Pentapartitioned Neutrosophic Q-Ideals of Q-Algebra

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Abstract: In this paper, we procure the idea of pentapartitioned neutrosophic Q-ideal of Q-algebra. Then, we formulate some definitions and results on it. Further, we furnish some suitable examples.

Keywords: *Pentapartitioned Neutrosophic Set*; *PN-Q-Algebra*; *PN-Q-Ideal*; *PN-Q-Sub-Algebra*.

1. Introduction

Iseki and Tanaka [15] presented the concept of BCK-algebra in the year 1978. Later on, Negger and Kim [26] established the notion of d-algebra by extending the idea of BCK-algebra. In the year 1999, Negger et al. [25] defined the d-ideal in d-algebra. The notion of fuzzy set (FS) theory was established by Zadeh [28] in the year 1965. Thereafter, Atanassov [4] introduced the idea of intuitionistic fuzzy set (IFS) theory by generalizing the concept of FS. In the year 2013 F. Smarandache [27] extended the neutrosophic set to refined [n-valued] neutrosophic set, and to refined neutrosophic logic, and to refined neutrosophic probability. The notion of fuzzy d-ideals of d-algebras was studied by Jun et. al. [17] in the year 2000. The idea of intuitionistic fuzzy d-algebra was presented by Jun et al. [16]. In the year 2017, the concept of intuitionistic fuzzy d-ideal of d-algebra was introduced by Hasan [12]. Hasan [13] also studied the intuitionistic fuzzy d-filter of d-algebra. The notion of Q-algebra was grounded by Neggers et. al. [24] in the year 2001. Thereafter, Abdullah and Jawad [1] studied some new types of ideals in Q-algebra. Mostafa et. al. [22] introduced the notion of fuzzy Q-ideals in Q-algebras. Mostafa et. al. [23] also studied the intuitionistic fuzzy Q-ideals of Q-algebra. In the year 2005, Smarandache [27] grounded the idea of

neutrosophic set by extending the IFS. Later on, the notion of neutrosophic *BCI/BCK*-algebras was presented by Agboola and Davvaz [2]. In the year 2016, Martina Jency and Arockiarani [21] established the notion of single valued neutrosophic ideals of *BCK*-algebras. In the year 2019, Mallick and Pramanik [20] presented the concept of pentapartitioned neutrosophic set and studied different operations on them. In this article, we procure the idea of pentapartitioned neutrosophic *Q*-ideals of *Q*-algebra.

The rest of the paper is designed as follows:

In section 2, we recall some preliminary definitions and results on *Q*-algebra, *Q*-ideal, fuzzy *Q*-algebra, fuzzy *Q*-ideal, intuitionistic fuzzy *Q*-algebra, intuitionistic fuzzy *Q*-ideal. In section-3, we introduce the notion of pentapartitioned neutrosophic *Q*-ideal of *Q*-algebra by generalizing the theory of intuitionistic fuzzy *Q*-ideal and neutrosophic *Q*-ideal. Further, we formulate some results on pentapartitioned neutrosophic *Q*-ideals of *Q*-algebra. In section 4, we conclude the work done in this article.

2. Relevant Definitions and Results:

Here we procure some basic definition and example which is needed for our work.

F. Smarandache[27] introduced the n-symbolic or numerical-Valued Refined Neutrosophic Logic

In general: T can be split into many types of truths: T_1, T_2, ..., T_p, and I into many types of indeterminacies: I_1, I_2, ..., I_r, and F into many types of falsities: F_1, F_2, ..., F_{s},, where all p, r, s $\geq$ 1 are integers, and $p + r + s = n$.

All subcomponents T_j, I_k, F_l are symbolic or numerical for $j \in \{1,2,…,p\}$, $k \in \{1,2,…,r\}$, and $l \in \{1,2,…,s\}$.

If at least one $I_k = T_j \wedge F_l$ = contradiction, we get again the Extenics.

We use five valued neutrosophic logic which is the particular case of the n-valued neutrosophic logic. The details for the *n*-valued neutrosophic logic one may refer to [27].

Definition 2.1.[20] Assume that W be a fixed set. A pentapartitioned neutrosophic set P over W is defined as follows:

$Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$, where $\hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)$ ($\in$]0,1[ ) are the truth, contradiction, ignorance, unknown and falsity membership values of each $c \in W$. So,

$$0 \leq \hat{A}_Y(c) + \hat{C}_Y(c) + \hat{E}_Y(c) + \check{D}_Y(c) + \hat{U}_Y(c) \leq 5.$$

Definition 2.3.[20] Suppose that $X = \{(c, \hat{A}_X(c), \hat{C}_X(c), \hat{E}_X(c), \check{D}_X(c), \hat{U}_X(c)) : c \in W\}$ and $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be two pentapartitioned neutrosophic sets over W. Then, $X \subseteq Y$ if and only if $\hat{A}_X(c) \leq \hat{A}_Y(c)$, $\hat{C}_X(c) \leq \hat{C}_Y(c)$, $\hat{E}_X(c) \geq \hat{E}_Y(c)$, $\check{D}_X(c) \geq \check{D}_Y(c)$, $\hat{U}_X(c) \geq \hat{U}_Y(c)$, for all $c \in W$.

Definition 2.4.[20] Suppose that $X = \{(c, \hat{A}_X(c), \hat{C}_X(c), \hat{E}_X(c), \check{D}_X(c), \hat{U}_X(c)) : c \in W\}$ and $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be two pentapartitioned neutrosophic sets over W. Then, $X \cap Y =$

$\{(c, \min\{\hat{A}_X(c), \hat{A}_Y(c)\}, \min\{\hat{C}_X(c), \hat{C}_Y(c)\}, \max\{\hat{E}_X(c), \hat{E}_Y(c)\}, \max\{\check{D}_X(c), \check{D}_Y(c)\}, \max\{\hat{U}_X(c), \hat{U}_Y(c)\})$: $c \in W\}$.

Definition 2.5.[20] Suppose that $X = \{(c, \hat{A}_X(c), \hat{C}_X(c), \hat{E}_X(c), \check{D}_X(c), \hat{U}_X(c)) : c \in W\}$ and $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be two pentapartitioned neutrosophic sets over *W*. Then, $X \cup Y = \{(c, \max\{\hat{A}_X(c), \hat{A}_Y(c)\}, \max\{\hat{C}_X(c), \hat{C}_Y(c)\}, \min\{\hat{E}_X(c), \hat{E}_Y(c)\}, \min\{\check{D}_X(c), \check{D}_Y(c)\}, \min\{\hat{U}_X(c), \hat{U}_Y(c)\})$: $c \in W\}$.

Definition 2.6.[20] Suppose that $X = \{(c, \hat{A}_X(c), \hat{C}_X(c), \hat{E}_X(c), \check{D}_X(c), \hat{U}_X(c)) : c \in W\}$ be a pentapartitioned neutrosophic set over *W*. Then, $X^c = \{(c, \hat{U}_X(c), \check{D}_X(c), 1-\hat{E}_X(c), \hat{C}_X(c), \hat{A}_X(c)): q \in W\}$.

Definition 2.1.[15] Suppose that *W* be a fixed set. Let 0 be a constant in W and $*$ be a binary operation defined on W. Then (W, $*$, 0) is called a BCK-algebra if the following holds:

(i) $((x * y) * (x * z)) * (z * y) = 0$,

(ii) $(x * (x * y)) * y = 0$,

(iii) $x * x = 0$,

(iv) $x * y = y * x = 0 \Rightarrow x = y$,

(v) $0 * x = 0$, for all $x, y, z \in X$.

Definition 2.2.[15] Let W be a BCK-algebra with binary operator $*$ and a constant 0. Then $I \subseteq W$ is called a BCK-ideal of W if the following holds:

(i) $0 \in I$;

(ii) $h * d \in I$ and $d \in I \Rightarrow h \in I$, $\forall h, d \in W$.

Definition 2.3.[22] Suppose that *W* be a fixed set. Let 0 be a constant in *W* and $*$ be a binary operation defined on W. Then (W, $*$, 0) is called a *Q*-algebra if the followings hold:

(i) $h * h = 0$, $\forall h \in W$

(ii) $0 * h = h = h * 0$, $\forall h \in W$

(iii) $(h * d) * e = (h * e) * d$, $\forall h, d, e \in W$.

Sometime, one can refer to $h \le d$ if and only if $h * d = 0$.

Definition 2.4.[24] Let (W, $*$, 0) be a *Q*-algebra. Then, (W, $*$, 0) is said to be commutative *Q*-algebra if $c * (c * d) = d * (d * c)$, $\forall c, d \in W$, and $d * (d * c)$ is denoted by $(c \wedge d)$.

Definition 2.5.[24] A *Q*-algebra W is called bounded if there exist $g \in W$ such that $h \le g$ for all $h \in W$, i.e. $h * g = 0$, $\forall h \in W$.

Definition 2.6.[24] Let W be a *Q*-algebra with binary operator $*$ and H $(\neq 0_N) \subseteq W$. Then, H is called a *Q*-sub-algebra of W, if h, d $\in$ A implies $h * d \in H$.

Definition 2.7.[1] Let W be a *Q*-algebra with binary operator $*$ and a constant 0. Then, $I \subseteq W$ is called a *Q*-ideal of W if the following holds:

(i) $0 \in I$;

(ii) $(h * d) * e \in I$ and $d \in I \Rightarrow h * e \in I$, for all $h, d, e \in W$.

Proposition 2.1.[1] Let (W, $*$, 0) be called a *Q*-algebra. Let I be a *Q*-ideal of W. Then, I be a BCK-ideal of W.

Definition 2.8.[3] A fuzzy set $Y = \{(c, T_Y(c)): c \in W\}$ over a BCK-algebra W is called the fuzzy BCK-ideal if the following two conditions holds:

(i) $T_Y(0) \ge T_Y(c)$, for all $c \in W$;

(ii) $T_Y(c) \ge \min\{T_Y((c * d), T_Y(d)\}$ for all $c, d \in W$.

Definition 2.9.[22] A fuzzy set Y={(c,T$_Y$(c)): c∈W} over a *Q*-algebra W is called the fuzzy *Q*-ideal if the following holds:

(i) T$_Y$(0) ≥ T$_Y$(c), for all c∈ W;

(ii) T$_Y$(c ∗ d) ≥ min {T$_Y$((c ∗ h) ∗ d), T$_Y$(h)} for all c, h, d∈W.

Lemma 2.1.[22] Let (W, ∗, 0) be a *Q*-algebra. If Y = {(c,T$_Y$(c)): c∈W} is a fuzzy *Q*-ideal of W, then it is also a fuzzy BCK-ideal of W.

Definition 2.10.[23] An intuitionistic fuzzy set Y={(c,T$_Y$(c),F$_Y$(c)): c∈W} over a *Q*-algebra W is called the intuitionistic fuzzy *Q*-ideal if it satisfies the following inequalities:

(i) T$_Y$(0)≥ T$_Y$(c), F$_Y$(0)≤ F$_Y$(c), for all c∈ W;

(ii) T$_Y$(c ∗ d) ≥ min{T$_Y$((c ∗ h) ∗ d), T$_Y$(h)};

(iii)F$_Y$(c ∗ d) ≤ max {F$_Y$((c ∗ h) ∗ d), F$_Y$(h)}.

Lemma 2.2.[23] Let Y={(c,T$_Y$(c),F$_Y$(c)): c∈W} be an intuitionistic fuzzy *Q*-ideal over a *Q*-algebra W. If c ∗ d≤ h, for all c, d, h ∈W, then T$_Y$(c) ≥ min{ T$_Y$(d), T$_Y$(h)} and F$_Y$(c) ≥ max{ F$_Y$(d), F$_Y$(h)}, for all c∈W.

Lemma 2.3.[23] Let Y={(c,T$_Y$(c),F$_Y$(c)): c∈W} be an intuitionistic fuzzy *Q*-ideal over a *Q*-algebra W. If c ≤ d, , for all c, d∈ W, then T$_Y$(c) ≥ T$_Y$(d) and F$_Y$(c) ≤ F$_Y$(d).

Lemma 2.4.[23] If Y={(c,T$_Y$(c),F$_Y$(c)): c∈W} be an intuitionistic fuzzy *Q*-ideal over a *Q*-algebra W, then the sets α-T$_Y$ = {c: c∈W, T$_Y$(c) ≥ α} and α-F$_Y$ = {c: c∈W, F$_Y$(c) ≤ α} are *Q*-ideal of *Q*-algebra W.

3. Pentapartitioned Neutrosophic *Q*-Ideals of *Q*-Algebra:

In this section, we procure the notion of pentapartitioned neutrosophic *Q*-ideal (PN-*Q*-Ideal) of pentapartitioned neutrosophic *Q*-algebra (PN-*Q*-Algebra). Then, we formulate some definitions and results on PN-*Q*-Ideal and PN-*Q*-Algebra.

Definition 3.1. Suppose that *W* be a *Q*-algebra and Y = {(c, Â$_Y$(c), Ĉ$_Y$(c), Ê$_Y$(c), Ď$_Y$(c), Û$_Y$(c)) : c ∈W} be a pentapartitioned neutrosophic set over W. Then, Y is said to be a pentapartitioned neutrosophic *Q*-algebra (PN-*Q*-algebra) if and only if the following holds:

(i)   Â$_Y$(c ∗ d) ≥ min{Â$_Y$((c ∗ h) ∗ d), Â$_Y$(h)};

(ii)   Ĉ$_Y$(c ∗ d) ≥ min{Ĉ$_Y$((c ∗ h) ∗ d), Ĉ$_Y$(h)};

(iii) Ê$_Y$(c ∗ d) ≤ max {Ê$_Y$((c ∗ h) ∗ d), Ê$_Y$(h);

(iv)   Ď$_Y$(c ∗ d) ≤ max {Ď$_Y$((c ∗ h) ∗ d), Ď$_Y$(h);

(v)   Û$_Y$(c ∗ d) ≤ max {Û$_Y$((c ∗ h) ∗ d), Û$_Y$(h)}, where c, d∈ W.

By the structure [(W, Y), ∗, 0], we denotes the PN-*Q*-algebra Y over W.

Theorem 3.1. If {Y$_i$: i∈Δ} be the collection of PN-*Q*-algebra's of W, then, $\bigcap_{i∈Δ} Y_i$ is also a PN-*Q*-algebra of W.

Proof. Assume that {Y$_i$: i∈Δ} be a family of PN-*Q*-algebras of W. It is clear that, $\bigcap_{i∈Δ} Y_i$ = {(c, ∧Â$_{Y_i}$(c), ∧Ĉ$_{Y_i}$(c), ∨Ê$_{Y_i}$(c), ∨Ď$_{Y_i}$(c), ∨Û$_{Y_i}$(c)) : c∈W}.

Now,

$$∧Â_{Y_i}(c ∗ d) = ∧\{Â_{Y_i}(c ∗ d): i∈Δ\}$$
$$≥ ∧\{min\{Â_{Y_i}((c ∗ h) ∗ d),\ Â_{Y_i}(h)\}\}$$
$$= min\{∧Â_{Y_i}((c ∗ h) ∗ d),\ ∧Â_{Y_i}(h)\}\}$$
$$⟹ ∧Â_{Y_i}(c ∗ d) ≥ min\{∧Â_{Y_i}((c ∗ h) ∗ d),\ ∧Â_{Y_i}(h)\}\}.$$

Now,

$\wedge\hat{C}_{Y_i}(c * d) = \wedge\{\hat{C}_{Y_i}(c * d): i \in \Delta\}$

$\geq \wedge\{\min\{\hat{C}_{Y_i}((c * h) * d), \hat{C}_{Y_i}(h)\}\}$

$= \min\{\wedge\hat{C}_{Y_i}((c * h) * d), \wedge\hat{C}_{Y_i}(h)\}\}$

$\Rightarrow \wedge\hat{C}_{Y_i}(c * d) \geq \min\{\wedge\hat{C}_{Y_i}((c * h) * d), \wedge\hat{C}_{Y_i}(h)\}\}.$

Now,

$\vee\hat{E}_{Y_i}(c * d) = \vee\{\hat{E}_{Y_i}(c * d): i \in \Delta\}$

$\geq \vee\{\min\{\hat{E}_{Y_i}((c * h) * d), \hat{E}_{Y_i}(h)\}\}$

$= \min\{\vee\hat{E}_{Y_i}((c * h) * d), \vee\hat{E}_{Y_i}(h)\}\}$

$\Rightarrow \vee\hat{E}_{Y_i}(c * d) \geq \min\{\vee\hat{E}_{Y_i}((c * h) * d), \vee\hat{E}_{Y_i}(h)\}\}.$

Now,

$\vee\check{D}_{Y_i}(c * d) = \vee\{\check{D}_{Y_i}(c * d): i \in \Delta\}$

$\geq \vee\{\min\{\check{D}_{Y_i}((c * h) * d), \check{D}_{Y_i}(h)\}\}$

$= \min\{\vee\check{D}_{Y_i}((c * h) * d), \vee\check{D}_{Y_i}(h)\}\}$

$\Rightarrow \vee\check{D}_{Y_i}(c * d) \geq \min\{\vee\check{D}_{Y_i}((c * h) * d), \vee\check{D}_{Y_i}(h)\}\}.$

Now,

$\vee\hat{U}_{Y_i}(c * d) = \vee\{\hat{U}_{Y_i}(c * d): i \in \Delta\}$

$\geq \vee\{\min\{\hat{U}_{Y_i}((c * h) * d), \hat{U}_{Y_i}(h)\}\}$

$= \min\{\vee\hat{U}_{Y_i}((c * h) * d), \vee\hat{U}_{Y_i}(h)\}\}$

$\Rightarrow \vee\hat{U}_{Y_i}(c * d) \geq \min\{\vee\hat{U}_{Y_i}((c * h) * d), \vee\hat{U}_{Y_i}(h)\}\}.$

Therefore, $\bigcap_{i \in \Delta} Y_i$ is also a PN-*Q*-algebra of W.

Definition 3.2. Suppose that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a pentapartitioned neutrosophic set over W. Then, Y is said to be a pentapartitioned neutrosophic *Q*-sub-algebra (PN-*Q*-sub-algebra) if and only if the following holds:

    (i) $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y(c), \hat{A}_Y(d)\}$;

    (ii) $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y(c), \hat{C}_Y(d)\}$;

    (iii) $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\}$;

    (iv) $\check{D}_Y(c * d) \leq \max\{\check{D}_Y(c), \check{D}_Y(d)\}$;

    (v) $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\}$; where c, d∈ W.

Theorem 3.2. Let Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a PN-*Q*-sub-algebra of a *Q*-algebra W. Then, the following holds:

    (i) $\hat{A}_Y(0) \geq \hat{A}_Y(c)$, for all c ∈W;

    (ii) $\hat{C}_Y(0) \geq \hat{C}_Y(c)$, for all c ∈W;

    (iii) $\hat{E}_Y(0) \leq \hat{E}_Y(c)$, for all c ∈W;

    (iv) $\check{D}_Y(0) \leq \check{D}_Y(c)$, fo$\hat{C}$r all c ∈W;

    (v) $\hat{U}_Y(0) \leq \hat{U}_Y(c)$, for all c ∈W.

Proof. Assume that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a PN-*Q*-sub-algebra of a *Q*-algebra W. Hence, $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y(c), \hat{A}_Y(d)\}$, $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y(c), \hat{C}_Y(d)\}$, $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\}$, $\check{D}_Y(c * d) \leq \max\{\check{D}_Y(c), \check{D}_Y(d)\}$, $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\}$, for all c, d∈ W.

Now we have,

$\hat{A}_Y(0) = \hat{A}_Y(c * c)$

$$\geq \min\{\hat{A}_Y(c), \hat{A}_Y(c)\}$$
$$= \hat{A}_Y(c)$$
$$\Rightarrow \hat{A}_Y(0) \geq \hat{A}_Y(c), \text{ for all } c \in W.$$
$$\hat{C}_Y(0) = \hat{C}_Y(c * c)$$
$$\geq \min\{\hat{C}_Y(c), \hat{C}_Y(c)\}$$
$$= \hat{C}_Y(c)$$
$$\Rightarrow \hat{C}_Y(0) \geq \hat{C}_Y(c), \text{ for all } c \in W.$$
$$\hat{E}_Y(0) = \hat{E}_Y(c * c)$$
$$\leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\}$$
$$= \hat{E}_Y(c)$$
$$\Rightarrow \hat{E}_Y(0) \leq \hat{E}_Y(c), \text{ for all } c \in W.$$
$$\check{D}_Y(0) = \check{D}_Y(c * c)$$
$$\leq \max\{\check{D}_Y(c), \check{D}_Y(d)\}$$
$$= \check{D}_Y(c)$$
$$\Rightarrow \check{D}_Y(0) \leq \check{D}_Y(c), \text{ for all } c \in W.$$
$$\hat{U}_Y(0) = \hat{U}_Y(c * c)$$
$$\leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\}$$
$$= \hat{U}_Y(c)$$
$$\Rightarrow \hat{U}_Y(0) \leq \hat{U}_Y(c), \text{ for all } c \in W.$$

Definition 3.3. A pentapartitioned neutrosophic set $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ over a Q-algebra W is said to be a pentapartitioned neutrosophic Q-ideal (PN-Q-ideal) if and only if the following inequalities holds:

(i) $\hat{A}_Y(0) \geq \hat{A}_Y(c)$ & $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y((c * h) * d), \hat{A}_Y(h)\}$, for all c, d, h$\in$W;

(ii) $\hat{C}_Y(0) \geq \hat{C}_Y(c)$ & $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y((c * h) * d), \hat{C}_Y(h)\}$, for all c, d, h$\in$W;

(iii) $\hat{E}_Y(0) \leq \hat{E}_Y(c)$ & $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y((c * h) * d), \hat{E}_Y(h)\}$, for all c, d, h$\in$W;

(iv) $\check{D}_Y(0) \leq \check{D}_Y(c)$ & $\check{D}_Y(c * d) \leq \max\{\check{D}_Y((c * h) * d), \check{D}_Y(h)\}$, for all c, d, h$\in$W;

(v) $\hat{U}_Y(0) \leq \hat{U}_Y(c)$ & $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y((c * h) * d), \hat{U}_Y(h)\}$, for all c, d, h$\in$W.

Remark 3.1. Every PN-Q-ideal of a Q-algebra W is also a PN-Q-sub-algebra.

Theorem 3.3. Suppose that $\{D_i : i \in \Delta\}$ be a family of PN-Q-ideals of Q-algebra W. Then, $\bigcap_{i \in \Delta} D_i$ is also a PN-Q-ideal of Q-algebra W.

Proof. Let $\{D_i : i \in \Delta\}$ be a family of PN-Q-ideals of Q-algebra W. Therefore,

(i) $\hat{A}_{D_i}(0) \geq \hat{A}_{D_i}(c)$ & $\hat{A}_{D_i}(c * d) \geq \min\{\hat{A}_{D_i}((c * h) * d), \hat{A}_{D_i}(h)\}$, for all c, d, h$\in$W and i$\in\Delta$;

(ii) $\hat{C}_{D_i}(0) \geq \hat{C}_{D_i}(c)$ & $\hat{C}_{D_i}(c * d) \geq \min\{\hat{C}_{D_i}((c * h) * d), \hat{C}_{D_i}(h)\}$, for all c, d, h$\in$W and i$\in\Delta$;

(iii) $\hat{E}_{D_i}(0) \leq \hat{E}_{D_i}(c)$ & $\hat{E}_{D_i}(c * d) \leq \max\{\hat{E}_{D_i}((c * h) * d), \hat{E}_{D_i}(h)\}$, for all c, d, h$\in$W and i$\in\Delta$;

(iv) $\check{D}_{D_i}(0) \leq \check{D}_{D_i}(c)$ & $\check{D}_{D_i}(c * d) \leq \max\{\check{D}_{D_i}((c * h) * d), \check{D}_{D_i}(h)\}$, for all c, d, h$\in$W and i$\in\Delta$;

(v) $\hat{U}_{D_i}(0) \leq \hat{U}_{D_i}(c)$ & $\hat{U}_{D_i}(c * d) \leq \max\{\hat{U}_{D_i}((c * h) * d), \hat{U}_{D_i}(h)\}$, for all c, d, h$\in$W and i$\in\Delta$.

Clearly, $\bigcap_{i \in \Delta} D_i = \{< c, \wedge\hat{A}_{D_i}(c), \wedge\hat{C}_{D_i}(c), \vee\hat{E}_{D_i}(c), \vee\check{D}_{D_i}(c), \vee\hat{U}_{D_i}(c) > : c \in W\}$.

Now, we have

$$\hat{A}_{D_i}(0) \geq \hat{A}_{D_i}(c), \text{ for all } c \in W \text{ and } i \in \Delta$$
$$\Rightarrow \wedge\hat{A}_{D_i}(0) \geq \wedge\hat{A}_{D_i}(c).$$

$\hat{C}_{D_i}(0) \geq \hat{C}_{D_i}(c)$, for all $c \in W$ and $i \in \Delta$

$\Rightarrow \wedge \hat{C}_{D_i}(0) \geq \wedge \hat{C}_{D_i}(c)$.

$\hat{E}_{D_i}(0) \leq \hat{E}_{D_i}(c)$, for all $c \in W$ and $i \in \Delta$

$\Rightarrow \vee \hat{E}_{D_i}(0) \leq \vee \hat{E}_{D_i}(c)$.

$\check{D}_{D_i}(0) \leq \check{D}_{D_i}(c)$, for all $c \in W$ and $i \in \Delta$

$\Rightarrow \vee \check{D}_{D_i}(0) \leq \vee \check{D}_{D_i}(c)$.

$\hat{U}_{D_i}(0) \leq \hat{U}_{D_i}(c)$, for all $c \in W$ and $i \in \Delta$

$\Rightarrow \vee \hat{U}_{D_i}(0) \leq \vee \hat{U}_{D_i}(c)$.

Further, we have

$\hat{A}_{D_i}(c * d) \geq \min \{\hat{A}_{D_i}((c * h) * d), \hat{A}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$.

$\Rightarrow \wedge \hat{A}_{D_i}(c * d) \geq \wedge \min \{\hat{A}_{D_i}((c * h) * d), \hat{A}_{D_i}(h)\}$

$\qquad\qquad = \min \{\wedge \hat{A}_{D_i}((c * h) * d), \wedge \hat{A}_{D_i}(h)\}$

$\Rightarrow \wedge \hat{A}_{D_i}(c * d) \geq \min \{\wedge \hat{A}_{D_i}((c * h) * d), \wedge \hat{A}_{D_i}(h)\}$.

$\hat{C}_{D_i}(c * d) \geq \min \{\hat{C}_{D_i}((c * h) * d), \hat{C}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$.

$\Rightarrow \wedge \hat{C}_{D_i}(c * d) \geq \wedge \min \{\hat{C}_{D_i}((c * h) * d), \hat{C}_{D_i}(h)\}$

$\qquad\qquad = \min \{\wedge \hat{C}_{D_i}((c * h) * d), \wedge \hat{C}_{D_i}(h)\}$

$\Rightarrow \wedge \hat{C}_{D_i}(c * d) \geq \min \{\wedge \hat{C}_{D_i}((c * h) * d), \wedge \hat{C}_{D_i}(h)\}$.

$\hat{E}_{D_i}(c * d) \leq \max \{\hat{E}_{D_i}((c * h) * d), \hat{E}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$.

$\Rightarrow \vee \hat{E}_{D_i}(c * d) \leq \vee \max \{\hat{E}_{D_i}((c * h) * d), \hat{E}_{D_i}(h)\}$

$\qquad\qquad = \max \{\vee \hat{E}_{D_i}((c * h) * d), \vee \hat{E}_{D_i}(h)\}$

$\Rightarrow \vee \hat{E}_{D_i}(c * d) \leq \max \{\vee \hat{E}_{D_i}((c * h) * d), \vee \hat{E}_{D_i}(h)\}$.

$\check{D}_{D_i}(c * d) \leq \max \{\check{D}_{D_i}((c * h) * d), \check{D}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$.

$\Rightarrow \vee \check{D}_{D_i}(c * d) \leq \vee \max \{\check{D}_{D_i}((c * h) * d), \check{D}_{D_i}(h)\}$

$\qquad\qquad = \max \{\vee \check{D}_{D_i}((c * h) * d), \vee \check{D}_{D_i}(h)\}$

$\Rightarrow \vee \check{D}_{D_i}(c * d) \leq \max \{\vee \check{D}_{D_i}((c * h) * d), \vee \check{D}_{D_i}(h)\}$.

$\hat{U}_{D_i}(c * d) \leq \max \{\hat{U}_{D_i}((c * h) * d), \hat{U}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$.

$\Rightarrow \vee \hat{U}_{D_i}(c * d) \leq \vee \max \{\hat{U}_{D_i}((c * h) * d), \hat{U}_{D_i}(h)\}$

$\qquad\qquad = \max \{\vee \hat{U}_{D_i}((c * h) * d), \vee \hat{U}_{D_i}(h)\}$

$\Rightarrow \vee \hat{U}_{D_i}(c * d) \leq \max \{\vee \hat{U}_{D_i}((c * h) * d), \vee \hat{U}_{D_i}(h)\}$.

Therefore, $\bigcap_{i \in \Delta} D_i$ is a PN-Q-ideal of Q-algebra W.

Corollary 3.1. Assume that $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be a PN-Q-ideal of a Q-algebra W. Then, Y is a neutrosophic BCK-ideal of the BCK-algebra W.

Theorem 3.4. Assume that $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be a PN-Q-ideal over a Q-algebra W. If c, h, d $\in$ W such that $c * h \leq d$, then $\hat{A}_Y(c) \geq \min \{\hat{A}_Y(h), \hat{A}_Y(d)\}$, $\hat{C}_Y(c) \geq \min \{\hat{C}_Y(h), \hat{C}_Y(d)\}$, $\hat{E}_Y(c) \leq \max \{\hat{E}_Y(h), \hat{E}_Y(d)\}$, $\check{D}_Y(c) \leq \max \{\check{D}_Y(h), \check{D}_Y(d)\}$ and $\hat{U}_Y(c) \leq \max \{\hat{U}_Y(h), \hat{U}_Y(d)\}$.

Proof. Let $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be a PN-Q-ideal over a Q-algebra W. Suppose that c, h, d $\in$ W such that $c * h \leq d$. Therefore, $(c * h) * d = 0$.

Now, we have

$\hat{A}_Y(c) = \hat{A}_Y(c * 0) \geq \min \{\hat{A}_Y((c * h) * 0), \hat{A}_Y(h)\}$

$\qquad\qquad = \min \{\hat{A}_Y((c * h)), \hat{A}_Y(h)\}$

$\qquad\qquad \geq \min \{\min \{\hat{A}_Y((c * d) * h), \hat{A}_Y(d)\}, \hat{A}_Y(h)\}$

$$= \min \{\hat{A}_Y((c * h) * d), \hat{A}_Y(d), \hat{A}_Y(h)\}$$
$$= \min \{\hat{A}_Y(0), \hat{A}_Y(d), \hat{A}_Y(h)\}$$
$$= \min \{\hat{A}_Y(h), \hat{A}_Y(d)\}$$

$\Rightarrow \hat{A}_Y(c) \geq \min \{ \hat{A}_Y(h), \hat{A}_Y(d)\}$.

$\hat{C}_Y(c) = \hat{C}_Y(c * 0) \geq \min \{\hat{C}_Y((c * h) * 0), \hat{C}_Y(h)\}$
$$= \min \{\hat{C}_Y((c * h)), \hat{C}_Y(h)\}$$
$$\geq \min \{\min \{\hat{C}_Y((c * d) * h), \hat{C}_Y(d)\}, \hat{C}_Y(h)\}$$
$$= \min \{\hat{C}_Y((c * h) * d), \hat{C}_Y(d), \hat{C}_Y(h)\}$$
$$= \min \{\hat{C}_Y(0), \hat{C}_Y(d), \hat{C}_Y(h)\}$$
$$= \min \{\hat{C}_Y(h), \hat{C}_Y(d)\}$$

$\Rightarrow \hat{C}_Y(c) \geq \min \{ \hat{C}_Y(h), \hat{C}_Y(d)\}$.

$\hat{E}_Y(c) = \hat{E}_Y(c * 0) \leq \max \{ \hat{E}_Y((c * h) * 0), \hat{E}_Y(h)\}$
$$= \max \{ \hat{E}_Y((c * h)), \hat{E}_Y(h)\}$$
$$\leq \max \{ \max\{ \hat{E}_Y((c * d) * h), \hat{E}_Y(d)\}, \hat{E}_Y(h)\}$$
$$= \max\{ \hat{E}_Y((c * h) * d), \hat{E}_Y(d), \hat{E}_Y(h)\}$$
$$= \max\{ \hat{E}_Y(0), \hat{E}_Y(d), \hat{E}_Y(h)\}$$
$$= \max\{ \hat{E}_Y(h), \hat{E}_Y(d)\}$$

$\Rightarrow \hat{E}_Y(c) \leq \max\{ \hat{E}_Y(h), \hat{E}_Y(d)\}$.

$\check{D}_Y(c) = \check{D}_Y(c * 0) \leq \max \{ \check{D}_Y((c * h) * 0), \check{D}_Y(h)\}$
$$= \max \{ \check{D}_Y((c * h)), \check{D}_Y(h)\}$$
$$\leq \max \{ \max\{ \check{D}_Y((c * d) * h), \check{D}_Y(d)\}, \check{D}_Y(h)\}$$
$$= \max\{ \check{D}_Y((c * h) * d), \check{D}_Y(d), \check{D}_Y(h)\}$$
$$= \max\{ \check{D}_Y(0), \check{D}_Y(d), \check{D}_Y(h)\}$$
$$= \max\{ \check{D}_Y(h), \check{D}_Y(d)\}$$

$\Rightarrow \check{D}_Y(c) \leq \max\{ \check{D}_Y(h), \check{D}_Y(d)\}$.

Further, we have

$\hat{U}_Y(c) = \hat{U}_Y(c * 0) \leq \max \{ \hat{U}_Y((c * h) * 0), \hat{U}_Y(h)\}$
$$= \max \{ \hat{U}_Y((c * h)), \hat{U}_Y(h)\}$$
$$\leq \max \{ \max\{ \hat{U}_Y((c * d) * h), \hat{U}_Y(d)\}, \hat{U}_Y(h)\}$$
$$= \max \{ \hat{U}_Y((c * h) * d), \hat{U}_Y(d), \hat{U}_Y(h)\}$$
$$= \max \{ \hat{U}_Y(0), \hat{U}_Y(d), \hat{U}_Y(h)\}$$
$$= \max \{ \hat{U}_Y(h), \hat{U}_Y(d)\}$$

$\Rightarrow \hat{U}_Y(c) \leq \max \{\hat{U}_Y(h), \hat{U}_Y(d)\}$.

Theorem 3.5. Let $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be a PN-*Q*-ideal over a *Q*-algebra W. If c, h $\in$ W such that $c \leq h$, then $\hat{A}_Y(c) \geq \hat{A}_Y(h)$, $\hat{C}_Y(c) \geq \hat{C}_Y(h)$, $\hat{E}_Y(c) \leq \hat{E}_Y(h)$, $\check{D}_Y(c) \leq \check{D}_Y(h)$ and $\hat{U}_Y(c) \leq \hat{U}_Y(h)$.

Proof. Assume that $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be a PN-*Q*-ideal over a *Q*-algebra W. Suppose c, h be two elements of W such that $c \leq h$. Therefore, $c * h = 0$.

Now, we have

$\hat{A}_Y(c) = \hat{A}_Y(c * 0) \geq \min \{\hat{A}_Y((c * h) * 0), \hat{A}_Y(h)\}$
$$= \min \{\hat{A}_Y((c * h)), \hat{A}_Y(h)\}$$

$$= \min \{\hat{A}_Y(0), \hat{A}_Y(h)\}$$
$$= \hat{A}_Y(h)$$

$\Rightarrow \hat{A}_Y(c) \geq \hat{A}_Y(h)$.

$\hat{C}_Y(c) = \hat{C}_Y(c * 0) \geq \min \{\hat{C}_Y((c * h) * 0), \hat{C}_Y(h)\}$

$$= \min \{\hat{C}_Y((c * h)), \hat{C}_Y(h)\}$$
$$= \min \{\hat{C}_Y(0), \hat{C}_Y(h)\}$$
$$= \hat{C}_Y(h)$$

$\Rightarrow \hat{C}_Y(c) \geq \hat{C}_Y(h)$.

$\hat{E}_Y(c) = \hat{E}_Y(c * 0) \leq \max \{ \hat{E}_Y((c * h) * 0), \hat{E}_Y(h)\}$

$$= \max \{ \hat{E}_Y((c * h)), \hat{E}_Y(h)\}$$
$$= \max \{ \hat{E}_Y(0), \hat{E}_Y(h)\}$$
$$= \hat{E}_Y(h)$$

$\Rightarrow \hat{E}_Y(c) \leq \hat{E}_Y(h)$.

$\check{D}_Y(c) = \check{D}_Y(c * 0) \leq \max \{ \check{D}_Y((c * h) * 0), \check{D}_Y(h)\}$

$$= \max \{ \check{D}_Y((c * h)), \check{D}_Y(h)\}$$
$$= \max \{ \check{D}_Y(0), \check{D}_Y(h)\}$$
$$= \check{D}_Y(h)$$

$\Rightarrow \check{D}_Y(c) \leq \check{D}_Y(h)$.

$\hat{U}_Y(c) = \hat{U}_Y(c * 0) \leq \max \{\hat{U}_Y((c * h) * 0), \hat{U}_Y(h)\}$

$$= \max \{\hat{U}_Y((c * h)), \hat{U}_Y(h)\}$$
$$= \max \{\hat{U}_Y(0), \hat{U}_Y(h)\}$$
$$= \hat{U}_Y(h)$$

$\Rightarrow \hat{U}_Y(c) \leq \hat{U}_Y(h)$.

Theorem 3.6. If Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c $\in$ W} is a PN-*Q*-sub-algebra over a *Q*-algebra W, then the sets α-$\hat{A}_Y$ = {c: c$\in$ W, $\hat{A}_Y(c) \geq$ α}, α-$\hat{C}_Y$ = {c: c$\in$ W, $\hat{C}_Y(c) \geq$ α}, α-$\hat{E}_Y$ = {c: c$\in$ W, $\hat{E}_Y(c) \leq$ α}, α-$\check{D}_Y$ = {c: c$\in$ W, $\check{D}_Y(c) \leq$ α} and α-$\hat{U}_Y$ = {c: c$\in$ W, $\hat{U}_Y(c) \leq$ α} are the *Q*-sub-algebra of W.

Proof. Suppose that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c $\in$ W} be a PN-*Q*-sub-algebra over a *Q*-algebra W. Therefore,

  (i) $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y(c), \hat{A}_Y(d)\}$;
  (ii) $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y(c), \hat{C}_Y(d)\}$;
  (iii) $\hat{E}_Y(c * d) \leq \max \{\hat{E}_Y(c), \hat{E}_Y(d)\}$;
  (iv) $\check{D}_Y(c * d) \leq \max \{\check{D}_Y(c), \check{D}_Y(d)\}$;
  (v) $\hat{U}_Y(c * d) \leq \max \{\hat{U}_Y(c), \hat{U}_Y(d)\}$; where c, d $\in$ W.

Let c, d $\in$ α-$\hat{A}_Y$. This implies, $\hat{A}_Y(c) \geq$ α, $\hat{A}_Y(d) \geq$ α.

Therefore, $\hat{A}_Y(c * d) \geq \min \{\hat{A}_Y(c), \hat{A}_Y(d)\} \geq \min\{$ α, α$\} \geq$ α.

Hence, α-$\hat{A}_Y$ = {c: c$\in$ W, $\hat{A}_Y(c) \geq$ α} is a *Q*-sub-algebra of W.

Let c, d $\in$ α-$\hat{C}_Y$. This implies, $\hat{C}_Y(c) \geq$ α, $\hat{C}_Y(d) \geq$ α.

Therefore, $\hat{C}_Y(c * d) \geq \min \{\hat{C}_Y(c), \hat{C}_Y(d)\} \geq \min\{$ α, α$\} \geq$ α.

Hence, α-$\hat{C}_Y$ = {c: c$\in$ W, $\hat{C}_Y(c) \geq$ α} is a *Q*-sub-algebra of W.

Let c, d $\in$ α-$\hat{E}_Y$. This implies, $\hat{E}_Y(c) \leq$ α, $\hat{E}_Y(d) \leq$ α.

Therefore, $\hat{E}_Y(c * d) \leq \max \{\hat{E}_Y(c), \hat{E}_Y(d)\} \leq \max\{$α, α$\} \leq$ α.

Hence, α-$\hat{E}_Y$ = {c: c$\in$ W, $\hat{E}_Y(c) \leq \alpha$} is a Q-sub-algebra of W.

Let c, d $\in \alpha$-$\check{D}_Y$. This implies, $\check{D}_Y(c) \leq \alpha$, $\check{D}_Y(d) \leq \alpha$.

Therefore, $\check{D}_Y(c * d) \leq$ max {$\check{D}_Y(c)$, $\check{D}_Y(d)$} $\leq$ max{α, α} $\leq \alpha$.

Hence, α-$\check{D}_Y$ = {c: c$\in$ W, $\check{D}_Y(c) \leq \alpha$} is a Q-sub-algebra of W.

Let c, d $\in \alpha$-$\hat{U}_Y$. This implies, $\hat{U}_Y(c) \leq \alpha$, $\hat{U}_Y(d) \leq \alpha$.

Therefore, $\hat{U}_Y(c * d) \leq$ max {$\hat{U}_Y(c)$, $\hat{U}_Y(d)$} $\leq$ max{ α, α} $\leq \alpha$.

Hence, α-$\hat{U}_Y$ = {c: c$\in$ W, $\hat{U}_Y(c) \leq \alpha$} is a Q-sub-algebra of W.

Theorem 3.7. Suppose that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c $\in$W} be a PN-Q-ideal of W, then the sets W($\hat{A}$)={c$\in$W: $\hat{A}_Y(c)$=$\hat{A}_Y(0)$}, W($\hat{C}$)={c$\in$W: $\hat{C}_Y(c)$=$\hat{C}_Y(0)$}, W($\hat{E}$)={c$\in$W: $\hat{E}_Y(c)$= $\hat{E}_Y(0)$}, W($\check{D}$)={c$\in$W: $\check{D}_Y(c)$= $\check{D}_Y(0)$}, and W($\hat{U}$)={c$\in$W: $\hat{U}_Y(c)$= $\hat{U}_Y(0)$} are Q-ideals of W.

Proof. Suppose that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c $\in$W} be a PN-Q-ideal of W. Therefore,

    (i) $\hat{A}_Y(0) \geq \hat{A}_Y(c)$ & $\hat{A}_Y(c * d) \geq$ min{$\hat{A}_Y((c * h) * d)$, $\hat{A}_Y(h)$}, for all c, d, h$\in$W;

    (ii) $\hat{C}_Y(0) \geq \hat{C}_Y(c)$ & $\hat{C}_Y(c * d) \geq$ min{$\hat{C}_Y((c * h) * d)$, $\hat{C}_Y(h)$}, for all c, d, h$\in$W;

    (iii) $\hat{E}_Y(0) \leq \hat{E}_Y(c)$ & $\hat{E}_Y(c * d) \leq$ max {$\hat{E}_Y((c * h) * d)$, $\hat{E}_Y(h)$}, for all c, d, h$\in$W;

    (iv) $\check{D}_Y(0) \leq \check{D}_Y(c)$ & $\check{D}_Y(c * d) \leq$ max {$\check{D}_Y((c * h) * d)$, $\check{D}_Y(h)$}, for all c, d, h$\in$W;

    (v) $\hat{U}_Y(0) \leq \hat{U}_Y(c)$ & $\hat{U}_Y(c * d) \leq$ max {$\hat{U}_Y((c * h) * d)$, $\hat{U}_Y(h)$}, for all c, d, h$\in$W.

Since, $\hat{A}_Y(0)$=$\hat{A}_Y(0)$, so 0 $\in$ W($\hat{A}$).

Since, $\hat{C}_Y(0)$= $\hat{C}_Y(0)$, so 0 $\in$ W($\hat{C}$).

Since, $\hat{E}_Y(0)$= $\hat{E}_Y(0)$, so 0 $\in$ W($\hat{E}$).

Since, $\check{D}_Y(0)$= $\check{D}_Y(0)$, so 0 $\in$ W($\check{D}$).

Since, $\hat{U}_Y(0)$= $\hat{U}_Y(0)$, so 0 $\in$ W($\hat{U}$).

Let (h * d) * e$\in$W($\hat{A}$) and d$\in$W($\hat{A}$). Therefore, $\hat{A}_Y((h * d) * e)$=$\hat{A}_Y(0)$ and $\hat{A}_Y(d)$=$\hat{A}_Y(0)$.

It is clear that $\hat{A}_Y(0) \geq \hat{A}_Y(h * e)$ (1)

Now, we have

$\hat{A}_Y(h * e) \geq$ min{$\hat{A}_Y((h * d) * e)$, $\hat{A}_Y(d)$}= min { $\hat{A}_Y(0)$, $\hat{A}_Y(0)$}= $\hat{A}_Y(0)$

$\Rightarrow \hat{A}_Y(h * e) \geq \hat{A}_Y(0)$ (2)

From (1) and (2), we get

$\hat{A}_Y(h * e)$ = $\hat{A}_Y(0)$.

This implies, h * e$\in$ W($\hat{A}$). Therefore, the set W($\hat{A}$)={c$\in$W: $\hat{A}_Y(c)$=$\hat{A}_Y(0)$} is a Q-ideal of W.

Similarly, it can be shown that, the sets W($\hat{C}$)={c$\in$W: $\hat{C}_Y(c)$= $\hat{C}_Y(0)$}, W($\hat{E}$)={c$\in$W: $\hat{E}_Y(c)$= $\hat{E}_Y(0)$}, W($\check{D}$)={c$\in$W: $\check{D}_Y(c)$= $\check{D}_Y(0)$} and W($\hat{U}$)={c$\in$W: $\hat{U}_Y(c)$= $\hat{U}_Y(0)$} are Q-ideals of W.

Theorem 3.8. Assume that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c $\in$W} be a PN-Q-ideal of Q-algebra W. Then, the fuzzy sets {(c, $\hat{A}_Y(c)$): c$\in$W}, {(c, $\hat{C}_Y(c)$): c$\in$W}, {(c,1- $\hat{E}_Y(c)$): c$\in$W}, {(c,1- $\check{D}_Y(c)$): c$\in$W}, {(c,1- $\hat{U}_Y(c)$): c$\in$W} are fuzzy Q-ideals of W.

Proof. Let Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c $\in$W} be a PN-Q-ideal of a Q-algebra W. Therefore,

    (i) $\hat{A}_Y(0) \geq \hat{A}_Y(c)$ & $\hat{A}_Y(c * d) \geq$ min{$\hat{A}_Y((c * h) * d)$, $\hat{A}_Y(h)$}, for all c, d, h$\in$W;

    (ii) $\hat{C}_Y(0) \geq \hat{C}_Y(c)$ & $\hat{C}_Y(c * d) \geq$ min{$\hat{C}_Y((c * h) * d)$, $\hat{C}_Y(h)$}, for all c, d, h$\in$W;

    (iii) $\hat{E}_Y(0) \leq \hat{E}_Y(c)$ & $\hat{E}_Y(c * d) \leq$ max {$\hat{E}_Y((c * h) * d)$, $\hat{E}_Y(h)$}, for all c, d, h$\in$W;

    (iv) $\check{D}_Y(0) \leq \check{D}_Y(c)$ & $\check{D}_Y(c * d) \leq$ max {$\check{D}_Y((c * h) * d)$, $\check{D}_Y(h)$}, for all c, d, h$\in$W;

    (v) $\hat{U}_Y(0) \leq \hat{U}_Y(c)$ & $\hat{U}_Y(c * d) \leq$ max {$\hat{U}_Y((c * h) * d)$, $\hat{U}_Y(h)$}, for all c, d, h$\in$W.

It is clear that, $\hat{A}_Y(0) \geq \hat{A}_Y(c)$ & $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y((c * h) * d), \hat{A}_Y(h)\}$, for all c, d, h$\in$W. Therefore, the fuzzy set {(c, $\hat{A}_Y(c)$): c$\in$W} is a fuzzy *Q*-ideal of W.

It is clear that, $\hat{C}_Y(0) \geq \hat{C}_Y(c)$ & $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y((c * h) * d), \hat{C}_Y(h)\}$, for all c, d, h$\in$W. Therefore, the fuzzy set {(c, $\hat{C}_Y(c)$): c$\in$W} is a fuzzy *Q*-ideal of W.

Now, for all c, d, h$\in$W,

$\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y((c * h) * d), \hat{E}_Y(h)\} \Rightarrow 1 - \hat{E}_Y(c) \geq \min\{1 - \hat{E}_Y((c * h) * d), 1 - \hat{E}_Y(h)\}$

and $\hat{E}_Y(0) \leq \hat{E}_Y(c) \Rightarrow 1 - \hat{E}_Y(0) \geq 1 - \hat{E}_Y(c)$.

Therefore, the fuzzy set {(c, $1 - \hat{E}_Y(c)$): c$\in$W} is a fuzzy *Q*-ideal of W.

Now, for all c, d, h$\in$W,

$\check{D}_Y(c * d) \leq \max\{\check{D}_Y((c * h) * d), \check{D}_Y(h)\} \Rightarrow 1 - \check{D}_Y(c) \geq \min\{1 - \check{D}_Y((c * h) * d), 1 - \check{D}_Y(h)\}$

and $\check{D}_Y(0) \leq \check{D}_Y(c) \Rightarrow 1 - \check{D}_Y(0) \geq 1 - \check{D}_Y(c)$.

Therefore, the fuzzy set {(c, $1 - \check{D}_Y(c)$): c$\in$W} is a fuzzy *Q*-ideal of W.

Further, for all c, d, h$\in$W,

$\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y((c * h) * d), \hat{U}_Y(h)\} \Rightarrow 1 - \hat{U}_Y(c) \geq \min\{1 - \hat{U}_Y((c * h) * d), 1 - \hat{U}_Y(h)\}$

and $\hat{U}_Y(0) \leq \hat{U}_Y(c) \Rightarrow 1 - \hat{U}_Y(0) \geq 1 - \hat{U}_Y(c)$.

Therefore, the fuzzy set {(c, $1 - \hat{U}_Y(c)$): c$\in$W} is a fuzzy *Q*-ideal of W.

4. Conclusions:

In this paper, we have established the notion of PN-*Q*-ideals of PN-*Q*-algebra. By defining PN-*Q*-ideals, we have formulated some results on PN-*Q*-algebra from the point of view of neutrosophic set. It is just the beginning of the concept of PN-*Q*-algebra. In the future, we hope that based on the notions of PN-*Q*-ideals many new investigations can be done.

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