NSS

Neutrosophic Sets and Systems, Vol. 41, 2021

University of New Mexico

Pairwise Pythagorean Neutrosophic P-spaces (with dependent neutrosophic components between T and F)

R.Jansi[1], K.Mohana[2]

[1]Research Scholar,Nirmala College for Women,Coimbatore; e-mail:mathematicsgasc@gmail.com

[2]Assistant Professor, Nirmala College for Women,Coimbatore; e-mail:riyaraju1116@gmail.com

Abstract. In this paper, we study the pairwise Pythagorean Neutrosophic (for shortly,Pairwise PN) bitopological spaces (with T and F are dependent neutrosophic components). We also study the pairwise PN P-spaces and the conditions under which PN bitopological spaces become pairwise PN P-spaces are investigated.

Keywords: PN bitopology, Pairwise PN P-spaces,PN Baire space.

1. Introduction

Fuzzy sets were introduced by Zadeh [17] and he discussed only membership function. The fuzzy topology concept was first introduced by C.L.Chang [3] in 1968.After the extensions of fuzzy set theory Atanassov [1] generalized this concept and introduced a new concept called intuitionistic fuzzy set (IFS). Yager [13] familiarized the model of Pythagorean fuzzy set. However, in the some practical problems, the sum of membership degree and non-membership degree to which an alternative satisfying attribute provided by decision maker(DM) may be bigger than 1, but their square sum is less than or equal to 1.

IFS was failed to deal with indeterminate and inconsistent information which exist in beliefs system, therefore, Smarandache [9] in 1995 introduced new concept known as neutrosophic set(NS) which generalizes fuzzy sets and intuitionistic fuzzy sets and so on. A neutrosophic set includes truth membership, falsity membership and indeterminacy membership. In 2006, F.Smarandache[9] introduced, for the first time, the degree of dependence (and consequently the degree of independence) between the components of the fuzzy set, and also between the components of the neutrosophic set. In 2016, the refined neutrosophic set was generalized to the degree of dependence or independence of subcomponents [10]. In neutrosophic set [10], if truth membership and falsity membership are dependent and indeterminacy is independent.Sometimes in real life, we face many problems which cannot be handled by using neutrosophic for example when $T_A(x) + I_A(x) + F_A(x) > 2$. Pythagorean neutrosophic sets [PN-sets]with T and F are dependent neutrosophic components [PNS] of condition is as their square sum does not exceeds 2.Jansi and Mohana[6]was studied about PN-sets.In 2003, A.K.Mishra [8] introduced the concept of P-spaces.The concept of P-spaces in fuzzy setting was defined by G.Balasubramanian[11].Almost P-spaces in classical topology was introduced by R.Levy[7] .

In this paper we study the pairwise PN P-spaces.Also we studied the conditions under which PN bitopological spaces become pairwise PN P-spaces are investigated.

2. preliminaries

Definition 2.1. (Pythagorean Fuzzy Set)[14] Let X be a non-empty set. A PF set A is an object of the form $A = \{(x, P_A(x), Q_A(x)) : x \in X\}$ where the function $P_A : X \to [0, 1]$ and $Q_A : X \to [0, 1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P, and $0 \leq (P_A(x))^2 + (Q_A(x))^2 \leq 1$ for each $x \in X$.

Definition 2.2. [10] Let X be a non-empty set. A neutrosophic set A on X is an object of the form: $A = \{(x, P_A(x), Q_A(x), R_A(x)) : x \in X\}$, Where $P_A(x), Q_A(x), R_A(x) \in [0, 1]$,$0 \leq P_A(x) + Q_A(x) + R_A(x) \leq 2$,for all x in X. $P_A(x)$ is the degree of membership, $Q_A(x)$ is the degree of inderminancy and $R_A(x)$ is the degree of non-membership. Here $P_A(x)$ and $R_A(x)$ are dependent components and $Q_A(x)$ is an independent components.

Definition 2.3. [6] (Pythagorean neutrosophic [PN]-sets (with T and F are dependent neutrosophic components))[13] Let X be a non-empty set. PN-set $A = \{(X, P_A(x), Q_A(x), R_A(x) : x \in X\}$ where $P_A : X \to [0, 1], Q_A : X \to [0, 1]$ and $R_A : X \to [0, 1]$ are the mappings such that $0 \leq P_A^2(x)+Q_A^2(x)+R_A^2(x) \leq 2$ and $P_A(x)$ denote the membership degree,$Q_A(x)$ denote the Indeterminacy and $R_A(x)$ denote the non-membership degree.Here P_A and R_A are dependent neutrosophic components and Q_A is an independent components.

Definition 2.4. [6] Let $A = (P_A, Q_A, R_A)$ and $B = (P_B, Q_B, R_B)$ be two PNSs,then their operations are defined as follows:
(1)$A \subseteq B$ if and if $P_A(x) \leq P_B(x), Q_A(x) \geq Q_B(x), R_A(x) \geq R_B(x)$
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(3) $A \cup B = \{(x, max(P_A, P_B), min(Q_A, Q_B), min(R_A, R_B) : x \in X\}$
(4)$A \cap B = \{(x, min(P_A, P_B), max(Q_A, Q_B), max(R_A, R_B) : x \in X\}$
(5)$A^C = \{(x, R_A, 1 - Q_A, P_A) : x \in X\}$.

3. Pairwise Pythagorean Neutrosophic [Pairwise PN] P-Spaces(with T and F are dependent neutrosophic components)

Definition 3.1. A Pythagorean neutrosophic (with T and F are dependent neutrosophic components) topology (PNT in Short) on X is a family $PN\tau$ of PNs in X satisfying the following axioms:

(1) $0_X, 1_X \in PN\tau$

(2)$G_1 \cap G_2 \in PN\tau$, for any $G_1, G_2 \in PN\tau$

(3) $\cup G_i \in PN\tau$ for any family $\{G_i/i \in J\} \subseteq PN\tau$.

In this case the pair $(X, PN\tau)$ is called a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components topological space (PNTS in Short) and any BPFTS in $PN\tau$ is known as a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components open set (PNOS in Short) in X.

The Complement A^c of a PNOS A in a PNTS $(X, PN\tau)$ is called a Pythagorean neutrosophic sets with T and F are dependent neutrosophic components closed set (PNCS in Short) in X.

Definition 3.2. Let $(X, PN\tau)$ be a PNTS and be a PN in X. Then the PN interior and closure of a PN closure are defined by

$$PNint(A) = \cup\{G/GisaPNOSinXandG \subseteq A\}$$

$$PNcl(A) = \cap\{K/KisaPNCSinXandA \subseteq K\}.$$

Note that for any PN A in $(X, PN\tau)$, we have $(PNcl(A))^c = PNint(A^c)$ and $(PNint(A))^c = PNcl(A^c)$.

Definition 3.3. A set X on which are defined two (arbitrary)PN topologies $PN\tau_1$ and $PN\tau_2$ is called PN bitopological spaces and denoted by $(X, PN\tau_1, PN\tau_2)$.
We shall write $PNint_{PN\tau_i}(A)$ and $PNcl_{PN\tau_i}(A)$ to mean respectively the PN interior and PN closure of PN set A with respect to the PN topology $PN\tau_i$ in $(X, PN\tau_1, PN\tau_2)$.

Definition 3.4. A PN A in a PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is called a pairwise PN open set if $A \in PN\tau_i(i = 1, 2)$.The complement of pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$ is called a pairwise PN closed set.

Definition 3.5. A PN A in a PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is called a pairwise PN dense set if $PNcl_{PN\tau_1}PNcl_{PN\tau_2}(A) = PNcl_{PN\tau_2}PNcl_{PN\tau_1}(A) = 1_X$ in $PNcl_{PN\tau_1}PNcl_{PN\tau_2}(A)$.

Definition 3.6. A PN A in a PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is called pairwise PN nowhere dense if $PNint_{PN\tau_1}PNcl_{PN\tau_2}(A) = PNint_{PN\tau_2}PNcl_{PN\tau_1}(A) = 0_X$ in $PNint_{PN\tau_1}PNcl_{PN\tau_2}(A)$.

Definition 3.7. Let $(X, PN\tau_1, PN\tau_2)$ be a PN bitopological space.A PN A in $(X, PN\tau_1, PN\tau_2)$ is called a pairwise PN first category set if $A = \bigcup_{i=1}^{\infty}(A_i)$, where (λ_i)'s are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.Any other PN set in $(X, PN\tau_1, PN\tau_2)$ is said to be a pairwise PN second category set in $(X, PN\tau_1, PN\tau_2)$.

Definition 3.8. If A is a pairwise PN first category set in a PN bitopological space $(X, PN\tau_1, PN\tau_2)$, then the PN A^c is called a pairwise PN residual set in $(X, PN\tau_1, PN\tau_2)$.

Definition 3.9. A PN A in a PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is called pairwise PN F_σ-set in $(X, PN\tau_1, PN\tau_2)$ if $A = \bigcup_{i=1}^{\infty}(A_i)$ where $(A_i)^c \in PN\tau_i$.

Definition 3.10. A PN A in a PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is called pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$ if $A = \bigcap_{i=1}^{\infty}(A_i)$ where $A_i \in PN\tau_i$.

Definition 3.11. A PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is called pairwise PN first category space if the PN set 1_X is a pairwise PN first category set in $(X, PN\tau_1, PN\tau_2)$.That is, $1_X = \bigcup_{i=1}^{\infty}(A_i)$, where A_i's are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.
Otherwise $(X, PN\tau_1, PN\tau_2)$ will be called a pairwise PN second category space.

Definition 3.12. A PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is called a pairwise PN Baire if $PNint_{PN\tau_i}(\bigcup_{k=1}^{\infty}(A_k))$
$= 0_X, (i = 1, 2)$, where A_k's are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.

Definition 3.13. A PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is called a pairwise PN P-space if every non-zero pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$ is a pairwise open set in $(X, PN\tau_1, PN\tau_2)$.That is, if $A = \bigcap_{k=1}^{\infty}(A_k)$,where $(A_k)'$s are pairwise PN open sets in $(X, PN\tau_1, PN\tau_2)$, then A is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.

Proposition 3.14. If A is a pairwise PN F_σ-set in a pairwise PN P-space $(X, PN\tau_1, PN\tau_2)$,then A is a pairwise PN closed set in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let A be a pairwise PN F_σ-set in $(X, PN\tau_1, PN\tau_2)$.Then A^c is a pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$.Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space, the pairwise PN G_δ-set (A^c) is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.
Hence A is a pairwise PN closed sets in $(X, PN\tau_1, PN\tau_2)$. □

Proposition 3.15. If the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space, then $PNcl_{PN\tau_i}(\bigcap_{k=1}^{\infty}(A_k)) = \bigcup_{k=1}^{\infty} PNcl_{PN\tau_i}(A_k), (i = 1, 2)$ where A_k's are pairwise PN closed sets in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let the PN sets (A_k)'s $(k = 1$ to $\infty)$ be pairwise PN closed sets in $(X, PN\tau_1, PN\tau_2)$. Then, $PNcl_{PN\tau_i}(A_k), (i = 1, 2)$ in $(X, PN\tau_1, PN\tau_2)$.Let $A = \bigcup_{k=1}^{\infty}(A_k)$.Then, A is a pairwise PN F_σ-set in $(X, PN\tau_1, PN\tau_2)$.Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space, by proposition 3.12, A is a pairwise PN closed sets in $(X, PN\tau_1, PN\tau_2)$.
Then, $PNcl_{PN\tau_i}(A) = A(i = 1, 2)$.
Now $PNcl_{PN\tau_i}(\bigcup_{k=1}^{\infty}(A_k)) = \bigcup_{k=1}^{\infty}(A_k) = \bigcup_{k=1}^{\infty} PNcl_{PN_tau_i}(A_k)$
and hence $PNcl_{A_i}(\bigcup_{k=1}^{\infty}(A_k)) = \bigcup_{k=1}^{\infty} PNcl_{PN\tau_i}(A_k)$, where (A_k)'s are pairwise PN closed sets in $(X, PN\tau_1, PN\tau_2)$. □

Proposition 3.16. If the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space, then $PNint_{PN\tau_i}(\bigcap_{k=1}^{\infty}(A_k)) = \bigcap_{k=1}^{\infty} PNint_{PN\tau_i}(A_k)$ where (A_k)'s are pairwise PN open sets in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let the PN sets (A_k)'s $(k = 1 to \infty)$ be pairwise PN open sets in $(X, PN\tau_1, PN\tau_2)$.
Then,$(A_k)^c$'s $(k = 1 to \infty)$ be pairwise PN closed sets in $(X, PN\tau_1, PN\tau_2)$.
Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space, by proposition 3.13,
$PNcl_{PN\tau_i}(\bigcup_{k=1}^{\infty}(A_k)^c) = \bigcup_{k=1}^{\infty} PNcl_{PN\tau_i}(A_k)^c$ in $(X, PN\tau_1, PN\tau_2)$.
Then, by definition 3.2, $PNcl_{PN\tau_i}(\bigcap_{k=1}^{\infty}(A_k)^c) = (\bigcup_{k=1}^{\infty}(PNint_{PN\tau_i}(A_k)))^c$and
hence $(PNint_{PN\tau_i}(\bigcap_{k=1}^{\infty}(A_k))^c = (\bigcap_{k=1}^{\infty} PNint_{PN\tau_i}(A_k))^c$.
Therefore $PNint_{PN\tau_i}(\bigcap_{k=1}^{\infty}(A_k) = \bigcap_{k=1}^{\infty}(A_k))$ where (A_k)'s are pairwise PN open sets in $(X, PN\tau_1, PN\tau_2)$. □

Example 3.17. Let $X = \{a, b\}$.The PN sets A_1, A_2 and A_3 are defined on X as follows:

A_1={(x,(0.2,0.3),(0.4,0.4),(0.7,0.6))}
A_2={(x,(0.8,0.6),(0.3,0.5),(0.4,0.1))}
A_3={(x,(0.2,0.1),(0.6,0.5),(0.9,0.7))}
A_4={(x,(0.6,0.2),(0.3,0.4),(0.5,0.5))}

Then, $PN\tau_1$=$\{0_X, A_1, A_2, A_3, A_1 \cup A_2, A_1 \cup A_3, A_2 \cup A_3, A_1 \cap A_2, A_1 \cap A_3, A_2 \cap A_3, A_1 \cup (A_2 \cap A_3), A_3 \cap (A_1 \cup A_2), A_2 \cup (A_1 \cap A_3), A_1 \cup A_2 \cup A_3, 1_X\}$ and
$PN\tau_2$=$\{0_X, A_1, A_3, A_4, A_1 \cup A_3, A_1 \cup A_4, A_3 \cup A_4, A_1 \cap A_3, A_3 \cap A_4, A_1 \cup (A_3 \cap A_4), A_4 \cap (A_1 \cup A_3), A_3 \cup (A_1 \cap A_4), A_1 \cup A_3 \cup A_4, 1_X\}$are PN topology on X.Now the PN sets $A_1, A_3, A_1 \cup A_3, A_1 \cap A_2, A_1 \cap A_3, A_1 \cup (A_2 \cap A_3), A_1 \cap A_4, A_3 \cap A_4, A_1 \cup (A_3 \cap A_4), A_3 \cap (A_1 \cup A_4)$ are pairwise PN open sets in $(X, PN\tau_1, PN\tau_2)$.Now the PN sets
$\alpha = A_1 \cap (A_1 \cap A_2) \cap (A_3 \cap (A_1 \cup A_2)) \cap (A_1 \cup (A_3 \cap A_4))$ and $\gamma = (A_1 \cup A_3) \cap (A_1 \cup (A_3 \cap A_4)) \cap (A_3 \cap (A_1 \cup A_4))$ are pairwise PN G_δ-sets in $(X, PN\tau_1, PN\tau_2)$ and $\alpha \in PN\tau_i (i = 1, 2)$ and $\gamma \in PN\tau_i (i = 1, 2)$ shows that the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space.

Definition 3.18. A PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is called a pairwise PN submaximal space if each pairwise PN dense set in $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.That is, if A is a pairwise PN dense in a PN bitopological space $(X, PN\tau_1, PN\tau_2)$, then $A \in PN\tau_i (i = 1, 2)$.

Proposition 3.19. If $(A_i)'s(i = 1to\infty)$ be pairwise PN dense sets in $(X, PN\tau_1, PN\tau_2)$.
Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN submaximal space and pairwise PN P-space $(X, PN\tau_1, PN\tau_2)$, then $\bigcap_{i=1}^{\infty}(A_i)$ is a pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let (A_i)'s $(i = 1to\infty)$ be pairwise PN dense sets in $(X, PN\tau_1, PN\tau_2)$.Since $(X, PN\tau_1, PN\tau_2)$ is a submaximal space, (A_i)'s are pairwise PN open sets in $(X, PN\tau_1, PN\tau_2)$.Then, $\bigcap_{i=1}^{\infty}(A_i)$ is a pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$.
Since the PN bitopologcal space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space, $\bigcap_{i=1}^{\infty}(A_i)$ is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$. □

Proposition 3.20. If each pairwise PN G_δ-set is a pairwise PN dense set in a pairwise PN submaximal space $(X, PN\tau_1, PN\tau_2)$, then $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space.

Proof. Let A be a pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$.By hypothesis, A is a pairwise PN dense set in $(X, PN\tau_1, PN\tau_2)$.Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN submaximal space,the pairwise PN dense set A is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.Hence the pairwise G_δ-set in $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.Thus the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space. □

Proposition 3.21. If $PNint_{PN\tau_i}PNint_{PN\tau_j}(A) = 0_X, (i, j = 1, 2 and i \neq j)$ where A is a pairwise PN F_σ-set in a pairwise PN submaximal space $(X, PN\tau_1, PN\tau_2)$,
then $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space.

Proof. Let A be a pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$.Then, A^c is a pairwise PN F_σ-set in $(X, PN\tau_1, PN\tau_2)$.
By hypothesis,
$PNint_{PN\tau_i}PNint_{PN\tau_j}(A^c) = 0_X, (i, j = 1, 2 and i \neq j)$ in $(X, PN\tau_1, PN\tau_2)$.
This implies that $(PNcl_{PN\tau_i}PNcl_{PN\tau_j}(A))^c = 0_X$ and thus
$PNcl_{PN\tau_i}PNcl_{PN\tau_j}(A) = 1_X$.
Hence A is a pairwise PN dense set in $(X, PN\tau_1, PN\tau_2)$.Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN submaximal space, the pairwise PN dense set A is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.Hence the pairwise PN G_δ-set A in $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.Therefore $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space. □

Proposition 3.22. If each pairwise PN F_σ-set is a pairwise PN nowhere dense set in a pairwise PN submaximal space $(X, PN\tau_1, PN\tau_2)$, then $(X, PN\tau_1, PN\tau_2)$ is a PN P-space.

Proof. Let A be a pairwise PN F_σ-set in a pairwise PN submaximal space $(X, PN\tau_1, PN\tau_2)$ such that $PNint_{PN\tau_i}PNcl_{PN\tau_j}(A) = 0_X, (i, j = 1, 2 and i \neq j)$.
But $PNint_{PN\tau_i}(A) \subseteq PNint_{PN\tau_i}PNcl_{PN\tau_j}(A)$, implies that
$PNint_{PN\tau_i}(A) \subseteq 0_X$.
That is, $PNint_{PN\tau_i}(A) = 0_X$.
Then, $PNint_{PN\tau_i}PNint_{PN\tau_j}(A) = PNint_{PN\tau_i}(A) = 0_X, (i, j = 1, 2 and i \neq j)$.
Thus, $PNint_{PN\tau_i}PNint_{PN\tau_j}(A) = 0_X$, for a pairwise PN F_σ-set A in a pairwise PN submaximal space $(X, PN\tau_1, PN\tau_2)$.
Then, by proposition 3.18, the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space. □

Proposition 3.23. If $PNcl_{PN\tau_i}PNint_{PN\tau_j}(A) = 1_X, (i, j = 1, 2 and i \neq j)$ for each pairwise G_σ-set A in a pairwise PN submaximal space $(X, PN\tau_1, PN\tau_2)$,then $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space.

Proof. Let A be a pairwise PN F_σ-set in the PN bitopological space $(X, PN\tau_1, PN\tau_2)$.
Then A^c is a pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$.
By hypothesis,
$PNcl_{PN\tau_i}PNint_{PN\tau_j}(A^c) = 1_X, (i, j = 1, 2 and i \neq j)$ in $(X, PN\tau_1, PN\tau_2)$.
This implies that $(PNint_{PN\tau_i}PNcl_{PN\tau_j}(A))^c = 1_X$ in $(X, PN\tau_1, PN\tau_2)$ and
hence $PNint_{PN\tau_i}PNcl_{PN\tau_j}(A) = 0_X$ in $(X, PN\tau_1, PN\tau_2)$.
Then A is a pairwise PN nowhere dense set in $(X, PN\tau_1, PN\tau_2)$.Thus, the pairwise PN F_σ-set A is a pairwise PN nowhere dense set in a pairwise PN submaximal space

$(X, PN\tau_1, PN\tau_2)$.Hence, by proposition 3.19, the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space. □

Proposition 3.24. If A is a pairwise PN residual set in a pairwise PN submaximal space $(X, PN\tau_1, PN\tau_2)$, then A is a pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let A be a pairwise PN residual set in a pairwise PN submaximal space $(X, PN\tau_1, PN\tau_2)$.Then, A^c is a pairwise PN first category set in $(X, PN\tau_1, PN\tau_2)$ and hence $A^c = \bigcup_{k=1}^{\infty}(A_k)$, where the PN (A_k)'s are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.Since (A_k)'s are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$, $PNint_{PN\tau_i}PNcl_{PN\tau_j}(A_k) = 0_X, (i, j = 1, 2 and i \neq j)$.
But $PNint_{PN\tau_i}(A_k) \subseteq PNint_{PN\tau_i}PNcl_{PN\tau_j}(A_k)$,
implies that $PNint_{PN\tau_i}(A_K) \subseteq 0_X$.That is, $PNint_{PN\tau_i}(A_k) = 0_X$.

Thus, $PNint_{PN\tau_i}PNint_{PN\tau_j}(A_k) = 0_X$ in $(X, PN\tau_1, PN\tau_2)$.
Then, $PNcl_{PN\tau_i}PNcl_{PN\tau_j}((A_k)^c) = (PNint_{PN\tau_i}PNint_{PN\tau_j}(A_k))^c = 1_X$.
Hence $(A_k)^c$'s are pairwise PN dense sets in $(X, PN\tau_1, PN\tau_2)$.Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN submaximal space, the pairwise PN dense sets $(A_k)^c$'s are pairwise PN open sets in $(X, PN\tau_1, PN\tau_2)$.
Then, $A_k)$'s are pairwise PN closed sets in $(X, PN\tau_1, PN\tau_2)$.Hence $A^c = \bigcup_{k=1}^{\infty}(A_k)$, where the PN (A_k)'s are pairwise PN closed sets in $(X, PN\tau_1, PN\tau_2)$,implies that A^c is a pairwise PN F_σ-set in $(X, PN\tau_1, PN\tau_2)$.Therefore A is a pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$. □

Proposition 3.25. If A is a pairwise PN residual set in a pairwise PN submaximal and pairwise PN P-space $(X, PN\tau_1, PN\tau_2)$, then A is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let A be a pairwise PN residual set in a pairwise PN submaximal space $(X, PN\tau_1, PN\tau_2)$.Since the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a PN submaximal space, by proposition 3.21,
A is a pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$.
Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space, the pairwise PN G_δ-set A is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.Therefore, the pairwise PN residual set A in a pairwise PN submaximal and pairwise PN P-space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$. □

Proposition 3.26. If A is a pairwise PN nowhere dense set in a pairwise submaximal space $(X, PN\tau_1, PN\tau_2)$, then A is a pairwise PN closed set in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let A be a pairwise PN nowhere dense set in the bitopological space $(X, PN\tau_1, PN\tau_2)$.
Then, $PNint_{PN\tau_i}PNcl_{PN\tau_j}(A) = 0_X, (i, j = 1, 2 and i \neq j)$.
But, $PNint_{PN\tau_i}(A) \subseteq PNint_{PN\tau_i}PNcl_{PN\tau_j}(A)$, implies that $PNint_{PN\tau_i}(A) \subseteq 0_X$.
That is, $PNint_{PN\tau_i}(A) = 0_X$.
Thus, $PNint_{PN\tau_i}PNint_{PN\tau_j}(A) = 0_X$,in $(X, PN\tau_1, PN\tau_2)$.
Then, $PNcl_{PN\tau_i}PNcl_{PN\tau_j}(A^c) = (PNint_{PN\tau_i}PNint_{PN\tau_j}(A))^c = 1_X$.
Hence A^c is a pairwise PN dense set in $(X, PN\tau_1, PN\tau_2)$.
Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN submaximal space, the pairwise PN dense set A^c is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.Thus, A is a pairwise PN closed set in $(X, PN\tau_1, PN\tau_2)$. □

Proposition 3.27. If (A_k)'s $(k = 1 to \infty)$ are pairwise PN nowhere dense sets in a pairwise PN submaximal and pairwise PN P-space $(X, PN\tau_1, PN\tau_2)$ such that $\bigcup_{k=1}^{\infty}(A_k) \neq 1_X$, then $PNcl_{PN\tau_i}(\bigcup_{k=1}^{\infty}(A_k)) = \bigcup_{k=1}^{\infty}(A_k)$ in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let (A_k)'s $(k = 1 to \infty)$ be pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.
Since the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN submaximal space, by proposition 3.23, (A_k)'s are pairwise PN closed sets in $(X, PN\tau_1, PN\tau_2)$ and hence $(A_k)^c$'s are pairwise PN open sets in $(X, PN\tau_1, PN\tau_2)$.
Let $A = \bigcap_{k=1}^{\infty}(A_k)^c$.Then, A is a non-zero pairwise PN G_δ-set in $(X, PN\tau_1, PN\tau_2)$.(For, if $A = 0_X$, then $\bigcap_{k=1}^{\infty}(A_k)^c = 0_X$, will imply that $\bigcup_{k=1}^{\infty}(A_k) = 1_X$, which is a contradiction to the hypothesis).Since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN P-space, the pairwise PN G_δ-set A is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$ and hence, $PNint_{PN\tau_i}(A) = A$.
This implies that $PNint_{PN\tau_i}(\bigcap_{k=1}^{\infty}(A_k)^c) = \bigcap_{k=1}^{\infty}(A_k)^c$.
Then, $(PNcl_{PN\tau_i}(\bigcup_{k=1}^{\infty}(A_k))^c = (\bigcup_{k=1}^{\infty}(A_k))^c$ in $(X, PN\tau_1, PN\tau_2)$.
Hence $PNcl_{PN\tau_i}(\bigcup_{k=1}^{\infty}(A_k) = \bigcup_{k=1}^{\infty}(A_k)$ in $(X, PN\tau_1, PN\tau_2)$. □

Proposition 3.28. If A is a pairwise PN first category set in a pairwise PN submaximal and pairwise PN P-space $(X, PN\tau_1, PN\tau_2)$, then A is a pairwise PN closed set in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let $A(\neq 1_X)$ be a pairwise PN first category set in $(X, PN\tau_1, PN\tau_2)$.Then, $A = \bigcup_{k=1}^{\infty}(A_k)$, where (A_k)'s are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.
Since the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN submaximal and pairwise PN P-space, by proposition 3.24, we have
$PNcl_{PN\tau_i}(\bigcup_{k=1}^{\infty}(A_k)) = \bigcup_{k=1}^{\infty}(A_k)$ in $(X, PN\tau_1, PN\tau_2)$ and hence
$PNcl_{PN\tau_i}(A) = A$.Thus the pairwise PN first category set A is a pairwise PN closed set in $(X, PN\tau_1, PN\tau_2)$. □

Theorem 3.29. Let $(X, PN\tau_1, PN\tau_2)$ be a PN bitopological space .Then the following are equivalent:

(i) $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN Baire space
(ii) $PNint_{PN\tau_i}(A) = 0_X, (i = 1, 2)$,for each pairwise PN first category set A in $(X, PN\tau_1, PN\tau_2)$.
(iii) $PNcl_{PN\tau_i}(A) = 1_X, (i = 1, 2)$,for each pairwise PN residual set A in $(X, PN\tau_1, PN\tau_2)$.

Proof. (i) ⇒ (ii) Let A be a pairwise PN first category set in $(X, PN\tau_1, PN\tau_2)$.
Then $A = \bigcup_{k=1}^{\infty}(A_k)$, where A_i's are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.
Now $PNint_{PN\tau_i}(A) = PNint_{PN\tau_i}(\bigcup_{k=1}^{\infty}(A_k)) = 0_X, (i = 1, 2)$, (since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN Baire space).
Therefore $PNint_{PN\tau_i}(A) = 0_X$, where A_k's are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.

(ii) ⇒ (iii).Let B be a pairwise PN residual set in $(X, PN\tau_1, PN\tau_2)$.
Then B^c is a pairwise PN first category set in $(X, PN\tau_1, PN\tau_2)$.
By hypothesis, $PNint_{PN\tau_i}(B^c) = 0_X, (i = 1, 2)$,
which implies that $(PNcl_{PN\tau_i}(B))^c = 0_X$.
Hence $PNcl_{PN\tau_i}(B) = 1_X, (i = 1, 2)$.

(iii) ⇒ (i).Let A be a pairwise PN first category set in $(X, PN\tau_1, PN\tau_2)$.Then $A = \bigcup_{k=1}^{\infty}(A_k)$, where A_k's are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.
Now nA is a pairwise PN first category set in $(X, PN\tau_1, PN\tau_2)$ implies that A^c is a pairwise PN residual set in $(X, PN\tau_1, PN\tau_2)$.
By hypothesis, we have $PNcl_{PN\tau_i}(A^c) = 1_X, (i = 1, 2)$,
which implies that $(PNint_{PN\tau_i}(A))^c = 0_X, (i = 1, 2)$.
Then $PNint_{PN\tau_i}(A) = 1_X$.That is, $PNint_{PN\tau_i}(\bigcup_{k=1}^{\infty}(A_k)) = 0_X$, where A_k's are pairwise PN nowhere dense sets in $(X, PN\tau_1, PN\tau_2)$.
Hence the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN Baire space. □

Proposition 3.30. If the PN bitopological space $PNint_{PN\tau_i}(A) = 0_X, (i = 1, 2)$,for each pairwise PN first category set A in $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN Baire space and pairwise PN submaximal space, then each pairwise PN first category set is a pairwise PN nowhere dense set in $PNint_{PN\tau_i}(A) = 0_X, (i = 1, 2)$,for each pairwise PN first category set A in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let A be a pairwise PN first category set in $(X, PN\tau_1, PN\tau_2)$.Since the PN bitopological space $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN submaximal and pairwise PN P-space, by proposition 3.25 , A is a pairwise PN closed set in $(X, PN\tau_1, PN\tau_2)$.Then $PNcl_{PN\tau_i}(A) = A, (i = 1, 2)$ in $(X, PN\tau_1, PN\tau_2)$.
Also since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN Baire space, by Theorem 3.26,
$PNint_{PN\tau_i}(A) = 0_X, (i = 1, 2)$, for the pairwise PN first category set A in $(X, PN\tau_1, PN\tau_2)$.
Now $PNint_{PN\tau_i}PNcl_{PN\tau_j}(A) = PNint_{PN\tau_i}(A) = 0_X,)i = 1, 2)$.
Therefore, the pairwise PN first category set A is a pairwise PN nowhere dense set in $(X, PN\tau_1, PN\tau_2)$. □

Proposition 3.31. If A is a pairwise PN residual set in a pairwise PN submaximal, pairwise PN Baire and pairwise PN P-space $(X, PN\tau_1, PN\tau_2)$,
then A is a pairwise PN open and pairwise PN dense set in $(X, PN\tau_1, PN\tau_2)$.

Proof. Let A be a pairwise PN residual set in $(X, PN\tau_1, PN\tau_2)$.
Since $(X, PN\tau_1, PN\tau_2)$ is a pairewise PN submaximal and pairwise PN P-space, by proposition 3.25, A is a pairwise PN open set in $(X, PN\tau_1, PN\tau_2)$.Also since $(X, PN\tau_1, PN\tau_2)$ is a pairwise PN Baire space, by Theorem 3.26,
$PNcl_{PN\tau_i}(A) = 1_X, (i = 1, 2)$, for the pairwise PN residual set A in $(X, PN\tau_1, PN\tau_2)$ and hence A is a pairwise PN open and a pairewise PN dense set in $(X, PN\tau_1, PN\tau_2)$. □

References

[1] K.T.Atanassov,Intuitionistic fuzzy sets,Fuzzy Sets Syst 20 (1986),87-96.

[2] P.Bosc,O.Pivert,On a fuzzy bipolar relational algebra, Information Sciences,219 (2013) 1-16.

[3] C.L.Chang, Fuzzy Topological Spaces,J.Math.Anal.Appl.,24 (1968),182-190.

[4] J.Chen,S.Li,S.Ma and X.Wang,m-Polar Fuzzy Sets:An Extension of Bipolar Fuzzy Sets,The Scientific World Journal, (2014) http://dx.doi.org/10.1155/2014/416530.

[5] D.Coker,An introduction to fuzzy Topological Spaces, Fuzzy Sets and Systems, 88 (1997), 81-89.

[6] R.Jansi,K.Mohana,Correlation Measure for Pythagorean Neutrosophic Fuzzy Sets with T and F are Dependent Neutrosophic Components ,Neutrosophic Sets and Systems, 30 (2019),202-212.

[7] R.Levy,Almost P-spaces,Canad.J.Math.,XXIX(2) (1977),284-288.

[8] A.K.Misra, A topological view of P-spaces,Gen.Tology Appl.,2(1972),349-362.

[9] F.Smarandache,A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set,Neutrosophic Probability; American Research Press: Rehoboth, DE, USA, 1999.

[10] F. Smarandache,Degree of dependence and independence of the (sub)components of fuzzy set and Neutrosophic set. Neutrosophic Sets Syst., 11(2016), 95-97.

[11] G.Thangaraj, G.Balasubramanian,On fuzzy basically disconnected spaces,J.Fuzzy Math, 9 (2001),103-110.

[12] W.Wang,Z.Wang, An approach to multiatrribute interval-valued intuitionistic fuzzy decision and Knowledge Discovery,3(2008),346-350.

[13] Xindong peng, Yong Yan, Some Results for Pythagorean Fuzzy Sets, International Journal of Intelligent Systems,30(2015), 1133-1160.

[14] R.R.Yager,Pythagorean fuzzy subsets,In:Proc Joint IFSA World Congress and NAFIPS Annual Meeting,Edmonton,Canada,2013,57-61.

[15] R.R.Yager,A.M.Abbasov, Pythagorean membership grades,complex numbers and decision making,Int J Intell Syst, 28 (2013),436-452.

[16] R.R.Yager, Pythagorean membership grades in multicriteria decision making,IEEE Trans Fuzzy Syst, 22(2014),958-965.

[17] L.A.Zadeh,Fuzzy sets,Inf Control 8(1965),338-353.

Received: Jan 10, 2021. Accepted: March 2, 2021.