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Application of Neutrosophic Vague Nano Topological Spaces

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Abstract: This paper intends to presents a new structure of nano topological space by using neutrosophic vague sets. Here we define the neutrosophic vague nano topological spaces, various properties and characterizations related to these sets are also examined. Because of the increased volume of information available to physicians from advanced medical technology, the obtained information of each symptom with respect to a disease may contain truth, falsity and indeterminacy information, by using this information an application is also discussed with neutrosophic vague nano topological space. In this application we identified the risk factors for the cause of stroke attack through the concept of neutrosophic vague nano topology.

Keywords: Neutrosophic vague nano topological space, neutrosophic vague lower approximation, neutrosophic vague upper approximation and neutrosophic vague boundary.

1. Introduction

Recently, several theories have been proposed to deal with uncertainty, imprecision and vagueness. The concept of fuzzy sets was introduced in 1965 by Zadeh[14]. Using this fuzzy sets in 1986 intuitionistic fuzzy sets was introduced by Atanassov[5]. The theory of vague sets was first proposed and developed as an extension of fuzzy set theory by Gau and Buehrer[6]. Then, Smarandache[13], introduces the neutrosophic elements T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^{-}0,1^{+}[$ is that the non-standard unit interval in 1998. Shawkat Alkhazaleh[12], in 2015 introduced and constructed the concept of neutrosophic vague set. The concept of nano topological space was introduced by Lellis Thivagar[8]. Intuitionistic fuzzy nano topological space was introduced by Ramachandran[11] in 2017.

Medical diagnosis is process of investigation of a person's symptoms on the basis of diseases. From modern medical technology, a large amount of information available to medical experts due to whom medical diagnosis contains uncertain, inconsistent, indeterminate information and this information are mandatory in medical diagnosis. A characterized relationship between a symptom and a disease is usually based on these uncertain, inconsistent information which leads to us for decision making in a medical diagnosis. Mostly diagnosis problems have pattern recognition on the basis of which medical experts make their decision. Medical diagnosis has successful practical applications in several areas such as telemedicine, space medicine and rescue services etc. where access of human means of diagnosis is a difficult task. Thus, starting from the early time of Artificial Intelligence, medical diagnosis has got full attention from both computer science and computer applicable mathematics research society. Abdel-Basset,et.al.,[1,2,3,4] formulated various multi-criteria decision-making approach.

In this paper we use the neutrosophic vague sets in nano topological space and defined the neutrosophic vague nano topological space and discussed some of its properties. Here we also use the neutrosophic vague nano topological space in real time application in multi decision problem such as medical diagnosis to identify the vital factors for the Stroke in patients.

2. Preliminaries:

Definition 2.1:[10] Let U^* be a non-empty finite set of objects called the universe and R^* be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U^*, R^*) is said to be the approximation space. Let $X^* \subseteq U^*$.

i) The lower approximation of X^* with respect to R^* is denoted by $L_{R^*}(X^*)$. That is, $L_{R^*}(X^*) = \bigcup_{x \in U^*}\{R^*(x): R^*(x) \subseteq X^*\}$, where denotes the equivalence class determined by x.

ii) The upper approximation of X^* with respect to R^* denoted by $U_{R^*}(X^*)$. That is, $U_{R^*}(X^*) = \bigcup_{x \in U^*}\{R^*(x): R^*(x) \cap X^* \neq \emptyset\}$.

iii) The boundary region of X^* with respect to R^* is denoted by $B_{R^*}(X^*)$. That is, $B_{R^*}(X^*) = U_{R^*}(X^*) - L_{R^*}(X^*)$.

Definition 2.2:[8] Let U^* be an universe, R^* be an equivalence relation on U^* and $\tau_{R^*}(X^*) = \{U^*, \emptyset, L_{R^*}(X^*), U_{R^*}(X^*), B_{R^*}(X^*)\}$ where $X^* \subseteq U^*$. $\tau_{R^*}(X^*)$ satisfies the following axioms:

i) X^* and $\emptyset \in \tau_{R^*}(X^*)$.

ii) The union of the elements of any sub-collection of $\tau_{R^*}(X^*)$ is in $\tau_{R^*}(X^*)$.

iii) The intersection of the elements of any finite sub-collection of $\tau_{R^*}(X^*)$ is in $\tau_{R^*}(X^*)$.

Definition 2.3:[12] A neutrosophic vague set A^*_{NV} (NVS in short) on the universe of discourse X^* written as $A^*_{NV}=\{\langle x;\hat{T}_{A^*_{NV}}(x);\hat{I}_{A^*_{NV}}(x);\hat{F}_{A^*_{NV}}(x)\rangle;x\in X^*\}$, whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_{A^*_{NV}}(x)=[T^-,T^+],\hat{I}_{A^*_{NV}}(x)=[I^-,I^+],\hat{F}_{A^*_{NV}}(x)=[F^-,F^+]$$

where,

i) $T^+=1-F^-$

ii) $F^+=1-T^-$ and

iii) $^-0\le T^-+I^-+F^-\le 2^+$.

Definition 2.4:[12] Let A^*_{NV} and B^*_{NV} be two NVSs of the universe U . If $\forall u_i\in U,\ \hat{T}_{A^*_{NV}}(u_i)\le\hat{T}_{B^*_{NV}}(u_i);\ \hat{I}_{A^*_{NV}}(u_i)\ge\hat{I}_{B^*_{NV}}(u_i);\ \hat{F}_{A^*_{NV}}(u_i)\ge\hat{F}_{B^*_{NV}}(u_i)$, then the NVS A^*_{NV} is included by B^*_{NV}, denoted by $A^*_{NV}\subseteq B^*_{NV}$, where $1\le i\le n$.

Definition 2.5:[12] The complement of NVS A^*_{NV} is denoted by $(A^*_{NV})^c$ and is defined by

$$\hat{T}^c_{A^*_{NV}}(x)=[1-T^+,1-T^-],\hat{I}^c_{A^*_{NV}}(x)=[1-I^+,1-I^-],\hat{F}^c_{A^*_{NV}}(x)=[1-F^+,1-F^-].$$

3. Neutrosophic Vague Nano Topological Spaces:

Definition 3.1: Let Ω be a non-empty set and $\mathcal{R}^*$ be an equivalence relation on Ω. Let Q be a neutrosophic vague set in Ω with the truth membership function $\hat{T}_Q$, the indeterminacy function $\hat{I}_Q$ and the false membership function $\hat{F}_Q$. The neutrosophic vague lower approximation, neutrosophic vague upper approximation and neutrosophic vague boundary of Q in the neutrosophic vague approximation space (Ω, $\mathcal{R}^*$) is denoted by NVLA(Q), NVUA(Q) and NVB(Q) are respectively defined as follows.

i) $NVLA(Q)=\{\langle x;\hat{T}_{LQ}(x);\hat{I}_{LQ}(x);\hat{F}_{LQ}(x)\rangle;y\in[x]_{\mathcal{R}^*},x\in\Omega\}$.

ii) $NVUA(Q)=\{\langle x;\hat{T}_{UQ}(x);\hat{I}_{UQ}(x);\hat{F}_{UQ}(x)\rangle;y\in[x]_{\mathcal{R}^*},x\in\Omega\}$.

iii) $NVB(Q)=NVUA(Q)-NVLA(Q)$.

Where $\hat{T}_{LQ}(x)=\wedge_{y\in[x]_{\mathcal{R}^*}}\hat{T}_Q(y)$; $\hat{I}_{LQ}(x)=\vee_{y\in[x]_{\mathcal{R}^*}}\hat{I}_Q(y)$; $\hat{F}_{LQ}(x)=\vee_{y\in[x]_{\mathcal{R}^*}}\hat{F}_Q(y)$

for neutrosophic vague set Q we have $\hat{T}_Q(y) = [T^-, T^+]$; $\hat{I}_Q(y) = [I^-, I^+]$; $\hat{F}_Q(y) = [F^-, F^+]$

so, $\hat{T}_{LQ}(x) = \left[\wedge_{y\in[x]_{\mathcal{R}^*}} T^-{}_Q(y), \wedge_{y\in[x]_{\mathcal{R}^*}} T^+{}_Q(y)\right]$;

$\hat{I}_{LQ}(x) = \left[\vee_{y\in[x]_{\mathcal{R}^*}} I^-{}_Q(y), \vee_{y\in[x]_{\mathcal{R}^*}} I^+{}_Q(y)\right]$ and

$\hat{F}_{LQ}(x) = \left[\vee_{y\in[x]_{\mathcal{R}^*}} F^-{}_Q(y), \vee_{y\in[x]_{\mathcal{R}^*}} F^+{}_Q(y)\right]$

And $\hat{T}_{UQ}(x) = \vee_{y\in[x]_{\mathcal{R}^*}} \hat{T}_Q(y)$; $\hat{I}_{UQ}(x) = \wedge_{\in[x]_{\mathcal{R}^*}} \hat{I}_Q(y)$; $\hat{F}_{UQ}(x) = \wedge_{y\in[x]_{\mathcal{R}^*}} \hat{F}_Q(y)$

for neutrosophic vague set Q we have $\hat{T}_Q(y) = [T^-, T^+]$; $\hat{I}_Q(y) = [I^-, I^+]$; $\hat{F}_Q(y) = [F^-, F^+]$

so, $\hat{T}_{UQ}(x) = \left[\vee_{y\in[x]_{\mathcal{R}^*}} T^-{}_Q(y), \vee_{y\in[x]_{\mathcal{R}^*}} T^+{}_Q(y)\right]$;

$\hat{I}_{UQ}(x) = \left[\wedge_{y\in[x]_{\mathcal{R}^*}} I^-{}_Q(y), \wedge_{y\in[x]_{\mathcal{R}^*}} I^+{}_Q(y)\right]$ and

$\hat{F}_{UQ}(x) = \left[\wedge_{y\in[x]_{\mathcal{R}^*}} F^-{}_Q(y), \wedge_{y\in[x]_{\mathcal{R}^*}} F^+{}_Q(y)\right]$

Definition 3.2: If (Ω, $\mathcal{R}^*$) is a neutrosophic vague approximation space and let $P, Q \subseteq \Omega$, then the following statements holds:

i) $NVLA(P) \subseteq P \subseteq NVUA(P)$.

ii) $NVLA(0_{NV}) = NVUA(0_{NV}) = 0_{NV}$ and $NVLA(1_{NV}) = NVUA(1_{NV}) = 1_{NV}$.

iii) $NVLA(P \cup Q) \supseteq NVLA(P) \cup NVLA(Q)$.

iv) $NVLA(P \cap Q) = NVLA(P) \cap NVLA(Q)$.

v) $NVUA(P \cup Q) = NVUA(P) \cup NVUA(Q)$.

vi) $NVUA(P \cap Q) \subseteq NVUA(P) \cap NVUA(Q)$.

vii) For $P \subseteq Q$, we have $NVLA(P) \subseteq NVLA(Q)$ and $NVUA(P) \subseteq NVUA(Q)$.

viii) $NVUA(P^c) = \left(NVLA(P)\right)^c$ and $NVLA(P^c) = \left(NVUA(P)\right)^c$.

ix) $NVUA\left(NVUA(P)\right) = NVLA\left(NVUU(P)\right) = NVUA(P)$.

x) $NVLA\left(NVLA(P)\right) = NVUA\left(NVLA(P)\right) = NVLA(P)$

Definition 3.3: Let Ω be universe, $\mathcal{R}^*$ be an equivalence relation on Ω and Q be a neutrosophic vague set in Ω and if $\tau_{\mathcal{R}^*}(Q) = \{0_{NV}, 1_{NV}, NVLA(Q), NVUA(Q), NVB(Q)\}$ where $Q \subseteq \Omega$. $\tau_{\mathcal{R}^*}(Q)$ satisfies the following axioms:

i) 0_{NV} and $1_{NV} \in \tau_{\mathcal{R}^*}(Q)$.

ii) The union of the elements of any sub-collection of $\tau_{\mathcal{R}^*}(Q)$ is in $\tau_{\mathcal{R}^*}(Q)$.

iii) The intersection of the elements of any finite sub-collection of $\tau_{\mathcal{R}^*}(Q)$ is in $\tau_{\mathcal{R}^*}(Q)$.

Then $\tau_{\mathcal{R}^*}(Q)$ is said to be neutrosophic vague nano topology. We call $(\Omega, \tau_{\mathcal{R}^*}(Q))$ as neutrosophic vague nano topological space. The elements of $\tau_{\mathcal{R}^*}(Q)$ are called as neutrosophic vague nano open sets.

Example 3.4: Let $\Omega = \{u, v, w, x, y, z\}$ be universe, $\mathcal{R}^*$ be an equivalence relation on Ω, so we have

$$\Omega = \left\{ \begin{matrix} \langle u: [0.6,0.9]; [0.2,0.5]; [0.1,0.4] \rangle, \langle v: [0.1,0.4]; [0.3,0.5]; [0.6,0.9] \rangle, \\ \langle w: [0.7,0.8]; [0.1,0.6]; [0.2,0.3] \rangle, \langle x: [0.2,0.6]; [0.5,0.7]; [0.4,0.8] \rangle, \\ \langle y: [0.3,0.7]; [0.6,0.8]; [0.3,0.7] \rangle, \langle z: [0.4,0.5]; [0.1,0.3]; [0.5,0.6] \rangle \end{matrix} \right\}$$

$\Omega/_{\mathcal{R}^*} = \{\{u, w\}, \{v\}, \{x, y\}, \{z\}\}$ is the $\mathcal{R}^*$ be an equivalence relation on Ω. Let $Q = \{u, v, w, x, z\}$ then

$$Q = \left\{ \begin{matrix} \langle u: [0.6,0.9]; [0.2,0.5]; [0.1,0.4] \rangle, \langle v: [0.1,0.4]; [0.3,0.5]; [0.6,0.9] \rangle, \\ \langle w: [0.7,0.8]; [0.1,0.6]; [0.2,0.3] \rangle, \langle x: [0.2,0.6]; [0.5,0.7]; [0.4,0.8] \rangle, \\ \langle z: [0.4,0.5]; [0.1,0.3]; [0.5,0.6] \rangle \end{matrix} \right\}$$

$$NVLA(Q) = \left\{ \begin{matrix} \langle u: [0.6,0.8]; [0.2,0.6]; [0.2,0.4] \rangle, \langle v: [0.1,0.4]; [0.3,0.5]; [0.6,0.9] \rangle, \\ \langle w: [0.6,0.8]; [0.2,0.6]; [0.2,0.4] \rangle, \langle x: [0.2,0.6]; [0.6,0.8]; [0.4,0.8] \rangle, \\ \langle z: [0.4,0.5]; [0.1,0.3]; [0.5,0.6] \rangle \end{matrix} \right\}$$

$$NVUA(Q) = \left\{ \begin{matrix} \langle u: [0.7,0.9]; [0.1,0.5]; [0.1,0.3] \rangle, \langle v: [0.1,0.4]; [0.3,0.5]; [0.6,0.9] \rangle, \\ \langle w: [0.7,0.9]; [0.1,0.5]; [0.1,0.3] \rangle, \langle x: [0.3,0.7]; [0.5,0.7]; [0.3,0.7] \rangle, \\ \langle z: [0.4,0.5]; [0.1,0.3]; [0.5,0.6] \rangle \end{matrix} \right\}$$

$$NVB(Q) = \left\{ \begin{matrix} \langle u: [0.2,0.4]; [0.4,0.8]; [0.6,0.8] \rangle, \langle v: [0.1,0.4]; [0.5,0.7]; [0.6,0.9] \rangle, \\ \langle w: [0.2,0.4]; [0.4,0.8]; [0.6,0.8] \rangle, \langle x: [0.3,0.7]; [0.5,0.7]; [0.3,0.7] \rangle, \\ \langle z: [0.4,0.5]; [0.7,0.9]; [0.5,0.6] \rangle \end{matrix} \right\}$$

Then $\tau_{\mathcal{R}^*}(Q) = \{0_{NV}, 1_{NV}, NVLA(Q), NVUA(Q), NVB(Q)\}$ is neutrosophic vague nano topology on Ω.

Definition 3.5: Let Ω be universe and let $Q \subseteq \Omega$, then the following statements holds:

i) If $NVLA(Q) = 0_{NV}$ and $NVUA(Q) = 1_{NV}$, then $\tau_{\mathcal{R}^*}(Q) = \{0_{NV}, 1_{NV}\}$, is neutrosophic vague nano discrete topology on Ω.

ii) If $NVLA(Q) = NVUA(Q) = Q$, then the neutrosophic vague nano topology is $\tau_{\mathcal{R}^*}(Q) = \{0_{NV}, 1_{NV}, NVLA(Q)\}$.

iii) If $NVLA(Q) = 0_{NV}$ and $NVUA(Q) \neq 1_{NV}$, then $\tau_{\mathcal{R}^*}(Q) = \{0_{NV}, 1_{NV}, NVUA(Q)\}$.

iv) If $NVLA(Q) \neq 0_{NV}$ and $NVUA(Q) = 1_{NV}$, then $\tau_{\mathcal{R}^*}(Q) = \{0_{NV}, 1_{NV}, NVLA(Q), NVB(Q)\}$.

v) If $NVLA(Q) \neq NVLA(Q)$ where $NVLA(Q) \neq 0_{NV}$ and $NVUA(Q) \neq 1_{NV}$, then $\tau_{\mathcal{R}^*}(Q) = \{0_{NV}, 1_{NV}, NVLA(Q), NVUA(Q), NVB(Q)\}$ is neutrosophic vague nano discrete topology on Ω.

Definition 3.5: Let $(\Omega, \tau_{\mathcal{R}^*}(Q))$ be neutrosophic vague nano topological space with respect to Q, where $Q \subseteq \Omega$ and if $A \subseteq \Omega$, then

i) The neutrosophic vague nano interior of the set *A* is defined as the union of all neutrosophic vague nano open subsets contained in *A*, and is denoted by $NV_Q\, int(A)$.

ii) The neutrosophic vague nano closure of the set *A* is defined as the intersection of all neutrosophic vague nano closed subsets containing *A*, and is denoted by $NV_Q\, cl(A)$.

Theorem 3.6: Let $(\Omega, \tau_{\mathcal{R}^*}(Q))$ be neutrosophic vague nano topological space with respect to Q, where $Q \subseteq \Omega$. Let A and B be two neutrosophic vague nano subsets of Ω. Then the following statements hold:

i) $NV_Q\, int(A) \subseteq A$.

ii) $A \subseteq NV_Q\, cl(A)$.

iii) *A* is neutrosophic vague nano closed if and only if $NV_Q\, cl(A) = A$.

iv) $NV_Q\, cl(0_{NV}) = 0_{NV}$ and $NV_Q\, cl(1_{NV}) = 1_{NV}$.

v) $A \subseteq B$ implies $NV_Q\, cl(A) \subseteq NV_Q\, cl(B)$.

vi) $NV_Q\, cl(A \cup B) = NV_Q\, cl(A) \cup NV_Q\, cl(B)$.

vii) $NV_Q\, cl(A \cap B) \subseteq NV_Q\, cl(A) \cap NV_Q\, cl(B)$.

viii) $NV_Q\, cl(NV_Q\, cl(A)) = NV_Q\, cl(A)$.

Proof:

i) By definition of neutrosophic vague nano interior we have, $NV_Q\, int(A) \subseteq A$.

ii) By definition of neutrosophic vague nano closure we have, $A \subseteq NV_Q\, cl(A)$.

iii) If A is neutrosophic vague nano closed set, then A is the smallest neutrosophic vague nano closed set containing itself and hence $NV_Q\, cl(A) = A$. Conversely, if $NV_Q\, cl(A) = A$, then A is the smallest neutrosophic vague nano closed set containing itself and hence A is neutrosophic vague nano closed set.

iv) Since 0_{NV} and 1_{NV} are neutrosophic vague nano closed in $(\Omega, \tau_{\mathcal{R}}(Q))$, so $NV_Q\, cl(0_{NV}) = 0_{NV}$ and $NV_Q\, cl(1_{NV}) = 1_{NV}$.

v) If $A \subseteq B$, since $B \subseteq NV_Q\, cl(B)$, then $A \subseteq NV_Q\, cl(B)$. That is, $NV_Q\, cl(B)$ is a neutrosophic vague nano closed set containing A. But $NV_Q\, cl(A)$ is the smallest neutrosophic vague nano closed set containing A. Therefore, $NV_Q\, cl(A) \subseteq NV_Q\, cl(B)$.

vi) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, $NV_Q\, cl(A) \subseteq NV_Q\, cl(A \cup B)$ and $NV_Q\, cl(B) \subseteq NV_Q\, cl(A \cup B)$. Therefore $NV_Q\, cl(A) \cup NV_Q\, cl(B) \subseteq NV_Q\, cl(A \cup B)$. By the fact that $A \cup B \subseteq NV_Q\, cl(A) \cup NV_Q\, cl(B)$, and since $NV_Q\, cl(A \cup B)$ is the smallest nano closed set containing $A \cup B$, so $NV_Q\, cl(A \cup B) \subseteq NV_Q\, cl(A) \cup NV_Q\, cl(B)$. Thus $NV_Q\, cl(A \cup B) = NV_Q\, cl(A) \cup NV_Q\, cl(B)$.

vii) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, $NV_Q\,cl(A \cap B) \subseteq NV_Q\,cl(A) \cap NV_Q\,cl(B)$.

viii) Since $NV_Q\,cl(A)$ is neutrosophic vague nano closed, $NV_Q\,cl\big(NV_Q\,cl(A)\big) = NV_Q\,cl(A)$.

Theorem 3.7: Let $\big(\Omega, \tau_{\mathcal{R}^*}(Q)\big)$ be neutrosophic vague nano topological space with respect to Q, where $Q \subseteq \Omega$ and if $A \subseteq \Omega$, then

i) $1_{NV} - NV_Q\,int(A) = NV_Q\,cl(1_{NV} - A)$.

ii) $1_{NV} - NV_Q\,cl(A) = NV_Q\,int(1_{NV} - A)$.

4. Real Time Application Of Neutrosophic Vague Nano Topological Space:

In this example we use the neutrosophic vague nano topology to find the vital factors of "Stroke" by using topological reduction of attributes in the data set.

We consider the following information table about patients various attributes such as Age, Sugar, Blood Pressure, Heart Disease, BMI, Family History, Smoking, Pain Killer Intake are taken as the data base set. From this data set we find the vital factor for causing stroke among patients. Here $V = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10}\}$ the set of patients and S={Age, Sugar, Blood Pressure, Heart Disease, BMI, Family History, Smoking, Pain Killer Intake} the set of factors that may lead to stroke. Table 1 gives the information of the patients and in Table 2 the patients are denoted using the neutrosophic vague sets.

Patients	Age	Sugar	Blood Pressure	Heart Disease	BMI	Family History	Smoking	Pain Killer Intake	Result
m_1	Very Old	Yes	Very High	Yes	Over Weight	Yes	No	No	Yes
m_2	Old	Yes	Very High	Yes	Over Weight	Yes	No	No	Yes
m_3	Old	No	High	No	Obese	Yes	Yes	Yes	No
m_4	Very Old	Yes	Very High	Yes	Over Weight	Yes	No	No	Yes
m_5	Very Old	Yes	Normal	Yes	Normal	No	No	No	No
m_6	Very Old	Yes	Very High	No	Normal	No	Yes	No	Yes

m_7	Very Old	No	High	No	Obese	Yes	Yes	Yes	No
m_8	Old	No	Normal	No	Obese	No	Yes	Yes	Yes
m_9	Old	No	High	No	Obese	Yes	Yes	Yes	Yes
m_{10}	Old	No	Normal	No	Obese	Yes	Yes	Yes	No

Table 1: Patient's possible attribute

Patients	**Neutrosophic Vague Sets**
m_1	$\{\langle[0.5,0.8];[0.1,0.2];[0.2,0.5]\rangle\}$
m_2	$\{\langle[0.6,0.7];[0.3,0.5];[0.3,0.4]\rangle\}$
m_3	$\{\langle[0.2,0.3];[0.5,0.7];[0.7,0.8]\rangle\}$
m_4	$\{\langle[0.6,0.9];[0.2,0.4];[0.1,0.4]\rangle\}$
m_5	$\{\langle[0.1,0.4];[0.3,0.6];[0.6,0.9]\rangle\}$
m_6	$\{\langle[0.6,0.7];[0.4,0.5];[0.3,0.4]\rangle\}$
m_7	$\{\langle[0.1,0.2];[0.7,0.9];[0.8,0.9]\rangle\}$
m_8	$\{\langle[0.7,0.9];[0.2,0.3];[0.1,0.3]\rangle\}$
m_9	$\{\langle[0.4,0.7];[0.5,0.8];[0.3,0.6]\rangle\}$
m_{10}	$\{\langle[0.1,0.5];[0.4,0.6];[0.5,0.9]\rangle\}$

Table 2: Neutrosophic Vague sets

Here $V = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10}\}$ the set of patients and S = {Age, Sugar, Blood Pressure, Heart Disease, BMI, Family History, Smoking, Pain Killer Intake} be the set of attributes that may cause stroke. In short the set is denoted by S = {AG, SU, BP, HD, BMI, FH, SM, PK}. The family of equivalence classes corresponding to S is given by $V/_{\mathcal{R}^*(S)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$.

Case 1: (Patients affected by Stroke)

Let $V = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10}\}$ the set of patients. Let $V/_{\mathcal{R}^*(S)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$ be an equivalence relation on V and $P^* = \{m_1, m_2, m_4, m_6, m_8, m_9\}$ be the set of patients affected by stroke. Then

$$NVLA(P^*) = \begin{Bmatrix} \langle m_1: [.5,.8]; [.2,.4]; [.2,.5] \rangle, \langle m_2: [.6,.7]; [.3,.5]; [.3,.4] \rangle, \\ \langle m_4: [.5,.8]; [.2,.4]; [.2,.5] \rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4] \rangle, \\ \langle m_8: [.7,.9]; [.2,.3]; [.1,.3] \rangle, \langle m_9: [.2,.3]; [.5,.8]; [.7,.8] \rangle \end{Bmatrix}$$

$$NVUA(P^*) = \begin{Bmatrix} \langle m_1: [.6,0.9]; [.1,.2]; [.1,.4] \rangle, \langle m_2: [.6,.7]; [.3,.5]; [.3,.4] \rangle, \\ \langle m_4: [.6,0.9]; [.1,.2]; [.1,.4] \rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4] \rangle, \\ \langle m_8: [.7,0.9]; [.2,.3]; [.1,.3] \rangle, \langle m_9: [.4,.7]; [.5,.7]; [.3,.6] \rangle \end{Bmatrix}$$

$$NVB(P^*) = \begin{Bmatrix} \langle m_1: [.2,.5]; [.6,.8]; [.5,.8] \rangle, \langle m_2: [.3,.4]; [.5,.7]; [.6,.7] \rangle, \\ \langle m_4: [.2,.5]; [.6,.8]; [.5,.8] \rangle, \langle m_6: [.3,.4]; [.5,.6]; [.6,.7] \rangle, \\ \langle m_8: [.1,.3]; [.7,.8]; [.7,.9] \rangle, \langle m_9: [.4,.7]; [.5,.7]; [.3,.6] \rangle \end{Bmatrix}$$

Therefore, $\tau_S(P^*) = \{0_{NV}, 1_{NV}, NVLA(P^*), NVUA(P^*), NVB(P^*)\}$ ----------- (1)

Step 1: Let $P^* = \{m_1, m_2, m_4, m_6, m_8, m_9\}$, when the attribute "Age" is removed from S, we have

$V/_{\mathcal{R}^*(S-AG)} = \{\{m_1, m_2, m_4\}, \{m_3, m_7, m_9\}, \{m_5\}, \{m_6\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(P^*) = \begin{Bmatrix} \langle m_1: [.5,.7]; [.3,.5]; [.3,.5] \rangle, \langle m_2: [.5,.7]; [.3,.5]; [.3,.5] \rangle, \\ \langle m_4: [.5,.7]; [.3,.5]; [.3,.5] \rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4] \rangle, \\ \langle m_8: [.7,.9]; [.2,.3]; [.1,.3] \rangle, \langle m_9: [.1,.2]; [.7,.9]; [.8,.9] \rangle \end{Bmatrix}$$

$$NVUA(P^*) = \begin{Bmatrix} \langle m_1: [.6,.9]; [.1,.2]; [.1,.4] \rangle, \langle m_2: [.6,.9]; [.1,.2]; [.1,.4] \rangle, \\ \langle m_4: [.6,.9]; [.1,.2]; [.1,.4] \rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4] \rangle, \\ \langle m_8: [.7,.9]; [.2,.3]; [.1,.3] \rangle, \langle m_9: [.4,.7]; [.5,.7]; [.3,.6] \rangle \end{Bmatrix}$$

$$NVB(P^*) = \begin{Bmatrix} \langle m_1: [.3,.5]; [.5,.7]; [.5,.7] \rangle, \langle m_2: [.3,.5]; [.5,.7]; [.5,.7] \rangle, \\ \langle m_4: [.3,.5]; [.5,.7]; [.5,.7] \rangle, \langle m_6: [.3,.4]; [.5,.6]; [.6,.7] \rangle, \\ \langle m_8: [.1,.3]; [.7,.8]; [.7,.9] \rangle, \langle m_9: [.4,.7]; [.5,.7]; [.3,.6] \rangle \end{Bmatrix}$$

Therefore, $\tau_S(P^*) = \{0_{NV}, 1_{NV}, NVLA(P^*), NVUA(P^*), NVB(P^*)\} \neq$ (1)

Step 2: Let $P^* = \{m_1, m_2, m_4, m_6, m_8, m_9\}$, when the attribute "Sugar" is removed from S, we have $V/_{\mathcal{R}^*(S-SU)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(P^*) = \begin{Bmatrix} \langle m_1: [.5,.8]; [.2,.4]; [.2,.5] \rangle, \langle m_2: [.6,.7]; [.3,.5]; [.3,.4] \rangle, \\ \langle m_4: [.5,.8]; [.2,.4]; [.2,.5] \rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4] \rangle, \\ \langle m_8: [.7,.9]; [.2,.3]; [.1,.3] \rangle, \langle m_9: [.2,.3]; [.5,.8]; [.7,.8] \rangle \end{Bmatrix}$$

$$NVUA(P^*) = \begin{Bmatrix} \langle m_1: [.6,.9]; [.1,.2]; [.1,.4]\rangle, \langle m_2: [.6,.7]; [.3,.5]; [.3,.4]\rangle, \\ \langle m_4: [.6,.9]; [.1,.2]; [.1,.4]\rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4]\rangle, \\ \langle m_8: [.7,.9]; [.2,.3]; [.1,.3]\rangle, \langle m_9: [.4,.7]; [.5,.7]; [.3,.6]\rangle \end{Bmatrix}$$

$$NVB(P^*) = \begin{Bmatrix} \langle m_1: [.2,.5]; [.6,.8]; [.5,.8]\rangle, \langle m_2: [.3,.4]; [.5,.7]; [.6,.7]\rangle, \\ \langle m_4: [.2,.5]; [.6,.8]; [.5,.8]\rangle, \langle m_6: [.3,.4]; [.5,.6]; [.6,.7]\rangle, \\ \langle m_8: [.1,.3]; [.7,.8]; [.7,.9]\rangle, \langle m_9: [.4,.7]; [.5,.7]; [.3,.6]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(P^*) = \{0_{NV}, 1_{NV}, NVLA(P^*), NVUA(P^*), NVB(P^*)\} = (1)$

Step 3: Let $P^* = \{m_1, m_2, m_4, m_6, m_8, m_9\}$, when the attribute "Blood Pressure" is removed from S, we have $V/_{\mathcal{R}^*(S-BP)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9, m_{10}\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}\}$. Then

$$NVLA(P^*) = \begin{Bmatrix} \langle m_1: [.5,.8]; [.2,.4]; [.2,.5]\rangle, \langle m_2: [.6,.7]; [.3,.5]; [.3,.4]\rangle, \\ \langle m_4: [.5,.8]; [.2,.4]; [.2,.5]\rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4]\rangle, \\ \langle m_8: [.7,.9]; [.2,.3]; [.1,.3]\rangle, \langle m_9: [.1,.3]; [.5,.8]; [.7,.9]\rangle \end{Bmatrix}$$

$$NVUA(P^*) = \begin{Bmatrix} \langle m_1: [.6,.9]; [.1,.2]; [.1,.4]\rangle, \langle m_2: [.6,.7]; [.3,.5]; [.3,.4]\rangle, \\ \langle m_4: [.6,.9]; [.1,.2]; [.1,.4]\rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4]\rangle, \\ \langle m_8: [.7,.9]; [.2,.3]; [.1,.3]\rangle, \langle m_9: [.4,.7]; [.4,.6]; [.3,.6]\rangle \end{Bmatrix}$$

$$NVB(P^*) = \begin{Bmatrix} \langle m_1: [.2,.5]; [.6,.8]; [.5,.8]\rangle, \langle m_2: [.3,.4]; [.5,.7]; [.6,.7]\rangle, \\ \langle m_4: [.2,.5]; [.6,.8]; [.5,.8]\rangle, \langle m_6: [.3,.4]; [.5,.6]; [.6,.7]\rangle, \\ \langle m_8: [.1,.3]; [.7,.8]; [.7,.9]\rangle, \langle m_9: [.4,.7]; [.4,.6]; [.3,.6]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(P^*) = \{0_{NV}, 1_{NV}, NVLA(P^*), NVUA(P^*), NVB(P^*)\} \neq (1)$

Step 4: Let $P^* = \{m_1, m_2, m_4, m_6, m_8, m_9\}$, when the attribute "Heart Disease" is removed from S, we have $V/_{\mathcal{R}^*(S-HD)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(P^*) = \begin{Bmatrix} \langle m_1: [.5,.8]; [.2,.4]; [.2,.5]\rangle, \langle m_2: [.6,.7]; [.3,.5]; [.3,.4]\rangle, \\ \langle m_4: [.5,.8]; [.2,.4]; [.2,.5]\rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4]\rangle, \\ \langle m_8: [.7,.9]; [.2,.3]; [.1,.3]\rangle, \langle m_9: [.2,.3]; [.5,.8]; [.7,.8]\rangle \end{Bmatrix}$$

$$NVUA(P^*) = \begin{Bmatrix} \langle m_1: [.6,.9]; [.1,.2]; [.1,.4]\rangle, \langle m_2: [.6,.7]; [.3,.5]; [.3,.4]\rangle, \\ \langle m_4: [.6,.9]; [.1,.2]; [.1,.4]\rangle, \langle m_6: [.6,.7]; [.4,.5]; [.3,.4]\rangle, \\ \langle m_8: [.7,.9]; [.2,.3]; [.1,.3]\rangle, \langle m_9: [.4,.7]; [.5,.7]; [.3,.6]\rangle \end{Bmatrix}$$

$$NVB(P^*) = \begin{Bmatrix} \langle m_1: [.2,.5]; [.6,.8]; [.5,.8]\rangle, \langle m_2: [.3,.4]; [.5,.7]; [.6,.7]\rangle, \\ \langle m_4: [.2,.5]; [.6,.8]; [.5,.8]\rangle, \langle m_6: [.3,.4]; [.5,.6]; [.6,.7]\rangle, \\ \langle m_8: [.1,.3]; [.7,.8]; [.7,.9]\rangle, \langle m_9: [.4,.7]; [.5,.7]; [.3,.6]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(P^*) = \{0_{NV}, 1_{NV}, NVLA(P^*), NVUA(P^*), NVB(P^*)\} = (1)$

Step 5: Let $P^* = \{m_1, m_2, m_4, m_6, m_8, m_9\}$, when the attribute "BMI" is removed from S, we have $V/_{\mathcal{R}^*(S-BMI)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(P^*) = \begin{Bmatrix} \langle m_1: [.5, .8]; [.2, .4]; [.2, .5]\rangle, \langle m_2: [.6, .7]; [.3, .5]; [.3, .4]\rangle, \\ \langle m_4: [.5, .8]; [.2, .4]; [.2, .5]\rangle, \langle m_6: [.6, .7]; [.4, .5]; [.3, .4]\rangle, \\ \langle m_8: [.7, .9]; [.2, .3]; [.1, .3]\rangle, \langle m_9: [.2, .3]; [.5, .8]; [.7, .8]\rangle \end{Bmatrix}$$

$$NVUA(P^*) = \begin{Bmatrix} \langle m_1: [.6, .9]; [.1, .2]; [.1, .4]\rangle, \langle m_2: [.6, .7]; [.3, .5]; [.3, .4]\rangle, \\ \langle m_4: [.6, .9]; [.1, .2]; [.1, .4]\rangle, \langle m_6: [.6, .7]; [.4, .5]; [.3, .4]\rangle, \\ \langle m_8: [.7, .9]; [.2, .3]; [.1, .3]\rangle, \langle m_9: [.4, .7]; [.5, .7]; [.3, .6]\rangle \end{Bmatrix}$$

$$NVB(P^*) = \begin{Bmatrix} \langle m_1: [.2, .5]; [.6, .8]; [.5, .8]\rangle, \langle m_2: [.3, .4]; [.5, .7]; [.6, .7]\rangle, \\ \langle m_4: [.2, .5]; [.6, .8]; [.5, .8]\rangle, \langle m_6: [.3, .4]; [.5, .6]; [.6, .7]\rangle, \\ \langle m_8: [.1, .3]; [.7, .8]; [.7, .9]\rangle, \langle m_9: [.4, .7]; [.5, .7]; [.3, .6]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(P^*) = \{0_{NV}, 1_{NV}, NVLA(P^*), NVUA(P^*), NVB(P^*)\} = (1)$

Step 6: Let $P^* = \{m_1, m_2, m_4, m_6, m_8, m_9\}$, when the attribute "Family History" is removed from S, we have $V/_{\mathcal{R}^*(S-FH)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8, m_{10}\}\}$. Then

$$NVLA(P^*) = \begin{Bmatrix} \langle m_1: [.5, .8]; [.2, .4]; [.2, .5]\rangle, \langle m_2: [.6, .7]; [.3, .5]; [.3, .4]\rangle, \\ \langle m_4: [.5, .8]; [.2, .4]; [.2, .5]\rangle, \langle m_6: [.6, .7]; [.4, .5]; [.3, .4]\rangle, \\ \langle m_8: [.1, .5]; [.4, .6]; [.5, .9]\rangle, \langle m_9: [.2, .3]; [.5, .8]; [.7, .8]\rangle \end{Bmatrix}$$

$$NVUA(P^*) = \begin{Bmatrix} \langle m_1: [.6, .9]; [.1, .2]; [.1, .4]\rangle, \langle m_2: [.6, .7]; [.3, .5]; [.3, .4]\rangle, \\ \langle m_4: [.6, .9]; [.1, .2]; [.1, .4]\rangle, \langle m_6: [.6, .7]; [.4, .5]; [.3, .4]\rangle, \\ \langle m_8: [.7, .9]; [.2, .3]; [.1, .3]\rangle, \langle m_9: [.4, .7]; [.5, .7]; [.3, .6]\rangle \end{Bmatrix}$$

$$NVB(P^*) = \begin{Bmatrix} \langle m_1: [.2, .5]; [.6, .8]; [.5, .8]\rangle, \langle m_2: [.3, .4]; [.5, .7]; [.6, .7]\rangle, \\ \langle m_4: [.2, .5]; [.6, .8]; [.5, .8]\rangle, \langle m_6: [.3, .4]; [.5, .6]; [.6, .7]\rangle, \\ \langle m_8: [.5, .9]; [.4, .6]; [.1, .5]\rangle, \langle m_9: [.4, .7]; [.5, .7]; [.3, .6]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(P^*) = \{0_{NV}, 1_{NV}, NVLA(P^*), NVUA(P^*), NVB(P^*)\} \neq (1)$

Step 7: Let $P^* = \{m_1, m_2, m_4, m_6, m_8, m_9\}$, when the attribute "Smoking" is removed from S, we have $V/_{\mathcal{R}^*(S-SM)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(P^*) = \begin{Bmatrix} \langle m_1: [.5, .8]; [.2, .4]; [.2, .5]\rangle, \langle m_2: [.6, .7]; [.3, .5]; [.3, .4]\rangle, \\ \langle m_4: [.5, .8]; [.2, .4]; [.2, .5]\rangle, \langle m_6: [.6, .7]; [.4, .5]; [.3, .4]\rangle, \\ \langle m_8: [.7, .9]; [.2, .3]; [.1, .3]\rangle, \langle m_9: [.2, .3]; [.5, .8]; [.7, .8]\rangle \end{Bmatrix}$$

$$NVUA(P^*) = \begin{Bmatrix} \langle m_1: [.6, .9]; [.1, .2]; [.1, .4]\rangle, \langle m_2: [.6, .7]; [.3, .5]; [.3, .4]\rangle, \\ \langle m_4: [.6, .9]; [.1, .2]; [.1, .4]\rangle, \langle m_6: [.6, .7]; [.4, .5]; [.3, .4]\rangle, \\ \langle m_8: [.7, .9]; [.2, .3]; [.1, .3]\rangle, \langle m_9: [.4, .7]; [.5, .7]; [.3, .6]\rangle \end{Bmatrix}$$

$$NVB(P^*) = \left\{ \begin{array}{l} \langle m_1: [.2, .5]; [.6, .8]; [.5, .8] \rangle, \langle m_2: [.3, .4]; [.5, .7]; [.6, .7] \rangle, \\ \langle m_4: [.2, .5]; [.6, .8]; [.5, .8] \rangle, \langle m_6: [.3, .4]; [.5, .6]; [.6, .7] \rangle, \\ \langle m_8: [.1, .3]; [.7, .8]; [.7, .9] \rangle, \langle m_9: [.4, .7]; [.5, .7]; [.3, .6] \rangle \end{array} \right\}$$

Therefore, $\tau_S(P^*) = \{0_{NV}, 1_{NV}, NVLA(P^*), NVUA(P^*), NVB(P^*)\} =$ (1)

Step 8: Let $P^* = \{m_1, m_2, m_4, m_6, m_8, m_9\}$, when the attribute "Pain Killer Intake" is removed from S, we have

$V/_{\mathcal{R}^*(S-PK)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(P^*) = \left\{ \begin{array}{l} \langle m_1: [.5, .8]; [.2, .4]; [.2, .5] \rangle, \langle m_2: [.6, .7]; [.3, .5]; [.3, .4] \rangle, \\ \langle m_4: [.5, .8]; [.2, .4]; [.2, .5] \rangle, \langle m_6: [.6, .7]; [.4, .5]; [.3, .4] \rangle, \\ \langle m_8: [.7, .9]; [.2, .3]; [.1, .3] \rangle, \langle m_9: [.2, .3]; [.5, .8]; [.7, .8] \rangle \end{array} \right\}$$

$$NVUA(P^*) = \left\{ \begin{array}{l} \langle m_1: [.6, .9]; [.1, .2]; [.1, .4] \rangle, \langle m_2: [.6, .7]; [.3, .5]; [.3, .4] \rangle, \\ \langle m_4: [.6, .9]; [.1, .2]; [.1, .4] \rangle, \langle m_6: [.6, .7]; [.4, .5]; [.3, .4] \rangle, \\ \langle m_8: [.7, .9]; [.2, .3]; [.1, .3] \rangle, \langle m_9: [.4, .7]; [.5, .7]; [.3, .6] \rangle \end{array} \right\}$$

$$NVB(P^*) = \left\{ \begin{array}{l} \langle m_1: [.2, .5]; [.6, .8]; [.5, .8] \rangle, \langle m_2: [.3, .4]; [.5, .7]; [.6, .7] \rangle, \\ \langle m_4: [.2, .5]; [.6, .8]; [.5, .8] \rangle, \langle m_6: [.3, .4]; [.5, .6]; [.6, .7] \rangle, \\ \langle m_8: [.1, .3]; [.7, .8]; [.7, .9] \rangle, \langle m_9: [.4, .7]; [.5, .7]; [.3, .6] \rangle \end{array} \right\}$$

Therefore, $\tau_S(P^*) = \{0_{NV}, 1_{NV}, NVLA(P^*), NVUA(P^*), NVB(P^*)\} =$ (1)

Thus CORE(S)={Age, Blood Pressure, Family History}.

Case 2: (Patients not affected by Stroke)

Let $V = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{10}\}$ the set of patients.

Let $V/_{\mathcal{R}^*(S)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$ be an equivalence relation on V and $Q^* = \{m_3, m_5, m_7, m_{10}\}$ be the set of patients not affected by stroke. Then

$$NVLA(Q^*) = \left\{ \begin{array}{l} \langle m_3: [.2, .3]; [.5, .8]; [.7, .8] \rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9] \rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9] \rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9] \rangle \end{array} \right\}$$

$$NVUA(Q^*) = \left\{ \begin{array}{l} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6] \rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9] \rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9] \rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9] \rangle \end{array} \right\}$$

$$NVB(Q^*) = \left\{ \begin{array}{l} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6] \rangle, \langle m_5: [.1, .4]; [.4, .7]; [.6, .9] \rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9] \rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9] \rangle \end{array} \right\}$$

Therefore, $\tau_S(Q^*) = \{0_{NV}, 1_{NV}, NVLA(Q^*), NVUA(Q^*), NVB(Q^*)\}$ ----------- (2)

Step 1: Let $Q^* = \{m_3, m_5, m_7, m_{10}\}$, when the attribute "Age" is removed from S, we have

$V/_{\mathcal{R}^*(S-AG)} = \{\{m_1, m_2, m_4\}, \{m_3, m_7, m_9\}, \{m_5\}, \{m_6\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(Q^*) = \begin{Bmatrix} \langle m_3: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_5: [.1,.4]; [.3,.6]; [.6,.9]\rangle, \\ \langle m_7: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_{10}: [.1,.5]; [.4,.6]; [.5,.9]\rangle \end{Bmatrix}$$

$$NVUA(Q^*) = \begin{Bmatrix} \langle m_3: [.4,.7]; [.5,.7]; [.3,.6]\rangle, \langle m_5: [.1,.4]; [.3,.6]; [.6,.9]\rangle, \\ \langle m_7: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_{10}: [.1,.5]; [.4,.6]; [.5,.9]\rangle \end{Bmatrix}$$

$$NVB(Q^*) = \begin{Bmatrix} \langle m_3: [.4,.7]; [.5,.7]; [.3,.6]\rangle, \langle m_5: [.1,.4]; [.4,.7]; [.6,.9]\rangle, \\ \langle m_7: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_{10}: [.1,.5]; [.4,.6]; [.5,.9]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(Q^*) = \{0_{NV}, 1_{NV}, NVLA(Q^*), NVUA(Q^*), NVB(Q^*)\} \neq$ (2)

Step 2: Let $Q^* = \{m_3, m_5, m_7, m_{10}\}$, when the attribute "Sugar" is removed from S, we have

$V/_{\mathcal{R}^*(S-SU)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$.

Then

$$NVLA(Q^*) = \begin{Bmatrix} \langle m_3: [.2,.3]; [.5,.8]; [.7,.8]\rangle, \langle m_5: [.1,.4]; [.3,.6]; [.6,.9]\rangle, \\ \langle m_7: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_{10}: [.1,.5]; [.4,.6]; [.5,.9]\rangle \end{Bmatrix}$$

$$NVUA(Q^*) = \begin{Bmatrix} \langle m_3: [.4,.7]; [.5,.7]; [.3,.6]\rangle, \langle m_5: [.1,.4]; [.3,.6]; [.6,.9]\rangle, \\ \langle m_7: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_{10}: [.1,.5]; [.4,.6]; [.5,.9]\rangle \end{Bmatrix}$$

$$NVB(Q^*) = \begin{Bmatrix} \langle m_3: [.4,.7]; [.5,.7]; [.3,.6]\rangle, \langle m_5: [.1,.4]; [.4,.7]; [.6,.9]\rangle, \\ \langle m_7: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_{10}: [.1,.5]; [.4,.6]; [.5,.9]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(Q^*) = \{0_{NV}, 1_{NV}, NVLA(Q^*), NVUA(Q^*), NVB(Q^*)\} =$ (2)

Step 3: Let $Q^* = \{m_3, m_5, m_7, m_{10}\}$, when the attribute "Blood Pressure" is removed from S, we have $V/_{\mathcal{R}^*(S-BP)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9, m_{10}\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}\}$. Then

$$NVLA(Q^*) = \begin{Bmatrix} \langle m_3: [.1,.3]; [.5,.8]; [.7,.9]\rangle, \langle m_5: [.1,.4]; [.3,.6]; [.6,.9]\rangle, \\ \langle m_7: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_{10}: [.1,0.3]; [.5,.8]; [.7,.9]\rangle \end{Bmatrix}$$

$$NVUA(Q^*) = \begin{Bmatrix} \langle m_3: [.4,.7]; [.4,.6]; [.3,.6]\rangle, \langle m_5: [.1,.4]; [.3,.6]; [.6,.9]\rangle, \\ \langle m_7: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_{10}: [.4,.7]; [.4,.6]; [.3,.6]\rangle \end{Bmatrix}$$

$$NVB(Q^*) = \begin{Bmatrix} \langle m_3: [.4,.7]; [.4,.6]; [.3,.6]\rangle, \langle m_5: [.1,.4]; [.4,.7]; [.6,.9]\rangle, \\ \langle m_7: [.1,.2]; [.7,.9]; [.8,.9]\rangle, \langle m_{10}: [.4,.7]; [.4,.6]; [.3,.6]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(Q^*) = \{0_{NV}, 1_{NV}, NVLA(Q^*), NVUA(Q^*), NVB(Q^*)\} \neq$ (2)

Step 4: Let $Q^* = \{m_3, m_5, m_7, m_{10}\}$, when the attribute "Heart Disease" is removed from S, we have $V/_{\mathcal{R}^*(S-HD)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(Q^*) = \begin{Bmatrix} \langle m_3: [.2, .3]; [.5, .8]; [.7, .8]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

$$NVUA(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

$$NVB(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.4, .7]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(Q^*) = \{0_{NV}, 1_{NV}, NVLA(Q^*), NVUA(Q^*), NVB(Q^*)\} = (2)$

Step 5: Let $Q^* = \{m_3, m_5, m_7, m_{10}\}$, when the attribute "BMI" is removed from S, we have

$V/_{\mathcal{R}^*(S-BMI)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(Q^*) = \begin{Bmatrix} \langle m_3: [.2, .3]; [.5, .8]; [.7, .8]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

$$NVUA(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

$$NVB(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.4, .7]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(Q^*) = \{0_{NV}, 1_{NV}, NVLA(Q^*), NVUA(Q^*), NVB(Q^*)\} = (2)$

Step 6: Let $Q^* = \{m_3, m_5, m_7, m_{10}\}$, when the attribute "Family History" is removed from S, we have $V/_{\mathcal{R}^*(S-FH)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8, m_{10}\}\}$. Then

$$NVLA(Q^*) = \begin{Bmatrix} \langle m_3: [.2, .3]; [.5, .8]; [.7, .8]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

$$NVUA(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.7, .9]; [.2, .3]; [.1, .3]\rangle \end{Bmatrix}$$

$$NVB(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.4, .7]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.5, .9]; [.4, .6]; [.1, .5]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(Q^*) = \{0_{NV}, 1_{NV}, NVLA(Q^*), NVUA(Q^*), NVB(Q^*)\} \neq (2)$

Step 7: Let $Q^* = \{m_3, m_5, m_7, m_{10}\}$, when the attribute "Smoking" is removed from S, we have

$V/_{\mathcal{R}^*(S-SM)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$. Then

$$NVLA(Q^*) = \begin{Bmatrix} \langle m_3: [.2, .3]; [.5, .8]; [.7, .8]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

$$NVUA(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

$$NVB(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.4, .7]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(Q^*) = \{0_{NV}, 1_{NV}, NVLA(Q^*), NVUA(Q^*), NVB(Q^*)\} =$ (2)

Step 8: Let $Q^* = \{m_3, m_5, m_7, m_{10}\}$, when the attribute "Pain Killer Intake" is removed from S, we have $V/_{\mathcal{R}^*(S-PK)} = \{\{m_1, m_4\}, \{m_2\}, \{m_3, m_9\}, \{m_5\}, \{m_6\}, \{m_7\}, \{m_8\}, \{m_{10}\}\}$.

Then

$$NVLA(Q^*) = \begin{Bmatrix} \langle m_3: [.2, .3]; [.5, .8]; [.7, .8]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

$$NVUA(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.3, .6]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

$$NVB(Q^*) = \begin{Bmatrix} \langle m_3: [.4, .7]; [.5, .7]; [.3, .6]\rangle, \langle m_5: [.1, .4]; [.4, .7]; [.6, .9]\rangle, \\ \langle m_7: [.1, .2]; [.7, .9]; [.8, .9]\rangle, \langle m_{10}: [.1, .5]; [.4, .6]; [.5, .9]\rangle \end{Bmatrix}$$

Therefore, $\tau_S(Q^*) = \{0_{NV}, 1_{NV}, NVLA(Q^*), NVUA(Q^*), NVB(Q^*)\} =$ (2)

Thus CORE(S)={Age, Blood Pressure, Family History}.

Conclusion: By applying neutrosophic vague nano topology in the field of medical diagnosis to identify the key factor that is most important for the patient we conclude that, from the Core of both the cases, we observe that "Age", "Blood Pressure", and "Family History" are the vital factors for the Stroke. So patients with these factors must take proper frequent medical check and care to prevent themselves from stroke.

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