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Some Properties of Q-Neutrosophic Ideals of Semirings

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Abstract. The intention of this paper is to introduce and study some properties of the ideals of semirings using the concept of Q-neutrosophic set.

Keywords: Semiring; Q-neutrosophic ideal; Cartesian Product; Composition.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [21] in 1965 to overcome the uncertainties in various problems in environment, economics, engineering etc. As an extension of it, Atanassov [7] introduced intuitionistic fuzzy set in 1986, where a degree of non-membership was considered besides the degree of membership of each element with (membership value + non-membership value) ≤ 1.
After that several generalizations such as, rough sets, vague sets, interval-valued sets etc. are considered as mathematical tools for dealing with uncertainties. In 2005, F. Smarandache introduced Neutrosophic set [19] in which he introduced the indeterminacy to intuitionistic fuzzy sets. So, the resultant can be taken as a tri-component logic which can be applied to non-standard analysis such as decision making (for example, result of games (win/tie/defeat), votes, from no/yes/NA), control theory etc.. Since then several researchers has applied this concept in many practical fields such as multi-criteria decision making, signal processing, disaster management etc.. Some of its recent applications can be found in [1–5, 9, 18, 20].
In 2011, Majumder [13] introduced and studied the concept of Q-fuzzifcation of ideals of Γ-semigroup. Akram et al [6], Lekkoksung [11, 12], Mandal [14], Qamar et al [15, 16] extended this concept in case of Γ-semigroup, ordered semigroups [10], ordered Γ-semiring, soft fields, group theory and investigated some important properties.
Motivated by this idea and combining the concept with neutrosophic set, in the paper we

have studied the ideal theory of semirings since it has several applications in graph theory, automata theory, mathematical modelling etc. [8].

2. Preliminaries

At first let us remember some definitions which will be used in the discussion of the paper.

Definition 2.1. A semiring is a nonempty set S on which two operations $+$ and $\cdot$ have been defined such that $(S, +)$ and $(S, \cdot)$ form monoid where $\cdot$ distributes over $+$ from any side.

Definition 2.2. A nonempty subset $X(\neq S)$ of semiring S is said to be an ideal if for all $x, y \in X$ and $p \in S$, $x + y \in X$, $px \in X$. Similarly we can define a right ideal also. An ideal of S is a nonempty subset which satisfies both properties of left ideal and right ideal.

Definition 2.3. A neutrosophic set N on the universe U is defined as $N = \{< u, A^T(u), A^I(u), A^F(u) >, u \in U\}$, where $A^T, A^I, A^F : U \rightarrow]^-0, 1^+[$ and $^-0 \leq A^T(u) + A^I(u) + A^F(u) \leq 3^+$. For practical purposes, it is difficult to consider $]^-0, 1^+[$. So, for studying neutrosophic set we consider the set which takes the value from the subset of [0, 1].

Definition 2.4. For a non-empty set Q, a mapping $\nu : S \times Q \longrightarrow [0, 1]$ is said to be a Q-fuzzy subset of S and $\nu_l = \{(s, q) \in S \times Q | \nu(s, q) \geq l\}$ where $l \in [0, 1]$ is its level subset.

3. Main Results

Definition 3.1. Let $\nu = (\nu^T, \nu^I, \nu^F)$ be a non empty neutrosophic subset of a semiring S. Then ν is called a Q-neutrosophic left ideal of S if

(i) $\nu^T(s_1 + s_2, p) \geq \min\{\nu^T(s_1, p), \nu^T(s_2, p)\}$, $\nu^T(s_1 s_2, p) \geq \nu^T(s_2, p)$
(ii) $\nu^I(s_1 + s_2, p) \geq \frac{\nu^I(s_1,p)+\nu^I(s_2,p)}{2}$, $\nu^I(s_1 s_2, p) \geq \nu^I(s_2, p)$
(iii) $\nu^F(s_1 + s_2, p) \leq \max\{\nu^F(s_1, p), \nu^F(s_2, p)\}$, $\nu^F(s_1 s_2, p) \leq \nu^F(s_2, p)$.

for all $s_1, s_2 \in S$ and $p \in Q$.

Theorem 3.2. *Any Q-neutrosophic set ν of a semiring S is a left ideal iff its level subsets $\nu^T_l := \{(x, p) \in S \times Q : \nu^T(x, p) \geq l,\ l \in [0, 1],\ p \in Q\}$, $\nu^I_l := \{(x, p) \in S \times Q : \nu^I(x, p) \geq l,\ l \in [0, 1]\}$ and $\nu^F_l := \{(x, p) \in S \times Q : \nu^F(x, p) \leq l,\ l \in [0, 1]\}$ are left ideals of $S \times Q$.*

Proof. Suppose ν of S is a Q-neutrosophic left ideal of S. Then anyone of ν^T, ν^I or ν^F is not equal to zero for some $(s, p) \in S \times Q$. Without loss of generality we consider, all of them are not equal to zero.

Suppose $a, b \in \nu_l = (\nu_l^T,\ \nu_l^I,\ \nu_l^F)$, $s \in S$ and $p \in Q$. Then

$$\begin{aligned} \nu^T(a+b,p) &\geq \min\{\nu^T(a,p), \nu^T(b,p)\} \geq \min\{l,l\} = l \\ \nu^I(a+b,p) &\geq \tfrac{\nu^I(a,p)+\nu^I(b,p)}{2} \geq \tfrac{l+l}{2} = l \\ \nu^F(a+b,p) &\leq \max\{\nu^F(a,p), \nu^F(b,p)\} \leq \max\{l,l\} = l \end{aligned}$$

which implies $(a+b,p) \in \nu_l^T, \nu_l^I, \nu_l^F$ i.e., $(a+b,p) \in \nu_l$. Also

$$\begin{aligned} \nu^T(sa,p) &\geq \nu^T(a,p) \geq l \\ \nu^I(sa,p) &\geq \nu^I(a,p) \geq l \\ \nu^F(sa,p) &\leq \nu^F(a,p) \leq l \end{aligned}$$

Hence $(sa,p) \in \nu_l$.

Therefore ν_l is a left ideal of S.

Conversely, let $\nu_l(\neq \phi)$ is a left ideal of $S \times Q$ and ν is not a Q-neutrosophic left ideal of S. Then for $a, b \in S$ and $p \in Q$ anyone of the following inequality will hold.

$$\begin{aligned} \nu^T(a+b,p) &< \min\{\nu^T(a,p), \nu^T(b,p)\} \\ \nu^I(a+b,p) &< \tfrac{\nu^I(a,p)+\nu^I(b,p)}{2} \\ \nu^F(a+b,p) &> \max\{\nu^F(a,p), \nu^F(b,p)\} \end{aligned}$$

For the first inequality, choose $l_1 = \frac{1}{2}[\nu^T(a+b,p)+\min\{\nu^T(a,p), \nu^T(b,p)\}]$. Then $\nu^T(a+b,p) < l_1 < \min\{\nu^T(a,p), \nu^T(b,p)\} \Rightarrow (a,p),(b,p) \in \nu_{l_1}^T$. but $(a+b,p) \notin \nu_{l_1}^T$ - contradiction.

For the second inequality, choose $t_2 = \frac{1}{2}[\nu^I(a+b,p) + \min\{\nu^I(a,p), \nu^I(b,p)\}]$. Then $\nu^I(a+b,p) < l_2 < \frac{\nu^I(a,p)+\nu^I(b,p)}{2} \Rightarrow (a,p),(b,p) \in \nu_{l_2}^I$. But $(a+b,p) \notin \nu_{l_2}^I$ - contradiction.

For the third inequality, choose $t_3 = \frac{1}{2}[\nu^F(a+b,p) + \max\{\nu^F(a,p), \nu^F(b,p)\}]$. Then $\nu^F(a+b,p) > l_3 > \max\{\nu^F(a,p), \nu^F(b,p)\} \Rightarrow (a,p),(b,p) \in \nu_{t_3}^F$ but $(a+b,p) \notin \nu_{p_3}^F$ - contradiction.

Hence the theorem. □

Definition 3.3. For two Q-neutrosophic subsets ν and σ of $S \times Q$, define their intersection by

$$(\nu^T \cap \sigma^T)(a,p) = \min\{\nu^T(a,p), \sigma^T(a,p)\}$$

$$(\nu^I \cap \sigma^I)(a,p) = \min\{\nu^I(a,p), \sigma^I(a,p)\}$$

$$(\nu^F \cap \sigma^F)(a,p) = \max\{\nu^F(a,p), \sigma^F(a,p)\}$$

for all $a \in S$ and $p \in Q$.

Proposition 3.4. *Intersection of any number of Q-neutrosophic left ideals of S is also a Q-neutrosophic left ideal.*

Proof. Assume that $\{\nu_i : c \in C\}$ be a collection of Q-neutrosophic left ideals of S and $a, b \in S$, $p \in Q$. Then

$$\begin{aligned}(\bigcap_{c\in C}\nu_c^T)(a+b,p) &= \inf_{c\in C}\nu_c^T(a+b,p) \geq \inf_{c\in C}\{\min\{\nu_c^T(a,p),\nu_c^T(b,p)\}\}\\ &= \min\{\inf_{c\in C}\nu_c^T(a,p),\inf_{c\in C}\nu_c^T(b,p)\}\\ &= \min\{(\bigcap_{c\in C}\nu_c^T)(a,p),(\bigcap_{c\in C}\nu_c^T)(b,p)\}\end{aligned}$$

$$\begin{aligned}(\bigcap_{c\in C}\nu_c^I)(a+b,p) &= \inf_{c\in C}\nu_c^I(a+b,p) \geq \inf_{c\in C}\frac{\nu_c^I(a,p)+\nu_c^I(b,p)}{2}\\ &= \frac{\inf_{c\in C}\nu_c^I(a,p)+\inf_{c\in C}\nu_c^I(b,p)}{2}\\ &= \frac{\bigcap_{c\in C}\nu_c^I(a,p)+\bigcap_{c\in C}\nu_c^I(b,p)}{2}\end{aligned}$$

$$\begin{aligned}(\bigcap_{c\in C}\nu_c^F)(a+b,p) &= \sup_{c\in C}\nu_c^F(a+b,p) \leq \sup_{c\in C}\{\max\{\nu_c^F(a,p),\nu_c^F(b,p)\}\}\\ &= \max\{\sup_{c\in C}\nu_c^F(a,p),\sup_{c\in C}\nu_c^F(b,p)\}\\ &= \max\{(\bigcap_{c\in C}\nu_c^F)(a,p),(\bigcap_{c\in C}\nu_c^F)(b,p)\}\end{aligned}$$

$$(\bigcap_{c\in C}\nu_c^T)(ab,p) = \inf_{c\in C}\nu_c^T(ab,p) \geq \inf_{c\in C}\nu_c^T(b,p) = (\bigcap_{c\in C}\nu_c^T)(b,p).$$
$$(\bigcap_{c\in C}\nu_c^I)(ab,p) = \inf_{c\in C}\nu_c^I(ab,p) \geq \inf_{c\in C}\nu_c^I(b,p) = (\bigcap_{c\in C}\nu_c^I)(b,p).$$
$$(\bigcap_{c\in C}\nu_c^F)(ab,p) = \sup_{c\in C}\nu_c^F(ab,p) \leq \sup_{c\in C}\nu_c^F(b,p) = (\bigcap_{c\in C}\nu_c^F)(b,p).$$

Therefore $\bigcap_{c\in C}\nu_c$ is a Q-neutrosophic left ideal of S. □

Definition 3.5. For two Q-neutrosophic subsets ν and σ of S, define their cartesian product by

$$(\nu^T \times \sigma^T)((a,b),p) = \min\{\nu^T(a,p),\sigma^T(b,p)\}$$
$$(\nu^I \times \sigma^I)((a,b),p) = \frac{\nu^I(a,p)+\sigma^I(b,p)}{2}$$
$$(\nu^F \times \sigma^F)((a,b),p) = \max\{\nu^F((a,p),\sigma^F(b,p)\}$$

$\forall a, b \in S,\ p \in Q.$

Theorem 3.6. *For two Q-neutrosophic left ideals ν and σ of S, $\nu \times \sigma$ is a Q-neutrosophic left ideal of $S \times S$.*

Proof. Let $(a_1, a_2), (b_1, b_2) \in S \times S$ and $p \in Q$. Now

$$\begin{aligned}(\nu^T \times \sigma^T)((a_1,a_2)+(b_1,b_2),p) &= (\nu^T \times \sigma^T)((a_1+b_1,a_2+b_2),p)\\ &= \min\{\nu^T(a_1+b_1,p),\sigma^T(a_2+b_2,p)\}\\ &\geq \min\{\min\{\nu^T(a_1,p),\nu^T(b_1,p)\},\min\{\sigma^T(a_2,p),\sigma^T(b_2,p)\}\}\\ &= \min\{\min\{\nu^T(a_1,p),\sigma^T(a_2,p)\},\min\{\nu^T(b_1,p),\sigma^T(b_2,p)\}\}\\ &= \min\{(\nu^T \times \sigma^T)((a_1,a_2),p),(\nu^T \times \sigma^T)((b_1,b_2),p)\}.\end{aligned}$$

$$\begin{aligned}(\nu^I\times\sigma^I)((a_1,a_2)+(b_1,b_2),p) &= (\nu^I\times\sigma^I)((a_1+b_1,a_2+b_2),p)\\ &= \tfrac{\nu^I(a_1+b_1,p)+\sigma^I(a_2+b_2,p)}{2}\\ &\geq \tfrac{1}{2}\{\tfrac{\nu^I(a_1,p)+\nu^I(b_1,p)}{2}+\tfrac{\sigma^I(a_2,p)+\sigma^I(b_2,p)}{2}\}\\ &= \tfrac{1}{2}\{\tfrac{\nu^I(a_1,p)+\sigma^I(a_2,p)}{2}+\tfrac{\nu^I(b_1,p)+\sigma^I(b_2,p)}{2}\}\\ &= \tfrac{1}{2}\{(\nu^I\times\sigma^I)((a_1,a_2),p)+(\nu^I\times\sigma^I)((b_1,b_2),p)\}.\end{aligned}$$

$$\begin{aligned}(\nu^F\times\sigma^F)((a_1,a_2)+(b_1,b_2),p) &= (\nu^F\times\sigma^F)((a_1+b_1,a_2+b_2),p)\\ &= \max\{\nu^F(a_1+b_1,p),\nu^F(a_2+b_2,p)\}\\ &\leq \max\{\max\{\nu^F(a_1,p),\nu^F(b_1,p)\},\max\{\sigma^F(a_2,p),\sigma^F(b_2,p)\}\}\\ &= \max\{\max\{\nu^F(a_1,p),\sigma^F(a_2,p)\},\max\{\nu^F(b_1,p),\sigma^F(b_2,p)\}\}\\ &= \max\{(\nu^F\times\sigma^F)((a_1,a_2),p),(\nu^F\times\sigma^F)((b_1,b_2),p)\}.\end{aligned}$$

$$\begin{aligned}(\nu^T\times\sigma^T)((a_1,a_2)(b_1,b_2),p) &= (\nu^T\times\sigma^T)((a_1b_1,a_2b_2),p)=\min\{\nu^T(a_1b_1,p),\sigma^T(a_2b_2,p)\}\\ &\geq \min\{\nu^T(b_1,p),\sigma^T(b_2,p)\}=(\nu^T\times\sigma^T)((b_1,b_2),p).\end{aligned}$$

$$\begin{aligned}(\nu^I\times\sigma^I)((a_1,a_2)(b_1,b_2),p) &= (\nu^I\times\sigma^I)((a_1b_1,a_2b_2),p)=\tfrac{\nu^I(a_1b_1,p)+\sigma^I(a_2b_2,p)}{2}\\ &\geq \tfrac{\nu^I(b_1,p)+\sigma^I(b_2,p)}{2}=(\nu^I\times\sigma^I)((b_1,b_2),p).\end{aligned}$$

$$\begin{aligned}(\nu^F\times\sigma^F)((a_1,a_2)(b_1,b_2),p) &= (\nu^F\times\sigma^F)((a_1b_1,a_2b_2),p)=\max\{\nu^F(a_1b_1,p),\sigma^F(a_2b_2,p)\}\\ &\leq \max\{\nu^F(b_1,p),\nu^F(b_2,p)\}=(\nu^F\times\sigma^F)(b_1,b_2,p).\end{aligned}$$

Therefore $\nu\times\sigma$ is a Q-neutrosophic left ideal of $S\times S$. □

Theorem 3.7. *A Q-neutrosophic set ν of S is a Q-neutrosophic left ideal iff $\nu\times\nu$ is a Q-neutrosophic left ideal of $S\times S$.*

Proof. If a Q-neutrosophic subset ν of S is a Q-neutrosophic left ideal then by Theorem 3.6, $\nu\times\nu$ is a Q-neutrosophic left ideal of $S\times S$.

Conversely, suppose $\nu\times\nu$ is a Q-neutrosophic left ideal of $S\times S$ and $a_1,a_2,b_1,b_2\in S$, $p\in Q$. Then

$$\begin{aligned}\min\{\nu^T(a_1+b_1,p),\nu^T(a_2+b_2,p)\} &= (\nu^T\times\nu^T)((a_1+b_1,a_2+b_2),p)\\ &= (\nu^T\times\nu^T)((a_1,a_2)+(b_1,b_2),p)\\ &\geq \min\{(\nu^T\times\nu^T)((a_1,a_2),p),(\nu^T\times\nu^T)((b_1,b_2),p)\}\\ &= \min\{\min\{\nu^T(a_1,p),\nu^T(a_2,p)\},\min\{\nu^T(b_1,p),\nu^T(b_2,p)\}\}.\end{aligned}$$

$$\begin{aligned}\tfrac{\nu^I(a_1+b_1,p)+\nu^I(a_2+b_2,p)}{2} &= (\nu^I\times\nu^I)((a_1+b_1,a_2+b_2),p)\\ &= (\nu^I\times\nu^I)((a_1,a_2)+(b_1,b_2),p)\\ &\geq \tfrac{(\nu^I\times\nu^I)((a_1,a_2),p)+(\nu^I\times\nu^I)((b_1,b_2),p)}{2}\\ &= \tfrac{1}{2}[\tfrac{\nu^I(a_1,p)+\nu^I(a_2,p)}{2}+\tfrac{\nu^I(b_1,p)+\nu^I(b_2,p)}{2}].\end{aligned}$$

$$\begin{aligned}\max\{\nu^F(a_1+b_1,p),\nu^F(a_2+b_2,p)\} &= (\nu^F\times\nu^F)((a_1+b_1,a_2+b_2),p)\\ &= (\nu^F\times\nu^F)((a_1,a_2)+(b_1,b_2),p)\\ &\leq \max\{(\nu^F\times\nu^F)((a_1,a_2),p),(\nu^F\times\nu^F)((b_1,b_2),p)\}\\ &= \min\{\max\{\nu^F(a_1,p),\nu^F(a_2,p)\},\max\{\nu^F(b_1,p),\nu^F(b_2,p)\}\}.\end{aligned}$$

Now, putting $a_1 = a, a_2 = 0, b_1 = b$ and $b_2 = 0$, in the above inequalities and noting that $\nu^T(0) \geq \nu^T(x)$, $\nu^I(0) = 0$ and $\nu^F(0) \leq \nu^F(x)$ for all $a \in S$ we obtain

$$\begin{aligned}&\nu^T(a+b,p) \geq \min\{\nu^T(a,p),\nu^T(b,p)\}\\ &\nu^I(a+b,p) \geq \tfrac{\nu^I(a,p)+\nu^I(b,p)}{2}\\ &\nu^F(a+b,p) \leq \max\{\nu^F(a,p),\nu^F(b,p)\}.\end{aligned}$$

Next, we have

$$\begin{aligned}\min\{\nu^T(a_1b_1),\nu^T(a_2b_2)\} &= (\nu^T\times\nu^T)(a_1b_1,a_2b_2) = (\nu^T\times\nu^T)((a_1,a_2)(b_1,b_2))\\ &\geq (\nu^T\times\nu^T)(b_1,b_2) = \min\{\nu^T(b_1),\nu^T(b_2)\}.\end{aligned}$$

$$\begin{aligned}\tfrac{\nu^I(a_1b_1,q)+\nu^I(a_2b_2,q)}{2} &= (\nu^I\times\nu^I)((a_1,a_2)(b_1,b_2),q)\\ &\geq (\nu^I\times\nu^I)((b_1,b_2),q)\\ &= \tfrac{\nu^I(b_1,q)+\nu^I(b_2,q)}{2}.\end{aligned}$$

$$\begin{aligned}\max\{\nu^F(a_1b_1,q),\nu^F(a_2b_2,q)\} &= (\nu^F\times\nu^F)((a_1b_1,a_2b_2),q) = (\nu^F\times\nu^F)((a_1,a_2)(b_1,b_2),q)\\ &\leq (\nu^F\times\nu^F)((b_1,b_2),q) = \max\{\nu^F(b_1,q),\nu^F(b_2,q)\}.\end{aligned}$$

Taking $a_1 = a, b_1 = b$ and $b_2 = 0$, we obtain

$$\begin{aligned}&\nu^T(ab,p) \geq \nu^T(b,p)\\ &\nu^I(ab,p) \geq \nu^I(b,p)\\ &\nu^F(ab,p) \leq \nu^F(b,p).\end{aligned}$$

Hence ν becomes a Q-neutrosophic left ideal of S. □

Definition 3.8. For two Q-neutrosophic sets ν and σ of a semiring S, define their composition by

$$\begin{aligned}\nu^T o\sigma^T(a,p) &= \sup_{a=\sum_{c=1}^{m} a_cb_c} \{\min_c\{\nu^T(a_c,p),\sigma^T(b_c,p)\}\}\\ &= 0, \text{ otherwise}\\ \nu^I o\sigma^I(a,p) &= \sup_{a=\sum_{c=1}^{m} a_cb_c} \textstyle\sum_{c=1}^{m} \frac{\nu^I(a_c,p)+\sigma^I(b_c,p)}{2m}\\ &= 0, \text{ otherwise}\end{aligned}$$

$$\nu^F o\sigma^F(a,p) = \inf_{a=\sum_{c=1}^{m} a_c b_c} \{\max_c\{\nu^F(a_c,p), \sigma^F(b_c,p)\}\}$$
$$= 0, \text{otherwise}$$

where $p \in Q$, $a, a_c, b_c \in S$, $m \in N$-the set of natural number.

Theorem 3.9. *For two Q-neutrosophic left ideals ν and σ of S, $\nu o\sigma$ also forms a Q-neutrosophic left ideal of S.*

Proof. Consider two Q-neutrosophic left ideals ν, σ of S with $a, b \in S$, $p \in Q$. If $(a+b,p)$ has the expression $(\sum_{i=1}^{m} a_c b_c, p)$, where $a_c, b_c \in S$ and $p \in Q$, then the proof is immediate from the definition. So, assume that $a+b$ can be expressed in the said form. Then

$$(\nu^T o\sigma^T)(a+b,p)$$
$$= \sup_{a+b=\sum_{c=1}^{m} a_c b_c} \{\min_c\{\nu^T(a_c,p), \sigma^T(b_c,p)\}\}$$
$$\geq \sup_{a=\sum_{c=1}^{m} c_c d_c,\, b=\sum_{c=1}^{m} e_c f_c} \{\min_c\{\nu^T(c_c,p), \sigma^T(d_c,p), \nu^T(e_c,p), \sigma^T(f_c,p)\}\}$$
$$= \min\{\sup_{a=\sum_{c=1}^{m} c_c d_c}\{\min_c\{\nu^T(c_c,p), \sigma^T(d_c,p)\}\}, \sup_{b=\sum_{c=1}^{m} e_c f_c}\{\min_c\{\nu^T(e_c,p), \sigma^T(f_c,p)\}\}\}$$
$$= \min\{(\nu^T o\sigma^T)(a,p), (\nu^T o\sigma^T)(b,p)\}$$

$$(\nu^I o\sigma^I)(a+b,p)$$
$$= \sup_{a+b=\sum_{c=1}^{m} a_c b_c} \sum_{c=1}^{m} \frac{\nu^I(a_c,p)+\sigma^I(b_c,p)}{2m}$$
$$\geq \sup_{a=\sum_{c=1}^{m} c_c d_c,\, b=\sum_{c=1}^{m} e_c f_c} \sum_{c=1}^{m} \frac{\nu^I(c_c,p)+\sigma^I(d_c,p)+\nu^I(e_c,p)+\sigma^I(f_c,p)}{2m}$$
$$\geq \frac{1}{2}\Big[\sup_{a=\sum_{c=1}^{m} c_c d_c} \sum_{c=1}^{m} \frac{\nu^I(c_c,p)+\sigma^I(d_c,p)}{2m}, \sup_{b=\sum_{c=1}^{m} e_c f_c} \sum_{c=1}^{m} \frac{\nu^I(e_c,p)+\sigma^I(f_c,p)}{2m}\Big]$$
$$= \frac{(\nu^I o\sigma^I)(a,p)+(\nu^I o\sigma^I)(b,p)}{2}$$

$$
\begin{aligned}
&(\nu^F o\sigma^F)(a+b,p)\\
&= \inf_{a+b=\sum_{c=1}^{m} a_c b_c} \{\max_c\{\nu^F(a_c,p),\sigma^F(b_c,p)\}\}\\
&\leq \inf_{a=\sum_{c=1}^{m} c_c d_c,\, b=\sum_{c=1}^{m} e_c f_c} \{\max_c\{\nu^F(c_c,p),\sigma^F(d_c,p),\nu^F(e_c,p),\sigma^F(f_c,p)\}\}\\
&= \max\{\inf_{a=\sum_{c=1}^{m} c_c d_c}\{\max_c\{\nu^F(c_c,p),\sigma^F(d_c p)\}\}, \inf_{b=\sum_{c=1}^{m} e_c f_c}\{\max_c\{\nu^F(e_c,p),\sigma^F(f_c,p)\}\}\}\\
&= \max\{(\nu^F o\sigma^F)(a,p),(\nu^F o\sigma^F)(b,p)\}
\end{aligned}
$$

$$
\begin{aligned}
(\nu^T o\sigma^T)(ab,p) &= \sup_{ab=\sum_{c=1}^{m} a_c b_c} \{\min_c\{\nu^T(a_c,p),\sigma^T(b_c,p)\}\}\\
&\geq \sup_{ab=\sum_{c=1}^{m} ae_c f_c} \{\min_c\{\nu^T(ae_c,p),\sigma^T(f_c,p)\}\}\\
&\geq \sup_{b=\sum_{c=1}^{m} e_c f_c} \{\min_c\{\nu^T(e_c,p),\sigma^T(f_c,p)\}\} = (\nu^T o\sigma^T)(b,p)
\end{aligned}
$$

$$
\begin{aligned}
(\nu^I o\sigma^I)(ab,p) &= \sup_{ab=\sum_{c=1}^{m} a_c b_c} \textstyle\sum_{c=1}^{m} \frac{\nu^I(a_c,p)+\sigma^I(b_c,p)}{2m}\\
&\geq \sup_{ab=\sum_{c=1}^{m} ae_c f_c} \textstyle\sum_{c=1}^{m} \frac{\nu^I(ae_c,p)+\sigma^I(f_c,p)}{2m}\\
&\geq \sup_{b=\sum_{c=1}^{m} e_c f_c} \textstyle\sum_{c=1}^{m} \frac{\nu^I(e_c,p)+\sigma^I(f_c,p)}{2m} = (\nu^I o\sigma^I)(b,p)
\end{aligned}
$$

$$
\begin{aligned}
(\nu^F o\sigma^F)(ab,p) &= \inf_{ab=\sum_{c=1}^{m} a_c b_c} \{\max_c\{\nu^F(a_c,p),\sigma^F(b_c,p)\}\}\\
&\leq \inf_{ab=\sum_{c=1}^{m} ae_c f_c} \{\max_c\{\nu^F(ae_c,p),\sigma^F(f_c,p)\}\}\\
&\leq \inf_{b=\sum_{c=1}^{m} e_c f_c} \{\max_c\{\nu^F(e_c,p),\sigma^F(f_c,p)\}\} = (\nu^F o\nu^F)(b,p)
\end{aligned}
$$

Therefore $\nu o \sigma$ is a Q-neutrosophic left ideal of S. □

Conclusion: In this paper, we have defined Q-neutrosophic ideals of a semiring and studied some its elementary properties. Here also we obtain its characterizations by label subset criteria, cartesian product and composition of two Q-neutrosophic ideals. Our next aim to extend the idea in case of Q-neutrosophic bi-ideals, Q-neutrosophic quasi-ideals and investigate some properties of regular semirings.

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