Neutrosophic Sets and Systems, Vol. 40, 2021
University of New Mexico

The safety assessment in dynamic conditions using interval neutrosophic sets

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Abstract: In this paper, the notions of three operators, Basic Belief Assignment Operator, Dynamic Basic Belief Assignment Operator, and Dynamic Weight Vector Operator in interval neutrosophic set are defined and presented. The procedure based on Dynamic Basic Belief Assignment and Dynamic Weight Vector using Dezert-Smarandache Theory is developed to solve the dynamic decision-making problems in a neutrosophic environment where criteria values take the form of interval neutrosophic numbers collected at various periods. Practical applications for validating the proposed method and assessing system safety are given taking an example from the marine industry. The results indicate that the proposed methodology provides a feasible solution for monitoring and enhancing the safety of systems working in complex and dynamically changing environment. The model can be applied to solve multicriteria decision-making problems in diversified areas that require dynamic data.

Keywords: Basic Belief Assignment Operator; Dezert-Smarandache Theory; Dynamic Basic Belief Assignment Operator; Dynamic Weight Vector; Evidential Reasoning; Interval Neutrosophic Number

1. Introduction

Multi-Criteria Decision Making (MCDM) involves either selecting the best alternative or prioritizing them after evaluating for the laid down criteria. MCDM takes the required data from records. In case the data are unreliable or scarce, experts' judgments are used for analysis. Such data contain a lot of uncertainty and hence conventional crisp techniques do not work. To overcome the limitation of crisp sets, Zadeh [1, 2] proposed the concept of a fuzzy set. The fuzzy sets were further extended to Interval Valued Fuzzy Set (IVFS) [3], Intuitionistic Fuzzy Set (IFS) [4], and Interval Valued Intuitionistic Fuzzy Set (IVIFS) [5]. The fuzzy sets are extensively used in solving MCDM problems [6-18]. But, none of the above fuzzy sets could explain the indeterminacy component associated with the membership of an element. The fuzzy sets cannot handle the possibility of the statement being true is 0.6, the statement being false is 0.4 and the statement not being sure is 0.3. Smarandache [19] developed the concept of neutrosophic sets where indeterminacy is explicitly characterized that overcome the prime limitation of fuzzy set. Neutrosophic set is defined as, a set A in a universal set X is characterized independently by a truth membership function $T_A(X)$, indeterminacy membership function $I_A(X)$, and falsity membership function $F_A(X)$, wherein

X are real or nonstandard subsets of $\left]^{-}0,1^{+}\right[$. In neutrosophic notation, the above example can be characterized as $A = \{\langle 0.6, 0.3, 0.4 \rangle\}$. To use neutrosophic sets in practical applications, Wang [20, 21] proposed the concept of a Single Valued Neutrosophic set (SVNS) and an Interval Neutrosophic set (INS). Neutrosophic sets have wide applications in decision-making problems [22-26]. Triangular neutrosophic numbers [27, 28], pentagonal fuzzy neutrosophic numbers [29-32], cylindrical neutrosophic numbers [33] are other forms of neutrosophic numbers used in solving MCDM problems. N-valued neutrosophic sets [34], bipolar neutrosophic sets [35], and neutrosophic refined sets [36] are also very popular among researchers. Neutrosophic sets are further generalized into plithogenic sets [37] which are currently used to solve real-life problems [38, 39].

Most of the MCDM problems are solved by taking static data that must be available in advance for assessment. But, most of the time we need to make decisions in dynamic conditions where scenarios change very often. Several techniques and methods have been proposed in the past to solve such dynamic decision-making problems [40-46]. Decision making in dynamic conditions requires a fusion of information gathered at different periods, different operating conditions, and even by different teams of experts [47]. Amongst the most popular theories of information fusion is the Dempster-Shafer theory of evidential reasoning [48]. But, this theory suffers from a major limitation under highly conflicting conditions and gives counter-intuitive results [49-51]. Dezert-Smarandache [52] proposed a new DSm rule of combination (DSmT). The classic DSm rule is simple and corresponds to the Free DSm model. Like D-S theory, the classic DSm rule exhibits the commutative and associative properties. It does not use the renormalization process and hence does not suffer from the problems faced by the D-S rule.

Neutrosophic PROMETHEE techniques [53], IoT based fog computing model [54], and neutrosophic analytical hierarchy process [55, 56] are effectively used to solve MCDM problems with fuzzy information. Neutrosophic sets in combination with rough sets are used to segregate and apply only the precise/complete data to enhance the quality of service in smart cities [57]. In this paper, a model is proposed to assess the safety of engineering systems in dynamic conditions. Decision-making in safety (risk) assessment is based on data collected from experts' ambiguous judgment. We have to rely on experts' judgments because the past data are either incomplete, imprecise, or not reliable. The neutrosophic sets are preferred in this study because they can very easily handle the hesitancy part of the experts' judgment. The third component of indeterminacy in the neutrosophic set eliminates the major limitation of a fuzzy set that cannot handle the hesitancy. The model used the INS because of its greater flexibility and precision over single valued neutrosophic sets. The fusion of information in dynamic conditions is done using DSmT of information fusion.

Three operators, Basic Belief Assignment Operator (BBAO), Dynamic Basic Belief Assignment Operator (DBBAO), and Dynamic weight Vector Operator (DWVO) are proposed in this study to get the basic belief assignments from Interval Neutrosophic Number (INN) and to combine the information in a dynamic environment. We have also suggested the utility of the proposed model to solve real-life problems.

1.1. The motivation for the study

Most of the multi-criteria decision-making problems are solved in static conditions where the data are available beforehand. But, in reality, there are situations when we need to use data collected in different periods. This requires the model to be robust which can be used dynamically and iteratively to ascertain the benefits of the actions taken. Moreover, we need to avoid uncertainty due to incomplete, imprecise, and missing data. Neutrosophic set has the potential to eliminate such uncertainty. In this paper, a model is proposed using neutrosophic numbers wherein the data collected in dynamic conditions can be suitably incorporated.

1.2. The novelty of the work

Neutrosophic sets are used to develop a model to assess the risk/safety of the system dynamically in a complex uncertain environment using an evidential reasoning approach. The primary purpose is to develop,

1. Basic Belief Assignment Operator (BBAO)
2. Dynamic Basic Belief Assignment Operator (DBBAO)
3. Dynamic Weight Vector Operator (DWVO)
4. A model using Dezert Smarandache's theory to solve the dynamic decision-making problems

2. Preliminaries

2.1. Neutrosophic Set

Smarandache [19] proposed and developed the concept of a neutrosophic set as an improvement of a fuzzy set. The neutrosophic sets become popular over fuzzy sets due to their indeterminacy component which handles the hesitancy efficiently and in a better way than even the highest level fuzzy set i.e. IVIFS. The neutrosophic set contains three independent components namely, the truth membership T, the Indeterminacy membership I, and the Falsity membership F. SVNS and INS help us represent the real world with uncertain, imprecise, incomplete, and inconsistent information.

2.2. Set Definition

Definition 2.1 [19]: Let U represent a universe of discourse. A neutrosophic set is:

$$A=\{\langle x:T_A(X),I_A(X),F_A(X),x\in U\rangle\}$$

Where $T_A(X),I_A(X),F_A(X),x\in[0,1]$ and

$$0^-\leq \sup(T_A(X))+\sup(I_A(X))+\sup(F_A(X))\leq 3^+$$

Definition 2.2 [47]: A Dynamic Single-Valued Neutrosophic Set (DSVNS) is:

$A=\{x\in U;x(T_x(t),I_x(t),F_x(t))\}$ for all $x\in A$:

$$T_x,I_x,F_x:[0,\infty)\rightarrow[0,1]$$

where T_x,I_x,F_x are continuous functions whose arguments is time (t).

A Dynamic Interval Valued Neutrosophic Set (DIVNS) is:

$x([T_x^L(t),T_x^U(t)],[I_x^L(t),I_x^U(t)],[F_x^L(t),F_x^U(t)])$ where $t\geq 0$

$T_x^L(t)<T_x^U(t),I_x^L(t)<I_x^U(t),F_x^L(t)<F_x^U(t)$ and

$$[T_x^L(t),T_x^U(t)],[I_x^L(t),I_x^U(t)],[F_x^L(t),F_x^U(t)]\subseteq[0,1]$$

In DIVNS, all intervals are changing w.r.t. time (t).

2.3. Set theoretic operations of DIVNS

Let us consider two DIVN numbers:

$$a(t)=\left\{\left\langle T_x^A(t_1),I_x^A(t_1),F_x^A(t_1)\right\rangle,\ldots,\left\langle T_x^A(t_k),I_x^A(t_k),F_x^A(t_k)\right\rangle\right\}$$

$$b(t)=\left\{\left\langle T_x^B(t_1),I_x^B(t_1),F_x^B(t_1)\right\rangle,\ldots,\left\langle T_x^B(t_k),I_x^B(t_k),F_x^B(t_k)\right\rangle\right\}$$

where $t=\{t_1,t_2,\ldots,t_k\}$ is a time sequence at each time $t_l, 1\le l\le k$

Definition 2.3 [47]: Addition of Dynamic Interval Valued Neutrosophic Numbers (DIVNN):

$$a(t)\oplus b(t)=\left\{\begin{array}{l}\left\langle T_x^A(t_1)+T_x^B(t_1)-T_x^A(t_1)\times T_x^B(t_1),I_x^A(t_1)\times I_x^B(t_1),F_x^A(t_1)\times F_x^B(t_1)\right\rangle,\ldots,\\ \left\langle T_x^A(t_k)+T_x^B(t_k)-T_x^A(t_k)\times T_x^B(t_k),I_x^A(t_k)\times I_x^B(t_k),F_x^A(t_k)\times F_x^B(t_k)\right\rangle\end{array}\right\} \quad (1)$$

Multiplication of DIVNN

$$a(t)\otimes b(t)=\left\{\begin{array}{l}\left\langle T_x^A(t_1)\times T_x^B(t_1),I_x^A(t_1)+I_x^B(t_1)-I_x^A(t_1)\times I_x^B(t_1),F_x^A(t_1)+F_x^B(t_1)-F_x^A(t_1)\times F_x^B(t_1)\right\rangle,\ldots,\\ \left\langle T_x^A(t_k)\times T_x^B(t_k),I_x^A(t_k)+I_x^B(t_k)-I_x^A(t_k)\times I_x^B(t_k),F_x^A(t_k)+F_x^B(t_k)-F_x^A(t_k)\times F_x^B(t_k)\right\rangle\end{array}\right\} \quad (2)$$

Scalar Multiplication of DIVNN

$$\alpha\times a(t)=\left\{\begin{array}{l}\left\langle 1-\left(1-T_x^A(t_1)\right)^{\alpha},I_x^A(t_1)^{\alpha},F_x^A(t_1)^{\alpha}\right\rangle,\ldots,\\ \left\langle 1-\left(1-T_x^A(t_k)\right)^{\alpha},I_x^A(t_k)^{\alpha},F_x^A(t_k)^{\alpha}\right\rangle\end{array}\right\} \quad (3)$$

Power of the DIVNN

$$a(t)^{\alpha}=\left\{\begin{array}{l}\left\langle T_x^A(t_1)^{\alpha},1-\left(1-I_x^A(t_1)\right)^{\alpha},1-\left(1-F_x^A(t_1)\right)^{\alpha}\right\rangle,\ldots,\\ \left\langle T_x^A(t_k)^{\alpha},1-\left(1-I_x^A(t_k)\right)^{\alpha},1-\left(1-F_x^A(t_k)\right)^{\alpha}\right\rangle\end{array}\right\} \quad (4)$$

2.4. Dezert-Smarandache Theory

Dezert-Smarandache [52] developed the theory of information fusion (DSmT) for dealing with imprecise, uncertain, and conflicting sources of information. It overcame three limitations of D-S theory i.e. accepting Shafer's model for the fusion problem under consideration which requires all hypotheses to be mutually exclusive and exhaustive, the third middle excluded principle, and the acceptance of Dempster's rule of combination as the framework for the combination of independent sources of information. DSmT starts with a free DSm model and is denoted as $M^f(\Theta)$, and considers Θ only as a frame of exhaustive elements, $\theta_i, i=1,\ldots,n$ which can potentially overlap. The free DSm model is commutative and associative.

Definition 2.4 [52]: Let $\Theta=\{\theta_1,\ldots,\theta_n\}$ be a finite set of n exhaustive elements. The hyper-power set D^{Θ} is defined as the set of all composite subsets built from elements of Θ with $\cup$ and $\cap$ operators such that

1. $\phi,\theta_1,\ldots,\theta_n\in D^{\Theta}$
2. If $A,B\in D^{\Theta}$, then $A\cap B\in D^{\Theta}$ and $A\cup B\in D^{\Theta}$
3. No other elements belong to D^{Θ}, except those obtained by rules 1 and 2.

When there is no constraint on the elements of the frame, the classic model is called free DSm model, $M^f(\Theta)$ of two independent sources of evidence over the same frame Θ with belief functions associated with generalized basic belief assignments $m_1(\cdot)$ and $m_2(\cdot)$ and is given by

$$\forall C \neq \phi \in D^{\Theta}, m_M f_{(\theta)}(C) \equiv m(C) = m_1(A) \oplus m_2(B) = \sum_{\substack{A,B \in D^{\Theta} \\ (A \cap B) = C}} m_1(A) m_2(B) \quad (5)$$

This rule is extended for $k \geq 2$ independent sources as,

$$\forall C \neq \phi \in D^{\Theta}, m_M f_{(\theta)}(C) \equiv m(C) = [m_1 \oplus \ldots \oplus m_k](C) = \sum_{\substack{x_1, x_2, \ldots, x_k \in D^{\Theta} \\ (x_1 \cap x_2 \cap \ldots \cap x_k) = A}} \prod_{i=1}^{k} m_i(X_i) \quad (6)$$

and $m_{M^{f(\theta)}}(\phi) = 0$

3. Basic Belief Assignment (BBA), Dynamic Basic Belief Assignment (DBBA) and Dynamic Weight Vector (DWV)

3.1. Basic Belief Assignment (BBA)

Consider an interval neutrosophic set. To use the neutrosophic number in the DSmT evidential reasoning approach, we need to convert the neutrosophic number into its corresponding BBA. BBA or mass function assigns evidence to a preposition. BBAO is proposed to transform the interval neutrosophic number into their corresponding BBA's i.e. $m(T), m(F)$ and $m(I)$.

$$m(\cdot) \equiv BBA(\cdot) = \frac{mean(\cdot)}{sum\_of\_the\_mean(\cdot)} \quad (7)$$

where mean $(\cdot)$ finds the mean of the neutrosophic component interval given by

$$mean(\cdot) = \frac{(\cdot)^L + (\cdot)^U}{2} \quad (8)$$

and $sum\_of\_the\_mean(\cdot)$ gives the summation of the means of all the three components of INS.

3.2. Dynamic Basic Belief Assignment (DBBA)

Consider $A = \{A_1, A_2, \ldots, A_v\}$, $C = \{C_1, C_2, \ldots, C_n\}$, and $D = \{D_1, D_2, \ldots, D_h\}$ be the sets of alternatives, criteria and decision makers [47]. For a decision maker $D_q; q = 1, \ldots, h$, the evaluation characteristic of an alternative $A_a; a = 1,\ldots, v$ on a criterion $C_p; p = 1,\ldots, n$ in time sequence $t_l = \{t_1, t_2, \ldots, t_k\}$ is represented by

$$X_{apq}(t_l) = \left\{ \left[T_{apq}^L\left(X_{t_l}\right), T_{apq}^U\left(X_{t_l}\right) \right], \left[I_{apq}^L\left(X_{t_l}\right), I_{apq}^U\left(X_{t_l}\right) \right], \left[F_{apq}^L\left(X_{t_l}\right), F_{apq}^U\left(X_{t_l}\right) \right] \right\} \tag{9}$$

DBBA for the above neutrosophic number is obtained by DBBAO and DSmT of information fusion. Since DSmT is closed on $\cup$ and $\cap$, so also truthness and falsity components are exclusive, both the belief components of $T \cup F$ and $T \cap F$ are assigned to $T \cup F$.

Dynamic basic belief mass,

$$m_{D_{ap}}(C) \equiv DBBAO(C) = \sum_{\substack{x_1, x_2, \ldots, x_l \in D^\Theta \\ x_1 \cap x_2 \cap \ldots \cap x_l = C}} \left[\prod_{l=1}^{k} \left[\sum_{\substack{x_1, x_2, \ldots, x_q \in D^\Theta \\ x_1 \cap x_2 \cap \ldots \cap x_q = C_a}} \left[\prod_{q=1}^{h} m_{lq}\left(X_{t_l}\right) \right] \right] \right] \tag{10}$$

for $a = 1,\ldots, v$ and $p = 1,\ldots, n$

3.3. Dynamic Weight Vector (DWV)

Decision-makers assess various alternatives w.r.t. assigned criteria. These criteria, in turn, are also evaluated to decide their importance by a group of decision-makers in different periods. These are generally expressed in linguistic terms. These are to be converted into neutrosophic numbers and aggregated to get the dynamic weight vector for information fusion. This is done by horizontal integration of neutrosophic numbers for all the decision-makers in all periods using DWVO.

Consider $C = \{C_1, C_2, \ldots, C_n\}$ and $D = \{D_1, D_2, \ldots, D_h\}$ be the sets of criteria and decision makers [47]. For a decision maker $D_q; q = 1,\ldots, h$, the evaluation characteristic of a criterion $C_p; p = 1,\ldots, n$ in time sequence $t_l = \{t_1, t_2, \ldots, t_k\}$ is represented by

$$X_{pq}(t_l) = \left\{ \left[T_{pq}^L\left(X_{t_l}\right), T_{pq}^U\left(X_{t_l}\right) \right], \left[I_{pq}^L\left(X_{t_l}\right), I_{pq}^U\left(X_{t_l}\right) \right], \left[F_{pq}^L\left(X_{t_l}\right), F_{pq}^U\left(X_{t_l}\right) \right] \right\} \tag{11}$$

The averaged aggregation is,

$$\overline{X}_p = \left\{ \left[\overline{T}_p^L(X), \overline{T}_p^U(X) \right], \left[\overline{I}_p^L(X), \overline{I}_p^U(X) \right], \left[\overline{F}_p^L(X), \overline{F}_p^U(X) \right] \right\} \tag{12}$$

where

$$\overline{T}_p(X) = \left[\left\langle 1 - \left[\prod_{i=1}^{k} \left[1 - \left[1 - \prod_{q=1}^{h} \left(1 - T_{iq}^L(X)\right)^{1/h} \right] \right] \right]^{1/k} \right\rangle, \left\langle 1 - \left[\prod_{i=1}^{k} \left[1 - \left[1 - \prod_{q=1}^{h} \left(1 - T_{iq}^U(X)\right)^{1/h} \right] \right] \right]^{1/k} \right\rangle \right]$$

(13)

$$\bar{I}_p(X)=\left[\left\langle\prod_{i=1}^{k}\left[\prod_{q=1}^{h}\left[I_{iq}^{L}(X)\right]^{1/h}\right]^{1/k}\right\rangle,\left\langle\prod_{i=1}^{k}\left[\prod_{q=1}^{h}\left[I_{iq}^{U}(X)\right]^{1/h}\right]^{1/k}\right\rangle\right] \quad (14)$$

and

$$\bar{F}_p(X)=\left[\left\langle\prod_{i=1}^{k}\left[\prod_{q=1}^{h}\left[F_{iq}^{L}(X)\right]^{1/h}\right]^{1/k}\right\rangle,\left\langle\prod_{i=1}^{k}\left[\prod_{q=1}^{h}\left[F_{iq}^{U}(X)\right]^{1/h}\right]^{1/k}\right\rangle\right] \quad (15)$$

The dynamic weight vector is a column vector $W=(w_d)_{n\times 1}$ and obtained by DWVO using the averaged aggregation,

$$\bar{w}_d \equiv DWVO(\bar{X}_p)=\frac{mean(\bar{T}_p(X))+mean(\bar{I}_p(X))+mean(\bar{F}_p(X))}{\sum_{p=1}^{n}\left[sum\_of\_the\_mean(\bar{X}_p)\right]} \quad (16)$$

4. Dynamic information fusion

Two methods are given below, one to dynamically evaluate and rank the alternatives and the second one to assess the safety of systems dynamically in a complex and uncertain environment.

4.1 Method to evaluate and rank the alternatives

Consider $A=\{A_1,A_2,...,A_v\}$, $C=\{C_1,C_2,...,C_n\}$, $D=\{D_1,D_2,...,D_h\}$ and $t=\{t_1,t_2,...,t_k\}$ be the sets of alternatives, criteria, decision-makers and periods. The proposed steps are:

Step 1: Let $'h'$ decision-makers evaluate $'v'$ alternatives w.r.t. $'n'$ criteria in $'k'$ periods as per the suitability ratings given in Table 1. Represent the evaluated characteristics in a matrix $\left(X_{apq}(t_l)\right)_{v\times k}$ given by,

$$X_{apq}(t_l)=\left\{\left[T_{apq}^{L}(X_{t_l}),T_{apq}^{U}(X_{t_l})\right],\left[I_{apq}^{L}(X_{t_l}),I_{apq}^{U}(X_{t_l})\right],\left[F_{apq}^{L}(X_{t_l}),F_{apq}^{U}(X_{t_l})\right]\right\} \quad (17)$$

$a=1,...,v$; $p=1,...,n$; $q=1,...,h$; $l=1,...,k$

Table 1. Suitability ratings as linguistic variables

Linguistic terms	INS
Very_Poor (Ve_Po)	([0.1, 0.2], [0.6, 0.7], [0.7, 0.8])
Poor (Po)	([0.2, 0.3], [0.5, 0.6], [0.6, 0.7])
Medium (Me)	([0.3, 0.5], [0.4, 0.6], [0.4, 0.5])
Good (Go)	([0.5, 0.6], [0.4, 0.5], [0.3, 0.4])
Very_Good (Ve_Go)	([0.6, 0.7], [0.2, 0.3], [0.2, 0.3])

Step 2: Applying DSmT on the evaluated characteristic matrix and using DBBAO, get the dynamic mass of an alternative $'a'$ for a criterion $'p'$ using Eq. (10).

Step 3: Let $'h'$ decision-makers evaluate $'n'$ criteria in $'k'$ periods as per their weights given in Table 2.

Table 2. Importance weights as linguistic variables

Linguistic terms	INS
Unimportant (U_IPA)	([0.1, 0.2], [0.4, 0.5], [0.6, 0.7])
Ordinary_Important (O_IPA)	([0.2, 0.4, [0.5, 0.6], [0.4, 0.5])
Important (IPA)	([0.4, 0.6], [0.4, 0.5], [0.3, 0.4])
Very_Important (V_IPA)	([0.6, 0.8], [0.3, 0.4], [0.2, 0.3])
Absolutely_Important (A_IPA)	([0.7, 0.9], [0.2, 0.3], [0.1, 0.2])

Step 4: Find the averaged aggregation of all the $'n'$ criteria as given by $'h'$ decision-makers in $'k'$ periods using Eq. (12).

Step 5: Calculate the dynamic weight vector using Eq. (16).

Step 6: Obtain the weighted dynamic basic belief assignments (m_{wD}) for all the alternatives from the dynamic basic belief assignments (m_D) and the dynamic weight vector $(\overline{w}_D)$ of the criteria.

$$m_{wD_{ap}}(X) = \overline{w}_d \times m_{D_{ap}}(X) \qquad \text{for } a = 1,\ldots,v \text{ and } p = 1,\ldots,n \tag{18}$$

Step 7: Synthesize the information using weighted dynamic basic belief assignments w.r.t. criteria and applying the classic DSmT of information fusion to get the dynamic belief masses for all the alternatives which are further normalized to get the final belief masses.

$$m_{D_a}(C) = \sum_{\substack{X_1, X_2, \ldots X_n \in D^{\Theta} \\ X_1 \cap X_2 \cap \ldots X_n = C}} \left[\prod_{p=1}^{n} m_{wd_a}(X) \right] \qquad \text{for } a = 1,\ldots,v \tag{19}$$

Step 8: To rank the alternatives and choose the best one, compare it with the ideal alternative using the similarity measure. The similarity measure proposed by Jiang [58] using the correlation coefficient of belief functions is used.

The flowchart of all the steps to evaluate and rank the alternatives is shown in Fig. 1.

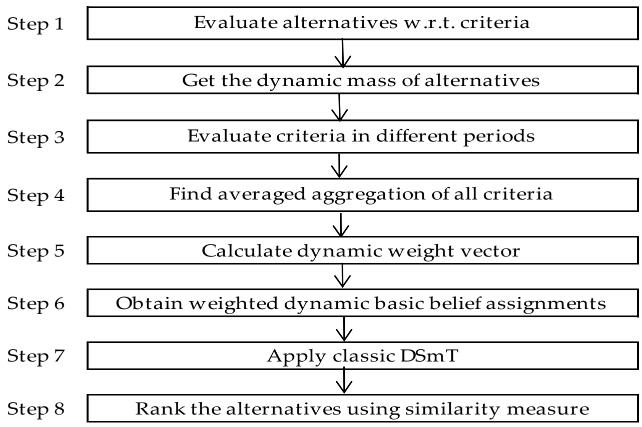

Fig.1. The flowchart to evaluate and rank the alternatives

Definition 4.1. [58]: Consider a discernment frame Θ of N elements. If we denote the mass of two pieces of evidence by m_1 and m_2, then the correlation coefficient is defined as,

$$r_{BPA}(m_1, m_2) = \frac{c(m_1, m_2)}{\sqrt{c(m_1, m_1), c(m_2, m_2)}} \quad (20)$$

where the correlation coefficient $r_{BPA} \in [0,1]$ and $c(m_1, m_2)$ is the degree of correlation denoted as:

$$c(m_1, m_2) = \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \quad (21)$$

and $i, j = 1, \dots, 2^n; A_i, A_j$ are the focal elements of mass and $|\cdot|$ is the cardinality of a subset.

The higher value of the correlation coefficient indicates that the belief masses are close to each other. The ideal and the best interval neutrosophic number is, $\alpha^* = \{\langle (1,1), (0,0), (0,0) \rangle\}$.

The correlation coefficient r_i calculated between α^* and any other INN is an unscaled distance. Higher the value of r_i indicates the two numbers are closer to each other. $r_i = 1$ indicates α^* is the same as the number. r_i can be normalized as,

$$\beta_i = \frac{r_i}{\sum_{i=1}^{4} r_i} \quad (22)$$

where, $\beta_i (i = 1,2,3,4)$ represents the degree of matching between α^* and the given neutrosophic number.

4.2 Method for assessing system safety

Consider $F = \{F_1, F_2, \dots, F_v\}$, $D = \{D_1, D_2, \dots, D_h\}$ and $t = \{t_1, t_2, \dots, t_k\}$ be the sets of failure modes of a system, decision-makers and periods. The proposed steps for assessing system safety are,

Step 1: Let $'h'$ decision-makers identify $'v'$ failure modes of a system.

Step 2: The decision-maker's views are collected on all the $'v'$ failure modes in $'k'$ periods as per the suitability ratings in linguistic terms from Table 1. The evaluated characteristic by $'q'$ decision-maker on failure mode $'a'$ in a period $'l'$ is represented in a matrix form as,

$$(X_{aq}(t_l))_{v \times k} = \{[T_{aq}^L(X_{t_l}), T_{aq}^U(X_{t_l})], [I_{aq}^L(X_{t_l}), I_{aq}^U(X_{t_l})], [F_{aq}^L(X_{t_l}), F_{aq}^U(X_{t_l})]\} \quad (23)$$

$a = 1,..., v$; $q = 1,..., h$; $l = 1,..., k$

Step 3: Horizontal integration is done using DBBAO and by applying DSmT on the evaluated characteristic matrix to get the dynamic mass of all the failure modes.

Step 4: Vertical integration of the dynamic masses of all the failure modes is done using DSmT to get the final dynamic mass of the system.

Step 5: The obtained dynamic mass of the system from step 4 above, is mapped back to the safety expressions of $'Poor'$, $'Average'$, $'Good'$ or using Eqs. (20) – (22). The mapping of dynamic mass with safety expressions gives a distributed assessment in combination of more than one safety expressions. Safety expressions in linguistic terms are shown in Table 3. The neutrosophic safety expressions are converted to their BBA's using BBAO to use the similarity measure.
The flowchart of all the steps for assessing system safety is shown in Fig. 2.

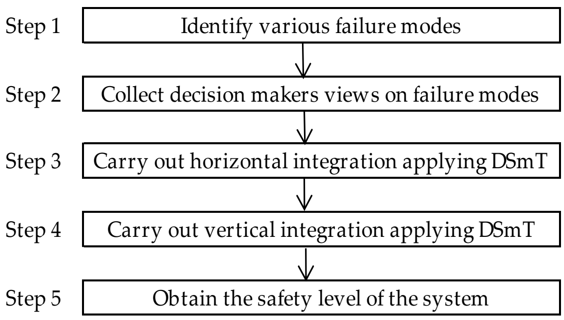

Fig.2. The flowchart for assessing system safety

Table 3. Safety expressions

Linguistic terms	INS
Poor (P)	([0.1, 0.2], [0.2, 0.3], [0.8, 0.9])
Average (A)	([0.4, 0.5, [0.4, 0.5], [0.6, 0.7])
Good (G)	([0.6, 0.7], [0.4, 0.5], [0.4, 0.5])
Excellent (E)	([0.8, 0.9], [0.2, 0.3], [0.1, 0.2])

5. Applications

Two numerical examples are discussed in this section, the first one to validate and demonstrate the proposed method. The second example shows the application of the proposed method to estimate the safety level of the systems on-board the ship.

Example 1: This example is taken from Thong et.al. [47] to evaluate lecturers' performance in the case study of ULIS-VNU. Consider five lecturers i.e. $A_1, A_2, ..., A_5$ and three decision-makers i.e. D_1, D_2, D_3. Five lecturers are evaluated with respect to 6 criteria: total publications (C_1), teaching student evaluations (C_2), personality characteristics (C_3), professional society (C_4), teaching experience (C_5), fluency of foreign language (C_6).

Suitability ratings as given by three decision-makers for lecturers versus defined criteria in three different periods are given in Table 4. Their dynamic basic belief assignments are shown at the right end in Table 4.

Table 4. Suitability ratings for lecturers

Criteria	Lecturers	Decision makers									Dynamic Basic Belief masses
		t_1			t_2			t_3			(T, F, TUF)
		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	
C_1	A_1	Me	Go	Go	Go	Go	Go	Go	Ve_Go	Go	(0.537456, 0.167772, 0.294773)
	A_2	Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	(0.677230, 0.089733, 0.233037)
	A_3	Me	Go	Go	Go	Go	Go	Go	Go	Ve_Go	(0.551952, 0.157914, 0.290134)
	A_4	Go	Me	Go	Go	Go	Go	Go	Go	Go	(0.506046, 0.189117, 0.304836)
	A_5	Me	Go	Me	Go	Go	Me	Go	Go	Go	(0.445630, 0.231638, 0.322731)
C_2	A_1	Go	Go	Go	Ve_Go	Go	Go	Go	Go	Go	(0.545188, 0.161556, 0.293256)
	A_2	Ve_Go	Go	Ve_Go	Me	Go	Go	Ve_Go	Go	Go	(0.587106, 0.137222, 0.275673)
	A_3	Ve_Go	Go	Go	Go	Me	Go	Go	Me	Go	(0.495164, 0.194219, 0.310617)
	A_4	Go	Go	Go	Go	Ve_Go	Go	Go	Go	Ve_Go	(0.592985, 0.134266, 0.272749)
	A_5	Ve_Go	Go	Go	Go	Ve_Go	Go	Go	Go	Me	(0.516687, 0.172812, 0.310501)
C_3	A_1	Ve_Go	Ve_Go	Go	Go	Ve_Go	Go	Go	Me	Go	(0.547366, 0.152625, 0.300009)
	A_2	Go	Ve_Go	Go	Ve_Go	Go	Ve_Go	Go	Go	Ve_Go	(0.639759, 0.107299, 0.252942)
	A_3	Go	Ve_Go	Ve_Go	Go	Go	Go	Go	Ve_Go	Go	(0.605431, 0.125957, 0.268611)
	A_4	Go	Go	Go	Ve_Go	Go	Go	Ve_Go	Go	Go	(0.577997, 0.142833, 0.279170)
	A_5	Ve_Go	Go	Go	Go	Ve_Go	Go	Go	Go	Go	(0.564545, 0.147920, 0.287535)
C_4	A_1	Me	Go	Me	Go	Go	Me	Me	Go	Me	(0.374181, 0.293782, 0.332038)
	A_2	Go	Me	Go	Go	Me	Go	Go	Me	Go	(0.456588, 0.224954, 0.318457)
	A_3	Go	Go	Go	Go	Go	Me	Go	Go	Ve_Go	(0.542148, 0.163564, 0.294288)
	A_4	Me	Po	Me	Go	Me	Me	Go	Go	Me	(0.335600, 0.325733, 0.338667)
	A_5	Me	Me	Po	Me	Me	Me	Me	Go	Me	(0.279417, 0.384679, 0.335904)
C_5	A_1	Me	Go	Me	Me	Go	Go	Go	Me	Go	(0.427180, 0.248386, 0.324434)
	A_2	Go	Ve_Go	Go	Ve_Go	Go	Go	Go	Ve_Go	Go	(0.597962, 0.130556, 0.271483)
	A_3	Go	Go	Me	Go	Go	Go	Go	Ve_Go	Go	(0.527769, 0.173730, 0.298501)
	A_4	Ve_Go	Go	Go	Ve_Go	Go	Go	Ve_Go	Go	Go	(0.597962, 0.130556, 0.271483)
	A_5	Go	Go	Go	Go	Go	Go	Go	Ve_Go	Go	(0.557417, 0.155493, 0.287090)
C_6	A_1	Ve_Go	Go	Go	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	(0.668533, 0.094153, 0.2237315)
	A_2	Go	Go	Go	Go	Ve_Go	Go	Go	Go	Ve_Go	(0.592985, 0.134266, 0.272749)
	A_3	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	(0.693488, 0.081472, 0.225040)
	A_4	Go	Ve_Go	Go	Go	Ve_Go	Go	Go	Go	Go	(0.564545, 0.147920, 0.287535)
	A_5	Go	Go	Go	Ve_Go	Go	Go	Go	Ve_Go	Go	(0.577997, 0.142833, 0.279170)

The evaluation of criteria by decision-makers as per their importance is shown in Table 5. The right end column of Table 5 shows the dynamic weight vector.

Table 5. Evaluation of criteria by decision makers

Criteria	Decision makers									Dynamic Weight vector
	t_1			t_2			t_3			
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	
C_1	IPA	IPA	IPA	IPA	V_IPA	IPA	V_IPA	IPA	V_IPA	0.166934
C_2	V_IPA	V_IPA	IPA	V_IPA	V_IPA	V_IPA	A_IPA	V_IPA	V_IPA	0.166570
C_3	IPA	IPA	V_IPA	IPA	IPA	V_IPA	V_IPA	IPA	V_IPA	0.167202
C_4	IPA	V_IPA	IPA	IPA	O_IPA	IPA	IPA	IPA	IPA	0.165894
C_5	IPA	IPA	IPA	V_IPA	IPA	V_IPA	IPA	IPA	IPA	0.166197
C_6	V_IPA	V_IPA	IPA	IPA	IPA	IPA	V_IPA	V_IPA	IPA	0.167202

The final normalized weighted dynamic belief masses of lecturers are given in Table 6. Table 7 gives the normalized correlation coefficients of all the alternatives w.r.t. the best and ideal neutrosophic number.

Table 6. Final normalized weighted dynamic belief masses

Lecturers	Normalized Weighted Dynamic Belief masses
A_1	(0.697808, 0.078287, 0.223905)
A_2	(0.760933, 0.050429, 0.188578)
A_3	(0.796668, 0.042129, 0.161202)
A_4	(0.701103, 0.077390, 0.221507)
A_5	(0.662146, 0.097506, 0.240348)

Table 7. Normalized correlation coefficients

Lecturers	Normalised correlation coefficients
$r_1(\alpha^*, A_1)$	0.198675
$r_2(\alpha^*, A_2)$	0.202459
$r_3(\alpha^*, A_3)$	0.204062
$r_4(\alpha^*, A_4)$	0.198903
$r_5(\alpha^*, A_5)$	0.195901

Referring to Table 7, the order of best performed lecturer to the least performed lecturer is $A_3 > A_2 > A_4 > A_1 > A_5$. The ranking order given by [47] is $A_2 > A_3 > A_4 > A_1 > A_5$. Except for the first two alternatives, the ranking order for the rest of other alternatives is in line with [47].

Example 2(a): An example from Ship is taken to illustrate how dynamically we can monitor the safety level of systems in a complex and uncertain environment using a neutrosophic set. Failure modes of Steering Gear on board ship are monitored periodically after maintenance and the safety level of the system is assessed. Steering Gear failure is common in the maritime industry and resulted in very serious accidents in the past causing major damage to the ship and its crew. This demands periodic maintenance to ensure and maintain the smooth functioning of the ship's steering gear. Two experts from the marine field (two Chief Engineers on the ship with sea sailing experience of over 20 years) were asked to analyze the steering gear system and identify the common failure modes of the system. Equal weights are assigned to the two experts. Experts identified five critical failure modes (Fig. 3) and their safety level using linguistic terms from Table 1 in two different periods. The evaluated characteristic matrix by experts in linguistic terms is given in Table 8.

Table 8. Evaluated characteristic matrix for failure modes

Failure Modes	Experts			
	t_1		t_2	
	D_1	D_2	D_1	D_2
F_1	Me	Me	Go	Go
F_2	Go	Go	Go	Go
F_3	Me	Go	Me	Go
F_4	Po	Po	Me	Me
F_5	Me	Me	Me	Go

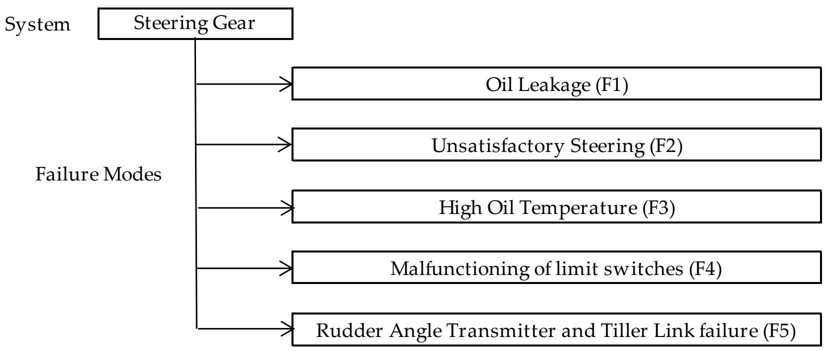

Fig.3. Steering Gear system with failure modes

Dynamic masses of all the failure modes are obtained by horizontal integration using DSmT and DBBAO. These are given in Table 9.

Table 9. Dynamic belief masses for the failure modes

Failure Modes	Dynamic Belief masses		
	m(T)	m(F)	m(T, F)
F_1	0.379971	0.281706	0.338324
F_2	0.473601	0.212394	0.314005
F_3	0.38612	0.283486	0.330394
F_4	0.194758	0.481636	0.323606
F_5	0.341035	0.323677	0.335288

Vertical integrating all the masses of failure mode using DSmT, we get the system's dynamic belief masses as,

$m(T)=0.325928,\ m(F)=0.340302$, and $m(T,F)=0.33377$

The safety score of the system is mapped back to the safety expressions using similarity measures. The safety level of the system obtained is,

$\beta_{poor}=0.23539$, $\beta_{Average}=0.266823$, $\beta_{Good}=0.265923$, $\beta_{Excellent}=0.231864$

From the above results, it is seen that the steering gear system is assessed as $'Average'$ with a belief of 26.68 %, as $'Good'$ with a belief of 26.59 %, as $'Poor'$ with a belief of 23.54 % and as $'Excellent'$ with a belief of 23.19%.

The result in graphical form is shown in Fig. 4.

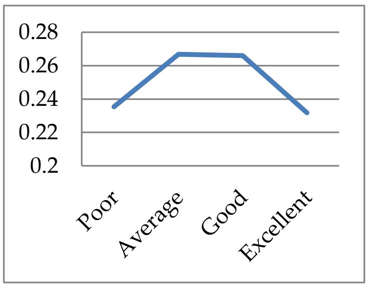

Fig. 4. System safety level

Example 2(b): The system safety level of the same example above is assessed in one more period after the regular maintenance. The two experts' views at time t_3 are given in Table 10.

Table 10. Evaluated characteristic matrix for failure modes at time t_3

Failure Modes	Experts	
	t_3	
	D_1	D_2
F1	Ve_Go	Go
F2	Ve_Go	Ve_Go
F3	Go	Ve_Go
F4	Go	Go
F5	Go	Ve_Go

System safety level after including the third period t_3 is,

$\beta_{poor}=0.190742$, $\beta_{Average}=0.255002$, $\beta_{Good}=0.277084$, $\beta_{Excellent}=0.277172$

The results show that after inclusion of the third period, the steering gear system is assessed as $'Excellent'$ with a belief of 27.72 %, as $'Good'$ with a belief of 27.71 %, as $'Average'$ with a belief of 25.50 % and as $'Poor'$ with a belief of 19.07%. With periodic maintenance of the system, the safety level can be improved. Fig. 5. shows the result in graphical form.

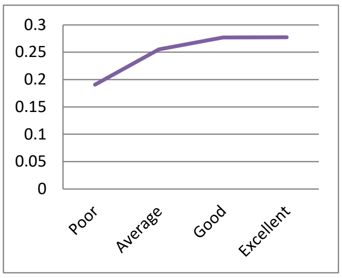

Fig. 5. System safety level (including the third period)

6. Conclusion

This paper proposed three operators Basic Belief Assignment Operator, Dynamic Basic Belief Assignment Operator (DBBAO), and Dynamic Weight Vector Operator (DWVO) to get Basic Belief Assignment (BBA), Dynamic Basic Belief Assignment (DBBA), and Dynamic Weight Vector (DWV) from the Interval Neutrosophic Number (INN). Methods are proposed with these operators in combination with Dezert-Smarandache Theory (DSmT) of information fusion to take decisions dynamically in the complex uncertain neutrosophic environments using INS. The feasibility and application of proposed methods are shown by examples from the marine industry. The method proposed can be used to monitor the systems' performance dynamically.

The main benefits of the proposed model are handling of fuzzy/vague data, converting the fuzzy data in their basic belief masses, combining the evidence using theory of information fusion and monitoring of the system periodically with different sets of data in dynamic conditions. Researchers can use this model to solve multi-criteria decision-making problems in various diversified research areas which requires data to be collected dynamically like autonomous ships, medical diagnostic support systems, weather forecasting, improving safety in transportation, etc. As future research, this model can be developed further using a plithogenic set which is an extension of a neutrosophic set.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare that they have no known conflicting interests or personal relationships that could have appeared to influence the work reported in this paper.

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Received: Sep 17, 2020. Accepted: Feb 6, 2021