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Weak LA-hypergroups; Neutrosophy, Enumeration and Redox Reaction

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Abstract. The main motivation of this article is to introduce the theme of Neutrosophic triplet(NT) H_v-LA-Groups. This inspiration is recieved from the structure of weak non-associative Neutrosophic triplet(NT) structures. For it, firstly, we define that each element x have left neut(x) and left anti(x) , which may or may not unique. We further introduce the notion of neutrosophic triplet Hv-LA-subgroups and neutrosophic weak homomorphism on NT H_v-LA-Group. Secondly, presented NT H_v-LA-Group and develop two Mathematica Packages which help to check the left invertive law, weak left invertive law and reproductive axiom. Finally established a numerical example to validate the proposed approach in chemistry using redox reactions.

Keywords: H$_v$ LA-groups, NT sets, Neutro weak homomorphism, Mathematica Packages, Chemical applications,

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1. Introduction

Neutrosophic logic: Neutrosophy is the new branch of philosophy that studies the origin and scope of neutralities, as well as their interaction with different ideational spectra. Smarandache used the idea of neutrosophic set. He defined the theme of t-membership, i- membership and f-membership, so neutrosophic logic generalize all previous versions, see [1], [2], [3]. Many researchers have studied neutrosophic cubic set, complex neutrosophic cubic set, N-cubic set and their applications in real life problems, see [52–55]. Further Abdel-Basset et. al., use neutrosophic set in different direction and discuss their use in real life probems [56–60] More about the neutrosophic algebraic structures we refer the reader [4–6] and [7–12]. For the NT groups see [13–18].

Hyperstructures theory: In 1934, Marty [19] introduced the theme of hyperstructures. More about the hyperstructures we refer the reader [20–22]. The idea of weak structure, which is known as H_v-structure is introduced by Vougiouklis [23], see also [24–31]. In 2007 Davvaz and Fotea mainly dedicated to the study of hyperring theory [32]. Davvaz and Vougiouklis [33], published recently a new book having title "A walk through weak hyperstructures, Hv-Structures" with some interesting applications of hyperstructures.

Left Invertive Structures: Kazim and Naseerudin [34] laid the idea of left almost semigroup (denoted by LA-semigroup). Afterwards, Mushtaq [35] and some other researcher, further worked in detail on the structure of LA-semigroup, see papers [36–42]. Hila and Dine [43] in 2011, furnished the idea of LA-semihypergroup. More detail can be seen in [44], [45], [46], [47], [48], [49], [50], [51].

Our Approach: This paper is the continuation of our published paper [18] and it consists of 6 sections. We arrange this work as: In section 2, we collected some of the relevant material after the introduction. In section 3, we give a new class of algebraic hyperstructure known as NT H_v-LA-Group, which is the main theme of LA-Group, LA-hypergroup, H_v-LA-Group. In NT H_v-LA-Group each element k have left neut(k) and left anti(k), which may or may not unique. We also define the neutro weak homomorphism on NT H_v-LA-Group. Moreover, we discuss many interesting properties of NT H_v-LA-Groups. In section 4, we provide the construction of NT H_v-LA-Groups with the two Mathematica Packages which help to check the left invertive law, weak left invertive law and reproductive axiom. In section 5, we present the application of propose structure in chemical reactions. In section 6, we end with the concluding remarks.

2. Preliminaries

In this section, we added some basic definition and result, which helped to prove the result of our proposed structure.

Definition 2.1. [44] "A hypergroupoid $(\aleph, \circ)$ is called LA-semihypergroup, if it satisfies the following law

$$(\flat_1 \circ \flat_2) \circ \flat_3 = (\flat_3 \circ \flat_2) \circ \flat_1 \text{ for all } \flat_1, \flat_2, \flat_3 \in \aleph.$$

"

Example 2.2. [44] "Let $\aleph = Z$ if we define $\flat_1 \circ \flat_2 = \flat_2 - \flat_1 + 3Z$, where $\flat_1, \flat_2 \in Z$. Then $(\aleph, \circ)$ become LA-semih ypergroup."

Definition 2.3. [24] "The hyperoperation $* : \aleph \times \aleph \longrightarrow P^*(\aleph)$ is called weakly associative hyperoperation (abbreviated as WASS) if for any $\flat_1, \flat_2, \flat_3 \in \aleph$

$$(\flat_1 * \flat_2) * \flat_3 \cap \flat_1 * (\flat_2 * \flat_3) \neq \phi$$

"

Definition 2.4. [24] "The hyperoperation is weakly commutative (abbreviated as COW) if for any $\flat_1, \flat_2 \in \aleph$

$$\flat_1 * \flat_2 \cap \flat_2 * \flat_1 \neq \phi$$

"

Definition 2.5. [47] "Let $\aleph$ be non-empty set and $*$ be hyperoperation on $\aleph$. Then $(\aleph, *)$ is called an $\aleph_v$-LA-semigroup, if it satisfies the weak left invertive law for all $\flat_1, \flat_2, \flat_3 \in \aleph$

$$(\flat_1 * \flat_2) * \flat_3 \cap (\flat_3 * \flat_2) * \flat_1 \neq \phi$$

"

Example 2.6. [47] "Let $\aleph = (0, \infty)$ we define $\flat_1 * \flat_2 = \left\{ \frac{\flat_2}{\flat_1 + 1}, \frac{\flat_2}{\flat_1} \right\}$ where $\flat_1, \flat_2 \in \aleph$. Then for all $\flat_1, \flat_2, \flat_3 \in \aleph$. Then for all $\flat_1, \flat_2, \flat_3 \in \aleph$ satisfies $(\flat_1 * \flat_2) * \flat_3 \cap (\flat_3 * \flat_2) * \flat_1 \neq \phi$. Hence $(\aleph, *)$ is an H_v-LA-semigroup."

3. Neutrosophic Triplet(NT) H$_v$-LA-Groups

In this section, we define a new class of hyper algebraic structure known as NT H$_v$-LA-group and discuss some results on NT H$_v$-LA-group.

Definition 3.1. Let $(\aleph, *)$ be a left (resp., right, pure left, pure right) NT set. Then $\aleph$ is called left (resp., right, pure left , pure right) NT H$_v$-LA-group, if it satisfies the following axioms,

(1) $(\aleph, *)$ is well defined,

(2) $(\aleph, *)$ satisfies the weak left invertive law, i.e, $(\flat_1 * \flat_2) * \flat_3 \cap (\flat_3 * \flat_2) * \flat_1 \neq \phi$ for all $\flat_1, \flat_2, \flat_3 \in \aleph$,

(3) $\aleph * \flat_1 = \aleph = \aleph * \flat_1$ for all $\flat_1 \in \aleph$.

Example 3.2. Let $\aleph = \{\flat_1, \flat_2, \flat_3\}$ be a finite set. The hyperoperation $*$ is defined in Table-1

$*$	$\flat_1$	$\flat_2$	$\flat_3$
$\flat_1$	$\flat_1$	$\{\flat_1, \flat_2\}$	$\{\flat_1, \flat_3\}$
$\flat_2$	$\flat_3$	$\{\aleph\}$	$\{\flat_1, \flat_2\}$
$\flat_3$	$\flat_2$	$\{\flat_1, \flat_3\}$	$\{\aleph\}$

Table-1, neutrosophic triplet H$_v$-LA-group

Here all elements of $\aleph$ satisfy the weak left invertive law. Also left invertive law is not hold in $\aleph$, i.e.

$$\aleph = (\flat_1 * \flat_2) * \flat_3 \neq (\flat_3 * \flat_2) * \flat_1 = \{\flat_1, \flat_2\}.$$

Alike, associative law is not hold in $\aleph$ i.e.

$$\aleph = (\flat_3 * \flat_3) * \flat_1 \neq \flat_3 * (\flat_3 * \flat_1) = \{\flat_1, \flat_3\}.$$

Even, weak associative law is not valid here

$$\{\flat_2\} = (\flat_2 * \flat_1) * \flat_1 \cap \flat_2 * (\flat_1 * \flat_1) = \{\flat_3\} = \phi.$$

Here $(\flat_1, \flat_1, \flat_1)$, $(\flat_2, \flat_1, \flat_2)$, $(\flat_3, \flat_1, \flat_3)$ are left NT sets. Hence $(\aleph, *)$ is a NT H_v-LA-group.

Proposition 3.3. *Let $(\aleph, *)$ be a pure right NT H_v-LA-group. Then $neut(\flat_1) * \flat_2 = neut(\flat_1) * \flat_3$ if $anti(\flat_1) * \flat_2 = anti(\flat_1) * \flat_3$ for all $\flat_1, \flat_2, \flat_3 \in \aleph$.*

Proof. Suppose $(\aleph, *)$ is a pure right NT H_v-LA-group and $anti(\flat_1) * \flat_2 = anti(\flat_1) * \flat_3$ for $\flat_1, \flat_2, \flat_3 \in \aleph$. Multiply $\flat_1$ to the left side of $(\flat_1 * anti(\flat_1) * \flat_2 = (\flat_1 * anti(\flat_1)) * \flat_3$,

$$(\flat_1 * anti(\flat_1)) * \flat_2 = (\flat_1 * anti(\flat_1)) * \flat_3$$

$$neut(\flat_1) * \flat_2 = neut(\flat_1) * \flat_3 \text{ (because } neut(\flat_1) = \flat_1 * anti(\flat_1)\text{)}.$$

Therefore, $neut(\flat_1) * \flat_2 = neut(\flat_1) * \flat_3$. □

Theorem 3.4. *Let $(\aleph, *)$ be a pure right NT H_v-LA-group. Then $neut(\flat_1) * neut(\flat_1) = neut(\flat_1)$.*

Proof. Consider $neut(\flat_1) * neut(\flat_1) = neut(\flat_1)$. Multiply first with $\flat_1$ to the right, i.e.,

$$(\flat_1 * (neut(\flat_1)) * neut(\flat_1) = \flat_1 * neut(\flat_1)$$

$$((\flat_1 * neut(\flat_1)) * neut(\flat_1)) = \flat_1$$

$$\flat_1 * neut(\flat_1) = \flat_1$$

$$\flat_1 = \flat_1.$$

This shows that $neut(\flat_1) * neut(\flat_1) = neut(\flat_1)$. □

Theorem 3.5. *Let $(\aleph, *)$ be a pure right NT H_v-LA-group. Then $neut(\flat_1) * anti(\flat_1) = anti(\flat_1)$.*

Proof. Let $(\aleph, *)$ be a pure right NT H$_v$-LA-group. Multiply $\flat_1$ to the left of both side $neut(\flat_1) * anti(\flat_1) = anti(\flat_1)$, i.e.

$$(\flat_1 * (neut(\flat_1)) * anti(\flat_1) = \flat_1 * anti(\flat_1)$$
$$\flat_1 * anti(\flat_1) = neut(\flat_1)$$
$$neut(\flat_1) = neut(\flat_1)$$
$$neut(\flat_1) = neut(\flat_1)$$

This shows that $neut(\flat_1) * anti(\flat_1) = anti(\flat_1)$. □

Theorem 3.6. *Let $(\aleph, *)$ be a pure left NT H$_v$-LA-group. Then $neut(anti(\flat_1)) = neut(\flat_1)$.*

Proof. Let $neut(anti(\flat_1)) = neut(\flat_1)$. If we put $anti(\flat_1) = \flat_2$, then

$$neut(\flat_2) = neut(\flat_1). \text{ Post multiply by } \flat_2$$
$$neut(\flat_2) * \flat_2 = neut(\flat_1) * \flat_2$$
$$\flat_2 = neut(\flat_1) * \flat_2$$
$$anti(\flat_1) = neut(\flat_1) * anti(\flat_1), \text{ as } \flat_2 = anti(\flat_1)$$
$$anti(\flat_1) = anti(\flat_1), \text{ By Theorem 3.5 } neut(\flat_1) * anti(\flat_1) = anti(\flat_1).$$

Hence $neut(anti(\flat_1)) = neut(\flat_1)$. □

Definition 3.7. A non-empty subset B of a left NT H$_v$-LA-group $(\aleph, *)$ is called a left NT H$_v$-LA-subgroup of $\aleph$, if B itself form NT H$_v$-LA-group under same hyperoperation defined in $\aleph$.

Example 3.8. Let $\aleph = \{\flat_1, \flat_2, \flat_3, \flat_4\}$ and the hyperoperation is defined in the Table-2

$*$	$\flat_1$	$\flat_2$	$\flat_3$	$\flat_4$
$\flat_1$	$\flat_1$	$\flat_2$	$\flat_3$	$\flat_4$
$\flat_2$	$\flat_3$	$\{\flat_1, \flat_3\}$	$\{\flat_2, \flat_3\}$	$\flat_4$
$\flat_3$	$\flat_2$	$\{\flat_1, \flat_3\}$	$\{\flat_1, \flat_3\}$	$\flat_4$
$\flat_4$	$\flat_4$	$\flat_4$	$\flat_4$	$\{\flat_1, \flat_2, \flat_3\}$

Table-2, neutrosophic triplet H$_v$-LA-group

Here $(\flat_1, \flat_1, \flat_1), (\flat_2, \flat_1, \flat_2), (\flat_3, \flat_2, \flat_2)$ and $(\flat_4, \flat_3, \flat_4)$ are NT sets. As all elements of $\aleph$ satisfy the weak left invertive law but $\aleph$ do not satisfies the left invertive law, associative law and weak associative law i.e.

$$\begin{aligned} \{\flat_1, \flat_3\} &= (\flat_2 * \flat_2) * \flat_3 \neq (\flat_3 * \flat_2) * \flat_2 = \{\flat_1, \flat_2, \flat_3\} \\ \text{and } \{\flat_1, \flat_3\} &= (\flat_2 * \flat_2) * \flat_3 \neq \flat_2 * (\flat_2 * \flat_3) = \{\flat_1, \flat_2, \flat_3\}. \\ \text{Also } \{\flat_2\} &= (\flat_2 * \flat_1) * \flat_1 \cap \flat_2 * (\flat_1 * \flat_1) = \{\flat_3\} = \phi. \end{aligned}$$

So $(\aleph, *)$ is a NT H_v-LA-group. Here $\flat = \{\flat_1, \flat_2, \flat_3\}$ is a NT H_v-LA-subgroup of $\aleph$.

Lemma 3.9. *If* $(\aleph, *)$ *is a NT H_v-LA group, then*

$$(\flat_1 * \flat_2) * (\flat_3 * \flat_4) \cap (\flat_1 * \flat_3) * (\flat_2 * \flat_4) \neq \phi,$$
hold for all $\flat_1, \flat_2, \flat_3, \flat_4 \in \aleph$.

Proof. Let

$$\begin{aligned} &(\flat_1 * \flat_2) * (\flat_3 * \flat_4) \\ &= (\flat_1 * \flat_2) * g, \text{ where } g = (\flat_3 * \flat_4) \\ &= (\flat_1 * \flat_2) * g \cap (g * \flat_2) * \flat_1 \text{ by the weak left invertive law} \\ &= (\flat_1 * \flat_2) * g \cap (g * \flat_2) * \flat_1 \text{ by the weak-left invertive law} \\ &= (\flat_1 * \flat_2) * g \cap \{(g * \flat_2) * \flat_1\} \text{ by the weak-left invertive law} \\ &= (\flat_1 * \flat_2) * (\flat_3 * \flat_4) \cap \{((\flat_3 * \flat_4) * \flat_2) * \flat_1\}, \text{ where } g = (\flat_3 * \flat_4) \\ &= (\flat_1 * \flat_2) * (\flat_3 * \flat_4) \cap \{\{((\flat_3 * \flat_4) * \flat_2) \cap (\flat_2 * \flat_4) * \flat_3\} * \flat_1\} \\ &= (\flat_1 * \flat_2) * (\flat_3 * \flat_4) \cap \{((\flat_3 * \flat_4) * \flat_2) * \flat_1\} \cap \{((\flat_2 * \flat_4) * \flat_3) * \flat_1\}\} \\ &= (\flat_1 * \flat_2) * (\flat_3 * \flat_4) \cap \left\{ \begin{array}{c} ((\flat_3 * \flat_4) * \flat_2) * \flat_1 \cap (\flat_1 * \flat_2) * (\flat_3 * \flat_4)\} \\ \cap\{((\flat_2 * \flat_4) * \flat_3) * \flat_1 \cap (\flat_1 * \flat_3) * (\flat_2 * \flat_4)\} \end{array} \right\} \to (1) \end{aligned}$$

Now

$$
\begin{aligned}
&(\flat_1 * \flat_3) * (\flat_2 * \flat_4) \\
&= (\flat_1 * \flat_3) * g, \text{ where } g = (\flat_2 * \flat_4) \\
&= (\flat_1 * \flat_3) * g \cap (g * \flat_3) * \flat_1 \text{ by the weak left invertive law} \\
&= (\flat_1 * \flat_3) * g \cap (g * \flat_3) * \flat_1 \text{ by the weak-left invertive law} \\
&= (\flat_1 * \flat_3) * g \cap \{(g * \flat_3) * \flat_1\} \text{ by the weak-left invertive law} \\
&= (\flat_1 * \flat_3) * (\flat_2 * \flat_4) \cap \{((\flat_2 * \flat_4) * \flat_3) * \flat_1\}, \text{ where } g = (\flat_2 * \flat_4) \\
&= (\flat_1 * \flat_3) * (\flat_2 * \flat_4) \cap \{\{((\flat_2 * \flat_4) * \flat_3) \cap (\flat_3 * \flat_4) * \flat_2\} * \flat_1\} \\
&= (\flat_1 * \flat_3) * (\flat_2 * \flat_4) \cap \{((\flat_2 * \flat_4) * \flat_3) * \flat_1\} \cap \{((\flat_3 * \flat_4) * \flat_2) * \flat_1\}\} \\
&= (\flat_1 * \flat_3) * (\flat_2 * \flat_4) \cap \left\{ \begin{array}{c} ((\flat_2 * \flat_4) * \flat_3) * \flat_1 \cap (\flat_1 * \flat_3) * (\flat_2 * \flat_4)\} \\ \cap\{((\flat_3 * \flat_4) * \flat_2) * \flat_1 \cap (\flat_1 * \flat_2) * (\flat_3 * \flat_4)\} \end{array} \right\} \to (2)
\end{aligned}
$$

From (1) and (2) we have $(\flat_1 * \flat_2) * (\flat_3 * \flat_4) \cap (\flat_1 * \flat_3) * (\flat_2 * \flat_4) \neq \phi$, hold for all $\flat_1, \flat_2, \flat_3, \flat_4 \in \aleph$. This law is known as weak medial law. □

Proposition 3.10. *Let $(\aleph, \circ)$ be a NT H_v-LA-group with left identity e and $\phi \neq A \subseteq \aleph$. If $(A \circ (A \circ \flat_1)) \circ \flat_2 \cap (A \circ (A \circ \flat_2)) \circ \flat_1 \neq \phi \ \forall \flat_1, \flat_2 \in \aleph$ and we define a hyperoperation $A_R^{\otimes}$ on $\aleph$ as $\flat_1 A_R^{\otimes} \flat_2 = (\flat_1 \circ \flat_2) \circ A$, then $(\aleph, A_R^{\otimes})$ become a NT H_v-LA-group.*

Proof. Let $\flat_1, \flat_2, \flat_3 \in \aleph$, we have

$$
\begin{aligned}
(\flat_1 A_R^{\otimes} \flat_2) A_R^{\otimes} \flat_3 &= ((\flat_1 \circ \flat_2) \circ A) A_R^{\otimes} \flat_3 \\
&= (((\flat_1 \circ \flat_2) \circ A) \circ \flat_3) \circ A \\
&= ((\flat_3 \circ A) \circ (\flat_1 \circ \flat_2)) \circ A \\
&= (A \circ (A \circ \flat_3)) \circ (\flat_2 \circ \flat_1) \\
&= \flat_2 \circ ((A \circ (A \circ \flat_3)) \circ \flat_1)
\end{aligned}
$$

and on the other hand

$$
\begin{aligned}
(\flat_3 A_R^{\otimes} \flat_2) A_R^{\otimes} \flat_1 &= ((\flat_3 \circ \flat_2) \circ A) A_R^{\otimes} \flat_1 \\
&= (((\flat_3 \circ \flat_2) \circ A) \circ \flat_3) \circ A \\
&= ((\flat_1 \circ A) \circ (\flat_3 \circ \flat_2)) \circ A \\
&= (A \circ (A \circ \flat_1)) \circ (\flat_2 \circ \flat_3) \\
&= \flat_2 \circ ((A \circ (A \circ \flat_1)) \circ \flat_3)
\end{aligned}
$$

but

$$\flat_2 \circ ((A \circ (A \circ \flat_3)) \circ \flat_1) \cap \flat_2 \circ ((A \circ (A \circ \flat_1)) \circ \flat_3) \neq \phi$$

for all $\flat_1, \flat_2, \flat_3 \in \aleph$. It follows that

$$(\flat_1 A_R^{\otimes} \flat_2) A_R^{\otimes} \flat_3 \cap (\flat_3 A_R^{\otimes} \flat_2) A_R^{\otimes} \flat_1 \neq \phi.$$

Next, we have

$$\flat_1 A_R^{\otimes} \aleph = (\flat_1 \circ \aleph) \circ A = \aleph \text{ also } H A_R^{\otimes} \flat_1 = (\aleph \circ \flat_1) \circ A = \aleph.$$

Hence $(\aleph, A_R^{\otimes})$ become an H_v-LA-group. □

Definition 3.11. Let $(\aleph_1, \circ)$ and $(\aleph_2, *)$ be two NT H_v-LA-groups. The map $f : \aleph_1 \longrightarrow \aleph_2$ is called neutro homomorphism, if for all $\flat_1, \flat_2 \in \aleph_1$, the following conditions hold,

1. $f(\flat_1 \circ \flat_2) \cap f(\flat_1) * f(\flat_2) \neq \phi$,
2. $f\left(neut\left(\flat_1\right)\right) \cap neut\left(f\left(\flat_1\right)\right) \neq \phi$,
3. $f\left(anti\left(\flat_1\right)\right) \cap anti\left(f\left(\flat_1\right)\right) \neq \phi$.

Example 3.12. Let $\aleph_1 = \{v_1, v_2, v_3\}$ and $\aleph_2 = \{\flat_1, \flat_2, \flat_3\}$ are two finite sets, where $(\aleph_1, *)$ and $(\aleph_2, \circ)$ are NT H_v-LA-groups, the hyperoperation is defined in following tables 3,4:

$*$	v_1	v_2	v_3
v_1	$\{v_1\}$	$\{v_2\}$	$\{v_3\}$
v_2	$\{v_3\}$	$\{v_1, v_2\}$	$\{v_2\}$
v_3	$\{v_2\}$	$\{v_3\}$	$\{v_3, v_1\}$

Table-3, neutrosophic triplet H_v**-LA-group**

and

$\circ$	$\flat_1$	$\flat_2$	$\flat_3$
$\flat_1$	$\flat_1$	$\{\flat_1, \flat_2\}$	$\{\flat_1, \flat_3\}$
$\flat_2$	$\flat_3$	$\{\aleph\}$	$\{\flat_1, \flat_2\}$
$\flat_3$	$\flat_2$	$\{\flat_1, \flat_3\}$	$\{\aleph\}$

Table-4, neutrosophic triplet H_v**-LA-group**

The mapping $f : \aleph_1 \longrightarrow \aleph_2$ is defined by $f(v_1) = \flat_1$, $f(v_2) = \flat_2$, $f(v_3) = \flat_3$. Then clearly f is a neutro homomorphism.

4. Construction Of Neutrosophic triplet(NT) H$_v$-LA-groups

In this section we provide the construction of NT H$_v$-LA-groups and develop two Mathematica Packages which help us to check the left invertive law, weak left invertive law and reproductive axiom.

Consider a finite set $\aleph$, such that $|\aleph| > 2$. Define the hyperoperation $\circ$ on $\aleph$ as follows

$$\flat_i \circ \flat_j = \left\{ \begin{array}{cc} \flat_j & \text{for } i = 1 \\ \flat_\flat \text{ for } j = 1 \text{and } \flat \equiv 2 - i \bmod |\aleph| \\ \aleph & \text{for } i = j, i \neq 1, j \neq 1 \\ \flat_i & \text{otherwise, for } i \prec j \text{ or } i \succ j \end{array} \right\}$$

and if $neut\,(\flat_i)$ and $anti\,(\flat_i)$ exist in $\aleph$. Then $\aleph$ under the hyperoperation $\circ$ forms a NT H$_v$-LA-group.

The above construction can be explained with the help of an example.

Example 4.1. Let $\aleph = \{\flat_1\flat_2, \flat_3\}$ under the binary hyperoperation $\circ$ defined in Table-5

$\circ$	$\flat_1$	$\flat_2$	$\flat_3$
$\flat_1$	$\flat_1$	$\flat_2$	$\flat_3$
$\flat_2$	$\flat_3$	$\aleph$	$\flat_2$
$\flat_3$	$\flat_2$	$\flat_3$	$\aleph$

Table-5, neutrosophic triplet H$_v$**-LA-group**

Here $(\flat_1, \flat_1, \flat_1), (\flat_2, \flat_1, \flat_2)$ and $(\flat_3, \flat_1, \flat_3)$ are NT set. One can see that $\circ$ satisfy the weak left invertive law, also $\circ$ is non-left invertive and non-associative i.e.

$$\aleph = (\flat_3 \circ \flat_3) \circ \flat_2 \neq (\flat_2 \circ \flat_3) \circ \flat_3 = \flat_2$$
$$\text{and } \aleph = (\flat_2 \circ \flat_2) \circ \flat_1 \neq \flat_2 \circ (\flat_2 \circ \flat_1) = \flat_2.$$

Also it is not WASS$(\flat_2 \circ \flat_1) \circ \flat_1 \cap \flat_2 \circ (\flat_1 \circ \flat_1) = \phi$. Hence $(\aleph, \circ)$ is a NT H$_v$-LA-group. The result of table can easily be generalized to n elements.

Remark 4.2. In NT H$_v$-LA-group, the property of H$_v$-LA-group can be checked by using the mathematica packages. The mathematica package(A) used to check the left invertive property and mathematica package(B) is used to check the weak non associative hypergroups. We paste the mathematica packages as under:

```
BeginPackage["LeftAlmostHyperGroupTest'"];
Clear["LeftAlmostHyperGroupTest'*"];
Begin["'Private'"]; Clear["LeftAlmostHyperGroupTest'Private'*"];
LeftAlmostHyperGroupTest[LookUpTable_List] :=
  Table[If[ReproductivityTest[LookUpTable[[j]]],
    If[LeftInvertiveTest[LookUpTable[[j]]], True, False],
    False], {j, 1, Length[LookUpTable]}];
LeftInvertiveTest[LookUpTable1_List] := Module[{i, j, k, len, test}, i = 1;
   j = 1;
   k = 1;
   test = True;
   len = Length[LookUpTable1];
   While[test && i ≤ len, test = Union[Flatten[Union[Extract[LookUpTable1,
          Distribute[{LookUpTable1[[i, j]], {k}}, List]]]]] == Union[Flatten[Union[
         Extract[LookUpTable1, Distribute[{LookUpTable1[[k, j]], {i}}, List]]]]];
    k = k + 1; If[k > len, k = 1; j = j + 1;
     If[j > len, i = i + 1; j = 1];];];
   Return[test]];
ReproductivityTest[LookUpTable1_List] :=
  Union[Apply[Union, LookUpTable1, 1]] == {Range[1, Length[LookUpTable1]]} &&
   Union[Apply[Union, Transpose[LookUpTable1], 1]] == {Range[1, Length[LookUpTable1]]};
End[];
EndPackage[];
```

Mathematica Package (A)

and

```
BeginPackage["WeakLeftAlmostHyperGroupTest'"];
Clear["WeakLeftAlmostHyperGroupTest'*"];
Begin["'Private'"]; Clear["WeakLeftAlmostHyperGroupTest'Private'*"];
WeakLeftAlmostHyperGroupTest[LookUpTable_List] :=
  Table[If[ReproductivityTest[LookUpTable[[j]]],
    If[WeakLeftInvertiveTest[LookUpTable[[j]]], True, False],
    False], {j, 1, Length[LookUpTable]}];
WeakLeftInvertiveTest[LookUpTable1_List] := Module[{i, j, k, len, test, ∅}, i = 1;
   j = 1; k = 1;
   test = True;
   len = Length[LookUpTable1];
   ∅ = NullSet;
   While[test && i ≤ len,
    test = Union[Flatten[Union[Extract[LookUpTable1, Distribute[{LookUpTable1[[i, j]],
           {k}}, List]]]]] ⋂ Union[Flatten[Union[Extract[LookUpTable1,
         Distribute[{LookUpTable1[[k, j]], {i}}, List]]]]] ≠ {};
    k = k + 1; If[k > len, k = 1; j = j + 1;
     If[j > len, i = i + 1; j = 1];];];
   Return[test]];
ReproductivityTest[LookUpTable1_List] :=
  Union[Apply[Union, LookUpTable1, 1]] == {Range[1, Length[LookUpTable1]]} &&
   Union[Apply[Union, Transpose[LookUpTable1], 1]] == {Range[1, Length[LookUpTable1]]};
End[];
EndPackage[];

WeakLeftAlmostHyperGroupTest.m;
```

Mathematica Package (B)

5. Application of Our proposed Structure

In the universe, the femininity, masculinity and neutrality exist. If we take the small particle, the small particle is an atom. The atom consists of three particle electrons, proton and neutron. So, from the above idea of the universe gave the concept of NT set. (Masculine, Neutral, feminine) and (Proton, Neutron, Electron) are the example of NT set.

There are three workers working in a factory. All three workers are disabled. The first worker has the right hand and no left hand. Factory made such a machine on which he can work with his right hand. The second worker has left hand but no right hand. Such a machine is made for him, on which he worked with his left hand. The third worker has an issue working with both of his hand. Such a machine is made for him, he works with his legs. All of these three worker's working performance is shown by the following Table-6.

$\circledast$	L	R	N
L	R	N	$\{L, N\}$
R	N	L	$\{R, N\}$
N	L	R	N

Table-6, neutrosophic triplet H_v**-LA-group**

In this table L represents the performance the worker, who work with his left hand. R represents the performance of the worker, who work with his right hand and N represents the performance of the worker, whose both hand are not functioning properly. Let $F = \{L, R, N\}$ be a finite set the hyperoperation is defined in the above table, and $(L, N, R), (R, N, L)$ and (N, L, L) are left NT set. $(F, \circledast)$ is a NT Hv-LA group.

5.1. *Chemical example of Neutrosophic Triplet(NT) H_v-LA- group*

The best example of NT H_v-LA-group in chemical reaction is a redox reaction.

Redox reaction: The chemical reaction in which one specie loss the electron and other specie gain the electron. Oxidation mean loss of electron. Reduction mean gain of electron. The redox reaction is a vital for biochemical reaction and industrial process. The electron transfer in cell and oxidation of glucose in the human body are the example of redox reaction. The reaction between hydrogen and fluorine is an example of redox reaction i.e.

$$\aleph_2 + F \longrightarrow 2\aleph F$$

$$\aleph_2 \longrightarrow 2\aleph^+ + 2e^- (\text{ Oxidation})$$

$$F_2 + 2e^- \longrightarrow 2F \text{ (Reduction)}$$

Each half reaction has standard reduction potential $\left(E^0\right)$ which is equal to the potential difference at equilibrium under the standard condition of an electrochemical cell in which the cathode reaction is half reaction considered and anode is a standard hydrogen electrode (SHE). For the redox reaction, the potential of cell is defined as

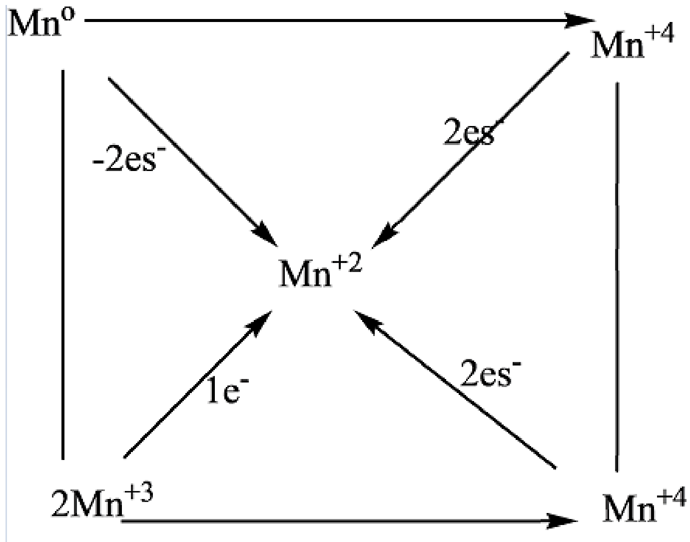
$$E^\circ cell = E^\circ{}_{cathode} - E^\circ{}_{anode}$$

where $E^\circ{}_{cathode}$ is the standard potential at the anode and $E^\circ{}_{cathode}$ is the standard potential at the cathode as given in the table of standard electrode potential. Now consider the redox reaction of Mn

$$Mn^0 + 2Mn^{+4} + 2Mn^{+3} \longrightarrow 3Mn^{+2} + 2Mn^{+4}$$

$$Mn^0 \longrightarrow Mn^{+2} + Mn^{+4} + 2e^- + 2Mn^{+3} + 2Mn^{+4}.$$

Manganese having a variable oxidation state of 0,+1, +2, +3, +4, +5, +6, +7. If we take $Mn^0, Mn^{+4}, Mn^{+3}, Mn^{+4}$ together we will get pure redox reaction. The flow chart is given as



Flow chart

Mn species with different oxidation state react with themselves. All possible reactions are presented in the following Table-7

$\oplus$	Mn^0	Mn^{+1}	Mn^{+2}	Mn^{+3}	Mn^{+4}
Mn^0	Mn^0	$\{Mn^0, Mn^{+1}\}$	$\{Mn^0, Mn^{+2}\}$	$\{Mn^0, Mn^{+3}\}$	$\{Mn^0, Mn^{+4}\}$
Mn^{+1}	$\{Mn^0, Mn^{+1}\}$	$\{Mn^0, Mn^{+2}\}$	$\{Mn^0, Mn^{+3}\}$	$\{Mn^{+2}\}$	$\{Mn^{+1}, Mn^{+4}\}$
Mn^{+2}	Mn^{+1}	$\{Mn^0, Mn^{+3}\}$	$\{Mn^{+1}, Mn^{+3}\}$	$\{Mn^{+1}, Mn^{+4}\}$	$\{Mn^{+2}, Mn^{+4}\}$
Mn^{+3}	$\{Mn^0, Mn^{+3}\}$	$\{Mn^{+1}, Mn^{+3}\}$	$\{Mn^{+2}, Mn^{+3}\}$	Mn^{+3}	$\{Mn^{+3}, Mn^{+4}\}$
Mn^{+4}	$\{Mn^0, Mn^{+4}\}$	$\{Mn^{+1}, Mn^{+4}\}$	$\{Mn^{+2}, Mn^{+4}\}$	$\{Mn^{+3}, Mn^{+4}\}$	Mn^{+4}

Table-7, All possible reactions

The standard reduction potentials (E^0) for conversion of each oxidation state to another are

$$E^0\left(Mn^{+4}/Mn^{+3}\right) = +0.95,$$
$$E^0\left(Mn^{+3}/Mn^{+2}\right) = +1.542,$$
$$E^0\left(Mn^{+2}/Mn^{+1}\right) = -0.59,$$
$$E^0\left(Mn^{+1}/Mn^{+0}\right) = 0.296.$$

If we replace

$$Mn^0 = \flat_1, Mn^{+1} = \flat_2, Mn^{+2} = \flat_3, Mn^{+3} = \flat_4, Mn^{+4} = \flat_5,$$

then we obtain the following Table-8

$\oplus$	$\flat_1$	$\flat_2$	$\flat_3$	$\flat_4$	$\flat_5$
$\flat_1$	$\{\flat_1\}$	$\{\flat_1,\flat_2\}$	$\{\flat_1,\flat_3\}$	$\{\flat_1,\flat_4\}$	$\{\flat_1,\flat_5\}$
$\flat_2$	$\{\flat_1,\flat_2\}$	$\{\flat_1,\flat_3\}$	$\{\flat_1,\flat_4\}$	$\{\flat_3\}$	$\{\flat_2,\flat_5\}$
$\flat_3$	$\{\flat_1,\flat_3\}$	$\{\flat_1,\flat_4\}$	$\{\flat_2,\flat_4\}$	$\{\flat_2,\flat_5\}$	$\{\flat_3,\flat_5\}$
$\flat_4$	$\{\flat_1,\flat_4\}$	$\{\flat_2,\flat_4\}$	$\{\flat_3,\flat_4\}$	$\{\flat_4\}$	$\{\flat_4,\flat_5\}$
$\flat_5$	$\{\flat_1,\flat_5\}$	$\{\flat_2,\flat_5\}$	$\{\flat_3,\flat_5\}$	$\{\flat_4,\flat_5\}$	$\{\flat_5\}$

Table-8, NT H_v-LA-group

As all elements of $\aleph$ satisfy the weak left invertive law but $\aleph$ do not satisfy the left invertive law, associative law and weak associative law

$$\{\flat_1,\flat_3\} = (\flat_2 \oplus \flat_2) \oplus \flat_1 \neq (\flat_1 \oplus \flat_2) \oplus \flat_2 = \{\flat_1,\flat_2,\flat_3\},$$

$$\{\flat_1,\flat_2,\flat_3,\flat_4\} = (\flat_2 \oplus \flat_2) \oplus \flat_3 \neq \flat_2 \oplus (\flat_2 \oplus \flat_3) = \{\flat_1,\flat_2,\flat_3\},$$

$$\text{and } (\flat_2 \oplus \flat_4) \oplus \flat_4 = \{\flat_2,\flat_5\} \cap \flat_3 = \flat_2 \oplus (\flat_4 \oplus \flat_4) = \phi$$

Here $(\flat_1,\flat_1,\flat_1)$, $(\flat_2,\flat_4,\flat_3)$, $(\flat_3,\flat_4,\flat_2)$, $(\flat_4,\flat_5,\flat_3)$ and $(\flat_5,\flat_4,\flat_4)$ are NT sets. Hence $(\aleph,\oplus)$ is a NT H_v-LA-group.

Remark 5.1. NT set, which helps the chemist to take the state of M_n which react or not react easily with other state or themselves. M_n^{+0} plays the role of neuta with different oxidation state and themselves. If the M_n have the same neuta and anti, it means that Mn having equal chances of loss or gain of electron.

6. Difference between the proposed work and existing methods

Our proposed structure has two main purpose,

1) This structure generalize the structure of groups, LA-groups, semigroups, LA-semigroup and as well as the hyper versions of above mentioned structures.

2) As NT set has the abelity to capture indeterminacy in a much better way so our proposed stricture of NT LA-semigroups can handle the uncertanity in a better way as we have seen in the Redox reaction.

7. Conclusions

In this article, we have studied and introduced NT H_v LA- groups. We presented some result on NT H_v LA-groups and construction of NT H_v-LA groups. We defined the neutro homomorphism on NT H_v LA groups. Also, we use the Mathematica packages to check the properties of left invertive and weak left invertive. Our defined structure have an interesting application in chemistry redox reaction.

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