_3 \times \ldots \times K_n$) = $\{< u, T_\Lambda(u), I_\Lambda(u), F_\Lambda(u) > : u \in$ Ű$\}$ where T, I, F are membership values for truthness, indeterminacy, and falsity respectively such that T, I, F: Ű $\rightarrow\ ]0^-,\ 1^+[$ and $0^- \leq T_\Lambda(u) + I_\Lambda(u) + F_\Lambda(u) \leq 3^+$.

Example 2.6 Assume that a person examines the attractiveness of a living house. Let Ű be a universe which consists of three choices Ű = $\{u_1,\ u_2\}$ and E = {έ$_1$, έ$_2$, έ$_3$} be a set of decision parameters. Then, the NHSS is given as

$$F_\Lambda = \{< \mathrm{u}_1, (έ_1\{0.4, 0.7, 0.5\}, έ_2\{0.8, 0.5, 0.3\}, έ_3\{0.6, 0.5, 0.9\}) >$$
$$< \mathrm{u}_2, (έ_1\{0.1,0.5,0.7\}, έ_2\{0.5, 0.6, 0.2\}, έ_3\{0.7, 0.4, 0.6\}) >\}$$

3. Generalized Aggregate Operators on Neutrosophic Hypersoft Set and Properties

In this section, we present the generalized aggregate operations on NHSS with examples. We prove commutative and associative laws by using proposed aggregate operators in the following section.

Definition 3.1

Let $F_\Lambda \in$ NHSS, then its complement, is written as $(F_\Lambda)^c = F^c(\Lambda)$ and defined as

$F^c(\Lambda) = \{< u, T(F^c(\Lambda)), I(F^c(\Lambda)), F(F^c(\Lambda)) > : u \in \mathrm{U}\}$ such that

$$T(F^c(\Lambda)) = 1 - T_\Lambda(u),$$
$$I(F^c(\Lambda)) = 1 - I_\Lambda(u),$$
$$F(F^c(\Lambda)) = 1 - F_\Lambda(u).$$

Example 3.2 Reconsider example 2.6

$$F^c(\Lambda) = \{< \mathrm{u}_1, (έ_1\{0.6, 0.3, 0.5\}, έ_2\{0.2, 0.5, 0.7\}, έ_3\{0.4, 0.5, 0.1\}) >$$
$$< \mathrm{u}_2, (έ_1\{0.9, 0.5, 0.3\}, έ_2\{0.5, 0.4, 0.8\}, έ_3\{0.3, 0.6, 0.4\}) >\}$$

Proposition 3.3

If $F_\Lambda \in$ NHSS, then $(F^c(\Lambda))^c = F_\Lambda$.

Proof

By using definition 3.1, we have

$$F^c(\Lambda) = \{< u, T(F^c(\Lambda)), I(F^c(\Lambda)), F(F^c(\Lambda)) > : u \in \mathrm{U}\}$$
$$= \{< u, 1 - T\ (F_\Lambda), 1 - I\ (F_\Lambda), 1 - F\ (F_\Lambda) > : u \in \mathrm{U}\},$$

Thus

$$(F^c(\Lambda))^c = \{< u, 1 - (1 - T(F_\Lambda)), 1 - (1 - I(F_\Lambda)), 1 - (1 - F(F_\Lambda)) > : u \in \mathrm{U}\},$$
$$(F^c(\Lambda))^c = \{< u, T(F_\Lambda), I(F_\Lambda), F(F_\Lambda) > : u \in \mathrm{U}\} = F_\Lambda.$$

Which completes the proof.

Definition 3.4 Extended Union of Two Neutrosophic Hypersoft Set

Let $F_{\Lambda_1},\ F_{\Lambda_2} \in$ NHSS, then their extended union is

$$T\ (F_{\Lambda_1} \cup F_{\Lambda_2}) = \begin{cases} T(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ T(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Max\left(T(F_{\Lambda_1}), T(F_{\Lambda_2})\right) & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$I\ (F_{\Lambda_1} \cup F_{\Lambda_2}) = \begin{cases} I(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ I(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\left(I(F_{\Lambda_1}), I(F_{\Lambda_2})\right) & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$F\ (F_{\Lambda_1} \cup F_{\Lambda_2}) = \begin{cases} F(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ F(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\left(F(F_{\Lambda_1}), F(F_{\Lambda_2})\right) & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

Example 3.5 Let U = $\{u_1, u_2, u_3, u_4\}$ be a universal set and E = $\{\acute{\varepsilon}_1, \acute{\varepsilon}_2, \acute{\varepsilon}_3, \acute{\varepsilon}_4\}$ be a set of decision parameters and F_{Λ_1} = $\{u_1, u_4\}$ and F_{Λ_2} = $\{u_2, u_4\}$

$$F_{\Lambda_1} = \{< u_1, (\acute{\varepsilon}_1\{0.4, 0.7, 0.5\}, \acute{\varepsilon}_2\{0.8, 0.5, 0.3\}, \acute{\varepsilon}_3\{0.6, 0.5, 0.9\}, \acute{\varepsilon}_4\{0.3, 0.7, 0.2\}) >$$
$$< u_4, (\acute{\varepsilon}_1\{0.4, 0.7, 0.2\}, \acute{\varepsilon}_2\{0.6, 0.5, 0.3\}, \acute{\varepsilon}_3\{0.8, 0.4, 0.7\}, \acute{\varepsilon}_4\{0.6, 0.4, 0.3\}) >\}$$
$$F_{\Lambda_2} = \{< u_2, (\acute{\varepsilon}_1\{0.7, 0.4, 0.6\}, \acute{\varepsilon}_2\{0.4, 0.6, 0.9\}, \acute{\varepsilon}_3\{0.7, 0.4, 0.6\}, \acute{\varepsilon}_4\{0.7, 0.6, 0.3\}) >$$
$$< u_4, (\acute{\varepsilon}_1\{0.6, 0.2, 0.7\}, \acute{\varepsilon}_2\{0.5, 0.7, 0.3\}, \acute{\varepsilon}_3\{0.4, 0.8, 0.5\}, \acute{\varepsilon}_4\{0.5, 0.6, 0.4\}) >\}$$
$$F_{\Lambda_1} \cup F_{\Lambda_2} = \{< u_1, (\acute{\varepsilon}_1\{0.4, 0.7, 0.5\}, \acute{\varepsilon}_2\{0.8, 0.5, 0.3\}, \acute{\varepsilon}_3\{0.6, 0.5, 0.9\}, \acute{\varepsilon}_4\{0.3, 0.7, 0.2\}) >$$
$$< u_2, (\acute{\varepsilon}_1\{0.7, 0.4, 0.6\}, \acute{\varepsilon}_2\{0.4, 0.6, 0.9\}, \acute{\varepsilon}_3\{0.7, 0.4, 0.6\}, \acute{\varepsilon}_4\{0.7, 0.6, 0.3\}) >$$
$$< u_4, (\acute{\varepsilon}_1\{0.6, 0.7, 0.7\}, \acute{\varepsilon}_2\{0.6, 0.7, 0.3\}, \acute{\varepsilon}_3\{0.8, 0.8, 0.7\}, \acute{\varepsilon}_4\{0.6, 0.6, 0.4\}) >\}$$

Proposition 3.6

Let F_{Λ_1}, F_{Λ_2} and F_{Λ_3} are NHSSs than

1. $\left(F_{\Lambda_1} \cup F_{\Lambda_2}\right) = \left(F_{\Lambda_2} \cup F_{\Lambda_1}\right)$ (Commutative law)
2. $\left(F_{\Lambda_1} \cup F_{\Lambda_2}\right) \cup F_{\Lambda_3} = F_{\Lambda_1} \cup \left(F_{\Lambda_2} \cup F_{\Lambda_3}\right)$ (Associative law)

Proof 1. In the following proof first two cases are trivial, we consider only the third case in this proposition

$\left(F_{\Lambda_1} \cup F_{\Lambda_2}\right) = \left\{< u, \left(max\left\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\right\}, min\left\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\right\}, min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}\right) >\right\}$

$= \left\{< u, \left(max\{T(F_{\Lambda_2}), T(F_{\Lambda_1})\}, min\left\{I(F_{\Lambda_2}), I(F_{\Lambda_1})\right\}, min\{F(F_{\Lambda_2}), F(F_{\Lambda_1})\}\right) >\right\}$

$= \left(F_{\Lambda_2} \cup F_{\Lambda_1}\right)$

Proof 2: Let F_{Λ_1}, F_{Λ_2} and F_{Λ_3} are NHSSs than

$F_{\Lambda_1} \cup F_{\Lambda_2} = \left\{< u, \left(Max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, Min\left\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\right\}, Min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}\right) >\right\}$

$\left(F_{\Lambda_1} \cup F_{\Lambda_2}\right) \cup F_{\Lambda_3} =$

$\{< u, \max\{\max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, T(F_{\Lambda_3})\}, \min\{\min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, I(F_{\Lambda_3})\}, \min\{\min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}, F(F_{\Lambda_3})\} >\}$

$= \left\{< u, \max\left\{T(F_{\Lambda_1}), T(F_{\Lambda_2}), T(F_{\Lambda_3})\right\}, \min\left\{\left\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\right\}, I(F_{\Lambda_3})\right\}, \min\left\{\left\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\right\}, F(F_{\Lambda_3})\right\} >\right\}$

$= \left\{< u, \max\left\{T(F_{\Lambda_1}), \max\left\{T(F_{\Lambda_2}), T(F_{\Lambda_3})\right\}\right\}, \min\left\{I(F_{\Lambda_1}), \min\left\{I(F_{\Lambda_2}), I(F_{\Lambda_3})\right\}\right\}, \min\left\{F(F_{\Lambda_1}), \min\left\{F(F_{\Lambda_2}), F(F_{\Lambda_3})\right\}\right\} >\right\}$

$= F_{\Lambda_1} \cup \left(F_{\Lambda_2} \cup F_{\Lambda_3}\right)$

Definition 3.7 Extended Intersection of Two Neutrosophic Hypersoft Set

Let F_{Λ_1}, F_{Λ_2} ∈ NHSS, then their extended intersection is

$$T\ (F_{\Lambda_1} \cap F_{\Lambda_2}) = \begin{cases} T(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ T(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\left(T(F_{\Lambda_1}), T(F_{\Lambda_2})\right) & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$I\ (F_{\Lambda_1} \cap F_{\Lambda_2}) = \begin{cases} I(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ I(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Max\left(I(F_{\Lambda_1}), I(F_{\Lambda_2})\right) & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$F\ (F_{\Lambda_1} \cap F_{\Lambda_2}) = \begin{cases} F(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ F(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Max\left(F(F_{\Lambda_1}), F(F_{\Lambda_2})\right) & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

Proposition 3.8 Let F_{Λ_1}, F_{Λ_2} and F_{Λ_3} are NHSSs than

1. $F_{\Lambda_1} \cap F_{\Lambda_2} = F_{\Lambda_2} \cap F_{\Lambda_1}$ (Commutative law)
2. $(F_{\Lambda_1} \cap F_{\Lambda_2}) \cap F_{\Lambda_3} = F_{\Lambda_1} \cap (F_{\Lambda_2} \cap F_{\Lambda_3})$ (Associative law)

Proof 1. Similar to Proposition 3.6.

Proposition 3.9 Let F_{Λ_1}, F_{Λ_2} are NHSSs then

1. $(F_{\Lambda_1} \cup F_{\Lambda_2})^c = F^c(\Lambda_1) \cap F^c(\Lambda_2)$
2. $(F_{\Lambda_1} \cap F_{\Lambda_1})^c = F^c(\Lambda_1) \cup F^c(\Lambda_2)$

Proof 1. Let F_{Λ_1} and F_{Λ_1} $\in$ NHSS, such as follows

$F_{\Lambda_1} = \{< \mathrm{u}, \{T(F_{\Lambda_1}), I(F_{\Lambda_1}), F(F_{\Lambda_1})\} >\}$ and $F_{\Lambda_2} = \{< \mathrm{u}, \{T(F_{\Lambda_2}), I(F_{\Lambda_2}), F(F_{\Lambda_2})\} >\}$

$(F_{\Lambda_1} \cup F_{\Lambda_2})^c = \left\{< \mathrm{u}, \left(max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, min\ \{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\right) >\right\}^c$

$= \{< \mathrm{u}, \left(min\{1 - T(F_{\Lambda_1}), 1 - T(F_{\Lambda_2})\}, max\ \{1 - I(F_{\Lambda_1}), 1 - I(F_{\Lambda_2})\}, max\{1 - F(F_{\Lambda_1}), 1 - F(F_{\Lambda_2})\right) >\}$

$= \{< \mathrm{u}, (min\{T(F^c(\Lambda_1)), T(F^c(\Lambda_2))\}, max\ \{I(F^c(\Lambda_1)), I(F^c(\Lambda_2))\}, max\{F(F^c(\Lambda_1)), F(F^c(\Lambda_2))\}) >\}$

$= F^c(\Lambda_1) \cap F^c(\Lambda_2)$

Proof 2. Similarly, we can prove 2.

Definition 3.10 OR-Operation of Two Neutrosophic Hypersoft Set

Let F_{Λ_1}, F_{Λ_2} $\in$ NHSS. Consider $k_1, k_2, k_3, \ldots, k_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $K_1, K_2, K_3, \ldots, K_n$ with $K_i \cap K_j = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \ldots n\}$ and their relation $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \Lambda$, then $F_{\Lambda_1} \vee F_{\Lambda_2} = F_{\Lambda_1 \times \Lambda_2}$, then

$$T\ (F_{\Lambda_1 \times \Lambda_2}) = Max\left(T(F_{\Lambda_1}), T(F_{\Lambda_2})\right),$$

$$I\ (F_{\Lambda_1 \times \Lambda_2}) = Min\left(I(F_{\Lambda_1}), I(F_{\Lambda_2})\right),$$

$$F\ (F_{\Lambda_1 \times \Lambda_2}) = Min\left(F(F_{\Lambda_1}), F(F_{\Lambda_2})\right).$$

Example 3.11 Reconsider example 3.5

$$\begin{aligned} F_{\Lambda_1} \vee F_{\Lambda_2} &= F_{\Lambda_1 \times \Lambda_2} \\ &= \{< (\mathrm{u}_1, \mathrm{u}_2), (\acute{\varepsilon}_1\{0.7, 0.4, 0.5\}, \acute{\varepsilon}_2\{0.8, 0.5, 0.3\}, \acute{\varepsilon}_3\{0.7, 0.4, 0.6\}, \acute{\varepsilon}_4\{0.7, 0.6, 0.2\}) > \\ &\quad < (\mathrm{u}_1, \mathrm{u}_4), (\acute{\varepsilon}_1\{0.6, 0.2, 0.5\}, \acute{\varepsilon}_2\{0.8, 0.5, 0.3\}, \acute{\varepsilon}_3\{0.6, 0.5, 0.5\}, \acute{\varepsilon}_4\{0.5, 0.6, 0.2\}) > \\ &\quad < (\mathrm{u}_4, \mathrm{u}_2), (\acute{\varepsilon}_1\{0.7, 0.4, 0.2\}, \acute{\varepsilon}_2\{0.6, 0.5, 0.3\}, \acute{\varepsilon}_3\{0.8, 0.4, 0.6\}, \acute{\varepsilon}_4\{0.7, 0.4, 0.3\}) > \\ &\quad < (\mathrm{u}_4, \mathrm{u}_4), (\acute{\varepsilon}_1\{0.6, 0.2, 0.2\}, \acute{\varepsilon}_2\{0.6, 0.5, 0.3\}, \acute{\varepsilon}_3\{0.8, 0.4, 0.5\}, \acute{\varepsilon}_4\{0.6, 0.4, 0.3\}) >\} \end{aligned}$$

Definition 3.12 AND-Operation of Two Neutrosophic Hypersoft Set

Let F_{Λ_1}, F_{Λ_2} $\in$ NHSS. Consider $k_1, k_2, k_3, \ldots, k_n$ for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are respectively the set $K_1, K_2, K_3, \ldots, K_n$ with $K_i \cap K_j = = \emptyset$, for $i \neq j$ and $i, j \epsilon \{1,2,3 \ldots n\}$ and their relation $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \Lambda$ then $F_{\Lambda_1} \wedge F_{\Lambda_2} = F_{\Lambda_1 \times \Lambda_2}$, then

$$T\ (F_{\Lambda_1 \times \Lambda_2}) = Min\left(T(F_{\Lambda_1}), T(F_{\Lambda_2})\right),$$

$$I\ (F_{\Lambda_1 \times \Lambda_2}) = Max\left(I(F_{\Lambda_1}), I(F_{\Lambda_2})\right),$$

$$\boldsymbol{F}\ (\boldsymbol{F}_{\Lambda_1 \times \Lambda_2}) = \boldsymbol{Max}\left(\boldsymbol{F}(\boldsymbol{F}_{\Lambda_1}), \boldsymbol{F}(\boldsymbol{F}_{\Lambda_2})\right).$$

Proposition 3.13 Let F_{Λ_1}, F_{Λ_2} are NHSSs then

1. $\left(F_{\Lambda_1} \vee F_{\Lambda_2}\right)^c = F^c(\Lambda_1) \wedge F^c(\Lambda_2)$
2. $\left(F_{\Lambda_1} \wedge F_{\Lambda_2}\right)^c = F^c(\Lambda_1) \vee F^c(\Lambda_2)$

Proof 1. Let F_{Λ_1} and F_{Λ_1} ∈ NHSS, such as follows

$F_{\Lambda_1} = \{< u_i, \{T(F_{\Lambda_1}), I(F_{\Lambda_1}), F(F_{\Lambda_1})\} >: u_i \in U\}$ and $F_{\Lambda_2} = \{< u_j, \{T(F_{\Lambda_2}), I(F_{\Lambda_2}), F(F_{\Lambda_2})\} >: u_j \in U\}$

By using definition 3.10 we get

$F_{\Lambda_1} \vee F_{\Lambda_2} = \{< (u_i, u_j), [e, \max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, \min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}] >\}$

$\left(F_{\Lambda_1} \vee F_{\Lambda_2}\right)^c = \{< (u_i, u_j), [e, 1 - \max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, 1 - \min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, 1 - \min\{F(F_{\Lambda_1}), 1 - F(F_{\Lambda_2})\}] >\}$

$\left(F_{\Lambda_1} \vee F_{\Lambda_2}\right)^c = \{< (u_i, u_j), [e, \min\{1 - T(F_{\Lambda_1}), 1 - T(F_{\Lambda_2})\}, \max\{1 - I(F_{\Lambda_1}), 1 - I(F_{\Lambda_2})\}, \max\{1 - F(F_{\Lambda_1}), 1 - F(F_{\Lambda_2})\}] >\}$

$\left(F_{\Lambda_1} \vee F_{\Lambda_2}\right)^c = \{< (u_i, u_j), [e, \min\{T(F^c(\Lambda_1)), T(F^c(\Lambda_2))\}, \max\{I(F^c(\Lambda_1)), I(F^c(\Lambda_2))\}, \max\{F(F^c(\Lambda_1)), F(F^c(\Lambda_2))\}] >\}$

Since

$F^c(\Lambda_1) = \{< u_i, \{T(F^c(\Lambda_1)), I(F^c(\Lambda_1)), F(F^c(\Lambda_1))\} >: u_i \in U\}$ and

$F^c(\Lambda_2) = \{< u_j, \{T(F^c(\Lambda_2)), I(F^c(\Lambda_2)), F(F^c(\Lambda_2))\} >: u_j \in U\}$

By using definition 3.12, we get

$F^c(\Lambda_1) \wedge F^c(\Lambda_2) = \{< (u_i, u_j), [e, \min\{T(F^c(\Lambda_1)), T(F^c(\Lambda_2))\}, \max\{I(F^c(\Lambda_1)), I(F^c(\Lambda_2))\}, \max\{F(F^c(\Lambda_1)), F(F^c(\Lambda_2))\}] >\}$

So

$\left(F_{\Lambda_1} \vee F_{\Lambda_2}\right)^c = F^c(\Lambda_1) \wedge F^c(\Lambda_2)$.

Similarly, we can prove 2.

4. Necessity and Possibility Operations

The necessity and possibility operations on NHSS with some properties are presented in the following section.

Definition 4.1 Necessity operation

Let F_Λ ∈ NHSS, then necessity operation on NHSS represented by $\oplus\ F_\Lambda$ and defined as follows

$\oplus\ F_\Lambda = \{< \mathrm{u}, \{T(F_\Lambda), I(F_\Lambda), 1 - T(F_\Lambda)\} >\}$ for all $u \in U$.

Example 4.2 Reconsider example 2.6

$$\oplus\ F_\Lambda = \{< \mathrm{u}_1, (\acute{\varepsilon}_1\{0.4, 0.7, 0.6\}, \acute{\varepsilon}_2\{0.8, 0.5, 0.2\}, \acute{\varepsilon}_3\{0.6, 0.5, 0.4\}) >$$
$$< \mathrm{u}_2, (\acute{\varepsilon}_1\{0.1, 0.5, 0.9\}, \acute{\varepsilon}_2\{0.5, 0.6, 0.5\}, \acute{\varepsilon}_3\{0.7, 0.4, 0.3\}) >\}$$

Proposition 4.3

1. $\oplus\ \left(F_{\Lambda_1} \cup F_{\Lambda_2}\right) = \oplus\ F_{\Lambda_2} \cup \oplus\ F_{\Lambda_1}$
2. $\oplus\ \left(F_{\Lambda_1} \cap F_{\Lambda_2}\right) = \oplus\ F_{\Lambda_2} \cap \oplus\ F_{\Lambda_1}$

Proof 1. Let $F_{\Lambda_1} \cup F_{\Lambda_2} = F_{\Lambda_3}$, then

$$T\ (F_{\Lambda_3}) = \begin{cases} T(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ T(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$I\ (F_{\Lambda_3}) = \begin{cases} I(F_{\Lambda_1}) & if\ u\ \in \Lambda_1 - \Lambda_2 \\ I(F_{\Lambda_2}) & if\ u\ \in \Lambda_2 - \Lambda_1 \\ Min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\} & if\ u\ \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$F\ (F_{\Lambda_3}) = \begin{cases} F(F_{\Lambda_1}) & if\ u\ \in \Lambda_1 - \Lambda_2 \\ F(F_{\Lambda_2}) & if\ u\ \in \Lambda_2 - \Lambda_1 \\ Min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\} & if\ u\ \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

By using the definition of necessity operation

$\oplus F_{\Lambda_3} = \{< u, \{\oplus T(F_{\Lambda_3}), \oplus I(F_{\Lambda_3}), \oplus F(F_{\Lambda_3})\} >: u \in U\}$, where

$$\oplus T\ (F_{\Lambda_3}) = \begin{cases} T(F_{\Lambda_1}) & if\ u\ \in \Lambda_1 - \Lambda_2 \\ T(F_{\Lambda_2}) & if\ u\ \in \Lambda_2 - \Lambda_1 \\ Max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\} & if\ u\ \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$\oplus I\ (F_{\Lambda_3}) = \begin{cases} I(F_{\Lambda_1}) & if\ u\ \in \Lambda_1 - \Lambda_2 \\ I(F_{\Lambda_2}) & if\ u\ \in \Lambda_2 - \Lambda_1 \\ Min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\} & if\ u\ \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$\oplus F\ (F_{\Lambda_3}) = \begin{cases} 1 - T(F_{\Lambda_1}) & if\ u\ \in \Lambda_1 - \Lambda_2 \\ 1 - T(F_{\Lambda_2}) & if\ u\ \in \Lambda_2 - \Lambda_1 \\ 1 - Max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\} & if\ u\ \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

Assume

$\oplus F_{\Lambda_1} = \{< u, \{T(F_{\Lambda_1}), I(F_{\Lambda_1}), 1 - T(F_{\Lambda_1})\} >: u \in U\}$

$\oplus F_{\Lambda_2} = \{< u, \{T(F_{\Lambda_2}), I(F_{\Lambda_2}), 1 - T(F_{\Lambda_2})\} >: u \in U\}$

$\oplus F_{\Lambda_1} \cup \oplus F_{\Lambda_2} = F_\delta$, where

$F_\delta = \{< u, \{T(F_\delta), I(F_\delta), F(F_\delta)\} >: u \in U\}$, such that

$$T\ (F_\delta) = \begin{cases} T(F_{\Lambda_1}) & if\ u\ \in \Lambda_1 - \Lambda_2 \\ T(F_{\Lambda_2}) & if\ u\ \in \Lambda_2 - \Lambda_1 \\ Max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\} & if\ u\ \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$I\ (F_\delta) = \begin{cases} I(F_{\Lambda_1}) & if\ u\ \in \Lambda_1 - \Lambda_2 \\ I(F_{\Lambda_2}) & if\ u\ \in \Lambda_2 - \Lambda_1 \\ Min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\} & if\ u\ \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$F\ (F_\delta) = \begin{cases} 1 - T(F_{\Lambda_1}) & if\ u\ \in \Lambda_1 - \Lambda_2 \\ 1 - T(F_{\Lambda_2}) & if\ u\ \in \Lambda_2 - \Lambda_1 \\ Min\{1 - T(F_{\Lambda_1}), 1 - T(F_{\Lambda_2})\} & if\ u\ \in \Lambda_1 \cap \Lambda_2 \end{cases} \text{ OR}$$

$$F\ (F_\delta) = \begin{cases} 1 - T(F_{\Lambda_1}) & if\ u\ \in \Lambda_1 - \Lambda_2 \\ 1 - T(F_{\Lambda_2}) & if\ u\ \in \Lambda_2 - \Lambda_1 \\ 1 - Max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\} & if\ u\ \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

Consequently $\oplus F_{\Lambda_3}$ and F_δ are same. So

$\oplus\ (F_{\Lambda_1} \cup F_{\Lambda_2}) = \oplus\ F_{\Lambda_2} \cup \oplus\ F_{\Lambda_1}$.

Similarly, we can prove 2.

Definition 4.4 Possibility operation

Let $F_\Lambda \in$ NHSS, then possibility operation on NHSS represented by $\otimes\ F_\Lambda$ and defined as follows

$\otimes\ F_\Lambda = \{< \mathrm{u}, \{1 - F(F_\Lambda), I(F_\Lambda), F(F_\Lambda)\} >\}$ for all $u \in U$.

Example 4.5 Reconsider the example 2.6

$$\otimes\ F_\Lambda = \{< \mathrm{u}_1, (\acute{\varepsilon}_1\{0.5, 0.7, 0.5\}, \acute{\varepsilon}_2\{0.7, 0.5, 0.3\}, \acute{\varepsilon}_3\{0.1, 0.5, 0.9\}) >$$

$$< \mathrm{u}_2, (\acute{\varepsilon}_1\{0.3, 0.5, 0.7\}, \acute{\varepsilon}_2\{0.8, 0.6, 0.2\}, \acute{\varepsilon}_3\{0.4, 0.4, 0.6\}) >\}$$

Proposition 4.6

1. $\otimes\ (F_{\Lambda_1} \cup F_{\Lambda_2}) = \otimes\ F_{\Lambda_2} \cup \otimes\ F_{\Lambda_1}$
2. $\otimes\ (F_{\Lambda_1} \cap F_{\Lambda_2}) = \otimes\ F_{\Lambda_2} \cap \otimes\ F_{\Lambda_1}$

Proof 1. Let $F_{\Lambda_1} \cup F_{\Lambda_2} = F_{\Lambda_3}$, then

$$T\ (F_{\Lambda_3}) = \begin{cases} T(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ T(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$I\ (F_{\Lambda_3}) = \begin{cases} I(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ I(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$F\ (F_{\Lambda_3}) = \begin{cases} F(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ F(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

By using the definition of necessity operation

$\otimes F_{\Lambda_3} = \{< \mathrm{u}, \{\otimes T(F_{\Lambda_3}), \otimes I(F_{\Lambda_3}), \otimes F(F_{\Lambda_3})\} >: u \in U\}$, where

$$\otimes\ T\ (F_{\Lambda_3}) = \begin{cases} 1 - F(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ 1 - F(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ 1 - Max\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$= \begin{cases} 1 - F(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ 1 - F(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\{1 - F(F_{\Lambda_1}), 1 - F(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$\otimes\ I\ (F_{\Lambda_3}) = \begin{cases} I(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ I(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$\otimes\ F\ (F_{\Lambda_3}) = \begin{cases} F(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ F(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

Assume

$\otimes F_{\Lambda_1} = \{< \mathrm{u}, \{1 - F(F_{\Lambda_1}), I(F_{\Lambda_1}), F(F_{\Lambda_1})\} >: u \in U\}$

$\otimes F_{\Lambda_2} = \{< \mathrm{u}, \{1 - F(F_{\Lambda_2}), I(F_{\Lambda_2}), F(F_{\Lambda_2})\} >: u \in U\}$

$\otimes F_{\Lambda_1} \cup \oplus F_{\Lambda_2} = F_\delta$, where

$F_\delta = \{< \mathrm{u}, \{T(F_\delta), I(F_\delta), F(F_\delta)\} >: u \in U\}$, such that

$$\otimes\ T\ (F_\delta) = \begin{cases} 1 - F(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ 1 - F(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ 1 - Max\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$\otimes\ I\ (F_\delta) = \begin{cases} I(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ I(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

$$\otimes\ F\ (F_\delta) = \begin{cases} F(F_{\Lambda_1}) & if\ u \in \Lambda_1 - \Lambda_2 \\ F(F_{\Lambda_2}) & if\ u \in \Lambda_2 - \Lambda_1 \\ Min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\} & if\ u \in \Lambda_1 \cap \Lambda_2 \end{cases}$$

Consequently $\otimes F_{\Lambda_3}$ and F_δ are same. So

$\otimes\ (F_{\Lambda_1} \cup F_{\Lambda_2}) = \otimes\ F_{\Lambda_2} \cup \otimes\ F_{\Lambda_1}$

Similarly, we can prove 2.

Proposition 4.7 Let F_{Λ_1} and $F_{\Lambda_2} \in$ NHSS, than we have the following

1. $\oplus(F_{\Lambda_1} \wedge F_{\Lambda_2}) = \oplus F_{\Lambda_1} \wedge \oplus F_{\Lambda_2}$
2. $\oplus(F_{\Lambda_1} \vee F_{\Lambda_2}) = \oplus F_{\Lambda_1} \vee \oplus F_{\Lambda_2}$
3. $\otimes\ (F_{\Lambda_1} \wedge F_{\Lambda_2}) = \otimes\ F_{\Lambda_1} \wedge \otimes\ F_{\Lambda_2}$
4. $\otimes\ (F_{\Lambda_1} \vee F_{\Lambda_2}) = \otimes\ F_{\Lambda_1} \vee \otimes\ F_{\Lambda_2}$

Proof 1. Assume $F_{\Lambda_1} \wedge F_{\Lambda_2} = F_{\Lambda_1 \times \Lambda_2}$, where $(u_i, u_j) \in \Lambda_1 \times \Lambda_2$

$F_{\Lambda_1 \times \Lambda_2} = \{< (u_i, u_j), [e, \min\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, \max\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \max\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}] >\}$

By using definition 4.1, we have

$\oplus(F_{\Lambda_1} \wedge F_{\Lambda_2}) = \{< (u_i, u_j), [e, \min\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, \max\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, 1 - \min\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}] >\}$

Since

$\oplus\ F_{\Lambda_1} = \{< \mathrm{u}, \{T(F_{\Lambda_1}), I(F_{\Lambda_1}), 1 - T(F_{\Lambda_1})\} >\}$, and

$\oplus\ F_{\Lambda_2} = \{< \mathrm{u}, \{T(F_{\Lambda_2}), I(F_{\Lambda_2}), 1 - T(F_{\Lambda_2})\} >\}$, then by using AND-operation, we get

$\oplus\ F_{\Lambda_1} \wedge \oplus\ F_{\Lambda_2} =$

$\{< (u_i, u_j), [e, \min\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, \max\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \max\{1 - T(F_{\Lambda_1}), 1 - T(F_{\Lambda_2})\}] >\}$

$= \{< (u_i, u_j), [e, \min\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, \max\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, 1 - \min\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}] >\}$

$= \oplus(F_{\Lambda_1} \wedge F_{\Lambda_2})$

Proof 2. Similar to Assertion 1.

Proof 3. Assume $F_{\Lambda_1} \wedge F_{\Lambda_2} = F_{\Lambda_1 \times \Lambda_2}$, where $(u_i, u_j) \in \Lambda_1 \times \Lambda_2$

$F_{\Lambda_1 \times \Lambda_2} = \{< (u_i, u_j), [e, \min\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, \max\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \max\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}] >\}$

By using definition 4.4, we have

$\otimes\ (F_{\Lambda_1} \wedge F_{\Lambda_2}) = \{< (u_i, u_j), [e, 1 - \max\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}, \max\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \max\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}] >\}$

Since

$\otimes\ F_{\Lambda_1} = \{< \mathrm{u}, \{1 - F(F_{\Lambda_1}), I(F_{\Lambda_1}), F(F_{\Lambda_1})\} >\}$, and

$\otimes\ F_{\Lambda_2} = \{< \mathrm{u}, \{1 - F(F_{\Lambda_2}), I(F_{\Lambda_2}), F(F_{\Lambda_2})\} >\}$, then by using AND-operation, we get

$\otimes\ F_{\Lambda_1} \wedge \oplus\ F_{\Lambda_2} =$

$\{< (u_i, u_j), [e, \min\{1 - F(F_{\Lambda_1}), 1 - F(F_{\Lambda_2})\}, \max\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \max\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}] >\}$

$= \{< (u_i, u_j), [e, 1 - \max\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}, \max\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \max\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}] >\}$

$= \otimes\ (F_{\Lambda_1} \wedge F_{\Lambda_2})$

Proof 4. Assume $F_{\Lambda_1} \vee F_{\Lambda_2} = F_{\Lambda_1 \times \Lambda_2}$, where $(u_i, u_j) \in \Lambda_1 \times \Lambda_2$

$F_{\Lambda_1 \times \Lambda_2} = \{< (u_i, u_j), [e, \max\{T(F_{\Lambda_1}), T(F_{\Lambda_2})\}, \min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}] >\}$

By using definition 4.4, we have

$\otimes\ (F_{\Lambda_1} \vee F_{\Lambda_2}) = \{< (u_i, u_j), [e, 1 - \min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}, \min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}] >\}$

Since

$\otimes\ F_{\Lambda_1} = \{< \mathrm{u}, \{1 - F(F_{\Lambda_1}), I(F_{\Lambda_1}), F(F_{\Lambda_1})\} >\}$, and

$\otimes\ F_{\Lambda_2} = \{< \mathrm{u}, \{1 - F(F_{\Lambda_2}), I(F_{\Lambda_2}), F(F_{\Lambda_2})\} >\}$, then by using OR-operation, we get

$\otimes\ F_{\Lambda_1} \vee \oplus\ F_{\Lambda_2} =$

$\{< (u_i, u_j), [e, \max\{1 - F(F_{\Lambda_1}), 1 - F(F_{\Lambda_2})\}, \min\{I(F_{\Lambda_1}), I(F_{\Lambda_2})\}, \min\{F(F_{\Lambda_1}), F(F_{\Lambda_2})\}] >\}$

$$= \{< (u_i, u_j), [e, 1 - \min\{F(F_{\wedge_1}), F(F_{\wedge_2})\}, \min\{I(F_{\wedge_1}), I(F_{\wedge_2})\}, \min\{F(F_{\wedge_1}), F(F_{\wedge_2})\}] >\}$$

$$= \otimes (\boldsymbol{F}_{\wedge_1} \wedge \boldsymbol{F}_{\wedge_2})$$

5. Conclusion

In this paper, we study neutrosophic hypersoft set with some basic definition. We proposed the generalized aggregate operators on neutrosophic hypersoft sets such as complement, extended union, extended intersection, And-operation, and Or-operation with their properties and proved the commutative and associative laws on NHSS by using extended union and extended intersection. Finally, the concept of necessity and possibility operations on NHSS with suitable numerical examples and properties are presented. For future trends, we can develop the distance-based similarity measure and will be used for decision making, medical diagnoses, pattern recognition, etc. We also develop the neutrosophic hypersoft matrices with its operations and properties by using proposed operations and use for decision making.

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