Neutrosophic Sets and Systems, Vol. 40, 2021
University of New Mexico

Neutrosophic Hypersoft Topological Spaces

D. Ajay [1,*] , J. Joseline Charisma [2]

[1] Assistant Professor, Department of Mathematics, Sacred Heart College (Autonomous), Tamilnadu, India; dajaypravin@gmail.com

[2] Ph. D Research Scholar, Department of Mathematics, Sacred Heart College (Autonomous), Tamilnadu, India; joecharish@gmail.com

* Correspondence: dajaypravin@gmail.com

Abstract: Hypersoft sets have gained more importance as a generalization of soft sets and have been investigated for possible extensions in many fields of mathematics. The main objective of this paper is to introduce Fuzzy Hypersoft Topology and study some of its properties such as neighbourhood of fuzzy hypersoft set, interior hypersoft set and closure fuzzy hypersoft set. Fuzzy hypersoft topology is then extended to Intuitionistic Hypersoft topology, Neutrosophic Hypersoft topology and its basic properties are discussed.

Keywords: Fuzzy Hypersoft Set, Hypersoft Set, Fuzzy Hypersoft Topology, Intuitionistic Hypersoft Topology, Neutrosophic Hypersoft Topology, Interior, Closure.

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1. Introduction

Zadeh [1] in 1965 presented the idea of fuzzy set theory, which has a very important role in solving problems by providing a suitable way for the expression of vague concepts by having membership. Computer scientists and mathematicians have studied and developed fuzzy set theory with widened applications in fuzzy logic, fuzzy topology, fuzzy control systems, etc. Also theories such as fuzzy probability, soft and rough set theories are used to solve these problems. A new approach for handling uncertainty, the idea of soft theory was presented by Molodtsov [2] in 1999. Now, there is a rapid growth of soft theory with applications in many fields. Several basic notions of soft set theory were defined by Maji et al. [3] while his works were improved in [4-7]. A combination of fuzzy sets and soft sets, named as fuzzy soft set theory, was presented by Maji et al. [8].

The idea of soft sets was generalized into hypersoft sets by Smarandache [9] by transforming the argument function F into a multi-argument function. He also introduced many results on hypersoft sets. Saqlain et al. [10] utilized this notion and proposed a generalized TOPSIS method for decision making. Neutrosophic sets [17], from their very introduction, have seen many such extensions and have been very successful in applications [18-29, 48-50]. In 2019, Rana et al. [11] introduced Plithogenic Fuzzy Hypersoft Set (PFHS) in matrix form and defined some operations on PFHS. Single and multi-valued Neutrosophic Hypersoft set were proposed by Saqlain et al. [12], who also defined tangent similarity measure for single-valued sets and an application of the same in a decision making scenario. In an another effort, Saqlain et al. [13] also introduced aggregation operators for neutrosophic hypersoft sets. A recent development in this area of research is the introduction of basic operations on hypersoft sets in which hypersoft points in different fuzzy environments are also introduced [14].

Fuzzy topology, a collection of fuzzy sets fulfilling the axioms was defined by Chang [15] in 1968. Fuzzy set theory was applied into topology by Chang and many topological notions were introduced in fuzzy setting such as convergence and compactness [30-32]. Then Intuitionistic fuzzy topological spaces were introduced and were developed further into many new concepts as separation axioms, categorical property, connectedness [33, 34, 37-39]. Neutrosophic topological spaces were introduced by Salma et al. and further concepts as connectedness, semi closed sets and generalized closed sets were developed [40-44]. Olgun developed the concept of Pythagorean topological spaces and recently Pythagorean nano topological spaces were introduced and advanced into concepts such as weak open sets [35, 36, 45-47]. The notion of fuzzy soft topological structure was coined by Tanay et al. and was further enquired [16, 51, 52]. This notion was applied to the advanced sets as intuitionistic and neutrosophic soft sets thus developed as Intuitionistic and Neutrosophic soft topological spaces [53-59].

In this paper, we define the concept of 'Fuzzy Hypersoft Topology' with the fuzzy hypersoft sets and we define some basic notions. A logical extension of this topology would necessarily be Intuitionistic and Neutrosophic Hypersoft topologies. Hence we propose Intuitionisitc and Neutrosophic Hypersoft topology in this paper. Following this we describe the basic definitions and concepts in second section and the third section contains the introduction of the base fuzzy hypersoft topological spaces with few properties. Fourth and fifth sections contain the extension of fuzzy hypersoft topological spaces which are intuitionistic and neutrosophic hypersoft topological spaces along with basic properties.

2. Preliminaries

Definition 2.1

Let V be the universe, $P(V)$ the power set of V and $E_1, E_2, E_3 \dots E_m$ be the parameters which are pairwise disjoint. Let A_l be the non-empty subset of E_l for each $l = 1, 2, \dots m$. A hypersoft set is the pair $(\Theta, A_1 \times A_2 \times \dots \times A_m)$ where

$$\Theta : A_1 \times A_2 \times \dots \times A_m \to P(V).$$

Simply, we write the symbols $\mathbb{E}$ for $E_1 \times E_2 \times \dots \times E_m$, $\mathfrak{J}$ for $A_1 \times A_2 \times \dots \times A_m$ and $\boldsymbol{a}$ for an element of $\mathfrak{J}$.

Definition 2.2

Let the fuzzy universe be V, $\mathfrak{A}$ a subset of $\mathbb{E}$. Then $(\Theta, \mathfrak{J})$ is called

1. a null fuzzy hypersoft set if for each parameter $\boldsymbol{a} \in \mathfrak{J}$, $\Theta(\boldsymbol{a})$ is 0.
2. an absolute fuzzy hypersoft set if for each parameter $\boldsymbol{a} \in \mathfrak{J}$, $\Theta(\boldsymbol{a})$ is V.

Definition 2.3 [14]

Let $(\Theta, \mathfrak{J})$ and $(\vartheta, \mathfrak{B})$ be two fuzzy hypersoft (FH) sets over V. Then union of $(\Theta, \mathfrak{J})$ and $(\vartheta, \mathfrak{B})$ is $(\xi, \mathfrak{G}) = (\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})$ with $\mathfrak{G} = G_1 \times G_2 \times \dots \times G_n$ where $G_k = A_k \cup B_k$ for $k = 1, 2, \dots n$ and ξ is defined by

$$\xi(\boldsymbol{a}) = \begin{cases} \Theta(\boldsymbol{a}) & if\ \boldsymbol{a} \in \mathfrak{J} - \mathfrak{B} \\ \vartheta(\boldsymbol{a}) & if\ \boldsymbol{a} \in \mathfrak{B} - \mathfrak{J} \\ \Theta(\boldsymbol{a}) \cup \vartheta(\boldsymbol{a}) & if\ \boldsymbol{a} \in \mathfrak{J} \cup \mathfrak{B} \\ 0 & else \end{cases} \qquad \text{where } \boldsymbol{a} = (G_1, G_2, \dots G_n) \in \mathfrak{G}.$$

Definition 2.4 [14]

Let $(\Theta, \mathfrak{J})$ and $(\vartheta, \mathfrak{B})$ be two FH sets. The intersection is denoted by $(\xi, \mathfrak{G}) = (\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B})$ where $\mathfrak{G} = G_1 \times G_2 \times \dots \times G_n$ where $G_k = A_k \cap B_k$ for $k = 1, 2, \dots n$.

$$\xi(\boldsymbol{a}) = \begin{cases} \Theta(\boldsymbol{a}) & if\ \boldsymbol{a} \in \mathfrak{J} - \mathfrak{B} \\ \vartheta(\boldsymbol{a}) & if\ \boldsymbol{a} \in \mathfrak{B} - \mathfrak{J} \\ \Theta(\boldsymbol{a}) \cap \vartheta(\boldsymbol{a}) & if\ \boldsymbol{a} \in \mathfrak{J} \cap \mathfrak{B} \end{cases} \quad \text{where } \boldsymbol{a} = (G_1, G_2, \ldots G_n) \in \mathfrak{G}.$$

Definition 2.5

Let $(\Theta, \mathfrak{J})$ and $(\vartheta, \mathfrak{B})$ be two FH sets. $(\Theta, \mathfrak{J})$ is called a FH subset of $(\vartheta, \mathfrak{B})$, if $\mathfrak{J} \subseteq \mathfrak{B}$ & $\Theta(\boldsymbol{a}) \subseteq \vartheta(\boldsymbol{a})$ for all $\boldsymbol{a} \in \mathfrak{J}$. We denote this by $(\Theta, \mathfrak{J}) \subseteq (\vartheta, \mathfrak{B})$ [14].

Definition 2.6

Let $(\Theta, \mathfrak{J})$ and $(\vartheta, \mathfrak{B})$ be two FH sets. $(\Theta, \mathfrak{J})$ & $(\vartheta, \mathfrak{B})$ are equal if and only if $(\Theta, \mathfrak{J}) \subseteq (\vartheta, \mathfrak{B})$ and $(\vartheta, \mathfrak{B}) \subseteq (\Theta, \mathfrak{J})$ [14].

3. Fuzzy Hypersoft Topological Space

In this section, we define the concept of "Fuzzy Hypersoft Topology". Let $E_1, E_2, E_3 \ldots E_n$ be the parameters of the universe V, the set of all fuzzy sets be $F(V)$, the collection of all FH sets over $V_{\mathbb{E}}$ (where $\mathbb{E} = E_1 \times E_2 \times E_3 \ldots \times E_n$) be $\mathfrak{P}(V, \mathbb{E})$.

Definition 3.1

Let $(\varrho, \mathfrak{X})$ be an element of $\mathfrak{P}(V, \mathbb{E})$ (where $\mathfrak{X} = \mathrm{X}_1 \text{ x } \mathrm{X}_2 \text{ x } \mathrm{X}_3 \ldots \text{ x } \mathrm{X}_\mathrm{n}$ with each X_i is a subset of E_i $(i = 1, 2 \ldots n)$, set of all fuzzy hypersoft (FH) subsets of $(\varrho, \mathfrak{X})$ be $P(\varrho, \mathfrak{X})$ and τ, a subcollection of $P(\varrho, \mathfrak{X})$.

(i) $\phi_{\mathfrak{X}}, (\varrho, \mathfrak{X}) \in \tau$
(ii) $(\Theta, \mathfrak{J}), (\vartheta, \mathfrak{B}) \in \tau \Longrightarrow (\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B}) \in \tau$
(iii) $\{(\Theta, \mathfrak{J})_l \mid l \in L\} \in \tau \Longrightarrow \bigcup_{l \in L} (\Theta, \mathfrak{J})_l \in \tau$

If the above axioms are satisfied then τ is fuzzy hypersoft topology (FHT) on $(\varrho, \mathfrak{X})$. $(\mathfrak{X}_\varrho, \tau)$ is called a fuzzy hypersoft topological space (FHTS). Every member of τ is called open fuzzy hypersoft set (OFHS). A fuzzy hypersoft set if called closed fuzzy hypersoft set (CFHS) if its complement is OFHS.

For example, $\{\phi_{\mathfrak{X}}, (\varrho, \mathfrak{X})\}$ and $P(\varrho, \mathfrak{X})$ are fuzzy hypersoft topology on $(\varrho, \mathfrak{X})$ and are called as indiscrete FHT and discrete FHT respectively.

Example 3.2

Let $V = \{x_1, x_2, x_3, x_4\}$ and the attributes be $E_1 = \{a_1, a_2\}$, $E_2 = \{a_3, a_4\}$ and $E_3 = \{a_5, a_6\}$. Then the fuzzy hypersoft set be

$$\left\{ \begin{array}{c} \left((a_1, a_3, a_5), \left\{\frac{x_2}{0.4}, \frac{x_4}{0.6}\right\}\right), \left((a_1, a_3, a_6), \left\{\frac{x_1}{0.7}\right\}\right), \left((a_1, a_4, a_5), \left\{\frac{x_1}{0.4}, \frac{x_2}{0.3}\right\}\right), \left((a_1, a_4, a_6), \left\{\frac{x_1}{0.5}, \frac{x_3}{0.7}\right\}\right), \\ \left((a_2, a_3, a_5), \left\{\frac{x_2}{0.3}, \frac{x_3}{0.5}\right\}\right), \left((a_2, a_3, a_6), \left\{\frac{x_3}{0.8}\right\}\right), \left((a_2, a_4, a_5), \left\{\frac{x_4}{0.9}\right\}\right), \left((a_2, a_4, a_6), \left\{\frac{x_2}{0.6}\right\}\right) \end{array} \right\}$$

Let us consider this fuzzy hypersoft as $(\varrho, \mathfrak{X})$. Then the subfamily

$\tau = \{\phi_{\mathfrak{X}}, (\varrho, \mathfrak{X}),$

$$\left\{\left((a_1, a_3, a_5), \left\{\frac{x_1}{0.3}, \frac{x_2}{0.6}\right\}\right), \left((a_2, a_3, a_5), \left\{\frac{x_2}{0.4}, \frac{x_3}{0.5}\right\}\right)\right\}, \left\{\left((a_1, a_3, a_5), \left\{\frac{x_2}{0.4}\right\}\right), \left((a_2, a_3, a_5), \left\{\frac{x_2}{0.3}, \frac{x_3}{0.5}\right\}\right)\right\},$$

$$\left\{\begin{array}{c}\left((a_1,a_3,a_5),\left\{\frac{x_1}{0.3},\frac{x_2}{0.6},\frac{x_4}{0.6}\right\}\right),\left((a_1,a_4,a_6),\left\{\frac{x_1}{0.5},\frac{x_3}{0.7}\right\}\right),\left((a_1,a_3,a_6),\left\{\frac{x_1}{0.7}\right\}\right),\left((a_1,a_4,a_5),\left\{\frac{x_1}{0.4},\frac{x_2}{0.3}\right\}\right),\left((a_2,a_3,a_5),\left\{\frac{x_2}{0.4},\frac{x_3}{0.5}\right\}\right),\\ \left((a_2,a_3,a_6),\left\{\frac{x_3}{0.8}\right\}\right),\left((a_2,a_4,a_5),\left\{\frac{x_4}{0.9}\right\}\right),\left((a_2,a_4,a_6),\left\{\frac{x_2}{0.6}\right\}\right)\end{array}\right\}$$

of $P(\varrho,\mathfrak{X})$ is a FHT on $(\varrho,\mathfrak{X})$.

Definition 3.3

Let τ be a FHT on $(\varrho,\mathfrak{X})\in\mathfrak{P}(V,\mathbb{E})$ and $(\vartheta,\mathfrak{B})$ be a FH set in $P(\varrho,\mathfrak{X})$. A FH set $(\Theta,\mathfrak{J})$ in $P(\varrho,\mathfrak{X})$ is a neighbourhood of FH set of $(\vartheta,\mathfrak{B})$ if and only if there exists an OFHS $(\xi,\mathfrak{C})$ such that $(\vartheta,\mathfrak{B})\subset(\xi,\mathfrak{C})\subset(\Theta,\mathfrak{J})$.

Theorem 3.4

A FH set $(\Theta,\mathfrak{J})$ in $P(\varrho,\mathfrak{X})$ is an OFHS if and only if $(\Theta,\mathfrak{J})$ is a neighbourhood of each FH set $(\vartheta,\mathfrak{B})$ contained in $(\Theta,\mathfrak{J})$.

Proof:

Consider an OFHS $(\Theta,\mathfrak{J})$ and any FH set $(\vartheta,\mathfrak{B})$ confined in $(\Theta,\mathfrak{J})$. Thus we have $(\vartheta,\mathfrak{B})\subset(\Theta,\mathfrak{J})\subset(\Theta,\mathfrak{J})$. Implies that $(\theta,\mathfrak{A})$ is a neighbourhood of $(\vartheta,\mathfrak{B})$.

Let $(\theta,\mathfrak{A})$ be a neighbourhood of each FH set confined in it. Since $(\Theta,\mathfrak{J})\subset(\Theta,\mathfrak{J})$, there exists an OFHS $(\xi,\mathfrak{C})$ such that $(\Theta,\mathfrak{J})\subset(\xi,\mathfrak{C})\subset(\Theta,\mathfrak{J})$. Thus $(\Theta,\mathfrak{J})=(\xi,\mathfrak{C})$, $(\Theta,\mathfrak{J})$ is OFHS.

Definition 3.5

Let $(\mathfrak{X}_\varrho,\tau)$ is called a FHTS on $(\varrho,\mathfrak{X})$ and $(\Theta,\mathfrak{J})$ be a FH set in $P(\varrho,\mathfrak{X})$. The neighbourhood system of $(\Theta,\mathfrak{J})$ relative to τ is the collection of all neighbourhood of $(\Theta,\mathfrak{J})$ and denoted by $HN_{(\Theta,\mathfrak{J})}$.

Theorem 3.6

If $HN_{(\Theta,\mathfrak{J})}$ is the neighbourhood systems of FH set $(\Theta,\mathfrak{J})$. Then,
1. Finite intersection of member of $HN_{(\Theta,\mathfrak{J})}$ belongs to $HN_{(\Theta,\mathfrak{J})}$.
2. Each FH set which has a member of $HN_{(\Theta,\mathfrak{J})}$ belongs to $HN_{(\Theta,\mathfrak{J})}$.

Proof

1. $(\vartheta,\mathfrak{B})$ and $(\xi,\mathfrak{C})\in HN_{(\Theta,\mathfrak{J})}$ then there exists $(\vartheta',\mathfrak{B}'),(\xi',\mathfrak{C}')\in\tau$ such that
$$(\Theta,\mathfrak{J})\subset(\vartheta',\mathfrak{B}')\subset(\vartheta,\mathfrak{B})\text{ and }(\Theta,\mathfrak{J})\subset(\xi',\mathfrak{C}')\subset(\xi,\mathfrak{C}).$$
Since $(\vartheta',\mathfrak{B}')\cap(\xi',\mathfrak{C}')\in\tau$ we get $(\theta,\mathfrak{A})\subset(\vartheta',\mathfrak{B}')\cap(\xi',\mathfrak{C}')\subset(\vartheta,\mathfrak{B})\cap(\xi,\mathfrak{C})$
Hence $(\vartheta,\mathfrak{B})\cap(\xi,\mathfrak{C})$ belongs to $HN_{(\Theta,\mathfrak{J})}$.

2. Let $(\vartheta,\mathfrak{B})\in HN_{(\Theta,\mathfrak{J})}$ and $(\xi,\mathfrak{C})$ be a FH set having $(\vartheta,\mathfrak{B})$.
Since $(\vartheta,\mathfrak{B})\in HN_{(\Theta,\mathfrak{J})}$ there exists an OFHS containing $(\vartheta',\mathfrak{B}')$ such that $(\Theta,\mathfrak{J})\subset(\vartheta',\mathfrak{B}')\subset(\vartheta,\mathfrak{B})$ it follows that $(\Theta,\mathfrak{J})\subset(\vartheta',\mathfrak{B}')\subset(\xi,\mathfrak{C})$. Thus $(\xi,\mathfrak{C})$ belongs to $HN_{(\Theta,\mathfrak{J})}$.

Definition 3.7

Let $(\mathfrak{X}_\varrho,\tau)$ is called a FHTS and $(\Theta,\mathfrak{J}),(\vartheta,\mathfrak{B})$ be FH set in $P(\varrho,\mathfrak{X})$ such that $(\vartheta,\mathfrak{B})\subset(\Theta,\mathfrak{J})$. Then $(\vartheta,\mathfrak{B})$ is said to be an interior fuzzy hypersoft set (IFHS) of $(\Theta,\mathfrak{J})$ if and only if $(\Theta,\mathfrak{J})$ is a neighbourhood of $(\vartheta,\mathfrak{B})$.

The union of whole IFHS of $(\Theta,\mathfrak{J})$ is named the interior of $(\Theta,\mathfrak{J})$ and denoted as $(\Theta,\mathfrak{J})^\circ$.

Theorem 3.8

Let $(\mathfrak{X}_{\varrho}, \tau)$ is called a FHTS and $(\Theta, \mathfrak{J})$, a FH set in $P(\varrho, \mathfrak{X})$. Then,

i) $(\Theta, \mathfrak{J})^{\circ}$ is open and $(\Theta, \mathfrak{J})^{\circ}$ is the biggest OFHS confined in $(\Theta, \mathfrak{J})$.
ii) $(\Theta, \mathfrak{J})$ is OFHS iff $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^{\circ}$.

Proof

i) Since $(\Theta, \mathfrak{J})^{\circ} = \cup \{(\vartheta, \mathfrak{B})/(\Theta, \mathfrak{J})\ is\ a\ neighbourhood\ of\ (\vartheta, \mathfrak{B})\}$, $(\Theta, \mathfrak{J})^{\circ}$ is itself an IFHS of $(\Theta, \mathfrak{J})$. Then there exists an OFHS $(\xi, \mathfrak{C})$ such that $(\Theta, \mathfrak{J})^{\circ} \subset (\xi, \mathfrak{C}) \subset (\Theta, \mathfrak{J})$. $(\xi, \mathfrak{C})$ is an IFHS of $(\Theta, \mathfrak{J})$, hence $(\xi, \mathfrak{C}) \subset (\Theta, \mathfrak{J})^{\circ}$. Thus $(\Theta, \mathfrak{J})^{\circ}$ is the largest OFHS enclosed in $(\Theta, \mathfrak{J})$.

ii) Let $(\Theta, \mathfrak{J})$ be an OFHS. Since $(\Theta, \mathfrak{J})^{\circ}$ is the IFHS of $(\Theta, \mathfrak{J})$, we have $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^{\circ}$.

Conversely if $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^{\circ}$ then $(\Theta, \mathfrak{J})$ is OFHS.

Definition 3.9

Let $(\mathfrak{X}_{\varrho}, \tau_1)$ and $(\mathfrak{X}_{\varrho}, \tau_2)$ be two FHTS. If each $(\Theta, \mathfrak{J}) \in \tau_1$ is in τ_2 then τ_2 is called the FH finer than τ_1 (or) τ_1 is FH coarser than τ_2.

Definition 3.10

Let $(\mathfrak{X}_{\varrho}, \tau)$ be a FHTS and $(\Theta, \mathfrak{J}) \in \mathfrak{P}(V, \mathbb{E})$. The fuzzy hypersoft closure (FHC) of $(\Theta, \mathfrak{J})$ is the intersection of all CFH sets that contains $(\Theta, \mathfrak{J})$ which is denoted by $\overline{(\Theta, \mathfrak{J})}$.

Thus, $\overline{(\Theta, \mathfrak{J})}$ is the smallest CFHS which has $(\Theta, \mathfrak{J})$ and $\overline{(\Theta, \mathfrak{J})}$ is CFHS.

Theorem 3.11

Let $(\mathfrak{X}_{\varrho}, \tau)$ be a FHTS and $(\Theta, \mathfrak{J}), (\vartheta, \mathfrak{B}) \in \mathfrak{P}(V, \mathbb{E})$.
Then,

(i) $(\Theta, \mathfrak{J}) \subseteq \overline{(\Theta, \mathfrak{J})}$
(ii) $\overline{\overline{(\Theta, \mathfrak{J})}} = \overline{(\Theta, \mathfrak{J})}$
(iii) If $(\Theta, \mathfrak{J}) \subset (\vartheta, \mathfrak{B})$, then $\overline{(\Theta, \mathfrak{J})} \subset \overline{(\vartheta, \mathfrak{B})}$.
(iv) $(\Theta, \mathfrak{J})$ is a CFHS iff $(\Theta, \mathfrak{J}) = \overline{(\Theta, \mathfrak{J})}$.
(v) $\overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})} = \overline{(\Theta, \mathfrak{J})} \cup \overline{(\vartheta, \mathfrak{B})}$

Proof

From the definition of FHC, the proof of (i) to (iii) is attained.
(iv) Let $(\Theta, \mathfrak{J})$ be CFHS. By (i) $(\Theta, \mathfrak{J}) \subseteq \overline{(\Theta, \mathfrak{J})}$. Since $\overline{(\Theta, \mathfrak{J})}$ is the minutest CFHS which has $(\Theta, \mathfrak{J})$, then $\overline{(\Theta, \mathfrak{J})} \subseteq (\Theta, \mathfrak{J})$. Thus $(\Theta, \mathfrak{J}) = \overline{(\Theta, \mathfrak{J})}$.

Conversely let, $(\Theta, \mathfrak{J}) = \overline{(\Theta, \mathfrak{J})}$. Since $(\Theta, \mathfrak{J})$ is CFHS, then $(\Theta, \mathfrak{J})$ is also CFHS.

(v) By (iv) $\overline{(\Theta, \mathfrak{J})}$, $\overline{(\vartheta, \mathfrak{B})} \subseteq \overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})}$. So $\overline{(\Theta, \mathfrak{J})} \cup \overline{(\vartheta, \mathfrak{B})} \subseteq \overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})}$.

Conversely by (i), $(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B}) \subseteq \overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})}$.

Since $(\Theta, \mathfrak{J})$, $(\vartheta, \mathfrak{B})$ are FH sets and $\overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})}$ is the minutest CFHS which has $(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})$, then $\overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})} \subseteq \overline{(\Theta, \mathfrak{J})} \cup \overline{(\vartheta, \mathfrak{B})}$

Thus the equality is obtained.

Theorem 3.12

Let $(\mathfrak{X}_\varrho, \tau)$ be a FHTS and $(\Theta, \mathfrak{J}), (\vartheta, \mathfrak{B}) \in \mathfrak{P}(V, \mathbb{E})$.
Then,

(i) $(\Theta, \mathfrak{J})^\circ \subseteq (\Theta, \mathfrak{J})$
(ii) $((\Theta, \mathfrak{J})^\circ)^\circ = (\Theta, \mathfrak{J})^\circ$
(iii) If $(\Theta, \mathfrak{J}) \subseteq (\vartheta, \mathfrak{B})$, then $(\Theta, \mathfrak{J})^\circ \subseteq (\vartheta, \mathfrak{B})^\circ$.
(iv) $(\Theta, \mathfrak{J})$ is OFHS iff $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^\circ$.
(v) $\big((\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B})\big)^\circ = (\Theta, \mathfrak{J})^\circ \cap (\vartheta, \mathfrak{B})^\circ$.

Proof

(i) – (iii) are obvious from definition of interior

(iv) Let $(\Theta, \mathfrak{J})$ be a OFHS, by (i) $(\Theta, \mathfrak{J})^\circ \subseteq (\Theta, \mathfrak{J})$. Since $(\Theta, \mathfrak{J})^\circ$ is the largest OFHS that is contained in $(\Theta, \mathfrak{J})$, then $(\Theta, \mathfrak{J}) \subseteq (\Theta, \mathfrak{J})^\circ$. Thus $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^\circ$

Conversely, let $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^\circ$ since $(\Theta, \mathfrak{J})^\circ$ is OFHS, $(\Theta, \mathfrak{J})$ is also OFHS.

(v) $(\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B}) \subseteq (\Theta, \mathfrak{J}), (\vartheta, \mathfrak{B})$. Thus by (iii) $\big((\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B})\big)^\circ \subseteq (\Theta, \mathfrak{J})^\circ \cap (\vartheta, \mathfrak{B})^\circ$.

Conversely by (i), $(\Theta, \mathfrak{J})^\circ \cap (\vartheta, \mathfrak{B})^\circ \subseteq (\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B})$. Since $(\Theta, \mathfrak{J})^\circ$, $(\vartheta, \mathfrak{B})^\circ$ are OFHS & $\big((\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B})\big)^\circ$ is the largest OFHS that has $(\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B})$, then $(\Theta, \mathfrak{J})^\circ \cap (\vartheta, \mathfrak{B})^\circ \subseteq \big((\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B})\big)^\circ$. Thus, the equality is achieved.

Definition 3.13

Let $(\mathfrak{X}_\varrho, \tau)$ be a FHTS and $\mathcal{B}$ be a subcollection of τ. If each element of τ can be written as the arbitrary union of few elements of $\mathcal{B}$, then $\mathcal{B}$ is called a fuzzy hypersoft basis (FHB) for the FHT τ.

Lemma 3.14

Let $(\mathfrak{X}_\varrho, \tau)$ be a FHTS and $\mathcal{B}$ be FHB for τ. Then τ is the collection of FH union of elements of $\mathcal{B}$.

Lemma 3.15

Let $(\mathfrak{X}_\varrho, \tau)$ and $(\mathfrak{X}_\varrho, \tau')$ be FHTS and $\mathcal{B}, \mathcal{B}'$ be FHB for τ and τ' respectively. If $\mathcal{B}' \subset \mathcal{B}$, then τ is FH finer than τ'.

Lemma 3.16

Let $\{(\vartheta^i, \mathfrak{B}_i)/i \in \mathrm{I}\}$ be a collection of FH sets corresponding to V, and $(\Theta, \mathfrak{J})$ be a FH over V. Then

(i) $\bigcup_{i\in \mathrm{I}} [(\Theta, \mathfrak{J}) \cap (\vartheta^i, \mathfrak{B}_i)] = (\Theta, \mathfrak{J}) \cap (\bigcup_{i\in \mathrm{I}} (\vartheta^i, \mathfrak{B}_i))$
(ii) $\bigcap_{i\in \mathrm{I}} [(\Theta, \mathfrak{J}) \cup (\vartheta^i, \mathfrak{B}_i)] = (\Theta, \mathfrak{J}) \cup (\bigcap_{i\in \mathrm{I}} (\vartheta^i, \mathfrak{B}_i))$

Proof

(i) Let $(\Theta, \mathfrak{J}) \cap (\vartheta^i, \mathfrak{B}_i) = (\xi, \mathfrak{C})$ where $\mathfrak{C} = \mathfrak{J} \cap \mathfrak{B}_i$. Then $\bigcup_{i\in \mathrm{I}} [(\Theta, \mathfrak{J}) \cap (\vartheta^i, \mathfrak{B}_i)] = (\xi', \mathfrak{C}')$ where $\mathfrak{C}' = \bigcup_{i\in \mathrm{I}} (\mathfrak{J} \cap \mathfrak{B}_i)$. let $\bigcup_{i\in \mathrm{I}} (\vartheta^i, \mathfrak{B}_i) = (\xi'', \mathfrak{C}'')$ where $\mathfrak{C}'' = \bigcup_{i\in \mathrm{I}} \mathfrak{B}_i$. Then $(\Theta, \mathfrak{J}) \cap \bigcup_{i\in \mathrm{I}} (\vartheta^i, \mathfrak{B}_i) = (\xi''', \mathfrak{C}''')$ where $\mathfrak{C}''' = (\mathfrak{J} \cap \mathfrak{C}'')$. Since $\mathfrak{J} \cap (\bigcup_{i\in \mathrm{I}} \mathfrak{B}) = \bigcup_{i\in \mathrm{I}} (\mathfrak{J} \cap \mathfrak{B}_i)$, we have

$$\mathfrak{C}' = \bigcup_{i\in \mathrm{I}} (\mathfrak{J} \cap \mathfrak{B}_i) \text{ and } \mathfrak{C}''' = \mathfrak{J} \cap (\bigcup_{i\in \mathrm{I}} \mathfrak{B}_i) = \bigcup_{i\in \mathrm{I}} (\mathfrak{J} \cap \mathfrak{B}_i).$$

Thus $(\Theta, \mathfrak{J}) \cap (\cup_{i\in I} (\vartheta^i, \mathfrak{B}_i)) = \cup_{i\in I} (\Theta, \mathfrak{J}) \cap (\vartheta^i, \mathfrak{B}_i)$.

(ii) Let $\cap_{i\in I} (\vartheta^i, \mathfrak{B}_i) = (\eta, \mathfrak{D})$ where $\mathfrak{D} = \cap_{i\in I} \mathfrak{B}_i$. Thus $(\Theta, \mathfrak{J}) \cup (\cap_{i\in I} (\vartheta^i, \mathfrak{B}_i)) = (\eta', \mathfrak{D}')$ where $\mathfrak{D}' = (\mathfrak{J} \cup \mathfrak{D})$. Now consider $(\Theta, \mathfrak{J}) \cup (\vartheta^i, \mathfrak{B}_i) = (\eta'', \mathfrak{D}'')$, where $\mathfrak{D}'' = (\mathfrak{J} \cup \mathfrak{B}_i)$. Then $\cap_{i\in I} ((\Theta, \mathfrak{J}) \cup (\vartheta^i, \mathfrak{B}_i)) = (\eta''', \mathfrak{D}''')$ where $\mathfrak{D}''' = \cap_{i\in I} (\mathfrak{D}'')$. Since, $\mathfrak{J} \cup (\cap_{i\in I} \mathfrak{B}) = \cap_{i\in I} (\mathfrak{J} \cup \mathfrak{B})$, we get

$$\mathfrak{D}' = (\mathfrak{J} \cup \mathfrak{D}) = \mathfrak{J} \cup (\cap_{i\in I} \mathfrak{B}_i) = \cap_{i\in I} (\mathfrak{J} \cup \mathfrak{B}_i) \text{ and } \mathfrak{D}''' = \cap_{i\in I} (\mathfrak{D}'') = \cap_{i\in I} (\mathfrak{J} \cup \mathfrak{B}).$$

Thus, $(\Theta, \mathfrak{J}) \cup (\cap_{i\in I} (\vartheta^i, \mathfrak{B}_i)) = \cap_{i\in I} ((\Theta, \mathfrak{J}) \cup (\vartheta^i, \mathfrak{B}_i))$.

Theorem 3.17

Let $(\mathfrak{X}_\varrho, \tau)$ be a FHTS and $(\Theta, \mathfrak{J}) \epsilon P(\varrho, \mathfrak{X})$ then the collection $\tau_{(\Theta,\mathfrak{J})} = \{(\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B})/(\vartheta, \mathfrak{B}) \in \tau\}$ is a FHT.

Proof

(i) Since $\phi_{\mathfrak{X}}, (\varrho, \mathfrak{X}) \in \tau$, $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J}) \cap (\varrho, \mathfrak{X})$ and $\phi_{\mathfrak{A}} = (\Theta, \mathfrak{J}) \cap \phi_{\mathfrak{X}}$, then $\phi_{\mathfrak{A}}, (\Theta, \mathfrak{J}) \in \tau_{(\Theta,\mathfrak{J})}$.

(ii) Consider $(\Theta_1, \mathfrak{J}_1), (\Theta_2\, \mathfrak{J}_2) \in \tau_{(\Theta,\mathfrak{J})}$. Then there exists $(\vartheta_i, \mathfrak{B}_i) \in \tau_{(\Theta,\mathfrak{J})}$ for each $i = 1{,}2$ such that $(\Theta_i, \mathfrak{J}_i) = (\Theta, \mathfrak{J}) \cap (\vartheta_i, \mathfrak{B}_i)$.

Thus, $(\Theta_1, \mathfrak{J}_1) \cap (\Theta_2, \mathfrak{J}_2) = [(\Theta, \mathfrak{J}) \cap (\vartheta_1, \mathfrak{B}_1)] \cap [(\Theta, \mathfrak{J}) \cap (\vartheta_2, \mathfrak{B}_2)]$
$= (\Theta, \mathfrak{J}) \cap [(\vartheta_1, \mathfrak{B}_1) \cap (\vartheta_2, \mathfrak{B}_2)]$

Since $[(\vartheta_1, \mathfrak{B}_1) \cap (\vartheta_2, \mathfrak{B}_2) \in \tau$, we have $(\Theta_1, \mathfrak{J}_1) \cap (\Theta_2, \mathfrak{J}_2) \in \tau_{(\Theta,\mathfrak{J})}$.

(iii) Let $\{(\vartheta, \mathfrak{B})_j / \mathrm{j} \in J\}$ be a subcollection of $\tau_{(\Theta,\mathfrak{J})}$. Then for each $j \in J$, there is a FH set $(\xi, \mathfrak{C})_j$ of τ such that $(\vartheta, \mathfrak{B})_j = (\Theta, \mathfrak{J}) \cap (\xi, \mathfrak{C})_j$.

Thus, $\cup_{j\in J} (\vartheta, \mathfrak{B})_j = \cup_{j\in J} ((\theta, \mathfrak{J}) \cap (\xi, \mathfrak{C})_j) = (\Theta, \mathfrak{J}) \cap (\cup_{j\in J} (\xi, \mathfrak{C})_j)$.

Since $\cup_{j\in J} (\xi, \mathfrak{C})_j \in \tau$, then $(\vartheta, \mathfrak{B})_j \in \tau_{(\Theta,\mathfrak{J})}$.

Definition 3.18

Let $(\mathfrak{X}_\varrho, \tau)$ be a FHTS and $(\Theta, \mathfrak{J}) \subset P(\varrho, \mathfrak{X})$. Then, the FHT $\tau_{(\Theta,\mathfrak{J})}$ as in Theorem 3.17 is called Fuzzy hypersoft subspace topology and $(\mathfrak{A}_\theta, \tau_{(\Theta,\mathfrak{J})})$ is called a fuzzy hypersoft subspace of $(\mathfrak{X}_\varrho, \tau)$.

4. Intuitionistic Hypersoft Topological Spaces

In this section, we define the concept of "Intuitionistic Hypersoft Topology". Let $E_1, E_2, E_3 \dots E_n$ be the parameters of the universe T, the set of all intuitionistic sets be $F(T)$, the collection of all intuitionistic hypersoft sets over $T_{\mathbb{E}}$ (where $\mathbb{E} = E_1 \times E_2 \times E_3 \dots \times E_n$) be $\mathfrak{P}(T, \mathbb{E})$.

Definition 4.1

Let $(\varrho, \mathfrak{H})$ be an element of $\mathfrak{P}(T, \mathbb{E})$ (where $\mathfrak{H} = \mathrm{H}_1 \times \mathrm{H} \times \mathrm{H}_3 \dots \times \mathrm{H}_\mathrm{n}$ with each Y_i is a subset of E_i $(i = 1, 2 \dots n)$, set of all intuitionistic hypersoft (IH) subsets of $(\varrho, \mathfrak{H})$ be $P(\varrho, \mathfrak{H})$ and τ, a subcollection of $P(\varrho, \mathfrak{H})$.

(i) $\phi_{\mathfrak{Y}}, (\varrho, \mathfrak{Y}) \in \tau$
(ii) $(\Theta, \mathfrak{J}), (\vartheta, \mathfrak{B}) \in \tau \Rightarrow (\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B}) \in \tau$
(iii) $\{(\Theta, \mathfrak{J})_l \mid l \in L\} \in \tau \Rightarrow \cup_{l\in L} (\Theta, \mathfrak{J})_l \in \tau$

If the above axioms are satisfied then τ is an intuitionistic hypersoft topology (IHT) on $(\varrho, \mathfrak{H})$ and $(\mathfrak{H}_\varrho, \tau)$ is called an intuitionistic hypersoft topological space (IHTS). Every member of τ is called an open intuitionistic hypersoft set (OIHS). An intuitionistic hypersoft set is called a closed intuitionistic hypersoft set (CIHS) if its complement is an OIHS.

Example 4.2

Let $T = \{y_1, y_2, y_3, y_4\}$ and the attributes be $E_1 = \{b_1, b_2\}$, $E_2 = \{b_3, b_4\}$ and $E_3 = \{b_5, b_6\}$. Then the intuitionitic hypersoft set be

$$\left\{\begin{array}{c}\left((b_1, b_3, b_5), \left\{\frac{y_2}{(0.4,0.3)}, \frac{y_4}{(0.6,0.2)}\right\}\right), \left((b_1, b_3, b_6), \left\{\frac{y_1}{(0.7,0.1)}\right\}\right), \left((b_1, b_4, b_5), \left\{\frac{y_1}{(0.4,0.4)}, \frac{y_2}{(0.3,0.2)}\right\}\right), \left((b_1, b_4, b_6), \left\{\frac{y_1}{(0.5,0.3)}, \frac{y_3}{(0.7,0.1)}\right\}\right), \\ \left((b_2, b_3, b_5), \left\{\frac{y_2}{(0.3,0.5)}, \frac{y_3}{(0.5,0.1)}\right\}\right), \left((b_2, b_3, b_6), \left\{\frac{y_3}{(0.8,01.)}\right\}\right), \left((b_2, b_4, b_5), \left\{\frac{y_4}{(0.9,0.1)}\right\}\right), \left((b_2, b_4, b_6), \left\{\frac{y_2}{(0.6,0.3)}\right\}\right)\end{array}\right\}$$

Let us consider this intuitionistic hypersoft set as $(\varrho, \mathfrak{H})$. Then the subfamily

$$\tau = \{\phi_{\mathfrak{H}}, (\varrho, \mathfrak{H}), \left\{\left((b_1, b_3, b_5), \left\{\frac{y_1}{(0.3,0.4)}, \frac{y_2}{(0.6,0.1)}\right\}\right), \left((b_2, b_3, b_5), \left\{\frac{y_2}{(0.4,0.3)}, \frac{y_3}{(0.5,0.3)}\right\}\right)\right\}, \{\left((b_1, b_3, b_5), \left\{\frac{y_2}{(0.4,0.3)}\right\}\right)$$
$$\left((b_2, b_3, b_5), \left\{\frac{y_2}{(0.3,0.5)}, \frac{y_3}{(0.5,0.3)}\right\}\right)\}, \{\left((b_1, b_3, b_5), \left\{\frac{y_1}{(0.3,0.4)}, \frac{y_2}{(0.6,0.1)}, \frac{y_4}{(0.6,0.2)}\right\}\right), \left((b_1, b_4, b_6), \left\{\frac{y_1}{(0.5,0.3)}, \frac{y_3}{(0.7,0.1)}\right\}\right),$$
$$\left((b_1, b_3, b_6), \left\{\frac{y_1}{(0.7,0.1)}\right\}\right), \left((b_1, b_4, b_5), \left\{\frac{y_1}{(0.4,0.4)}, \frac{y_2}{(0.3,0.2)}\right\}\right), \left((b_2, b_3, b_5), \left\{\frac{y_2}{(0.4,0.3)}, \frac{y_3}{(0.5,0.1)}\right\}\right), \left((b_2, b_3, b_6), \left\{\frac{y_3}{(0.8,0.1)}\right\}\right),$$
$$\left((b_2, b_4, b_5), \left\{\frac{y_4}{(0.9,0.1)}\right\}\right), \left((b_2, b_4, b_6), \left\{\frac{y_2}{(0.6,0.3)}\right\}\right)\}\}$$

of $P(\varrho, \mathfrak{H})$ is a IHT on $(\varrho, \mathfrak{H})$.

Definition 4.3

Let τ be an IHT on $(\varrho, \mathfrak{H}) \in \mathfrak{P}(T, \mathbb{E})$ and $(\vartheta, \mathfrak{B})$ be an IH set in $P(\varrho, \mathfrak{H})$. An IH set $(\Theta, \mathfrak{J})$ in $P(\varrho, \mathfrak{H})$ is a neighbourhood of IH set of $(\vartheta, \mathfrak{B})$ if and only if there exists an OIHS $(\xi, \mathfrak{C})$ such that $(\vartheta, \mathfrak{B}) \subset (\xi, \mathfrak{C}) \subset (\Theta, \mathfrak{J})$.

Theorem 4.4

An IH set $(\Theta, \mathfrak{J})$ in $P(\varrho, \mathfrak{H})$ is an OIHS if and only if $(\Theta, \mathfrak{J})$ is a neighbourhood of each IH set $(\vartheta, \mathfrak{B})$ contained in $(\Theta, \mathfrak{J})$.

Proof:

Consider an OIHS $(\Theta, \mathfrak{J})$ and any IH set $(\vartheta, \mathfrak{B})$ confined in $(\Theta, \mathfrak{J})$. Thus we have $(\vartheta, \mathfrak{B}) \subset (\Theta, \mathfrak{J}) \subset (\Theta, \mathfrak{J})$. This implies that $(\theta, \mathfrak{A})$ is a neighbourhood of $(\vartheta, \mathfrak{B})$.

Let $(\theta, \mathfrak{A})$ be a neighbourhood of each IH set confined in it. Since $(\Theta, \mathfrak{J}) \subset (\Theta, \mathfrak{J})$, there exists an OIHS $(\xi, \mathfrak{C})$ such that $(\Theta, \mathfrak{J}) \subset (\xi, \mathfrak{C}) \subset (\Theta, \mathfrak{J})$. Thus $(\Theta, \mathfrak{J}) = (\xi, \mathfrak{C})$, $(\Theta, \mathfrak{J})$ is OIHS.

Definition 4.5

Let $(\mathfrak{H}_\varrho, \tau)$ be called an IHTS on $(\varrho, \mathfrak{H})$ and $(\Theta, \mathfrak{J})$ be an IH set in $P(\varrho, \mathfrak{H})$. The neighbourhood system of $(\Theta, \mathfrak{J})$ relative to τ is the collection of all neighbourhoods of $(\Theta, \mathfrak{J})$ and is denoted by $HNN_{(\Theta,\mathfrak{J})}$.

Theorem 4.6

If $HNN_{(\Theta,\mathfrak{J})}$ is the neighbourhood systems of IH set $(\Theta, \mathfrak{J})$. Then,

1. Finite intersection of member of $HNN_{(\Theta,\mathfrak{J})}$ belongs to $HNN_{(\Theta,\mathfrak{J})}$.
2. Each IH set which has a member of $HNN_{(\Theta,\mathfrak{J})}$ belongs to $HNN_{(\Theta,\mathfrak{J})}$.

Proof

1. $(\vartheta,\mathfrak{B})$ and $(\xi,\mathfrak{C}) \in HNN_{(\Theta,\mathfrak{J})}$ then there exists $(\vartheta',\mathfrak{B}'),(\xi',\mathfrak{C}') \in \tau$ such that

$$(\Theta,\mathfrak{J}) \subset (\vartheta',\mathfrak{B}') \subset (\vartheta,\mathfrak{B}) \text{ and } (\Theta,\mathfrak{J}) \subset (\xi',\mathfrak{C}') \subset (\xi,\mathfrak{C}).$$

Since $(\vartheta',\mathfrak{B}') \cap (\xi',\mathfrak{C}') \in \tau$ we get $(\theta,\mathfrak{A}) \subset (\vartheta',\mathfrak{B}') \cap (\xi',\mathfrak{C}') \subset (\vartheta,\mathfrak{B}) \cap (\xi,\mathfrak{C})$

Hence $(\vartheta,\mathfrak{B}) \cap (\xi,\mathfrak{C})$ belongs to $HNN_{(\Theta,\mathfrak{J})}$.

2. Let $(\vartheta,\mathfrak{B}) \in HNN_{(\Theta,\mathfrak{J})}$ and $(\xi,\mathfrak{C})$ be a IH set having $(\vartheta,\mathfrak{B})$.

Since $(\vartheta,\mathfrak{B}) \in HNN_{(\Theta,\mathfrak{J})}$ there exists an OIHS containing $(\vartheta',\mathfrak{B}')$ such that $(\Theta,\mathfrak{J}) \subset (\vartheta',\mathfrak{B}') \subset (\vartheta,\mathfrak{B})$ it follows that $(\Theta,\mathfrak{J}) \subset (\vartheta',\mathfrak{B}') \subset (\xi,\mathfrak{C})$. Thus $(\xi,\mathfrak{C})$ belongs to $HNN_{(\Theta,\mathfrak{J})}$.

Definition 4.7

Let $(\mathfrak{H}_\varrho,\tau)$ be an IHTS and $(\Theta,\mathfrak{J}),(\vartheta,\mathfrak{B})$ be an IH set in $P(\varrho,\mathfrak{H})$ such that $(\vartheta,\mathfrak{B}) \subset (\Theta,\mathfrak{J})$. Then $(\vartheta,\mathfrak{B})$ is said to be an interior intuitionistic hypersoft set (IIHS) of $(\Theta,\mathfrak{J})$ if and only if $(\Theta,\mathfrak{J})$ is a neighbourhood of $(\vartheta,\mathfrak{B})$.

The union of whole IIHS of $(\Theta,\mathfrak{J})$ is named the interior of $(\Theta,\mathfrak{J})$ and is denoted as $(\Theta,\mathfrak{J})^\circ$.

Theorem 4.8

Let $(\mathfrak{H}_\varrho,\tau)$ be an IHTS and $(\Theta,\mathfrak{J})$, an IH set in $P(\varrho,\mathfrak{H})$. Then,

i) $(\Theta,\mathfrak{J})^\circ$ is open and $(\Theta,\mathfrak{J})^\circ$ is the biggest OIHS confined in $(\Theta,\mathfrak{J})$.
ii) $(\Theta,\mathfrak{J})$ is OIHS iff $(\Theta,\mathfrak{J}) = (\Theta,\mathfrak{J})^\circ$.

Proof

i) Since $(\Theta,\mathfrak{J})^\circ = \bigcup\{(\vartheta,\mathfrak{B})/(\Theta,\mathfrak{J}) \text{ is a neighbourhood of } (\vartheta,\mathfrak{B})\}$, $(\Theta,\mathfrak{J})^\circ$ is itself an IIHS of $(\Theta,\mathfrak{J})$. Then there exists an OIHS $(\xi,\mathfrak{C})$ such that $(\Theta,\mathfrak{J})^\circ \subset (\xi,\mathfrak{C}) \subset (\Theta,\mathfrak{J})$. $(\xi,\mathfrak{C})$ is an IIHS of $(\Theta,\mathfrak{J})$, hence $(\xi,\mathfrak{C}) \subset (\Theta,\mathfrak{J})^\circ$. Thus $(\Theta,\mathfrak{J})^\circ$ is the largest OIHS enclosed in $(\Theta,\mathfrak{J})$.

ii) Let $(\Theta,\mathfrak{J})$ be an OIHS. Since $(\Theta,\mathfrak{J})^\circ$ is the IIHS of $(\Theta,\mathfrak{J})$, we have $(\Theta,\mathfrak{J}) = (\Theta,\mathfrak{J})^\circ$.

Conversely if $(\Theta,\mathfrak{J}) = (\Theta,\mathfrak{J})^\circ$ then $(\Theta,\mathfrak{J})$ is OIHS.

Definition 4.9

Let $(\mathfrak{H}_\varrho,\tau_1)$ and $(\mathfrak{H}_\varrho,\tau_2)$ be two IHTS. If each $(\Theta,\mathfrak{J}) \in \tau_1$ is in τ_2 then τ_2 is called the IH finer than τ_1 (or) τ_1 is IH coarser than τ_2.

Definition 4.10

Let $(\mathfrak{H}_\varrho,\tau)$ be a IHTS and $(\Theta,\mathfrak{J}) \in \mathfrak{P}(T,\mathbb{E})$. The intuitionistic hypersoft closure (IHC) of $(\Theta,\mathfrak{J})$ is the intersection of all CIH sets that contains $(\Theta,\mathfrak{J})$ which is denoted by $\overline{(\Theta,\mathfrak{J})}$.

Thus, $\overline{(\Theta,\mathfrak{J})}$ is the smallest CIHS which has $(\Theta,\mathfrak{J})$ and $\overline{(\Theta,\mathfrak{J})}$ is CIHS.

Theorem 4.11

Let $(\mathfrak{H}_\varrho,\tau)$ be an IHTS and $(\Theta,\mathfrak{J}),(\vartheta,\mathfrak{B}) \in \mathfrak{P}(T,\mathbb{E})$.

Then,

(i) $(\Theta,\mathfrak{J}) \subseteq \overline{(\Theta,\mathfrak{J})}$
(ii) $\overline{\overline{(\Theta,\mathfrak{J})}} = \overline{(\Theta,\mathfrak{J})}$
(iii) If $(\Theta,\mathfrak{J}) \subset (\vartheta,\mathfrak{B})$, then $\overline{(\Theta,\mathfrak{J})} \subset \overline{(\vartheta,\mathfrak{B})}$.
(iv) $(\Theta,\mathfrak{J})$ is a CIHS iff $(\Theta,\mathfrak{J}) = \overline{(\Theta,\mathfrak{J})}$.
(v) $\overline{(\Theta,\mathfrak{J}) \cup (\vartheta,\mathfrak{B})} = \overline{(\Theta,\mathfrak{J})} \cup \overline{(\vartheta,\mathfrak{B})}$

Proof

From the definition of IHC, the proof of (i) to (iii) is attained.
(iv) Let $(\Theta,\mathfrak{J})$ be CIHS. By (i) $(\Theta,\mathfrak{J}) \subseteq \overline{(\Theta,\mathfrak{J})}$. Since $\overline{(\Theta,\mathfrak{J})}$ is the minutest CIHS which has $(\Theta,\mathfrak{J})$, then $\overline{(\Theta,\mathfrak{J})} \subseteq (\Theta,\mathfrak{J})$. Thus $(\Theta,\mathfrak{J}) = \overline{(\Theta,\mathfrak{J})}$.

Conversely let, $(\Theta,\mathfrak{J}) = \overline{(\Theta,\mathfrak{J})}$. Since $(\Theta,\mathfrak{J})$ is CIHS, then $(\Theta,\mathfrak{J})$ is also a CIHS.

(v) By (iv) $\overline{(\Theta,\mathfrak{J})}$, $\overline{(\vartheta,\mathfrak{B})} \subseteq \overline{(\Theta,\mathfrak{J}) \cup (\vartheta,\mathfrak{B})}$. So $\overline{(\Theta,\mathfrak{J})} \cup \overline{(\vartheta,\mathfrak{B})} \subseteq \overline{(\Theta,\mathfrak{J}) \cup (\vartheta,\mathfrak{B})}$.

Conversely by (i), $(\Theta,\mathfrak{J}) \cup (\vartheta,\mathfrak{B}) \subseteq \overline{(\Theta,\mathfrak{J}) \cup (\vartheta,\mathfrak{B})}$.

Since $(\Theta,\mathfrak{J})$, $(\vartheta,\mathfrak{B})$ are IH sets and $\overline{(\Theta,\mathfrak{J}) \cup (\vartheta,\mathfrak{B})}$ is the minutest CIHS which has $(\Theta,\mathfrak{J}) \cup (\vartheta,\mathfrak{B})$, then $\overline{(\Theta,\mathfrak{J}) \cup (\vartheta,\mathfrak{B})} \subseteq \overline{(\Theta,\mathfrak{J})} \cup \overline{(\vartheta,\mathfrak{B})}$

Thus the equality is obtained.

Theorem 4.12

Let $(\mathfrak{H}_{\varrho}, \tau)$ be an IHTS and $(\Theta,\mathfrak{J}), (\vartheta,\mathfrak{B}) \in \mathfrak{P}(T,\mathbb{E})$.
Then,
(i) $(\Theta,\mathfrak{J})^{\circ} \subseteq (\Theta,\mathfrak{J})$
(ii) $((\Theta,\mathfrak{J})^{\circ})^{\circ} = (\Theta,\mathfrak{J})^{\circ}$
(iii) If $(\Theta,\mathfrak{J}) \subseteq (\vartheta,\mathfrak{B})$, then $(\Theta,\mathfrak{J})^{\circ} \subseteq (\vartheta,\mathfrak{B})^{\circ}$.
(iv) $(\Theta,\mathfrak{J})$ is OIHS iff $(\Theta,\mathfrak{J}) = (\Theta,\mathfrak{J})^{\circ}$.
(v) $\big((\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})\big)^{\circ} = (\Theta,\mathfrak{J})^{\circ} \cap (\vartheta,\mathfrak{B})^{\circ}$.

Proof

(i) – (iii) are obvious from the definition of interior

(iv) Let $(\Theta,\mathfrak{J})$ be a OIHS, by (i) $(\Theta,\mathfrak{J})^{\circ} \subseteq (\Theta,\mathfrak{J})$. Since $(\Theta,\mathfrak{J})^{\circ}$ is the largest OIHS that is contained in $(\Theta,\mathfrak{J})$, then $(\Theta,\mathfrak{J}) \subseteq (\Theta,\mathfrak{J})^{\circ}$. Thus $(\Theta,\mathfrak{J}) = (\Theta,\mathfrak{J})^{\circ}$

Conversely, let $(\Theta,\mathfrak{J}) = (\Theta,\mathfrak{J})^{\circ}$ since $(\Theta,\mathfrak{J})^{\circ}$ is OIHS, $(\Theta,\mathfrak{J})$ is also OIHS.

(v) $(\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B}) \subseteq (\Theta,\mathfrak{J}), (\vartheta,\mathfrak{B})$. Thus by (iii) $\big((\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})\big)^{\circ} \subseteq (\Theta,\mathfrak{J})^{\circ} \cap (\vartheta,\mathfrak{B})^{\circ}$.

Conversely by (i) $(\Theta,\mathfrak{J})^{\circ} \cap (\vartheta,\mathfrak{B})^{\circ} \subseteq (\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})$. Since $(\Theta,\mathfrak{J})^{\circ}$, $(\vartheta,\mathfrak{B})^{\circ}$ are OIHS & $\big((\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})\big)^{\circ}$ is the largest OIHS that has $(\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})$, then $(\Theta,\mathfrak{J})^{\circ} \cap (\vartheta,\mathfrak{B})^{\circ} \subseteq \big((\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})\big)^{\circ}$. Thus, the equality is achieved.

5. Neutrosophic Hypersoft Topological Spaces

In this section, we define the concept of "Neutrosophic Hypersoft Topology". Let $E_1, E_2, E_3 \dots E_n$ be the parameters of the universe K, the set of all neutrosophic sets be $F(K)$, the collection of all neutrosophic hypersoft sets over $K_{\mathbb{E}}$ (where $\mathbb{E} = E_1 \times E_2 \times E_3 \dots \times E_n$) be $\mathfrak{P}(K,\mathbb{E})$.

Definition 5.1

Let $(\varrho, \mathfrak{Y})$ be an element of $\mathfrak{P}(K, \mathbb{E})$ (where $\mathfrak{Y} = \mathrm{Y}_1 \times Y_2 \times \mathrm{Y}_3 \ldots \times \mathrm{Y}_n$ with each Y_i is a subset of E_i $(i = 1, 2 \ldots n)$, set of all neutrosophic hypersoft (NH) subsets of $(\varrho, \mathfrak{Y})$ be $P(\varrho, \mathfrak{Y})$ and τ, a subcollection of $P(\varrho, \mathfrak{Y})$.

(i) $\phi_{\mathfrak{Y}}, (\varrho, \mathfrak{Y}) \in \tau$
(ii) $(\Theta, \mathfrak{J}), (\vartheta, \mathfrak{B}) \in \tau \Longrightarrow (\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B}) \in \tau$
(iii) $\{(\Theta, \mathfrak{J})_l \mid l \in L\} \in \tau \Longrightarrow \bigcup_{l \in L} (\Theta, \mathfrak{J})_l \in \tau$

If the above axioms are satisfied then τ is neutrosophic hypersoft topology (NHT) on $(\varrho, \mathfrak{Y})$. $(\mathfrak{Y}_\varrho, \tau)$ is called a neutrosophic hypersoft topological space (NHTS). Every member of τ is called open neutrosophic hypersoft set (ONHS). A neutrosophic hypersoft set if called closed fuzzy hypersoft set (CNHS) if its complement is ONHS.

Example 5.2

Let $K = \{z_1, z_2, z_3, z_4\}$ and the attributes be $E_1 = \{c_1, c_2\}$, $E_2 = \{c_3, c_4\}$ and $E_3 = \{c_5, c_6\}$. Then the neutrosophic hypersoft set be

$$\left\{\begin{array}{c}\left((c_1, c_3, c_5), \left\{\frac{z_2}{(0.4,0.3,0.4)}, \frac{z_4}{(0.6,0.2,0.4)}\right\}\right), \left((c_1, c_3, c_6), \left\{\frac{z_1}{(0.7,0.1,0.4)}\right\}\right), \left((c_1, c_4, c_5), \left\{\frac{z_1}{(0.4,0.4,0.6)}, \frac{z_2}{(0.3,0.2,0.8)}\right\}\right), \\ \left((c_1, c_4, c_6), \left\{\frac{z_1}{(0.5,0.3,0.4)}, \frac{z_3}{(0.7,0.1,0.6)}\right\}\right), \left((c_2, c_3, c_5), \left\{\frac{z_2}{(0.3,0.5,0.6)}, \frac{z_3}{(0.5,0.1,0.4)}\right\}\right), \\ , \left((c_2, c_3, c_6), \left\{\frac{z_3}{(0.8,0.1,0.6)}\right\}\right), \left((c_2, c_4, c_5), \left\{\frac{z_4}{(0.9,0.6,0.4)}\right\}\right), \left((c_2, c_4, c_6), \left\{\frac{z_2}{(0.6,0.3,0.7)}\right\}\right)\end{array}\right\}$$

Let us consider this neutrosophic hypersoft as $(\varrho, \mathfrak{Y})$. Then the subfamily

$$\tau = \{\phi_{\mathfrak{Y}}, (\varrho, \mathfrak{Y}), \left\{\left((c_1, c_3, c_5), \left\{\frac{z_1}{(0.3,0.4,0.5)}, \frac{z_2}{(0.6,0.1,0.7)}\right\}\right), \left((c_2, c_3, c_5), \left\{\frac{z_2}{(0.4,0.3,0.6)}, \frac{z_3}{(0.5,0.3,0.7)}\right\}\right)\right\}, \{\left((c_1, c_3, c_5), \left\{\frac{z_2}{(0.4,0.1,0.7)}\right\}\right)$$
$$\left((c_2, c_3, c_5), \left\{\frac{z_2}{(0.3,0.3,0.6)}, \frac{z_3}{(0.5,0.1,0.7)}\right\}\right)\}, \left((c_1, c_3, c_5), \left\{\frac{z_1}{(0.3,0.4,0.5)}, \frac{z_2}{(0.6,0.3,0.4)}, \frac{z_4}{(0.6,0.2,0.4)}\right\}\right), \left((c_1, c_4, c_6), \left\{\frac{z_1}{(0.5,0.3,0.4)}, \frac{z_3}{(0.7,0.1,0.6)}\right\}\right),$$
$$\left((c_1, c_3, c_6), \left\{\frac{z_1}{(0.7,0.1,0.4)}\right\}\right), \left((c_1, c_4, c_5), \left\{\frac{z_1}{(0.4,0.4,0.6)}, \frac{z_2}{(0.3,0.2,0.8)}\right\}\right), \left((c_2, c_3, c_5), \left\{\frac{z_2}{(0.4,0.5,0.6)}, \frac{z_3}{(0.5,0.3,0.4)}\right\}\right),$$
$$\left((c_2, c_3, c_6), \left\{\frac{z_3}{(0.8,0.1,0.6)}\right\}\right), \left((c_2, c_4, c_5), \left\{\frac{z_4}{(0.9,0.6,0.4)}\right\}\right), \left((c_2, c_4, c_6), \left\{\frac{z_2}{(0.6,0.3,0.7)}\right\}\right)\}\}$$

of $P(\varrho, \mathfrak{Y})$ is a NHT on $(\varrho, \mathfrak{Y})$.

Definition 5.3

Let τ be a NHT on $(\varrho, \mathfrak{Y}) \in \mathfrak{P}(K, \mathbb{E})$ and $(\vartheta, \mathfrak{B})$ be a NH set in $P(\varrho, \mathfrak{Y})$. A FH set $(\Theta, \mathfrak{J})$ in $P(\varrho, \mathfrak{Y})$ is a neighbourhood of NH set of $(\vartheta, \mathfrak{B})$ if and only if there exists an ONHS $(\xi, \mathfrak{C})$ such that $(\vartheta, \mathfrak{B}) \subset (\xi, \mathfrak{C}) \subset (\Theta, \mathfrak{J})$.

Theorem 5.4

A NH set $(\Theta, \mathfrak{J})$ in $P(\varrho, \mathfrak{Y})$ is an ONHS if and only if $(\Theta, \mathfrak{J})$ is a neighbourhood of each NH set $(\vartheta, \mathfrak{B})$ contained in $(\Theta, \mathfrak{J})$.

Proof:

Consider an ONHS $(\Theta, \mathfrak{J})$ and any NH set $(\vartheta, \mathfrak{B})$ confined in $(\Theta, \mathfrak{J})$. Thus we have $(\vartheta, \mathfrak{B}) \subset (\Theta, \mathfrak{J}) \subset (\Theta, \mathfrak{J})$. Implies that $(\theta, \mathfrak{A})$ is a neighbourhood of $(\vartheta, \mathfrak{B})$.

Let $(\theta, \mathfrak{A})$ be a neighbourhood of each NH set confined in it. Since $(\Theta, \mathfrak{J}) \subset (\Theta, \mathfrak{J})$, there exists an ONHS $(\xi, \mathfrak{C})$ such that $(\Theta, \mathfrak{J}) \subset (\xi, \mathfrak{C}) \subset (\Theta, \mathfrak{J})$. Thus $(\Theta, \mathfrak{J}) = (\xi, \mathfrak{C})$, $(\Theta, \mathfrak{J})$ is ONHS.

Definition 5.5

Let $(\mathfrak{Y}_\varrho, \tau)$ is called a NHTS on $(\varrho, \mathfrak{Y})$ and $(\Theta, \mathfrak{J})$ be a NH set in $P(\varrho, \mathfrak{Y})$. The neighbourhood system of $(\Theta, \mathfrak{J})$ relative to τ is the collection of all neighbourhood of $(\Theta, \mathfrak{J})$ and denoted by $HNN_{(\Theta,\mathfrak{J})}$.

Theorem 5.6

If $HNN_{(\Theta,\mathfrak{J})}$ is the neighbourhood systems of NH set $(\Theta, \mathfrak{J})$. Then,

1. Finite intersection of member of $HNN_{(\Theta,\mathfrak{J})}$ belongs to $HNN_{(\Theta,\mathfrak{J})}$.
2. Each NH set which has a member of $HNN_{(\Theta,\mathfrak{J})}$ belongs to $HNN_{(\Theta,\mathfrak{J})}$.

Proof

1. $(\vartheta, \mathfrak{B})$ and $(\xi, \mathfrak{C}) \in HNN_{(\Theta,\mathfrak{J})}$ then there exists $(\vartheta', \mathfrak{B}'), (\xi', \mathfrak{C}') \in \tau$ such that

$$(\Theta, \mathfrak{J}) \subset (\vartheta', \mathfrak{B}') \subset (\vartheta, \mathfrak{B}) \text{ and } (\Theta, \mathfrak{J}) \subset (\xi', \mathfrak{C}') \subset (\xi, \mathfrak{C}).$$

Since $(\vartheta', \mathfrak{B}') \cap (\xi', \mathfrak{C}') \in \tau$ we get $(\theta, \mathfrak{A}) \subset (\vartheta', \mathfrak{B}') \cap (\xi', \mathfrak{C}') \subset (\vartheta, \mathfrak{B}) \cap (\xi, \mathfrak{C})$

Hence $(\vartheta, \mathfrak{B}) \cap (\xi, \mathfrak{C})$ belongs to $HNN_{(\Theta,\mathfrak{J})}$.

2. Let $(\vartheta, \mathfrak{B}) \in HNN_{(\Theta,\mathfrak{J})}$ and $(\xi, \mathfrak{C})$ be a NH set having $(\vartheta, \mathfrak{B})$.

Since $(\vartheta, \mathfrak{B}) \in HNN_{(\Theta,\mathfrak{J})}$ there exists an ONHS containing $(\vartheta', \mathfrak{B}')$ such that $(\Theta, \mathfrak{J}) \subset (\vartheta', \mathfrak{B}') \subset (\vartheta, \mathfrak{B})$ it follows that $(\Theta, \mathfrak{J}) \subset (\vartheta', \mathfrak{B}') \subset (\xi, \mathfrak{C})$. Thus $(\xi, \mathfrak{C})$ belongs to $HNN_{(\Theta,\mathfrak{J})}$.

Definition 5.7

Let $(\mathfrak{Y}_\varrho, \tau)$ is called a NHTS and $(\Theta, \mathfrak{J}), (\vartheta, \mathfrak{B})$ be NH set in $P(\varrho, \mathfrak{Y})$ such that $(\vartheta, \mathfrak{B}) \subset (\Theta, \mathfrak{J})$. Then $(\vartheta, \mathfrak{B})$ is said to be an interior neutrosophic hypersoft set (INHS) of $(\Theta, \mathfrak{J})$ if and only if $(\Theta, \mathfrak{J})$ is a neighbourhood of $(\vartheta, \mathfrak{B})$.

The union of whole INHS of $(\Theta, \mathfrak{J})$ is named the interior of $(\Theta, \mathfrak{J})$ and denoted as $(\Theta, \mathfrak{J})^\circ$.

Theorem 5.8

Let $(\mathfrak{Y}_\varrho, \tau)$ is called a NHTS and $(\Theta, \mathfrak{J})$, a NH set in $P(\varrho, \mathfrak{Y})$. Then,

i) $(\Theta, \mathfrak{J})^\circ$ is open and $(\Theta, \mathfrak{J})^\circ$ is the biggest ONHS confined in $(\Theta, \mathfrak{J})$.
ii) $(\Theta, \mathfrak{J})$ is ONHS iff $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^\circ$.

Proof

i) Since $(\Theta, \mathfrak{J})^\circ = \cup \{(\vartheta, \mathfrak{B})/(\Theta, \mathfrak{J}) \text{ is a neighbourhood of } (\vartheta, \mathfrak{B})\}$, $(\Theta, \mathfrak{J})^\circ$ is itself an INHS of $(\Theta, \mathfrak{J})$. Then there exists an ONHS $(\xi, \mathfrak{C})$ such that $(\Theta, \mathfrak{J})^\circ \subset (\xi, \mathfrak{C}) \subset (\Theta, \mathfrak{J})$. $(\xi, \mathfrak{C})$ is an INHS of $(\Theta, \mathfrak{J})$, hence $(\xi, \mathfrak{C}) \subset (\Theta, \mathfrak{J})^\circ$. Thus $(\Theta, \mathfrak{J})^\circ$ is the largest ONHS enclosed in $(\Theta, \mathfrak{J})$.

ii) Let $(\Theta, \mathfrak{J})$ be an ONHS. Since $(\Theta, \mathfrak{J})^\circ$ is the INHS of $(\Theta, \mathfrak{J})$, we have $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^\circ$.

Conversely if $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^\circ$ then $(\Theta, \mathfrak{J})$ is ONHS.

Definition 5.9

Let $(\mathfrak{Y}_{\varrho}, \tau_1)$ and $(\mathfrak{Y}_{\varrho}, \tau_2)$ be two NHTS. If each $(\Theta, \mathfrak{J}) \in \tau_1$ is in τ_2 then τ_2 is called the NH finer than τ_1 (or) τ_1 is NH coarser than τ_2.

Definition 5.10

Let $(\mathfrak{Y}_{\varrho}, \tau)$ be a NHTS and $(\Theta, \mathfrak{J}) \in \mathfrak{P}(K, \mathbb{E})$. The neutrosophic hypersoft closure (NHC) of $(\Theta, \mathfrak{J})$ is the intersection of all CNH sets that contains $(\Theta, \mathfrak{J})$ which is denoted by $\overline{(\Theta, \mathfrak{J})}$.

Thus, $\overline{(\Theta, \mathfrak{J})}$ is the smallest CNHS which has $(\Theta, \mathfrak{J})$ and $\overline{(\Theta, \mathfrak{J})}$ is CNHS.

Theorem 5.11

Let $(\mathfrak{Y}_{\varrho}, \tau)$ be a NHTS and $(\Theta, \mathfrak{J}), (\vartheta, \mathfrak{B}) \in \mathfrak{P}(K, \mathbb{E})$.

Then,

(i) $(\Theta, \mathfrak{J}) \subseteq \overline{(\Theta, \mathfrak{J})}$

(ii) $\overline{\overline{(\Theta, \mathfrak{J})}} = \overline{(\Theta, \mathfrak{J})}$

(iii) If $(\Theta, \mathfrak{J}) \subset (\vartheta, \mathfrak{B})$, then $\overline{(\Theta, \mathfrak{J})} \subset \overline{(\vartheta, \mathfrak{B})}$.

(iv) $(\Theta, \mathfrak{J})$ is a CNHS iff $(\Theta, \mathfrak{J}) = \overline{(\Theta, \mathfrak{J})}$.

(v) $\overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})} = \overline{(\Theta, \mathfrak{J})} \cup \overline{(\vartheta, \mathfrak{B})}$

Proof

From the definition of NHC, the proof of (i) to (iii) is attained.

(iv) Let $(\Theta, \mathfrak{J})$ be CNHS. By (i) $(\Theta, \mathfrak{J}) \subseteq \overline{(\Theta, \mathfrak{J})}$. Since $\overline{(\Theta, \mathfrak{J})}$ is the minutest CNHS which has $(\Theta, \mathfrak{J})$, then $\overline{(\Theta, \mathfrak{J})} \subseteq (\Theta, \mathfrak{J})$. Thus $(\Theta, \mathfrak{J}) = \overline{(\Theta, \mathfrak{J})}$.

Conversely let, $(\Theta, \mathfrak{J}) = \overline{(\Theta, \mathfrak{J})}$. Since $(\Theta, \mathfrak{J})$ is CNHS, then $(\Theta, \mathfrak{J})$ is also CNHS.

(v) By (iv) $\overline{(\Theta, \mathfrak{J})}$, $\overline{(\vartheta, \mathfrak{B})} \subseteq \overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})}$. So $\overline{(\Theta, \mathfrak{J})} \cup \overline{(\vartheta, \mathfrak{B})} \subseteq \overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})}$.

Conversely by (i), $(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B}) \subseteq \overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})}$.

Since $(\Theta, \mathfrak{J})$, $(\vartheta, \mathfrak{B})$ are NH sets and $\overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})}$ is the minutest CNHS which has $(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})$, then $\overline{(\Theta, \mathfrak{J}) \cup (\vartheta, \mathfrak{B})} \subseteq \overline{(\Theta, \mathfrak{J})} \cup \overline{(\vartheta, \mathfrak{B})}$

Thus the equality is obtained.

Theorem 5.12

Let $(\mathfrak{Y}_{\varrho}, \tau)$ be a NHTS and $(\Theta, \mathfrak{J}), (\vartheta, \mathfrak{B}) \in \mathfrak{P}(K, \mathbb{E})$.

Then,

(i) $(\Theta, \mathfrak{J})^{\circ} \subseteq (\Theta, \mathfrak{J})$

(ii) $((\Theta, \mathfrak{J})^{\circ})^{\circ} = (\Theta, \mathfrak{J})^{\circ}$

(iii) If $(\Theta, \mathfrak{J}) \subseteq (\vartheta, \mathfrak{B})$, then $(\Theta, \mathfrak{J})^{\circ} \subseteq (\vartheta, \mathfrak{B})^{\circ}$.

(iv) $(\Theta, \mathfrak{J})$ is ONHS iff $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^{\circ}$.

(v) $\left((\Theta, \mathfrak{J}) \cap (\vartheta, \mathfrak{B})\right)^{\circ} = (\Theta, \mathfrak{J})^{\circ} \cap (\vartheta, \mathfrak{B})^{\circ}$.

Proof

(i) – (iii) are obvious from definition of interior

(iv) Let $(\Theta, \mathfrak{J})$ be a ONHS, by (i) $(\Theta, \mathfrak{J})^{\circ} \subseteq (\Theta, \mathfrak{J})$. Since $(\Theta, \mathfrak{J})^{\circ}$ is the largest ONHS that is contained in $(\Theta, \mathfrak{J})$, then $(\Theta, \mathfrak{J}) \subseteq (\Theta, \mathfrak{J})^{\circ}$. Thus $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^{\circ}$

Conversely, let $(\Theta, \mathfrak{J}) = (\Theta, \mathfrak{J})^{\circ}$ since $(\Theta, \mathfrak{J})^{\circ}$ is ONHS, $(\Theta, \mathfrak{J})$ is also ONHS.

(v) $(\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B}) \subseteq (\Theta,\mathfrak{J}),(\vartheta,\mathfrak{B})$. Thus by (iii) $\left((\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})\right)^{\circ} \subseteq (\Theta,\mathfrak{J})^{\circ} \cap (\vartheta,\mathfrak{B})^{\circ}$.

Conversely by (i), $(\Theta,\mathfrak{J})^{\circ} \cap (\vartheta,\mathfrak{B})^{\circ} \subseteq (\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})$. Since $(\Theta,\mathfrak{J})^{\circ}$, $(\vartheta,\mathfrak{B})^{\circ}$ are ONHS & $\left((\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})\right)^{\circ}$ is the largest ONHS that has $(\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})$, then $(\Theta,\mathfrak{J})^{\circ} \cap (\vartheta,\mathfrak{B})^{\circ} \subseteq \left((\Theta,\mathfrak{J}) \cap (\vartheta,\mathfrak{B})\right)^{\circ}$. Thus, the equality is achieved.

6. Conclusion

Herein we have defined fuzzy hypersoft topology and few basic properties have also been presented. In addition fuzzy hypersoft topology is extended to intuitionistic hypersoft, neutrosophic hypersoft topology along with some of its basic properties. In future, many properties of topological spaces can be extended to fuzzy hypersoft, intuitionistic and neutrosophic hypersoft topological spaces.

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Received: Sep 20, 2020. Accepted: Feb 9, 2021