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# Decision-Making Approach Based on Correlation Coefficient with its Properties Under Interval-Valued Neutrosophic hypersoft set environment

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**Abstract:** The correlation coefficient between two variables plays an essential part in statistics. In addition, the preciseness in the assessment of correlation relies on information from the set of discourse. The data collected for various statistical studies is full of ambiguities. In this article, we investigated some fundamental concepts that strengthen the current research structure, such as soft sets, hypersoft sets, neutrosophic hypersoft set (NHSS), and interval-valued neutrosophic hypersoft set (IVNHSS). The IVNHSS is an extension of the interval-valued neutrosophic soft set. The main objective of this paper is to develop the concept of correlation and weighted correlation coefficients for IVNHSS. We also, discuss the desirable properties of correlation and weighted correlation coefficients under the IVNHSS environment in the following research. Also, develop a decision-making technique based on the proposed correlation coefficient. Through the developed methodology, a technique for solving decision-making concerns is planned. Moreover, an application of the projected methods is presented for the selection of a medical superintendent in a public hospital.

**Keywords:** Hypersoft set, NHSS, IVNHSS, correlation coefficient, weighted correlation coefficient

## 1. Introduction

Correlation plays a vital role in statistics and engineering; through correlation analysis, the joint relationship of two variables can be used to evaluate the interdependence of two variables. Although probabilistic methods have been applied to various practical engineering problems, there are still some obstacles to probabilistic strategies. For example, the probability of the process depends on the large amount of data collected, which is random. However, large complex systems have many fuzzy uncertainties, so it is difficult to obtain accurate probability events. Therefore, due to limited quantitative information, results based on probability theory do not always provide useful information for experts. In addition, in practical applications, sometimes there is not enough data to correctly process standard statistical data. Due to the aforementioned obstacles, results based on probability theory are not always available to experts. Therefore, probabilistic methods are usually insufficient to resolve such inherent uncertainties in the data. Many researchers in the world have proposed and suggested different methods to solve problems that contain uncertainty. First, Zadeh developed the concept of a fuzzy set (FS) [1] to solve those problems that contain uncertainty and ambiguity. It can be seen that in some cases, FS cannot solve this situation. To overcome such situations, Turksen [2] proposed the idea of interval-valued fuzzy sets (IVFS). In some cases, we must carefully consider membership as a non-member value in the proper representation of objects that cannot be processed by FS or IVFS under conditions of uncertainty. To overcome these difficulties, Atanasov proposed the idea of intuitionistic fuzzy sets (IFSs) [3]. The theory proposed by Atanassov only deals with insufficient data due to membership and non-membership values, but IFS cannot deal with incompatible and imprecise information.

Molodtsov [4] proposed a general mathematical tool to deal with uncertain, ambiguous, and undefined substances, called soft sets (SS). Maji et al. [5] Expanded the work of SS and developed some operations with properties. In [6], they also use SS theory to make decisions. Ali etc. [7] Modified the Maji method of SS and developed some new operations with its properties. By using different operators, they proved De Morgan's laws [8] under the SS environment. Cagman and Enginoglu [9] proposed the concept of soft matrices with operations and discussed their properties. They also introduced a decision-making method to solve problems that contain uncertainty. In [10], they modified the operation proposed by Molodtsov's SS. Maji et al. [11] proposed the concept of fuzzy soft set (FSS) by combining FS and SS. They also proposed an Intuitionistic Fuzzy Soft Set (IFSS) with basic operations and attributes [12]. Atanassov and Gargov [13] extended the theory of IFS and established a new concept called Interval Valued Intuitionistic Fuzzy Set (IVIFS). Zulqarnain et al. [14] utilized the intuitionistic fuzzy soft matrices for disease diagnosis. Yang et al. [15] proposed the concept of interval-valued fuzzy soft sets with operations (IVFSS) and proved some important results by combining IVFS and SS, and they also used the developed concepts for decision-making. Jiang et al. [16] proposed the concept of interval-valued intuitionistic fuzzy soft sets (IVIFSS) by extending IVIFS. They also proposed the necessity and possibility operations for IVIFSS with their properties. Zulqarnain and Saeed [17] developed some operations for interval-valued fuzzy soft matrix (IVFSM) and proposed a decision-making technique to solve the decision making problem. They also applied the IVFSM for decision making [18], a comparison among fuzzy soft matrices and IVFSM in [19]. Ma and Rani [20] constructed an algorithm based on IVIFSS and used the developed algorithm for decision-making. Zulqarnain et al. [21] developed the aggregation operators for IVIFSS. They also extended the TOPSIS technique under IVIFSS and utilized the presented approach to solving multi-attribute decision making problem. Zulqarnain et al. [22] utilized fuzzy TOPSIS to solve the multi-criteria decision-making (MCDM) problem.

Maji [23] offered the idea of a neutrosophic soft set (NSS) with necessary operations and properties. The idea of the possibility NSS was developed by Karaaslan [24] and introduced a possibility of neutrosophic soft decision-making method to solve those problems which contain uncertainty based on And-product. Broumi [25] developed the generalized NSS with some operations and properties and used the proposed concept for decision making. To solve MCDM problems with single-valued Neutrosophic numbers (SVNNs) presented by Deli and Subas in [26], they constructed the concept of cut sets of SVNNs. Based on the correlation of IFS, the term CC of SVNSs [27] was introduced. In [28] the idea of simplified NSs introduced with some operational laws and aggregation operators such as weighted arithmetic and weighted geometric average operators. They constructed an MCDM method on the base of proposed aggregation operators. Zulqarnain et al. [29] presented the generalized version of neutrosophic TOPSIS and utilized the considered technique to solve the MCDM problem. Hung and Wu [30] proposed the centroid method to calculate the CC of IFSs and extended the proposed method to IVIFS. Bustince and Burillo [31] introduced the correlation and CC of IVIFS and proved the decomposition theorems on the correlation of IVIFS. Hong [32] and Mitchell [33] also established the CC for IFSs and IVIFSs respectively. Garg and Arora introduced the correlation measures on IFSS and constructed the TOPSIS technique on developed correlation measures [34]. Huang and Guo [35] gave an improved CC on IFS with their properties, they also established the coefficient of IVIFS. Singh et al. [36] developed the one- and two- parametric generalization of CC on IFS and used the proposed technique in multi-attribute group decision-making problems. Zulqarnain et al. [37] proposed the aggregation operators for Pythagorean fuzzy soft sets and developed a decision-making approach to solving multi-criteria decision making problems. Sometimes experts considered the sub-attributes of the given attributes in the decision-making process. In such situations, all the above-discussed theories cannot provide any information to experts about sub-attributes of the given attributes.

To overcome the above-mentioned limitations Smarandche [38] extended the concept of soft sets to hypersoft sets (HSS) by replacing function F of one parameter to multi-parameter (sub-attributes) function defined on the cartesian product of n different attributes. The established HSS is more flexible than soft sets and more suitable for decision-making environments. He also presented the further extension of HSS, such as crisp HSS, fuzzy HSS, intuitionistic fuzzy HSS, neutrosophic HSS, and plithogenic HSS. Nowadays, the HSS theory and its extensions rapidly progress, many researchers developed different operators and properties based on HSS and its extensions [39-42]. Abdel-Basset et al. [43] plithogenic set theory was used to eliminate uncertainty and to evaluate the financial performance of the manufacturing industry. They then used the VIKOR and TOPSIS methods to determine the weight of the financial ratio using the AHP method to achieve this goal. Abdel-Basset et al. [44] presented an effective combination of plithogenic aggregate operations and quality feature deployment procedures. The advantage of this combination is to improve accuracy, as a result, summarizes the decision-makers. Zulqarnain et al. [45] extended the TOPSIS technique to an intuitionistic fuzzy hypersoft set and developed some aggregation operators under-considered environment. They also established a decision-making approach based on developed TOPSIS to solve the MADM problem.

Basset et al. [46] proposed the type 2 neutrosophic numbers with some operational laws. They also developed the aggregation operators for type 2 neutrosophic numbers and developed the decision-making technique based on developed operators to solve the MADM problem. Basset et al. [47] established the AHP and VIKOR methods for neutrosophic numbers and utilized them for supplier selection. Basset et al. [48] presented the robust ranking technique under a neutrosophic environment for the green supplier chain management. Basset et al. [49] presented a neutrosophic multi-criteria decision-making technique to aid the patient and physician to know if a patient is suffering from heart failure Samarandche's NHSS is unable to solve those problems where the truthness, indeterminacy, and falsity object of any sub-attribute is given in interval form. We know that generally, the values vary, for example, medical experts generate the report of any patient we can observe that the HP level of blood varies from 0-17.5, these values can not be handled by NHSS. To handle the above-discussed environment we need to develop IVNHSS. The developed IVNHSS competently deals with uncertain problems comparative to NHSS and other existing studies. The main objective of this research is to introduce CC and WCC for IVNHSS.

The following research is organized as follows: In Section 2, we review some basic definitions used in the following sequels, such as SS, NSS, NHSS, and IVNHSS, etc. Section 3, established the notions of CC and WCC under IVNHSS and discussed their desirable properties. An algorithm and decision-making method developed in section 4 is based on the proposed CC. We also used the established approach to solve decision making problems in an uncertain environment. Finally, the conclusion is made in section 5.

## 2. Preliminaries

In this section, we recollect some basic definitions which are helpful to build the structure of the following manuscript such as soft set, hypersoft set, and neutrosophic hypersoft set.

**Definition 2.1 [4]**

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathsf{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathsf{A})$ is called a soft set over $\mathcal{U}$ and its mapping is given as

$$\mathcal{F}: \mathsf{A} \rightarrow \mathcal{P}(\mathcal{U})$$

It is also defined as:

$$(\mathcal{F},\mathsf{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}): e \in \mathcal{E}, \mathcal{F}(e) = \emptyset \; if \; e \notin \mathsf{A}\}$$

**Definition 2.2 [38]**

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3,..., k_n\}$,(n ≥ 1) be a set of attributes and set $K_i$ a set of corresponding sub-attributes of $k_i$ respectively with $K_i \cap K_j$ = φ for $n \geq 1$ for each $i, j \epsilon \{1,2,3 \ldots n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} = \{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of multi-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta$, and $\gamma \in \mathbb{N}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}})$ is said to be HSS over $\mathcal{U}$ and its mapping is defined as

$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}} \rightarrow \mathcal{P}(\mathcal{U})$.

It is also defined as

$(\mathcal{F}, \ddot{\mathsf{A}}) = \{\breve{a}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a}): \breve{a} \in \ddot{\mathsf{A}}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{a}) \in \mathcal{P}(\mathcal{U})\}$

**Definition 2.3 [38]**

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3,..., k_n\}$,(n ≥ 1) be a set of attributes and set $K_i$ a set of corresponding sub-attributes of $k_i$ respectively with $K_i \cap K_j$ = φ for $n \geq 1$ for each $i, j \epsilon \{1,2,3 \ldots n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}} = \{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta$, and $\gamma \in \mathbb{N}$ and $NS^{\mathcal{U}}$ be a collection of all neutrosophic subsets over $\mathcal{U}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}})$ is said to be NHSS over $\mathcal{U}$ and its mapping is defined as

$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}} \rightarrow NS^{\mathcal{U}}$.

It is also defined as

$(\mathcal{F}, \ddot{\mathsf{A}}) = \{(\breve{a}, \mathcal{F}_{\ddot{\mathsf{A}}}(\breve{a})): \breve{a} \in \ddot{\mathsf{A}}, \mathcal{F}_{\ddot{\mathsf{A}}}(\breve{a}) \in NS^{\mathcal{U}}\}$, where $\mathcal{F}_{\ddot{\mathsf{A}}}(\breve{a}) = \{\langle \delta, \sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta), \gamma_{\mathcal{F}(\breve{a})}(\delta) \rangle: \delta \in \mathcal{U}\}$, where $\sigma_{\mathcal{F}(\breve{a})}(\delta)$, $\tau_{\mathcal{F}(\breve{a})}(\delta)$, and $\gamma_{\mathcal{F}(\breve{a})}(\delta)$ represent the truth, indeterminacy, and falsity grades of the attributes such as $\sigma_{\mathcal{F}(\breve{a})}(\delta), \tau_{\mathcal{F}(\breve{a})}(\delta), \gamma_{\mathcal{F}(\breve{a})}(\delta) \in [0,1]$, and $0 \leq \sigma_{\mathcal{F}(\breve{a})}(\delta) + \tau_{\mathcal{F}(\breve{a})}(\delta) + \gamma_{\mathcal{F}(\breve{a})}(\delta) \leq 3$.

**Example 2.4**

Consider the universe of discourse $\mathcal{U} = \{\delta_1, \delta_2\}$ and $\mathfrak{L} = \{\ell_1 = Teaching\ methdology, \ell_2 = Subjects, \ell_3 = Classes\}$ be a collection of attributes with following their corresponding attribute values are given as teaching methodology = $L_1 = \{a_{11} = project\ base, a_{12} = class\ discussion\}$, Subjects = $L_2 = \{a_{21} = Mathematics, a_{22} = Computer\ Science, a_{23} = Statistics\}$, and Classes = $L_3 = \{a_{31} = Masters, a_{32} = Doctorol\}$. Let $\ddot{\mathsf{A}} = L_1 \times L_2 \times L_3$ be a set of attributes

$\ddot{\mathsf{A}} = L_1 \times L_2 \times L_3 = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}, a_{23}\} \times \{a_{31}, a_{32}\}$

$$= \begin{Bmatrix} (a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), (a_{11}, a_{23}, a_{31}), (a_{11}, a_{23}, a_{32}), \\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32}), (a_{12}, a_{23}, a_{31}), (a_{12}, a_{23}, a_{32}), \end{Bmatrix}$$

$\ddot{\mathsf{A}} = \{\breve{a}_1, \breve{a}_2, \breve{a}_3, \breve{a}_4, \breve{a}_5, \breve{a}_6, \breve{a}_7, \breve{a}_8, \breve{a}_9, \breve{a}_{10}, \breve{a}_{11}, \breve{a}_{12}\}$

Then the NHSS over $\mathcal{U}$ is given as follows

$(\mathcal{F}, \ddot{\mathsf{A}})$ =

$$\begin{Bmatrix} (\breve{a}_1, (\delta_1, (.6, .3, .8)), (\delta_2, (.9, .3, .5))), (\breve{a}_2, (\delta_1, (.5, .2, .7)), (\delta_2, (.7, .1, .5))), (\breve{a}_3, (\delta_1, (.5, .2, .8)), (\delta_2, (.4, .3, .4))), \\ (\breve{a}_4, (\delta_1, (.2, .5, .6)), (\delta_2, (.5, .1, .6))), (\breve{a}_5, (\delta_1, (.8, .4, .3)), (\delta_2, (.2, .3, .5))), (\breve{a}_6, (\delta_1, (.9, .6, .4)), (\delta_2, (.7, .6, .8))), \\ (\breve{a}_7, (\delta_1, (.6..5, .3)), (\delta_2, (.4, .2, .8))), (\breve{a}_8, (\delta_1, (.8, .2, .5)), (\delta_2, (.6, .8, .4))), (\breve{a}_9, (\delta_1, (.7, .4, .9)), (\delta_2, (.7..3, .5))), \\ (\breve{a}_{10}, (\delta_1, (.8, .4, .6)), (\delta_2, (.7, .2, .9))), (\breve{a}_{11}, (\delta_1, (.8, .4, .5)), (\delta_2, (.4, .2, .5))), (\breve{a}_5, (\delta_1, (.7, .5, .8)), (\delta_2, (.7, .5, .9))) \end{Bmatrix}$$

**Definition 2.5 [42]**

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3,..., k_n\}$,(n ≥ 1) be a set of attributes and set $K_i$ a set of corresponding sub-attributes of $k_i$ respectively with $K_i \cap K_j = \varphi$ for $n \geq 1$ for each $i, j \in \{1,2,3 \ldots n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\text{A}} = \{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and $\alpha, \beta$, and $\gamma \in \mathbb{N}$ and $IVNS^{\mathcal{U}}$ be a collection of all interval-valued neutrosophic subsets over $\mathcal{U}$. Then the pair ($\mathcal{F}$, $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\text{A}}$) is said to be IVNHSS over $\mathcal{U}$ and its mapping is defined as

$\mathcal{F}$: $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\text{A}} \rightarrow IVNS^{\mathcal{U}}$.

It is also defined as

$(\mathcal{F}, \ddot{\text{A}}) = \{(\breve{a}_k, \mathcal{F}_{\ddot{\text{A}}}(\breve{a}_k)): \breve{a}_k \in \ddot{\text{A}}, \mathcal{F}_{\ddot{\text{A}}}(\breve{a}_k) \in NS^{\mathcal{U}}\}$, where $\mathcal{F}_{\ddot{\text{A}}}(\breve{a}) = \left\{\langle \delta, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta), \tau_{\mathcal{F}(\breve{a}_k)}(\delta), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta)\rangle : \delta \in \mathcal{U}\right\}$, where $\sigma_{\mathcal{F}(\breve{a}_k)}(\delta)$, $\tau_{\mathcal{F}(\breve{a}_k)}(\delta)$, and $\gamma_{\mathcal{F}(\breve{a}_k)}(\delta)$ represent the interval truth, indeterminacy, and falsity grades of the attributes such as $\sigma_{\mathcal{F}(\breve{a}_k)}(\delta) = \left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta), \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta)\right]$, $\tau_{\mathcal{F}(\breve{a}_k)}(\delta) = \left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta), \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta)\right]$, $\gamma_{\mathcal{F}(\breve{a}_k)}(\delta) = \left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta), \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta)\right]$, where $\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta), \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta)$, $\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta), \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta)$, $\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta), \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta) \subseteq [0, 1]$, and $0 \leq \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta) \leq 3$.

Simply an interval-valued neutrosophic hypersoft number (IVNHSN) can be expressed as $\boldsymbol{\mathcal{F}} = \left\{\left[\boldsymbol{\sigma}^{\ell}_{\mathcal{F}(\breve{a}_k)}(\boldsymbol{\delta}), \boldsymbol{\sigma}^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\boldsymbol{\delta})\right], \left[\boldsymbol{\tau}^{\ell}_{\mathcal{F}(\breve{a}_k)}(\boldsymbol{\delta}), \boldsymbol{\tau}^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\boldsymbol{\delta})\right], \left[\boldsymbol{\gamma}^{\ell}_{\mathcal{F}(\breve{a}_k)}(\boldsymbol{\delta}), \boldsymbol{\gamma}^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\boldsymbol{\delta})\right]\right\}$, where $0 \leq \boldsymbol{\sigma}^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\boldsymbol{\delta}) + \boldsymbol{\tau}^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\boldsymbol{\delta}) + \boldsymbol{\gamma}^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\boldsymbol{\delta}) \leq 3$.

## 3. Correlation Coefficient for Interval-Valued Neutrosophic Hypersoft Set

In this section, the concept of correlation coefficient and weighted correlation coefficient on NHSS has been proposed with some basic properties.

**Definition 3.1**

Let $(\mathcal{F}, \ddot{\text{A}}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G}, \ddot{\text{Ѫ}}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVNHSSs defined over a universe of discourse $\mathcal{U}$. Then, the informational interval neutrosophic energies of $(\mathcal{F}, \ddot{\text{A}})$ and $(\mathcal{G}, \ddot{\text{Ѫ}})$ can be described as follows:

$$\varsigma_{IVNHSS}(\mathcal{F}, \ddot{\text{A}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right) \quad (1)$$

$$\varsigma_{IVNHSS}(\mathcal{G}, \ddot{\text{Ѫ}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right). \quad (2)$$

**Definition 3.2**

Let $(\mathcal{F}, \ddot{\text{A}}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G}, \ddot{\text{Ѫ}}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVNHSSs defined over a universe of discourse $\mathcal{U}$. Then, the correlation measure between $(\mathcal{F}, \ddot{\text{A}})$ and $(\mathcal{G}, \ddot{\text{Ѫ}})$ can be described as follows:

$\boldsymbol{\mathcal{C}}_{\boldsymbol{IVNHSS}}((\boldsymbol{\mathcal{F}}, \ddot{\text{A}}), (\boldsymbol{\mathcal{G}}, \ddot{\text{Ѫ}})) =$

$$\sum_{k=1}^{m}\sum_{i=1}^{n}\begin{pmatrix}\boldsymbol{\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i) +} \\ \boldsymbol{\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)}\end{pmatrix} \tag{3}$$

**Proposition 3.3**

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVNHSSs and $\mathcal{C}_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}}))$ be a correlation between them, then the following properties hold.

1. $\mathcal{C}_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) = \varsigma_{IVNHSS}(\mathcal{F},\ddot{A})$
2. $\mathcal{C}_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) = \varsigma_{IVNHSS}(\mathcal{G},\ddot{\mathbb{A}})$

**Proof:** The proof is trivial.

**Definition 3.4**

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVNHSSs, then correlation coefficient between them given as $\delta_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}}))$ and expressed as follows:

$$\delta_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) = \frac{\mathcal{C}_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}}))}{\sqrt{\varsigma_{IVNHSS}(\mathcal{F},\ddot{A})} * \sqrt{\varsigma_{IVNHSS}(\mathcal{G},\ddot{\mathbb{A}})}} \tag{4}$$

$$\boldsymbol{\delta_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}}))} = \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\begin{pmatrix}\boldsymbol{\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i) +} \\ \boldsymbol{\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)}\end{pmatrix}}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\boldsymbol{\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)}\right)^2\right)}\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\boldsymbol{\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)}\right)^2 + \left(\boldsymbol{\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)}\right)^2\right)}} \tag{5}$$

**Proposition 3.5**

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i, \left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right], \left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVNHSSs, then CC satisfies the following properties

1. $0 \le \delta_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) \le 1$
2. $\delta_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) = \delta_{IVNHSS}((\mathcal{G},\ddot{\mathbb{A}}), (\mathcal{F},\ddot{A}))$
3. If $(\mathcal{F},\ddot{A}) = (\mathcal{G},\ddot{\mathbb{A}})$, that is $\forall\ i,\ k,\ \sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, then $\delta_{NHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) = 1$.

**Proof 1.** $\delta_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) \ge 0$ is trivial, here we only need to prove that $\delta_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) \le 1$.

From equation 3, we have

$$\delta_{IVNHSS}((\mathcal{F},\ddot{A}), (\mathcal{G},\ddot{\mathbb{A}})) = \sum_{k=1}^{m}\sum_{i=1}^{n}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\end{pmatrix}$$

$$= \sum_{k=1}^{m}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1) + \tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\end{pmatrix}$$

$$+$$

$$\sum_{k=1}^{m}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2) + \tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\end{pmatrix}$$

$$+ \\ \vdots \\ +$$

$$\sum_{k=1}^{m}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n) + \tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\end{pmatrix}$$

$\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$

$$=$$

$$\left\{\begin{matrix}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_1) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_1) + \tau^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \tau^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_1) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_1) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_1) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_1)\end{pmatrix} + \\ \begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_1) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_1) + \tau^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \tau^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_1) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_1) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_1) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_1)\end{pmatrix} \\ + \\ \vdots \\ + \\ \begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_1) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_1) + \tau^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \tau^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_1) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_1) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_1) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_1)\end{pmatrix}\end{matrix}\right\}$$

$$+$$

$$\left\{\begin{matrix}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_2) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_2) + \tau^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \tau^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_2) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_2) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_2) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_2)\end{pmatrix} + \\ \begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_2) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_2) + \tau^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \tau^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_2) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_2) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_2) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_2)\end{pmatrix} \\ + \\ \vdots \\ + \\ \begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_2) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_2) + \tau^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \tau^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_2) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_2) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_2) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_2)\end{pmatrix}\end{matrix}\right\}$$

$$+ \\ \vdots \\ +$$

$$\left\{\begin{matrix}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_n) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_n) + \tau^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \tau^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_n) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_n) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_n) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_n)\end{pmatrix} + \\ \begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_n) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_n) + \tau^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \tau^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_n) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_n) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_n) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_n)\end{pmatrix} \\ + \\ \vdots \\ + \\ \begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \sigma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_n) + \sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_n) + \tau^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \tau^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_n) + \tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_n) + \\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \gamma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_n) + \gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_n)\end{pmatrix}\end{matrix}\right\}$$

$$= \sum_{k=1}^{m}\left(\begin{array}{c}\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)+\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)\\ +\cdots+\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\end{array}\right)+$$
$$\sum_{k=1}^{m}\left(\begin{array}{c}\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)+\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)\\ +\cdots+\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\end{array}\right)$$
$$\sum_{k=1}^{m}\left(\begin{array}{c}\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)+\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)\\ +\cdots+\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\end{array}\right)$$

By using Cauchy-Schwarz inequality

$$\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda}))^2 \leq$$
$$\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\\ +\left(\left(\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\\ +\left(\left(\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\end{array}\right\}$$
$$\times\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\\ +\left(\left(\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\\ +\left(\left(\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\end{array}\right\}$$

$$\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda}))^2 \leq$$
$$\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)+\left(\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)+\left(\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)$$
$$\times\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)+\left(\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)+\left(\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)$$

$$\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda}))^2 \leq \varsigma_{IVNHSS}(\mathcal{F},\ddot{A}) \times \varsigma_{IVNHSS}(\mathcal{G},\ddot{\Lambda}).$$

Therefore, $\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda}))^2 \leq \varsigma_{IVNHSS}(\mathcal{F},\ddot{A}) \times \varsigma_{IVNHSS}(\mathcal{G},\ddot{\Lambda})$. Hence, by using definition 3.4, we have

$\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) \leq 1$. So, $0 \leq \delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) \leq 1$.

**Proof 2.** The proof is obvious.

**Proof 3.** From equation 5, we have

$$\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) = \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\end{array}\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)}}$$

As we know that

$\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$. We get

$\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) =$

$$\frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}}$$

$\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) = 1$

Thus, prove the required result.

**Definition 3.6**

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\dddot{A}) = \left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVNHSSs. Then, their correlation coefficient is given as $\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))$ and defined as follows:

$$\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) = \frac{\mathcal{C}_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))}{max\{\varsigma_{IVNHSS}(\mathcal{F},\ddot{A}),\varsigma_{IVNHSS}(\mathcal{G},\dddot{A})\}} \quad (6)$$

$\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) =$

$$\frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\end{pmatrix}}{max\left\{\begin{matrix}\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right),\\ \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\end{matrix}\right\}} \quad (7)$$

**Proposition 3.7**

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\dddot{A}) = \left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVNHSSs. Then, CC satisfies the following properties

1. $0 \le \delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) \le 1$
2. $\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) = \delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))$
3. If $(\mathcal{F},\ddot{A}) = (\mathcal{G},\dddot{A})$, that is $\forall\ i,\ k,\ \sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$. Then, $\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) = 1$.

**Proof 1.** $\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) \ge 0$ is trivial, here we only need to prove that $\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) \le 1$.

From equation 3, we have

$\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) =$

$$\sum_{k=1}^{m}\sum_{i=1}^{n}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\end{pmatrix}$$

$$=$$

$$\sum_{k=1}^{m}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\end{pmatrix}$$

$$+$$

$$\sum_{k=1}^{m}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\end{pmatrix}$$

$$+$$
$$\vdots$$
$$+$$

$$\sum_{k=1}^{m}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\end{pmatrix}$$

$$\delta^{1}_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))$$

$$=$$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_1)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_1)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_1)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_1)+\tau^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_1)*\tau^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_1)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_1)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_1)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_1)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_1)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_1)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_1)\end{array}\right)+\\ \left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_1)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_1)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_1)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_1)+\tau^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_1)*\tau^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_1)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_1)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_1)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_1)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_1)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_1)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_1)\end{array}\right)\\ +\\ \vdots\\ +\\ \left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_1)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_1)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_1)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_1)+\tau^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_1)*\tau^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_1)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_1)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_1)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_1)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_1)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_1)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_1)\end{array}\right)\end{array}\right\}$$

$$+$$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_2)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_2)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_2)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_2)+\tau^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_2)*\tau^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_2)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_2)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_2)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_2)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_2)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_2)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_2)\end{array}\right)+\\ \left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_2)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_2)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_2)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_2)+\tau^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_2)*\tau^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_2)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_2)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_2)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_2)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_2)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_2)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_2)\end{array}\right)\\ +\\ \vdots\\ +\\ \left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_2)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_2)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_2)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_2)+\tau^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_2)*\tau^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_2)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_2)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_2)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_2)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_2)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_2)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_2)\end{array}\right)\end{array}\right\}$$

$$+$$

$$\vdots$$

$$+$$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_n)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_n)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_n)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_n)+\tau^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_n)*\tau^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_n)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_n)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_n)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_1)}(\delta_n)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_1)}(\delta_n)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_1)}(\delta_n)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_1)}(\delta_n)\end{array}\right)+\\ \left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_n)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_n)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_n)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_n)+\tau^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_n)*\tau^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_n)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_n)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_n)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_2)}(\delta_n)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_2)}(\delta_n)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_2)}(\delta_n)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_2)}(\delta_n)\end{array}\right)\\ +\\ \vdots\\ +\\ \left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_n)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_n)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_n)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_n)+\tau^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_n)*\tau^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_n)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_n)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_n)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_m)}(\delta_n)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_m)}(\delta_n)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_m)}(\delta_n)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_m)}(\delta_n)\end{array}\right)\end{array}\right\}$$

$$=$$

$$\sum_{k=1}^{m}\left(\begin{array}{c}\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)+\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)\\ +\cdots+\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\end{array}\right)+$$

$$\sum_{k=1}^{m}\left(\begin{array}{c}\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)+\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)\\ +\cdots+\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\end{array}\right)$$

$$\sum_{k=1}^{m}\left(\begin{array}{c}\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)+\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)\\ +\cdots+\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\end{array}\right)$$

By using Cauchy-Schwarz inequality

$$\delta^{1}_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))^2 \le$$

$$\sum_{k=1}^{m}\left\{\begin{array}{l}\left(\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\\ +\left(\left(\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\\ +\left(\left(\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\end{array}\right\}$$

$$\times\sum_{k=1}^{m}\left\{\begin{array}{l}\left(\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\\ +\left(\left(\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\\ +\left(\left(\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2\right)+\left(\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2\right)+\cdots+\left(\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right)\right)\end{array}\right\}$$

$$\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))^2\le$$
$$\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)+\left(\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)+\left(\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)$$
$$\times\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)+\left(\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)+\left(\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)$$

$\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))^2\le\varsigma_{IVNHSS}(\mathcal{F},\ddot{A})\times\varsigma_{IVNHSS}(\mathcal{G},\ddot{\mathbb{A}})$.

Therefore, $\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))^2\le\varsigma_{IVNHSS}(\mathcal{F},\ddot{A})\times\varsigma_{IVNHSS}(\mathcal{G},\ddot{\mathbb{A}})$. Hence, by using definition 3.4, we have

$\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))\le 1$. So, $0\le\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))\le 1$.

**Proof 2.** The proof is obvious.

**Proof 3.** From equation 5, we have

$$\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))=\frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\end{array}\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)}}$$

As we know that

$\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)=\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$. We get

$$\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))=\frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}}$$

$\delta^1_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))=1$

Thus, prove the required result.

**Definition 3.8**

Let $(\mathcal{F},\ddot{A})=\left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right)\mid\delta_i\in\mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}})=\left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right)\mid\delta_i\in\mathcal{U}\right\}$ be two IVNHSSs. Then, their weighted correlation coefficient is given as $\delta_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$ and defined as follows:

$$\delta_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))=\frac{\mathcal{C}_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))}{\sqrt{\varsigma_{WIVNHSS}(\mathcal{G},\ddot{\mathbb{A}})}*\sqrt{\varsigma_{WIVNHSS}(\mathcal{G},\ddot{\mathbb{A}})}}\qquad(8)$$

$\delta_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) =$

$$\frac{\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\end{array}\right)\right)}{\sqrt{\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)}\sqrt{\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)}} \quad (9)$$

**Definition 3.9**

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVNHSSs. Then, their weighted correlation coefficient is given as $\delta^{1}_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$ and defined as follows:

$$\delta^{1}_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = \frac{\mathcal{C}_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))}{max\{\varsigma_{WIVNHSS}(\mathcal{F},\ddot{A}),\varsigma_{WIVNHSS}(\mathcal{G},\ddot{\mathbb{A}})\}} \quad (10)$$

$\delta^{1}_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) =$

$$\frac{\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\begin{array}{c}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\end{array}\right)\right)}{max\left\{\begin{array}{c}\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\left(\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)\right),\\ \sum_{k=1}^{m}\dot{\omega}_k\left(\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\left(\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)\right)\end{array}\right\}} \quad (11)$$

If we consider $\Omega = \{\frac{1}{m}, \frac{1}{m},\ldots, \frac{1}{m}\}$ and $\gamma = \{\frac{1}{n}, \frac{1}{n},\ldots, \frac{1}{n}\}$, then $\delta_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$ and $\delta^{1}_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$ are reduced to $\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$ and $\delta^{1}_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$ respectively.

**Proposition 3.10**

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i,\left[\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right],\left[\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i),\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right]\right) \mid \delta_i \in \mathcal{U}\right\}$ be two IVNHSSs, then WCC between satisfies the following properties

1. $0 \leq \delta_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) \leq 1$
2. $\delta_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = \delta_{WIVNHSS}((\mathcal{G},\ddot{\mathbb{A}}),(\mathcal{F},\ddot{A}))$
3. If $(\mathcal{F},\ddot{A}) = (\mathcal{G},\ddot{\mathbb{A}})$, that is $\forall\ i,\ k,\ \sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, then $\delta_{WIVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = 1$.

**Proof 1.** Similar to proposition 3.5.

## 4. Application of Correlation Coefficient for Decision Making Under IVNHSS Environment

In this section, we proposed the algorithm based on CC under IVNHSS and utilize the proposed approach for decision making in real-life problems.

### 4.1 Algorithm for Correlation Coefficient under IVNHSS

Step 1. Pick out the set containing sub-attributes of parameters.

Step 2. Construct the IVNHSS according to experts in form of IVNHSNs.

Step 3. Find the informational interval neutrosophic energies for IVNHSS.

Step 4. Calculate the correlation between IVNHSSs by using the following formula

$\boldsymbol{\mathcal{C}_{IVNHSS}}((\boldsymbol{\mathcal{F}},\ddot{A}),(\boldsymbol{\mathcal{G}},\ddot{\mathbb{A}})) =$

$$\sum_{k=1}^{m}\sum_{i=1}^{n}\begin{pmatrix}\sigma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\sigma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\\ \gamma^{\ell}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\ell}_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma^{\mathcal{U}}_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma^{\mathcal{U}}_{\mathcal{G}(\breve{a}_k)}(\delta_i)\end{pmatrix}$$

Step 5. Calculate the CC between IVNHSSs by using the following formula

$$\delta_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = \frac{\mathcal{C}_{IVNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))}{\sqrt{\varsigma_{IVNHSS}(\mathcal{F},\ddot{A})}*\sqrt{\varsigma_{IVNHSS}(\mathcal{G},\ddot{\mathbb{A}})}}$$

Step 6. Choose the alternative with a maximum value of CC.

Step 7. Analyze the ranking of the alternatives.

A flowchart of the above-presented algorithm can be seen in figure 1.

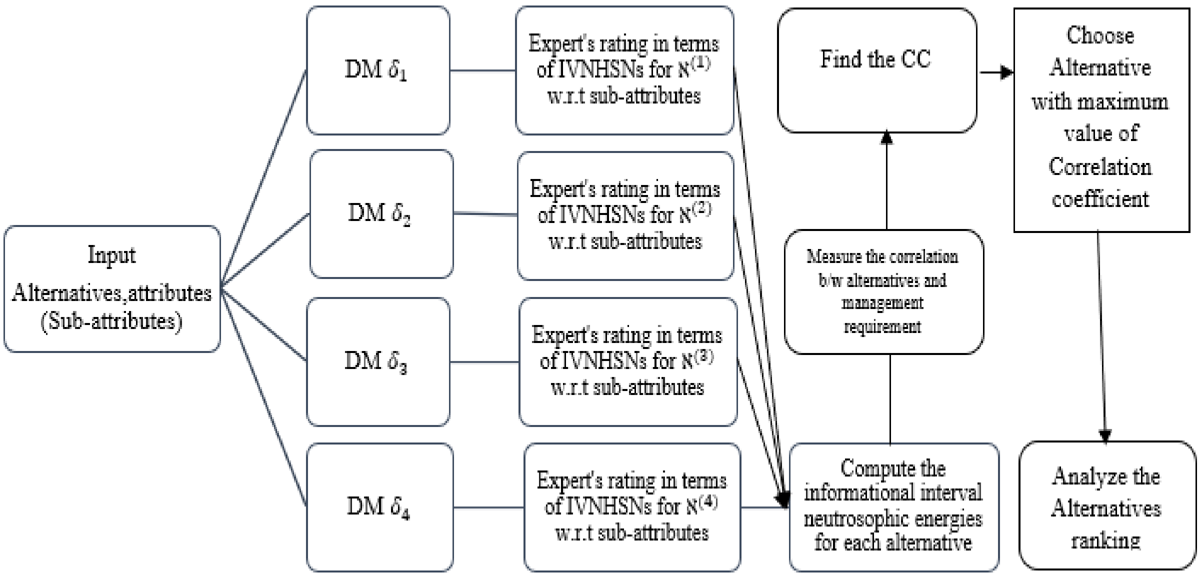


Figure 1: Flowchart for correlation coefficient under IVNHSS

### 4.1 Problem Formulation and Application of IVNHSS For Decision Making

Ministry of health advertises for the one vacant position of medical superintendent (MS) in hospital. Several medical experts apply for the post of MS, but referable probabilistic along with experience simply four experts are considered for further evaluation such as $\aleph = \{\aleph^1, \aleph^2, \aleph^3, \aleph^4\}$ be a set of alternatives. The secretary of the health department hires a committee of four decision-makers (DM) $\mathcal{U} = \{\delta_1, \delta_2, \delta_3, \delta_4\}$ for the selection of MS. The team of DM decides the criteria (attributes) for the selection of MS position such as $\mathfrak{L} = \{\ell_1 = Experiance, \ell_2 = Dealing\ skills, \ell_3 = Qualification\}$ be a collection of attributes and their corresponding sub-attribute are given as Experience = $\ell_1$ = $\{a_{11} = more\ than\ 20, a_{12} = less\ than\ 20\}$, Dealing skills = $\ell_2$ = $\{a_{21} = public\ dealing, a_{22} = Staff\ dealing\}$, and Qualification = $\ell_3$ = $\{a_{31} = Doctoral\ degree\ in\ medical\ education, a_{32} = Masters\ degree\ in\ medical\ education\}$. Let $\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3$ be a set of sub-attributes

$\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3 = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}\} \times \{a_{31}, a_{32}\}$

$= \begin{Bmatrix}(a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}),\\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32})\end{Bmatrix}$, $\mathfrak{L}' = \{\breve{a}_1, \breve{a}_2, \breve{a}_3, \breve{a}_4, \breve{a}_5, \breve{a}_6, \breve{a}_7, \breve{a}_8\}$ be a set of all multi sub-attributes. Each DM will evaluate the ratings of each alternative in the form of IVNHSNs under the considered multi sub-attributes. The developed method to find the best alternative is as follows.

#### 4.1.1. Application of IVNHSS For Decision Making

Assume $\aleph = \{\aleph^1, \aleph^2, \aleph^3, \aleph^4\}$ be a set of alternatives who are shortlisted for interview and $\mathfrak{L} = \{\ell_1 = Experiance, \ell_2 = Dealing\ skills, \ell_3 = Qualification\}$ be a set of parameters for the selection of MS. Experience = $\ell_1$ = $\{a_{11} = more\ than\ 20, a_{12} = \ less\ than\ 20\}$, Dealing skills = $\ell_2$ = $\{a_{21} = public\ dealing, a_{22} = \ Staff\ dealing\}$ , and Qualification = $\ell_3$ = $\{a_{31} = Doctoral\ degree\ in\ medical\ education, a_{32} = \ Masters\ degree\ in\ medical\ education\}$. Let $\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3$ be a set of sub-attributes. The health ministry defines a criterion for the selection of MS for all alternatives in terms of IVNHSNs given in Table 1.

**Table 1.** Decision Matrix of Concerning Department

| $\wp$ | $\breve{a}_1$ | $\breve{a}_2$ | $\breve{a}_3$ | $\breve{a}_4$ | $\breve{a}_5$ | $\breve{a}_6$ | $\breve{a}_7$ | $\breve{a}_8$ |
|---|---|---|---|---|---|---|---|---|
| $\delta_1$ | ([. 3,.5], [. 2,.4], [. 2,.6]) | ([. 2,.3], [. 5,.7], [. 1,.3]) | ([.5,.6], [.1,.3], [.4,.6]) | ([.2,.4], [.3,.5], [.3,.6]) | ([.2,.3], [.2,.4], [.4,.5]) | ([.4,.6], [.1,.3], [.2,.4]) | ([.6,.7], [.2,.3], [.3,.4]) | ([.4,.5], [.5,.8], [.1,.2]) |
| $\delta_2$ | ([.5,.6], [.1,.3], [.4,.6]) | ([.5,.7], [.1,.2], [.4,.6]) | ([.2,.4], [.3,.4], [.2,.5]) | ([.6,.8], [.1,.2], [.3,.5]) | ([.4,.6], [.4,.5], [.3,.5]) | ([.3,.5], [.4,.5], [.1,.3]) | ([.1,.2], [.5,.8], [.2,.4]) | ([.5,.7], [.1,.2], [.5,.6]) |
| $\delta_3$ | ([.2,.4], [.5,.6], [.4,.6]) | ([.2,.4], [.3,.4], [.2,.5]) | ([.4 .6], [.2,.3], [.1,.4]) | ([.2,.5], [.2,.3], [.1,.6]) | ([.3,.4], [.2,.5], [.5,.7]) | ([.3,.5], [.4,.5], [.1,.3]) | ([.2,.4], [.7,.8], [.1,.2]) | ([.1,.2], [.7,.8], [.2,.3]) |
| $\delta_4$ | ([.2,.3], [.5,.7], [.1,.3]) | ([.3,.4], [.2,.5], [.5,.7]) | ([.2,.4], [.3,.5], [.3,.6]) | ([.5,.7], [.1,.2], [.4,.6]) | ([.4,.6], [.1,.3], [.2,.4]) | ([.1,.2], [.5,.8], [.2,.4]) | ([.2,.4], [.3,.4], [.2,.5]) | ([.5,.6], [.1,.3], [.4,.6]) |

**Table 2.** Decision Matrix for Alternative $\aleph^{(1)}$

| $\aleph^{(1)}$ | $\breve{a}_1$ | $\breve{a}_2$ | $\breve{a}_3$ | $\breve{a}_4$ | $\breve{a}_5$ | $\breve{a}_6$ | $\breve{a}_7$ | $\breve{a}_8$ |
|---|---|---|---|---|---|---|---|---|
| $\delta_1$ | ([.2,.4], [.4,.5], [.3,.4]) | ([.3,.4], [.4,.5], [.2,.5]) | ([.3,.6], [.2,.3], [.1,.2]) | ([.2,.4], [.4,.6], [.1,.2]) | ([.1,.3], [.6,.7], [.2,.3]) | ([.4,.5], [.2,.5], [.2,.3]) | ([.6,.7], [.1,.2], [.2,.3]) | ([.4,.6], [.2,.3], [.4,.5]) |
| $\delta_2$ | ([.3,.4], [.2,.5], [.5,.7]) | ([.4,.7], [.1,.2], [.1,.2]) | ([.4,.5], [.2,.5], [.1,.2]) | ([.5,.7], [.1,.2], [.2,.4]) | ([.6,.8], [.1,.2], [.1,.5]) | ([.2,.4], [.7,.8], [.1,.2]) | ([.2,.4], [.3,.5], [.3,.6]) | ([.3,.4], [.4,.5], [.2,.4]) |
| $\delta_3$ | ([.5,.6], [.2,.3], [.4,.5]) | ([.5,.7], [.1,.2], [.2,.4]) | ([.7,.8], [.1,.2], [.2,.4]) | ([.1,.3], [.1,.5], [.2,.5]) | ([.1,.4], [.2,.4], [.1,.2]) | ([.2,.5], [.2,.4], [.3,.5]) | ([.3,.5], [.2,.4], [.4,.6]) | ([.5,.7], [.1,.2], [.5,.6]) |
| $\delta_4$ | ([.3,.5], [.3,.4], [.6,.7]) | ([.2,.4], [.3,.4], [.2,.5]) | ([.2,.4], [.7,.8], [.1,.2]) | ([.4,.7], [.1,.2], [.1,.2]) | ([.5,.6], [.2,.3], [.4,.5]) | ([.2,.4], [.3,.5], [.3,.6]) | ([.4,.6], [.2,.3], [.4,.5]) | ([.1,.3], [.1,.5], [.2,.5]) |

**Table 3.** Decision Matrix for Alternative $\aleph^{(2)}$

| $\aleph^{(2)}$ | $\breve{a}_1$ | $\breve{a}_2$ | $\breve{a}_3$ | $\breve{a}_4$ | $\breve{a}_5$ | $\breve{a}_6$ | $\breve{a}_7$ | $\breve{a}_8$ |
|---|---|---|---|---|---|---|---|---|
| $\delta_1$ | ([.2,.4], [.4,.6], [.4,.5]) | ([.2,.3], [.4,.6], [.3,.5]) | ([.1,.2], [.6,.8], [.2,.5]) | ([.4,.5], [.2,.5], [.1,.2]) | ([.2,.3], [.4,.6], [.3,.5]) | ([.1,.2], [.6,.8], [.2,.5]) | ([.7,.8], [.1,.2], [.2,.3]) | ([.1,.3], [.6,.7], [.2,.5]) |
| $\delta_2$ | ([.4,.5], [.2,.5], [.1,.2]) | ([.5,.7], [.1,.2], [.2,.4]) | ([.1,.3], [.6,.7], [.2,.6]) | ([.1,.4], [.2,.5], [.4,.6]) | ([.1,.4], [.2,.4], [.1,.2]) | ([.1,.2], [.2,.5], [.4,.6]) | ([.1,.4], [.2,.5], [.4,.6]) | ([.1,.4], [.2,.5], [.4,.6]) |
| $\delta_3$ | ([.3,.4], [.2,.6], [.4,.6]) | ([. 2,.4], [. 3,.4], [. 2,.5]) | ([.4,.5], [.2,.5], [.1,.2]) | ([.1,.2], [.2,.5], [.4,.6]) | ([.3,.5], [.3,.5], [.6,.7]) | ([.3,.5], [.3,.5], [.6,.7]) | ([.1,.2], [.2,.5], [.4,.6]) | ([.5,.7], [.1,.2], [.2,.4]) |
| $\delta_4$ | ([.2,.4], [.4,.5], [.6,.8]) | ([.3,.5], [.3,.5], [.6,.7]) | ([.1,.2], [.2,.5], [.4,.6]) | ([.1,.4], [.2,.4], [.1,.2]) | ([.4,.5], [.2,.5], [.1,.2]) | ([.1,.2], [.2,.5], [.4,.6]) | ([.4,.5], [.2,.5], [.1,.2]) | ([.1,.2], [.2,.5], [.4,.6]) |

**Table 4.** Decision Matrix for Alternative $\aleph^{(3)}$

| $\aleph^{(3)}$ | $\breve{a}_1$ | $\breve{a}_2$ | $\breve{a}_3$ | $\breve{a}_4$ | $\breve{a}_5$ | $\breve{a}_6$ | $\breve{a}_7$ | $\breve{a}_8$ |
|---|---|---|---|---|---|---|---|---|
| $\delta_1$ | ([.6,.7], [.1,.2], [.3,.5]) | ([.6,.8], [.1,.2], [.2,.3]) | ([.6,.7], [.3,.5], [.1,.2]) | ([.7,.8], [.1,.2], [.2,.5]) | ([.6,.7], [.1,.2], [.1,.2]) | ([.5,.8], [.1,.2], [.2,.4]) | ([.1,.3], [.6,.7], [.2,.5]) | ([.7,.8], [.1,.2], [.2,.3]) |
| $\delta_2$ | ([.5,.7], [.3,.4], [.2,.3]) | ([.5,.7], [.2,.5], [.2,.3]) | ([.5,.6], [.3,.4], [.1,.2]) | ([.7,.8], [.3,.5], [.1,.3]) | ([.1,.2], [.2,.5], [.4,.6]) | ([.1,.4], [.2,.5], [.4,.6]) | ([.4,.6], [.2,.3], [.1,.2]]) | ([.4,.6], [.2,.3], [.1,.2]) |
| $\delta_3$ | ([. 2,.4], [. 3,.4], [. 2,.5]) | ([.4,.7], [.2,.3], [.3,.7]) | ([.4,.6], [.2,.3], [.1,.2]) | ([.3,.5], [.3,.5], [.6,.7]) | ([.6,.8], [.1,.2], [.1,.2]) | ([.7,.8], [.1,.2], [.2,.4]) | ([.1,.2], [.2,.5], [.4,.6]) | ([.6,.8], [.1,.2], [.1,.3]) |
| $\delta_4$ | ([.6,.8], [.3,.4], [.1,.2]) | ([.5,.7], [.1,.2], [.4,.5]) | ([.1,.2], [.2,.5], [.4,.6]) | ([.5,.6], [.3,.4], [.1,.2]) | ([. 2,.4], [. 3,.4], [. 2,.5]) | ([.1,.3], [.6,.7], [.2,.5]) | ([.7,.8], [.1,.2], [.2,.5]) | ([.4,.6], [.2,.3], [.1,.2]]) |

**Table 5.** Decision Matrix for Alternative $\aleph^{(4)}$

| $\aleph^{(4)}$ | $\breve{a}_1$ | $\breve{a}_2$ | $\breve{a}_3$ | $\breve{a}_4$ | $\breve{a}_5$ | $\breve{a}_6$ | $\breve{a}_7$ | $\breve{a}_8$ |
|---|---|---|---|---|---|---|---|---|

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{\delta_1}$ | ([.3,.5],[.2,.4],[.1,.2]) | ([.3,.6],[.1,.2],[.4,.7]) | ([.4,.7],[.3,.4],[.2,.3]) | ([.7,.8],[.2,.4],[.3,.5]) | ([.5,.7],[.3,.4],[.2,.4]) | ([.4,.6],[.2,.5],[.3,.4]) | ([.2,.3],[.5,.7],[.2,.4]) | ([.5,.7],[.2,.4],[.3,.5]) |
| $\boldsymbol{\delta_2}$ | ([.4,.5],[.5,.7],[.2,.4]) | ([.4,.7],[.3,.5],[.2,.4]) | ([.5,.8],[.3,.4],[.2,.3]) | ([.2,.4],[.2,.3],[.4,.5]) | ([.3,.5],[.2,.3],[.3,.5]) | ([.2,.4],[.2,.3],[.3,.6]) | ([.5,.8],[.3,.6],[.2,.3]) | ([.4,.6],[.2,.3],[.1,.2]]) |
| $\boldsymbol{\delta_3}$ | ([. 2,.4],[. 3,.4],[. 2,.5]) | ([.4,.6],[.2,.3],[.3,.5]) | ([.3,.5],[.3,.5],[.1,.2]) | ([.3,.5],[.4,.6],[.6,.7]) | ([.5,.7],[.1,.2],[.4,.5]) | ([.4,.6],[.3,.5],[.1,.2]) | ([.6,.7],[.1,.2],[.3,.5]) | ([.2,.5],[.2,.3],[.4,.6]) |
| $\boldsymbol{\delta_4}$ | ([.1,.2],[.2,.5],[.4,.6]) | ([.5,.7],[.2,.4],[.1,.3]) | ([.3,.5],[.2,.5],[.1,.3]) | ([.4,.6],[.2,.5],[.3,.4]) | ([.5,.8],[.3,.4],[.2,.3]) | ([.4,.6],[.2,.3],[.1,.2]]) | ([.4,.7],[.3,.5],[.2,.4]) | ([. 2,.4],[. 3,.4],[. 2,.5]) |

By using Tables 1-5, compute the correlation coefficient between $\delta_{IVNHSS}(\wp, \aleph^{(1)})$, $\delta_{IVNHSS}(\wp, \aleph^{(2)})$, $\delta_{IVNHSS}(\wp, \aleph^{(3)})$, $\delta_{IVNHSS}(\wp, \aleph^{(4)})$ by using equation 5 given as follows:
$\delta_{IVNHSS}(\wp, \aleph^{(1)})$ = .99701, $\delta_{IVNHSS}(\wp, \aleph^{(2)})$ = .99822, $\delta_{IVNHSS}(\wp, \aleph^{(3)})$ = .99986, and $\delta_{IVNHSS}(\wp, \aleph^{(4)})$ = .99759. This shows that $\delta_{IVNHSS}(\wp, \aleph^{(3)}) > \delta_{IVNHSS}(\wp, \aleph^{(2)}) > \delta_{IVNHSS}(\wp, \aleph^{(4)}) > \delta_{IVNHSS}(\wp, \aleph^{(1)})$. It can be seen from this ranking alternative $\aleph^{(3)}$ is the most suitable alternative. Therefore $\aleph^{(3)}$ is the best alternative for the vacant position of associate professor, the ranking of other alternatives given as $\aleph^{(3)} > \aleph^{(2)} > \aleph^{(4)} > \aleph^{(1)}$. Graphical results of alternatives ratings can be seen in figure 2.

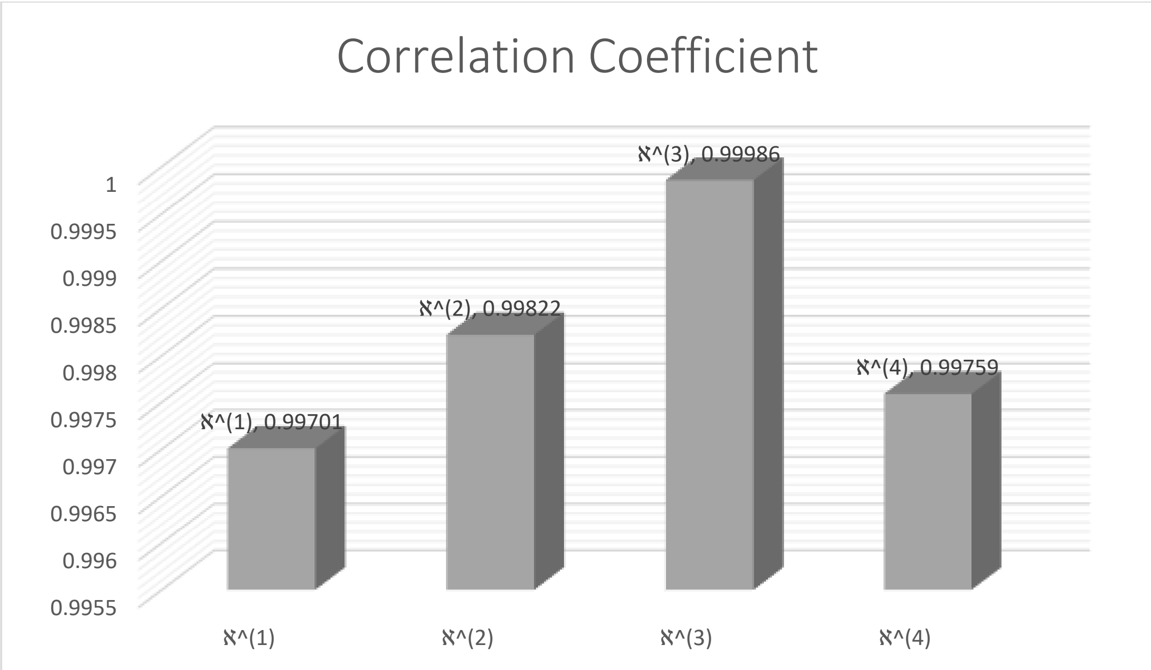


Figure 2: Alternatives rating based on correlation coefficient under IVNHSS

## 5. Conclusion

The interval-valued neutrosophic hypersoft set is a novel concept that is an extension of the interval-valued neutrosophic soft set. In this manuscript, we studied some basic concepts which were necessary to build the structure of the article. We introduced the correlation and weighted correlation coefficients under the IVNHSS environment. Some basic properties based on developed CC under IVNHSS were also introduced. A decision-making approach has been developed based on the established correlation coefficient and presented an algorithm under IVNHSS. Finally, a numerical illustration has been described to solve the decision-making problem by using the proposed technique. In the future, the correlation coefficient, the TOPSIS method based on correlation coefficient under IVNHSS can be presented. Future research will concentration on presenting numerous other operators under the IVNHSS environment to solve decision-making issues. Many other structures such as topological, algebraic, ordered structures, etc. can be developed and discussed under-considered environment.


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