Triangular Neutrosophic Topology

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Abstract: In this current article, the primary focus is to develop the concept of neutrosophic topology as triangular neutrosophic topology. We extend the neutrosophic set operations, we newly introduce the defuzzification, examples, and basics to clear the concept and new possibilities. Finally, we also investigate some properties such as neutrosophic exterior, neutrosophic subspace, neutrosophic boundary, and neutrosophic closure.

Keywords: Neutrosophic set, Triangular neutrosophic topology, MCDM, neutrosophic interior, neutrosophic exterior, neutrosophic subspace, and neutrosophic boundary.

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1. Introduction

Mathematicians, researchers, and analysts from all over the world work hard to develop new strategies to overcome decision-making problems in a different scenario. It widely uses in medicine, MCDM, engineering, and other math fields. The most challenging issue is to deal with critical situations such as theoretical stuff. To handle vagueness and uncertain situations in both practical and theoretical problems, researchers introduce theories like fuzzy, and neutrosophic sets. The neutrosophic set [1] is based on Truth, indeterminacy, and falsity, it is more effective than crisp and fuzzy.

The idea of triangular neutrosophic numbers was the new development in the line of neutrosophic. Smarandache [2,3] introduce the concept of neutrosophic sets. Un these concepts we have a degree of membership of truth, degree of indeterminacy, and degree of falsity. Many researchers [4-7] do their best and introduce new possibilities such as topology on neutrosophic sets. Other efforts of researchers [8-12] are also remarkable.

We put forward the concept of neutrosophic topology and design new operations and possibilities. MCDM is a wide and developed field and other researchers [13-32] put this concept forward. Triangular neutrosophic numbers plays important role in multi-criteria decision-making, and now it enhances the new evolutions in topology. We also introduce a decent way of defuzzification to make it useable in topological spaces. We also discuss neutrosophic interior, neutrosophic exterior, and other important things in this article.

The idea of neutrosophic is not limited and researchers in recent times proposed, Triangular neutrosophic numbers, Trapezoidal neutrosophic numbers, Pentagonal neutrosophic numbers, Hexagonal neutrosophic numbers, Heptagonal neutrosophic numbers. Moreover, the Octagonal neutrosophic numbers, Nonagonal neutrosophic numbers were presented and publisher by us [33-34]. By using neutrosophic techniques, researchers overcome real life problems [35-39].

2 Preliminaries

The related definitions are given below.

Definition 2.1: We define neutrosophic set $\dot{A}$ over universe of discourse as:

$$\dot{A} = \{\langle \dot{x}, \mu_A(\dot{x}), \sigma_A(\dot{x}), \Upsilon_A(x)\rangle : \dot{x} \in \dot{X}\}$$

Where $\mu_A, \sigma_A, \Upsilon_A : X \to]\dot{0}^-, 1^+[$ and $0^- \leq \mu_A(\dot{x}) + \sigma_A(\dot{x}) + \Upsilon_A(x) \leq 3^+$, the value of neutrosophic sets takes from standard and non-standard of $]0^-, 1^+[$. If we consider the value from real life problem, it will become hard to use value of neutrosophic sets takes from standard and non-standard of $]0^-, 1^+[$. For our convenient we take the value from the subset of [0,1]. $\dot{N}(\dot{X})$.

Definition 2.2: Apply formula $\frac{\acute{a}+\acute{b}+\acute{c}}{3}$ and here, $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$. So,

- (Inclusive) If $\mu_{\dot{A}}(\dot{x}) \leq \mu_{\dot{B}}(\dot{x})$, $\sigma_A(\dot{x}) \geq \sigma_B(\dot{x})$, $\Upsilon_{\dot{A}}(x) \geq \Upsilon_{\dot{B}}(x)$ for every $\dot{x} \in \dot{X}$, then neutrosophic subset can be define as $\dot{A} \sqsubseteq \dot{B}$ where, $\dot{A}$ neutrosophic subset of $\dot{B}$ and then neutrosophic subset can be define as $\dot{B} \sqsubseteq \dot{A}$ where, $\dot{B}$ neutrosophic subset of $\dot{A}$.

- (Equality) if $\dot{A} \sqsubseteq \dot{B}$ and $\dot{B} \sqsubseteq \dot{A}$ then $\dot{A} = \dot{B}$.

- (Intersection) The intersection in neutrosophic sense can be defined as: $\dot{A} \sqcap \dot{B}$ and defined:

  $$\dot{A} \sqcap \dot{B} = \{\dot{x}, \mu_{\dot{A}}(\dot{x}) \wedge \mu_{\dot{B}}(\dot{x}), \sigma_A(\dot{x}) \vee \sigma_B(\dot{x}), \Upsilon_{\dot{A}}(x) \vee \Upsilon_{\dot{B}}(x) : \dot{x} \in \dot{X}\}$$

- (Union) The union in neutrosophic sense can be define as

  $$\dot{A} \sqcup \dot{B} = \{\dot{x}, \mu_{\dot{A}}(\dot{x}) \vee \mu_{\dot{B}}(\dot{x}), \sigma_A(\dot{x}) \wedge \sigma_B(\dot{x}), \Upsilon_{\dot{A}}(x) \wedge \Upsilon_{\dot{B}}(x) : \dot{x} \in \dot{X}\}$$

- (Compliment) The compliment $\dot{A}^c$ in neutrosophic sense can be defined as:

  $$\dot{A}^c = \{\langle \dot{x}, \Upsilon_{\dot{A}}(x), 1 - \sigma_A(\dot{x}), \mu_{\dot{A}}(\dot{x})\rangle : \dot{x} \in \dot{X}\}$$

- (Universal set) It can be defined as: $\mu_{\dot{A}}(\dot{x}) = 1, \sigma_A(\dot{x}) = 0, \Upsilon_{\dot{A}}(x) = 0$ for all $\dot{x} \in \dot{X}$.

- (Empty set) It can be defined as: $\mu_{\dot{A}}(\dot{x}) = 0, \sigma_A(\dot{x}) = 1, \Upsilon_{\dot{A}}(x) = 1$ for all $\dot{x} \in \dot{X}$. We can denote it as $\emptyset$.

Example 1: A triangular neutrosophic problem is given below:

$$\dot{A} = \{\langle \dot{x}, (0.2,0.4,0.6)(0.3,0.5,0.8)(0.2,0.8,0.8)\rangle, \langle y, (0.4,0.7,0.2)(0.6,0.7,0.2)(0.4,0.5,0.6)\rangle\}$$

$$\dot{B} = \{\langle \dot{x}, (0.8,02,0.3)(0.4,0.6,0.1)(0.7,0.3,0.5)\rangle, \langle \dot{y}, (0.1,0.2,0.4)(0.4,0.2,0.7)(0.1,0.2,0.4)\rangle\}$$

$$\dot{C} = \{\langle \dot{x}, (0.9,0.3,0.1)(0.2,0.6,0.3)(0.7,0.5,0.6)\rangle, \langle \dot{y}, (0.1,0.9,0.5)(0.2,0.4,0.6)(0.5,0.4,0.9)\rangle\}$$

Apply formula $\frac{\acute{a}+\acute{b}+\acute{c}}{3}$,

Let $\dot{X} = \{\dot{x}, \dot{y}\}$ and $\dot{A}, \dot{B}, C^{\cdot} \in \dot{N}(\dot{x})$ then:

$$\dot{A} = \{\langle \dot{x}, 0.4,0.5,0.6\rangle, \langle \dot{y}, 0.4,0.3,0.5\rangle\}$$

$$\dot{B} = \{\langle \dot{x}, 0.4,0.3,0.6\rangle, \langle \dot{y}, 0.2,0.4,0.3\rangle\}$$

$$\dot{C} = \{\langle \dot{x}, 0.4,0.3,0.6\rangle, \langle \dot{y}, 0.5,0.4,0.6\rangle\}$$

We have $A^{\cdot} \sqsubseteq \dot{B}$.

Neutrosophic union of $\dot{B}$ *and* $\dot{C}$ as:

$$\dot{B} \sqcup \dot{C} = \{\langle \dot{x}, (0.4 \vee 0.4), (0.3 \wedge 0.3), (0.6 \wedge 0.6)\rangle, \langle y, (0.\dot{2} \vee 0.5), (0.4 \wedge 0.4), (0.3 \wedge 0.6)\rangle\}$$

$$\dot{B} \sqcup \dot{C} = \{\langle \dot{x}, 0.4,0.3,0.6\rangle, \langle \dot{y}, 0.5,0.4,0.3\rangle\}$$

The intersection in neutrosophic sense of $\dot{A}$ and $\dot{C}$

$$\dot{A} \sqcap \dot{C} = \{\langle \dot{x}, (0.4 \wedge 0.4), (0.5 \vee 0.3), (0.6 \vee 0.6)\rangle, \langle y, (0.\dot{4} \wedge 0.5), (0.3 \vee 0.4), (0.5 \vee 0.6)\rangle\}$$

$$\dot{A} \sqcap \dot{C} = \{\langle \dot{x}, 0.4,0.5,0.6\rangle, \langle \dot{y}, 0.4,0.4,0.6\rangle\}$$

The complement in neutrosophic sense of $\dot{C}$ is

$$\dot{C}^c = \{\langle \dot{x}, 0.4,0.3,0.6\rangle, \langle \dot{y}, 0.5,0.4,0.6\rangle\}^c$$

$$\dot{C}^c = \{\langle x, \dot{0}.6, 1 - 0.3,0.4\rangle, \langle \dot{y}, 0.6,1 - 0.4,0.5\rangle\}$$

$\dot{C}^c = \{\langle x, \dot{0}.6, 0.7, 0.4\rangle, \langle \dot{y}, 0.6, 0.6, 0.5\rangle\}$

Theorem 1 Let $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$. Then

- $\dot{A} \sqcap \dot{A} = A\ \dot{a}nd\ \dot{A} \sqcup \dot{A} = \dot{A}$
- $\dot{A} \sqcap \dot{B} = \dot{B} \sqcap \dot{A}\ and\ \dot{B} \sqcup \dot{A} = \dot{A} \sqcup \dot{B}$
- $\dot{A} \sqcap \varphi = \varphi\ and\ \dot{A} \sqcap \dot{X} = \dot{A}$
- $\dot{A} \sqcup \varphi = \varphi\ and\ \dot{A} \sqcup \dot{X} = \dot{X}$
- $\dot{A} \sqcap (B \sqcap \dot{C}) = (\dot{A} \sqcap B) \dot{\sqcap} \dot{C}\ and\ \dot{A} \sqcup (B \sqcup \dot{C}) = (\dot{A} \sqcup B) \dot{\sqcup} \dot{C}$
- $(\dot{A}^c)^c = \dot{A}$

Theorem 2 Let $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$. Then

- $(\sqcap_{i\in I} \dot{A}_i)^c = \sqcap_{i\in I} \dot{A}_i{}^c$
- $(\sqcup_{i\in I} \dot{A}_i)^c = \sqcup_{i\in I} \dot{A}_i{}^c$

Theorem 3 Let $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$. Then

- $\dot{B} \sqcap (\sqcup_{i\in I} \dot{A}_i) = \sqcup_{i\in I} (\dot{B} \sqcap \dot{A}_i)$
- $\dot{B} \sqcup (\sqcap_{i\in I} \dot{A}_i) = \sqcap_{i\in I} (\dot{B} \sqcup \dot{A}_i)$

3 Triangular neutrosophic topological spaces

Definition 3.1 Let $\dot{\tau} \subseteq \dot{N}(\dot{X})$, then $\dot{\tau}$ as neutrosophic topology on $\dot{X}$

- $\dot{X}\ and\ \varphi \in \dot{\tau}$.
- The union and intersection of any number of neutrosophic sets in $\dot{\tau}$ belong to $\dot{\tau}$.

The pair $(\dot{X}, \dot{\tau})$ mentioned as neutrosophic topology space over $\dot{X}$.

Definition 3.2 If $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over $\dot{X}$ then,

- $\dot{\phi}\ and\ \dot{X}\ as\ neutrosophic\ closed\ sets\ over\ \dot{X}.$

- The union and intersection of any two neutrosophic closed sets is a neutrosophic closed sets over $\dot{X}$.

Example 2: Let $\dot{X} = \{a.b\} and\ \dot{A} \in \dot{N}(\dot{X})$ so,

$A = \{\langle a, 0.4, \dot{0}.6, 0.\dot{8}\rangle, \langle b, 0.3, \dot{0}.5, 0.7\rangle\}$

Hence, $\dot{\tau} = \{\phi, \dot{X}, \dot{A}\}$ is neutrosophic topology on $\dot{X}$.

Example 3: Let $\dot{X} = \{a.b\} and\ \dot{A} \in \dot{N}(\dot{X})$ so,

$\dot{A} = \{\langle \dot{x}, (0.2,0.4,0.6)(0.3,0.5,0.8)(0.2,0.8,0.8)\rangle, \langle y, (0.4,0.7,0.2)(0.6,0.7,0.2)(0.4,0.5,0.6)\rangle\}$

$\dot{B} = \{\langle \dot{x}, (0.8,02,0.3)(0.4,0.6,0.1)(0.7,0.3,0.5)\rangle, \langle \dot{y}, (0.1,0.2,0.4)(0.4,0.2,0.7)(0.1,0.2,0.4)\rangle\}$

Apply formula $\frac{\acute{a}+\acute{b}+\acute{c}}{3}$,

$\dot{A} = \{\langle \dot{x}, 0.4,0.5,0.6\rangle, \langle \dot{y}, 0.4,0.3,0.5\rangle\}$

$\dot{B} = \{\langle \dot{x}, 0.4,0.3,0.6\rangle, \langle \dot{y}, 0.2,0.4,0.3\rangle\}$

Then, $\dot{\tau}_1 = \{\phi, \dot{X}, \dot{A}\}$ and $\dot{\tau}_2 = \{\phi, \dot{X}, \dot{A}\}$ are neutrosophic topology on $\dot{X}$. Here, $\dot{\tau}_1 \cup \dot{\tau}_1 = \{\phi, \dot{X}, \dot{A}, \dot{B}\}$ is not neutrosophic on $\dot{X}$. The reason is that: $\dot{A} \sqcap \dot{B} \notin \dot{\tau}_1 \cup \dot{\tau}_1$. Hence, it's not a neutrosophic topological space over $\dot{X}$.

Theorem 4: If $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over $\dot{X}$ and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then:

i. $\dot{int}(\emptyset) = \dot{\phi}\ and\ int\ (\dot{X}) = (\dot{X})$

ii. $\dot{int}(A) \sqsubseteq \dot{A}$

iii. $\dot{A}\ is\ neutrosophic\ open\ if\ and\ only\ if\ \dot{A} = int(\dot{A})$.

iv. $int(int(\dot{A})) = int(\dot{A})$.

v. $\dot{A} \sqsubseteq \dot{B}\ implies\ \dot{int}(A) \sqsubseteq int(\dot{B})$.

vi. $\dot{int}(A) \sqcup int(\dot{B}) \sqsubseteq int(\dot{A} \sqcup \dot{B})$.

vii. $int(\dot{A} \sqcap \dot{B}) = int(\dot{A}) \sqcap int(\dot{B})$.

Proof: i. and ii. are obvious.

iii. $\dot{A}$ is neutrosophic open set over $\dot{X}$, as well as, $\dot{A}$ is itself a neutrosophic set over $\dot{X}$ which also contain $\dot{A}$. The largest neutrosophic open set contain in $\dot{A}$ is $\dot{A}$ and $int(\dot{A})$=$\dot{A}$. Conversely, int $(\dot{A})$=$\dot{A}$ hence, $\dot{A} \in \dot{\tau}$.

iv. If $int(\dot{A})$=$\dot{B}. so, int(\dot{B}) = \dot{B}$ from above, $int\left(int(\dot{A})\right) = int(\dot{A})$.

v. As, $\dot{A} \sqsubseteq \dot{B}$. As $int(\dot{A}) \sqsubseteq \dot{A} \sqsubseteq \dot{B}. as\ int(\dot{A})$ is a neutrosophic subset of $\dot{B}$. So, $int(\dot{A}) \sqsubseteq int(\dot{B})$.

vi. It's clear $\dot{A} \sqsubseteq \dot{A} \sqcup \dot{B}\ and\ \dot{B} \sqsubseteq \dot{A} \sqcup \dot{B}$ thus, $int(\dot{A}) \sqsubseteq int(\dot{A} \sqcup \dot{B})\ and\ int(\dot{B}) \sqsubseteq int(\dot{A} \sqcup \dot{B})$ hence $int(\dot{A}) \sqcup int(\dot{B}) \sqsubseteq int(\dot{A} \sqcup \dot{B})$ by above.

Vii. If $(\dot{A} \sqcap \dot{B}) \sqsubseteq int(\dot{A})\ and\ (\dot{A} \sqcap \dot{B}) \sqsubseteq int(\dot{B})$ by above, so, $(\dot{A} \sqcap \dot{B}) \sqsubseteq int(\dot{A}) \sqcap int(\dot{B})$. Also, $int(\dot{A}) \sqsubseteq \dot{A}\ and\ int(\dot{B}) \sqsubseteq \dot{B}$ we have, $int(\dot{A}) \sqcap int(\dot{B}) \sqsubseteq \dot{A} \sqcap \dot{B}$. These make $(\dot{A} \sqcap \dot{B}) = int(\dot{A}) \sqcap int(\dot{B})$.

Example 4: Let $\dot{X} = \{\dot{x}, \dot{y}\}$ and $\dot{A}, \dot{B}, \dot{C} \in \dot{N}(\dot{x})$ then:

$$\dot{A} = \{\langle \dot{x}, (0.3,0.3,0.3)(0.3,0.3,0.3)(0.3,0.3,0.3)\rangle, \langle y, (0.5,0.5,0.5)(0.5,0.5,0.5)(0.5,0.5,0.5)\rangle\}$$

$$\dot{B} = \{\langle \dot{x}, (0.4,04,0.4)(0.4,0.4,0.4)(0.4,0.4,0.4)\rangle, \langle \dot{y}, (0.7,0.7,0.7)(0.7,0.7,0.7)(0.7,0.7,0.7)\rangle\}$$

$$\dot{C} = \{\langle \dot{x}, (0.2,0.2,0.2)(0.2,0.2,0.2)(0.2,0.2,0.2)\rangle, \langle \dot{y}, (0.6,0.6,0.6)(0.6,0.6,0.6)(0.6,0.6,0.6)\rangle\}$$

Apply formula $\frac{\dot{a}+\dot{b}+\dot{c}}{3}$,

$$\dot{A} = \{\langle \dot{x}, 0.3,0.3,0.3\rangle, \langle \dot{y}, 0.5,0.5,0.5\rangle\}$$

$\dot{B} = \{\langle \dot{x}, 0.4,0.4,0.4\rangle, \langle \dot{y}, 0.7,0.7,0.7\rangle\}$

$\dot{C} = \{\langle \dot{x}, 0.2,0.2,0.2\rangle, \langle \dot{y}, 0.6,0.6,0.6\rangle\}$

Then, $\dot{\tau} = \{\dot{\emptyset}, \dot{X}, \dot{A}\}$ is soft topological space over $\dot{X}$. $int(\dot{B}) = \dot{\phi}, int(\dot{C}) = \dot{\phi}$ and $(\dot{B} \sqcup \dot{C}) = \dot{A}$. Moreover, $int(\dot{B}) \sqcup int(\dot{C}) \neq int(\dot{B} \sqcup \dot{C})$.

Theorem 5: If $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over $\dot{X}$ and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then:

1) $cl(\emptyset) = \emptyset \ and \ cl(\dot{X}) = \dot{X}$.

2) $\dot{A} \sqsubseteq cl(\dot{A})$.

3) $\dot{A}$ can be consider as neutrosophic closed set if and only if $\dot{A} = cl(\dot{A})$.

4) $cl\left(cl(\dot{A})\right) = cl(\dot{A})$.

5) $\dot{A} \sqsubseteq \dot{B}$ implies as $cl(\dot{A}) \sqsubseteq cl(\dot{B})$

6) $cl(\dot{A} \sqcup \dot{B}) = cl(\dot{A}) \sqcup cl(\dot{B})$.

7) $cl(\dot{A} \sqcap \dot{B}) \sqsubseteq cl(\dot{A}) \sqcap cl(\dot{B})$.

Proof: 1, 2, 6, and 7 are clear, as well as done previously above.

3) Suppose that $\dot{A}$ is neutrosophic closed set over $\dot{X}$, here $\dot{A}$ contain $\dot{A}$ and it is itself closed set over $\dot{X}$. $\dot{A}$ can be consider as smallest neutrosophic closed set contains $\dot{A}$ such as $A \dot{=} cl(\dot{A})$. Conversely, $A \dot{=} cl(\dot{A}) \ as \ \dot{A}$ is small one neutrosophic closed set over $\dot{X}$ contains $\dot{A}$.

4) by above case, $A \dot{=} cl(\dot{A})$, $\dot{A}$ is neutrosophic closed set.

5) $\dot{A} \sqsubseteq \dot{B}$. We can clearly see every neutrosophic closed super set of $\dot{B}$ is also neutrosophic closed super set of $\dot{A}$. Hence, $cl(\dot{A}) \sqsubseteq cl(\dot{B})$.

Example 5: Let $\dot{X} = \{\dot{x}, \dot{y}\}$ and $\dot{A}, \dot{B}, \dot{C} \in \dot{N}(\dot{x})$ then:

$\dot{A}$
$= \{\langle \dot{x}, (0.3,0.3,0.3)(0.3,0.3,0.3)(0.3,0.3,0.3)\rangle, \langle y, (0.5,0.5,0.5)(0.5,0.5,0.5)(0.5,0.5,0.5)\rangle\}$

$\dot{B}$
$= \{\langle \dot{x}, (0.2,0.2,0.2)(0.2,0.2,0.2)(0.2,0.2,0.2)\rangle, \langle \dot{y}, (0.6,0.6,0.6)(0.6,0.6,0.6)(0.6,0.6,0.6)\rangle\}$

Apply formula $\frac{\dot{a}+\dot{b}+\dot{c}}{3}$,

$\dot{A} = \{\langle \dot{x}, 0.3,0.3,0.3\rangle, \langle \dot{y}, 0.5,0.5,0.5\rangle\}$

$\dot{B} = \{\langle \dot{x}, 0.2,0.2,0.2\rangle, \langle \dot{y}, 0.6,0.6,0.6\rangle\}$

Then,

$\dot{\tau} = \{\dot{\emptyset}, \dot{X}, \dot{A}, \dot{B}, \dot{A} \sqcap \dot{B}, \dot{A} \sqcup \dot{B}\}$

After taking the compliment,

$\{\dot{\emptyset}^c, \dot{X}^c, \dot{A}^c, \dot{B}^c, (\dot{A} \sqcap \dot{B})^c, (\dot{A} \sqcup \dot{B})^c\}$

Therefore,

$\dot{A}^c = \{\langle \dot{x}, 0.6,0.7,0.6\rangle, \langle \dot{y}, 0.5,0.5,0.5\rangle\}$

$\dot{B}^c = \{\langle \dot{x}, 0.2,0.8,0.2\rangle, \langle \dot{y}, 0.6,0.4,0.6\rangle\}$

$(\dot{A} \sqcap \dot{B})^c = \{\langle \dot{x}, 0.2,0.7,0.3\rangle\langle y, 0.6,\dot{0}.5,0.5\rangle\}$

$(\dot{A} \sqcup \dot{B})^c = \{\langle \dot{x}, 0.2,0.8,0.3\rangle, \langle y, 0.5,\dot{0}.5,0.6\rangle\}$

$\dot{A} \sqcap \dot{B} = \{\langle \dot{x}, 0.2,0.8,0.3\rangle, \langle y, 0.5,\dot{0}.5,0.6\rangle\}$

$cl(\dot{A}) = \dot{X}$

$cl(\dot{B}) = \dot{X}$

$cl(A \sqcap B) \dot{=} (\dot{A} \sqcup \dot{B})^c$

$cl(A \sqcap B) \sqsubseteq \dot{cl}(\dot{A}) \sqcap cl(\dot{B})$.

Theorem 6: Let, $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over $\dot{X}$ and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then:

i. $(\dot{fr}(\dot{A}))^c = ext(\dot{A}) \sqcup int(\dot{A})$.

ii. $cl(\dot{A}) = int(\dot{A}) \sqcup \dot{fr}(\dot{A})$.

Proof. $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$. Then,

Here we have,

$(\dot{fr}(\dot{A}))^c = (cl(\dot{A}) \sqcap \dot{fr}(\dot{A}^c))^c$

$(\dot{fr}(\dot{A}))^c = (cl(\dot{A}))^c \sqcup (\dot{fr}(\dot{A}^c))^c$

$(\dot{fr}(\dot{A}))^c = (cl(\dot{A}))^c \sqcup (\dot{int}(\dot{A}^c))^c$

$ext(\dot{A}) \sqcup int(\dot{A})$

$int(\dot{A}) \sqcup \dot{fr}(\dot{A}) = int(\dot{A}) \sqcup (cl(\dot{A}) \sqcap \dot{fr}(\dot{A}^c))$

$int(\dot{A}) \sqcup \dot{fr}(\dot{A}) = int(\dot{A}) \sqcup (cl(\dot{A}) \sqcap (\dot{int}(\dot{A}) \sqcup fr(\dot{A}^c))$

$int(\dot{A}) \sqcup \dot{fr}(\dot{A}) = cl(\dot{A}) \sqcap (int(\dot{A}) \sqcup int(\dot{A}))^c$

$int(\dot{A}) \sqcup \dot{fr}(\dot{A}) = cl(\dot{A}) \sqcap \dot{X}$

$int(\dot{A}) \sqcup \dot{fr}(\dot{A}) = cl(\dot{A})$.

Theorem 7: Let, $(\dot{X}, \dot{\tau})$ be neutrosophic topological space over $\dot{X}$ and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then:

i. $\dot{fr}(\dot{A}) \sqcap int(\dot{A}) = \emptyset$

ii. $\dot{fr}(int(\dot{A})) \sqsubseteq \dot{fr}(\dot{A})$

Proof: $\dot{A} \in \dot{N}(\dot{X})$. then,

i. is clear.
To prove ii. Let,

$\dot{fr}(int(\dot{A})) = cl(int(\dot{A})) \sqcap cl(int(\dot{A}))$

$\dot{fr}(int(\dot{A})) = cl(int(\dot{A})) \sqcap \dot{fr}(\dot{A}^c)$

$\dot{fr}(int(\dot{A})) = cl(\dot{A}) \sqcap \dot{fr}(\dot{A}^c)$

$\dot{f}r\left(int(\dot{A})\right) \sqsubseteq \dot{f}r(\dot{A})$

Definition: Let, $(\dot{X}, \dot{\tau})$ be neutrosophic topological space and $\dot{Y}$ is non empty subset of $\dot{X}$. Neutrosophic relative topology as:

$\dot{\tau}_Y = \{\dot{A} \sqcap \dot{Y} : \dot{A} \in \dot{\tau}\}$

$$Y(\dot{x}) = \begin{cases} \langle 1,0,0 \rangle & \dot{x} \in \dot{Y} \\ \langle 0,1,1 \rangle & otherwise \end{cases}$$

Hence, $(\dot{X}, \dot{\tau}_Y)$ as neutrosophic subspace of $(\dot{X}, \dot{\tau})$.

Example 6: Let $\dot{X} = \{\dot{a}, \dot{b}, \dot{c}\}$, $\dot{Y} = \{\dot{a}, \dot{b}\} \subseteq \dot{X}$ and $\dot{A}, \dot{B} \in \dot{N}(\dot{X})$ then,

$$\dot{A} = \{\langle \dot{x}, (0.3,0.5,0.7)(0.2,0.6,0.1)(0.4,0.6,0.7)\rangle, \langle y, (0.2,0.5,0.9)(0.3,0.1,0.8)(0.5,0.2,0.3)\rangle\}$$

$$\dot{B} = \{\langle \dot{x}, (0.2,0.6,0.4)(0.6,0.2,0.7)(0.2,0.1,0.6)\rangle, \langle \dot{y}, (0.6,0.8,0.7)(0.7,0.6,0.3)(0.1,0.5,0.6)\rangle\}$$

Apply formula $\frac{\dot{a}+\dot{b}+\dot{c}}{3}$,

$\dot{A} = \{\langle \dot{x}, 0.5,0.3,0.6\rangle, \langle \dot{y}, 0.5,0.4,0.3\rangle\}$

$\dot{B} = \{\langle \dot{x}, 0.4,0.5,0.3\rangle, \langle \dot{y}, 0.7,0.5,0.4\rangle\}$

Thus,

$\dot{\tau} = \{\dot{\emptyset}, \dot{X}, \dot{A}, \dot{B}, \dot{A} \sqcap \dot{B}, \dot{A} \sqcup \dot{B}\}$

As neutrosophic topology on $\dot{X}$. As well as,

$\dot{\tau}_Y = \{\dot{\emptyset}, \dot{Y}, \dot{C}, \dot{M}, \dot{L}, \dot{K}\}$ such that, $\dot{C} = \dot{Y} \sqcap \dot{A}$. $\dot{M} = \dot{Y} \sqcap \dot{B}$, $\dot{L} = \dot{Y} \sqcap (\dot{A} \sqcap \dot{B})$ and $\dot{K} = \dot{Y} \sqcap (\dot{A} \sqcup \dot{B})$.

Conclusion

In this current article, we rearrange the neutrosophic set operations and design triangular neutrosophic topology on the structure of neutrosophic topology with defuzzification. We introduce some properties linked to operations. We also clear the neutrosophic topology structure of neutrosophic sets. Moreover, we believe, with these new approaches, the researcher will able to enhance new possibilities in neutrosophic topology.

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