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Composite Neutrosophic Finite Automata

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Abstract. The idea behind the neutrosophic set is we can connect the concept by dynamics of opposite interacts and its neutral that are uncertain and get common parts. Automata theory is beneficial to solve computational complexity problem and also it is an influential mathematical modeling tool in computer science. Inspired by the concepts of neutrosophic sets and automata theory, here, we are introducing and discussing the algebraic concept of neutrosophic finite automata based on the paper [10]. Generally, composite machines can be achieved by the output of the one machine that will be used as input for another machines. This paper introduced the concept of composite automata under the environment of the neutrosophic set and also examined the box function between the composite neutrosophic finite automata.

Keywords: automata theory, stable, composite, box function, neutrosophic set

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1. Introduction

Smarandache [27, 28] has proposed an idea of neutrosophic sets which was extending from fuzzy sets. Neutrosophic sets have membership values lies in $]0^-, 1^+[$, the nonstandard unit interval [23] which includes the degree of truth, indeterminacy, and falsity. It is a device for handling the computational complexity of real-life and scientific problems whereas the fuzzy set has limited sources to depict it. The neutrosophic sets are different from intuitionistic fuzzy sets, it is because the neutrosophic set degree of indeterminacy can be defined independently since it is quantified explicitly. Aftermath, there are lots of research works done in various fields such as algebraic structures [5,21,29], topological structures [8,20,24], control theory [17,18,36], decision-making [2, 3, 14, 22, 34], medical [1, 25, 35] and smart product-service system [4].

Generally, computational complexity problems are solved by the automata theory. It has a wide application in computer science and discrete mathematics which is also used to study the behavior of dynamical discrete systems. Fuzzy automata emerge from the inclusion of fuzzy logic into automata theory. Fuzzy finite automata are beneficial to model uncertainties which inherent in many applications [6]. Wee [33] and Santos [26] first introduced the theory of fuzzy finite automata to deal with the notions frequently encountered in the study of natural languages such as vagueness and imprecision. Malik et al [16] introduced a considerably simpler notion of a fuzzy finite state machine that is almost identical to fuzzy finite automatons and greatly contributed to the algebraic study of the fuzzy automaton and fuzzy languages. In addition, several researchers contributed to the development of the theory of fuzzy automata ([11]). Fuzzy finite automata with output offer further inclination in providing output compare to one without outputs. For each assigning input, the machine will generate output and its value is a function of the current state and the current input. Verma and Tiwari [32] recently introduced and studied the concepts of state distinguishability, input-distinguishability, and output completeness of states of a crisp deterministic fuzzy automaton with output function based on [7].

In recent years neutrosophic sets and systems have become an area of interest for many researchers in different areas because it can provide a practical way to address real-world problems more efficiently along with indeterminacy naturally especially in the realm of decision-making. Neutrosophic automata is a newer model, which is extended from a fuzzy automata theory. The neutrosophic set idea was incorporated in automata theory by many researchers in different forms such as finite state machine and its switchboard machine was introduced by under the concept of interval neutrosophic sets [30] and single-valued neutrosophic sets [31]. Further, the finite automata theory has been extended by the concept of general fuzzy automata under the environment of neutrosophic sets, which is called as neutrosophic general finite automata [12]. In addition, the concept of distinguishability and inverse of neutrosophic finite automata was introduced by Kavikumar et al. in [10]. However, still, there are many algebraic structures of neutrosophic automata theory that haven't been studied yet especially automaton with output. Hence, it is important to study more algebraic structures on neutrosophic automata theory with outputs. Therefore, our motive is to study and introduce the concept of composite neutrosophic finite automata which we can obtain by using the outputs of one automaton as inputs to another automaton.

2. Preliminaries

Definition 2.1. Let X be a universe of discourse. The neutrosophic set is an object having the form $A = \{\prec x, \delta_1(x), \delta_2(x), \delta_3(x) \succ |\forall x \in X\}$ where the functions can be defined by $\delta_1, \delta_2, \delta_3 : X \to]0,1[$ and δ_1 is the degree of membership or truth, δ_2 is the degree of indeterminancy and δ_3 is the degree of non-membership or false of the element $x \in X$ to the set A with the condition $\delta_1(x) + \delta_2(x) + \delta_3(x) \leq 3$.

Let X be a universe of discourse and λ is a neutrosophic subset of X. A map $\lambda : X \to L$, where L is a lattice-ordered monoid. The definition of lattice-ordered monoid is as follows:

Definition 2.2. An algebra $\mathbb{L} = (L, \leq, \wedge, \vee, \bullet, 0, 1)$ is called a lattice-ordered monoid if

1. $\mathbb{L} = (L, \leq, \wedge, \vee, 0, 1)$ is a lattice with the least element 0 and the element element 1.
2. $(L, \bullet, 1)$ is a monoid with 1 identity $1 \in L$ such that $a, b, c \in L$.
   - (a) $a \bullet 0 = 0 \bullet a = 0$,
   - (b) $a \leq b \Rightarrow a \bullet x \leq b \bullet b, \forall x \in L$,
   - (c) $a \bullet (b \vee c) = (a \bullet b) \vee (b \bullet c)$ and $(b \vee c) \bullet a = (b \bullet a) \vee (c \bullet a)$.

Throughout, we work with a lattice-ordered monoid $\mathbb{L}$ so that the monoid $(L, \bullet, 1)$ satisfies the left cancellation law. A neutrosophic finite automaton with outputs (in short; neutrosophic finite automata (NFA)) has considered with neutrosophic transition function and neutrosophic output function.

Definition 2.3. A NFA is a five-tuple $\mathbb{M} = (Q, \Sigma, Z, \delta, \sigma)$, where Q is a finite non-empty set of states, Σ is a finite set of input alphabet, Z is a finite set of output alphabet, δ is a neutrosophic subset of $Q \times \Sigma \times Q$ which represents neutrosophic transition function, and σ is a neutrosophic subset of $Q \times \Sigma \times Z$ which represents neutrosophic output function.

Definition 2.4. Let $\mathbb{M} = (Q, \Sigma, Z, \delta, \sigma)$ be a NFA.

1. $Q = \{q_1, q_2, \ldots, q_n\}$, is a finite set of states,
2. $\Sigma = \{x_1, x_2, \ldots, x_n\}$, is a finite set of input symbols,
3. $Z = \{y_1, y_2, \ldots, y_n\}$, is a finite set of output symbols,
4. Let $\delta = \prec \delta_1, \delta_2, \delta_3 \succ$ is a neutrosophic subset of $Q \times \Sigma \times Q$ such that the neutrosophic transition function $\delta : A \times \Sigma \times Q \to L \times L \times L$ is defined as follows: $\forall q_i, q_j \in Q$ and $x_1, x_2 \in \Sigma$,

$$\delta_1(q_i, \Lambda, q_j) = \begin{cases} 1 & \text{if } q_i = q_j \\ 0 & \text{if } q_i \neq q_j \end{cases}$$

$$\delta_2(q_i, \Lambda, q_j) = \begin{cases} 0 & \text{if } q_i = q_j \\ 1 & \text{if } q_i \neq q_j \end{cases}$$

$$\delta_3(q_i, \Lambda, q_j) = \begin{cases} 0 & \text{if } q_i = q_j \\ 1 & \text{if } q_i \neq q_j \end{cases}$$

and

$$\delta_1(q_i, x_1x_2, q_j) = \bigvee_{r\in Q} \{\delta_1(q_i, x_1, r) \wedge \delta_1(r, x_2, q_j)\}$$

$$\delta_2(q_i, x_1x_2, q_j) = \bigwedge_{r\in Q} \{\delta_2(q_i, x_1, r) \vee \delta_2(r, x_2, q_j)\}$$

$$\delta_3(q_i, x_1x_2, q_j) = \bigwedge_{r\in Q} \{\delta_3(q_i, x_1, r) \vee \delta_3(r, x_2, q_j)\}$$

(5) Let $\sigma = \prec \sigma_1, \sigma_2, \sigma_3 \succ$ is a neutrosophic subset of $Q \times \Sigma \times Z$ such that the neutrosophic output function $\sigma : Q \times \Sigma \times Z \to L \times L \times L$ is defined as follows: $\forall q_i, q_j \in Q, x_1, x_2 \in \Sigma$ and $y_1, y_2 \in Z$,

$$\sigma_1(q_i, x_1, q_j) = \begin{cases} 1 & \text{if } x_1 = y_1 = \Lambda \\ 0 & \text{if } x_1 = \Lambda, y_1 \neq \Lambda \text{ or } x_1 \neq \Lambda, y_1 = \Lambda \end{cases}$$

$$\sigma_2(q_i, x_1, q_j) = \begin{cases} 0 & \text{if } x_1 = y_1 = \Lambda \\ 1 & \text{if } x_1 = \Lambda, y_1 \neq \Lambda \text{ or } x_1 \neq \Lambda, y_1 = \Lambda \end{cases}$$

$$\sigma_3(q_i, x_1, q_j) = \begin{cases} 0 & \text{if } x_1 = y_1 = \Lambda \\ 1 & \text{if } x_1 = \Lambda, y_1 \neq \Lambda \text{ or } x_1 \neq \Lambda, y_1 = \Lambda \end{cases}$$

and

$$\sigma_1(q_i, x_1x_2, y_1y_2) = \sigma_1(q_i, x_1, y_1) \bullet \bigvee_{r\in Q} \{\delta_1(q_i, x_1, r) \wedge \sigma_1(r, x_2, y_2)\}$$

$$\sigma_2(q_i, x_1x_2, y_1y_2) = \sigma_2(q_i, x_1, y_1) \bullet \bigwedge_{r\in Q} \{\delta_2(q_i, x_1, r) \vee \sigma_2(r, x_2, y_2)\}$$

$$\sigma_3(q_i, x_1x_2, y_1y_2) = \sigma_3(q_i, x_1, y_1) \bullet \bigwedge_{r\in Q} \{\delta_3(q_i, x_1, r) \vee \sigma_3(r, x_2, y_2)\}$$

3. Composite Neutrosophic Finite Automata

This section is interested in the concept of composite finite automata under the environment of neutrosophic sets.

Definition 3.1. For $i \leq n$, let $\mathbb{M}_i = (Q_i, \Sigma_i, Z_i, \delta^i, \sigma^i)$ be NFA's. Let $\mathbb{M}_T = \mathbb{M}_1 \to \mathbb{M}_2 \to \cdots \to \mathbb{M}_n$ be a composite NFA, where $(q_1, q_2, \ldots, q_n) = q_T \in Q_T$ and each $q_i \in Q_i$ if

(1) $Z_i \subseteq \Sigma_{i+1}$, for $i \leq n-1$.

(2) let $\{(x_T \in \Sigma_T \Rightarrow x_1 \in \Sigma_1)(y_T \in Z_T \Rightarrow y_n \in Z_n) | \sigma_1^1(q_1, x_T, y_1) > 0, \sigma_2^1(q_1, x_T, y_1) < 1, \sigma_3^1(q_1, x_T, y_1) < 1, \text{ for } i = 1\}$ then define

$$\delta_1^T\left[(q_1, q_2, \ldots, q_n), x_T, (q_1', q_2', \ldots, q_n')\right] = \begin{cases} \delta_1^1(q_1, x_1, q_1') > 0 & \text{for } i = 1, \\ \delta_1^i(q_i, (\sigma_1^i(q_i, y_{i-1}, y_i)), q_i') & \text{for } i > 1. \end{cases}$$

,

$$\delta_2^T\left[(q_1, q_2, \ldots, q_n), x_T, (q_1', q_2', \ldots, q_n')\right] = \begin{cases} \delta_2^1(q_1, x_1, q_1') < 1 & \text{for } i = 1, \\ \delta_2^i(q_i, (\sigma_2^i(q_i, y_{i-1}, y_i)), q_i') & \text{for } i > 1. \end{cases}$$

$$\delta_3^T\left[(q_1,q_2,\ldots,q_n),x_T,(q_1',q_2',\ldots,q_n')\right]=\begin{cases}\delta_3^1(q_1,x_1,q_1')<1 & \text{for } i=1,\\ \delta_3^i(q_i,(\sigma_3^i(q_i,y_{i-1},y_i)),q_i') & \text{for } i>1.\end{cases}$$

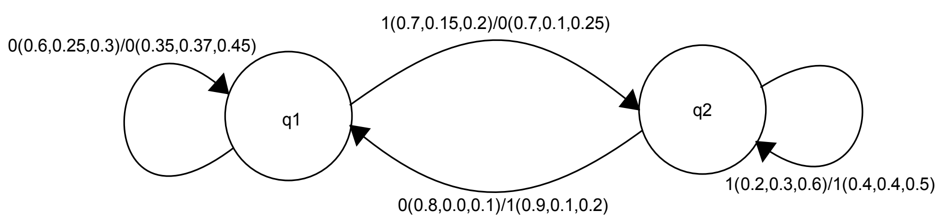
and

$$\sigma_1^T((q_1,q_2,\ldots,q_n),x_T,y_n)=\begin{cases}1 & \text{if } x_T=y_n=\Lambda\\ 0 & \text{if either } x_T\neq\Lambda \text{ and } y_n=\Lambda \text{ or } x_T=\Lambda \text{ and } y_n\neq\Lambda\end{cases}$$

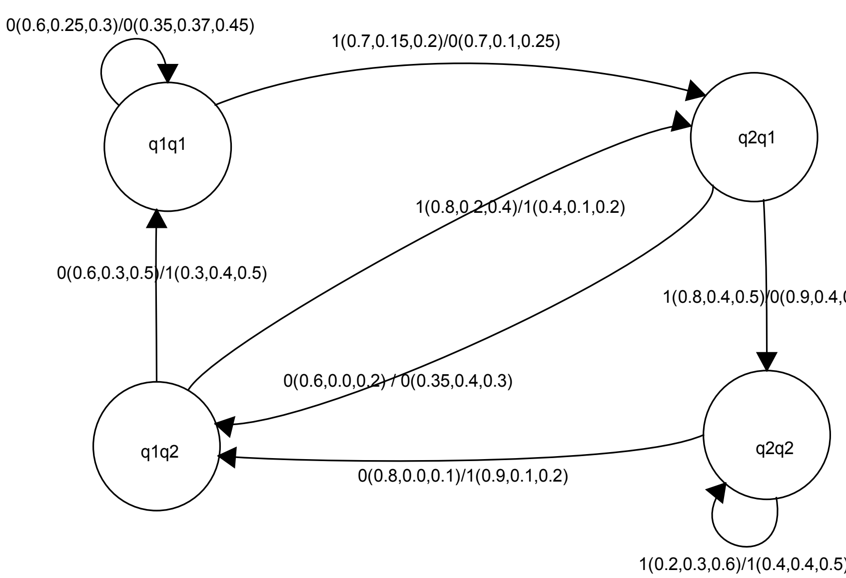
$$\sigma_2^T((q_1,q_2,\ldots,q_n),x_T,y_n)=\begin{cases}0 & \text{if } x_T=y_n=\Lambda\\ 1 & \text{if either } x_T\neq\Lambda \text{ and } y_n=\Lambda \text{ or } x_T=\Lambda \text{ and } y_n\neq\Lambda\end{cases}$$

$$\sigma_3^T((q_1,q_2,\ldots,q_n),x_T,y_n)=\begin{cases}0 & \text{if } x_T=y_n=\Lambda\\ 1 & \text{if either } x_T\neq\Lambda \text{ and } y_n=\Lambda \text{ or } x_T=\Lambda \text{ and } y_n\neq\Lambda\end{cases}$$

Example 3.2. Let $\mathbb{M}=(Q,\Sigma,Z,\delta,\sigma)$ is a NFA, where $Q=\{q_1,q_2\}$, $\Sigma=\{a,b\}$ and $Z=\{0,1\}$ and the transition diagram is given below:



Now, we define the composite NFA, $\mathbb{M}_T=\mathbb{M}\to\mathbb{M}$ and its transition diagram is given below:



Then the output for input $x_T=1001$ is $y_T=0010$.

Definition 3.3. Let $\mathbb{M} = (Q, \Sigma, Z, \delta, \sigma)$ be a NFA. A non-empty set of states $Q_A \subseteq \mathbb{M}$ is said to be stable if

$$\delta_1(q, x, p) > 0, \delta_2(q, x, p) < 1, \delta_3(q, x, p) < 1,$$

for all $q, p \in Q_A$ and $x \in \Sigma$.

Definition 3.4. Two NFA's $\mathbb{M}_1 = (Q_1, \Sigma_1, Z_1, \delta^1, \sigma^1)$ and $\mathbb{M}_2 = (Q_2, \Sigma_2, Z_2, \delta^2, \sigma^2)$ are said to be homomorphism if $\alpha\left[\delta^1(q, x, p)\right] = \delta^2(\alpha(q), \beta(x), \alpha(p))$ and $\sigma^1(q, x, y) \leq \sigma^2(\alpha(q), \beta(x), \gamma(y))$, $\forall q, p \in Q_1$, $x \in \Sigma_1$ and $y \in Z_1$, where the mapping $\alpha : Q_1 \to Q_2$, $\beta : \Sigma_1 \to \Sigma_2$ and $\gamma : Z_1 \to Z_2$ are monoid homomorphisms. Moreover, two NFA's are said to be isomorphism when the mapping α, β and γ are bijective.

Lemma 3.5. *Let $\mathbb{M}_1 = (Q_1, \Sigma_1, Z_1, \delta^1, \sigma^1)$, $\mathbb{M}_2 = (Q_2, \Sigma_2, Z_2, \delta^2, \sigma^2)$ and $\mathbb{M}_3 = (Q_3, \Sigma_3, Z_3, \delta^3, \sigma^3)$ be NFA's. Then $\mathbb{M}_1 \to (\mathbb{M}_2 \to \mathbb{M}_3)$ and $(\mathbb{M}_1 \to \mathbb{M}_2) \to \mathbb{M}_3$ are isomorphic.*

Proof. Since one neutrosophic finite automaton outputs are used as the another neutrosophic finite automaton inputs and omit the parentheses as follows $\mathbb{M}_1 \to \mathbb{M}_2 \to \mathbb{M}_3$. Now, we have an initial inputs for $\mathbb{M}_1$ and its outputs will become an input of $\mathbb{M}_2$. Then, the outputs of $\mathbb{M}_2$ will be an input of $\mathbb{M}_3$. In this manner, $\mathbb{M}_1 \to (\mathbb{M}_2 \to \mathbb{M}_3)$ and $(\mathbb{M}_1 \to \mathbb{M}_2) \to \mathbb{M}_3$ are isomorphic.

Remark 3.6. Lemma 3.5 can be easily extend to four or more NFA's.

Lemma 3.7. *Let $\mathbb{M}_i = (Q_i, \Sigma_i, Z_i, \delta^i, \sigma^i)$, where $i = 1, 2, \ldots, n$, be NFA's. If $\mathbb{M}_1 \to \mathbb{M}_2 \to \cdots \to \mathbb{M}_n$ is a composite NFA if and only if $\mathbb{M}_n$ is a NFA.*

Proof. Assume that $\mathbb{M}_1 \to \mathbb{M}_2 \to \cdots \to \mathbb{M}_n$ is a composite NFA. Then, by lemma 3.5, it is clear that $\mathbb{M}_n$ is a NFA. Conversely, since $\mathbb{M}_n$ is a NFA, the input of $\mathbb{M}_n$ is a output of the $\mathbb{M}_{n-1}$, so in this manner, $\mathbb{M}_1 \to \mathbb{M}_2 \to \cdots \to \mathbb{M}_n$ is a composite NFA.

Definition 3.8. A NFA $\mathbb{M} = (Q, \Sigma, Z, \delta, \sigma)$ is called free if $\forall q_i \in Q$, $x \in \Sigma$ $\exists$ $y \in Z$ such that

$$\sigma_1(q_i, x, y) > 0, \quad \sigma_2(q_i, x, y) < 1, \quad \text{and} \quad \sigma_3(q_i, x, y) < 1.$$

Theorem 3.9. *For each positive integer $i \leq n$, let $\mathbb{M}_i$ is a free NFA, then $\mathbb{M}_1 \to \mathbb{M}_2 \to \cdots \to \mathbb{M}_n$ is a composite NFA.*

Proof. Suppose $\mathbb{M}_i$, $i = 1, 2, \ldots, n$ is a NFA. Let $q, p \in Q_1$ and $x_1 \in \Sigma_1$ and $y_1 \in Z_1$. We prove the theorem by induction on $|i| = n$.

If $n = 1$, then $\mathbb{M}_1$ is a free NFA. Now, we have

$$\sigma_1^1(q_1, x_1, y_1) > 0, \quad \sigma_2^1(q_1, x_1, y_1) < 1, \quad \text{and} \quad \sigma_3^1(q_1 x_1, y_1) < 1,$$

since $\delta_1^1(q_1, x_1, p_1) > 0, \delta_2^1(q_1, x_1, p_1) < 1$ and $\delta_3^1(q_1, x_1, p_1) < 1$. This implies that $\mathbb{M}_1$ is a composite NFA. Hence, the theorem is true for $n = 1$.

Suppose the result is true for all $x_i \in \Sigma_i$ and $y_i \in Z_i$ such that $|i| = n-1$. Let $Z_i \subseteq \Sigma_{i+1}$ for $i \leq n-1$, $n > 1$, so that $\mathbb{M}_{n-1}$ is a free NFA. Now, we have,

$$\sigma_1^{n-1}(q_{n-1}, x_{n-1}, y_{n-1}) > 0, \quad \sigma_2^{n-1}(q_{n-1}, x_{n-1}, y_{n-1}) < 1, \quad \text{and} \quad \sigma_3^{n-1}(q_{n-1}, x_{n-1}, y_{n-1}) < 1.$$

Then by Definition 3.1, we have

$$\delta_1^n(q_n, y_{n-1}, p_n) > 0, \quad \delta_2^n(q_n, y_{n-1}, p_n) < 1 \quad \text{and} \quad \delta_3^n(q_n, y_{n-1}, p_n) < 1.$$

By the induction hypothesis and consider $y_{n-1} = x_n$, then we have

$$\delta_1^n(q_n, x_n, p_n) > 0, \quad \delta_2^n(q_n, x_n, p_n) < 1 \quad \text{and} \quad \delta_3^n(q_n, x_n, p_n) < 1.$$

This implies that, for $x_n \in \Sigma_n$ there exists $y_n \in Z_n$ such that

$$\sigma_1^n(q_n, x_n, y_n) > 0, \quad \sigma_2^n(q_n, x_n, y_n) < 1, \quad \text{and} \quad \sigma_3^n(q_n, x_n, y_n) < 1.$$

Hence, the theorem is true for induction.

Remark 3.10. The converse of Theorem 3.9 is not true since the outputs of composite NFA need not be satisfy the condition of free NFA.

Definition 3.11. Let $\mathbb{M}_1 = (Q_1, \Sigma_1, Z_1, \delta^1, \sigma^1)$ and $\mathbb{M}_2 = (Q_2, \Sigma_2, Z_2, \delta^2, \sigma^2)$ be NFA's. A box function β of $(\mathbb{M}_1, \mathbb{M}_2)$ is satisfy the following conditions, where $\beta : Q_1 \to Q_2$ such that

(1) $\Sigma_1 \subseteq Z_2$

(2) for all $q, p \in Q_1$ and $x \in \Sigma_1$ there exists $y \in Z_1$ such that

$$\beta\left[\delta^1(q, x, p)\right] = \delta^2\left[\beta(q), \sigma^1(q, x, y), \beta(p)\right].$$

Definition 3.12. Let $\mathbb{M}_i = (Q_i, \Sigma_i, Z_i, \delta^i, \sigma^i)$, i=1,2,...,n, be NFA's. To each box functions β_i of $(\mathbb{M}_i, \mathbb{M}_{i+1})$ for $1 \leq i \leq n-1$, there is a corresponding sub NFA $\mathbb{N}(\beta_1, \beta_2, \ldots, \beta_{n-1})$ of $\mathbb{M}_T = \mathbb{M}_1 \to \mathbb{M}_2 \to \cdots \to \mathbb{M}_n$.

Proposition 3.13. *Let* $\mathbb{M}_T = (Q_T, \Sigma_T, Z_T, \delta^T, \sigma^T)$ *be a composite NFA and* $\mathbb{N} = (Q_N, \Sigma_N, Z_N, \delta^N, \sigma^N) \subseteq \mathbb{M}$*, where* $Q_N = \{(q_1, q_2, \ldots, q_n) | q_1 \in \mathbb{M}$ *and* $q_i = \beta_{i-1}(q_{i-1})$ *for* $i > 1\}$*. If* Q_T *is stable, then* $\mathbb{N}$ *is a compositie NFA.*

Proof. Let $q = (q_1, \ldots, q_n), q' = (q'_1, \ldots, q'_n) \in Q_N, x_T \in \Sigma_T$ and $y_i \in Z_T$. Then, by definition 3.1 and $y_{i-1} = x_i$. Since $Q_N \subseteq Q_T$, it is enough to prove that Q_N is stable, for each $i > 1$. Then

$$\begin{aligned}\delta_1^i(q_i, x_i, q'_i) &= \delta_1^i\left[\beta_{i-1}(q_{i-1}), (\sigma_1^{i-1}(q_{i-1}, y_{i-2}, y_{i-1})), \beta_{i-1}(q'_{i-1})\right] \\ &= \beta_{i-1}\left[\delta_1^{i-1}(q_{i-1}, x_{i-1}, q'_{i-1})\right], \text{since } \beta_{i-1} \text{ is a box function of } (\mathbb{M}_{i-1}, \mathbb{M}_i), \\ &= \delta_1^{i-1}\left[\beta_{i-1}(q_{i-1}), x_{i-1}, \beta_{i-1}(q'_{i-1})\right]\end{aligned}$$

This implies that $\delta_1^{i-1}\left[\beta_{i-1}(q_{i-1}), x_{i-1}, \beta_{i-1}(q'_{i-1})\right]$ is stable, since $\delta_1^{i-1}(q_{i-1}, x_{i-1}, q'_{i-1})$ is stable. Hence, Q_N is stable. Therefore, $\mathbb{N}$ is a composite NFA.

Theorem 3.14. *Let $\mathbb{M}_1 = (Q_1, \Sigma_1, Z_1, \delta^1, \sigma^1)$ and $\mathbb{M}_2 = (Q_2, \Sigma_2, Z_2, \delta^2, \sigma^2)$ be two NFA's and let $\mathbb{H}$ be a NFA with inputs Σ_H which generating inputs set for Σ_1. Suppose $Z_1 \subseteq \Sigma_2$ and for all $p, q \in Q_1$, $x_1 \in \Sigma_H$, the map $\beta : Q_1 \to Q_2$ such that $\beta[\delta^1(q, x_1, p)] = \delta^2[\beta(q), \sigma^1(q, x_1, y_1), \beta(p)]$. Then β is a box function of* $(\mathbb{M}_1, \mathbb{M}_2)$.

Proof. We will prove the result by mathematical induction on the generated set of inputs Σ_H. For $n = 1$, let $x_1 \in \Sigma_H$ the result follows from 3.11.

For $n = 2$, let $x_1, x_2 \in \Sigma_H$ and $q, p \in Q_1$, then

$$\begin{aligned}
\beta\left[\delta^1(q, x_1x_2, p)\right] &= \beta\left[\bigvee_{r\in Q_1}\left\{\delta^1(q, x_1, r) \wedge \delta^1(r, x_2, p)\right\}\right] \\
&= \bigvee_{r\in Q_1}\left\{\beta(\delta^1(q, x_1, r)) \wedge \beta(\delta^1(r, x_2, p))\right\} \\
&= \bigvee_{\beta(r)\in Q_2}\left\{\delta^2(\beta(q), \sigma^1(q, x_1, y_1), \beta(r)) \wedge \delta^2(\beta(r), \sigma^1(q, x_2, y_2), \beta(p))\right\} \\
&= \delta^2\left[\beta(q), \sigma^1(q, x_1, y_1) \bullet \sigma^1(q, x_2, y_2), \beta(p)\right] \\
&= \delta^2\left[\beta(q), \sigma^1(q, x_1x_2, y_1y_2), \beta(p)\right]
\end{aligned}$$

If the induction continues for any finite sequence of inputs such as $n > 2$ for each $x_i \in \Sigma_H$, the results follows by induction. Hence β is a box function of $(\mathbb{M}_1, \mathbb{M}_2)$.

4. Conclusions

The main focus of this paper is to study the algebraic automata theory based on the concept of neutrosophic sets. Thus, this investigation contributes a small portion to algebraic automata theory such as composite neutrosophic finite automata which is established by outputs of one automaton as the inputs of another automaton. The future study will be concerned with similar concepts but the approaches are based on the combination of N-fuzzy structures [9,13] and type-2 fuzzy structures [15,19] under the environment of neutrosophic sets [27,28].

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