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Statistical Similarity Analysis Based on Neutrosophic Interval Probability

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Abstract: The roughness of the rock joints has a significant influence on the mechanical properties and deformation behavior of the rock masses. Due to the significant heterogeneity of the surface roughness of rock joints, there may be considerable variabilities, which makes it difficult to accurately determine the roughness. The evaluation of the anisotropic characteristics of joint roughness based on classical probability and statistics cannot reflect the vague, incomplete, imprecise, and indeterminate information of the roughness in different orientations. In this original study, we first propose the generalized Dice similarity measures based on neutrosophic interval statistical number (NISN) for evaluating the similarity of the roughness in different orientations. This method was applied to determine the similarity between the roughness along the sliding direction and each measurement direction. The research results show that this method can effectively determine the similarity between the roughness in different orientations. It may help determine the roughness along the potential sliding direction based on the roughness obtained in the direction with better measurement conditions.

Keywords: neutrosophic interval probability (NIP); neutrosophic interval statistical number (NISN); joint roughness coefficient (JRC); similarity measure

1. Introduction

The joint roughness coefficient (JRC) is one of the important parameters for evaluating the shear strength of rock joints and the stability of rock mass [1-5]. The effect of anisotropy on surface roughness has been proven to be an inherent characteristic of rock joints [6]. In addition, anisotropy can be seen everywhere in rock engineering, and the roughness of rock joints changes directionally, which is a crucial source of anisotropy behavior. Du et al. [7] measured 2180 joint profiles and statistically analyzed the roughness coefficients. The result indicates that the roughness coefficients of type I fractures (joints) and type III fractures (faults) are anisotropic. The anisotropy classification method proposed by Belen [8] showed that the roughness has a strong anisotropy. At present, many studies have been proposed for investigating the anisotropy of rock joints, and the anisotropy in roughness was proved to have a significant effect on the anisotropic shear strength of joints [9-13].

To describe the complex joint morphology, it is necessary to effectively quantify the anisotropic characteristics of the roughness of the joint surface. Chen et al. [14] proposed a geostatistical method to analyze the anisotropy and scale effect of rock joints. Ge et al. [15] believed that the roughness evaluation results in different directions of rock joints are very different, and it is recommended to select reasonable parameters in practice in combination with the sliding direction or the seepage direction. The anisotropy of the joint roughness is an important subject in the field of geotechnical engineering. In many practical situations, because the rock joints are located in different positions of the rock mass, and the empirical estimates obtained by experts along a certain direction are usually inaccurate, the potential sliding directions are different. Therefore, it is always difficult to calculate or provide a definite JRC value. While the neutrosophic number originally proposed by Smarandache [16-18] can express the incomplete and indeterminate information. It can better express the JRC with incomplete and uncertain information under an anisotropy environment. Recently, Ye et al. [19] proposed an approach to study the JRC with incomplete and indeterminate information using neutrosophic functions. Chen et al. [20] introduced a neutrosophic statistical method of JRC neutrosophic numbers for effectively analyzing the scale effect and anisotropy of JRC values, which was also applied by Muhammad [21] for JRC investigation. Smarandache [22] provided the interval function and some basic definitions of neutrosophic probability. Then, the neutrosophic interval statistical number (NISN) and NIP proposed by Chen et al. [23] are made up of both neutrosophic numbers and interval probability. However, there are few studies on the anisotropy of JRC based on the neutrosophy theory.

In the field, the situation of the rock joint surface is very complicated. Taking Laxiwa Hydropower Station as an example [24], there are many joints filled with calcite, but only a few countertops are exposed. Thus, it may be very difficult to measure the roughness of rock joints along some orientations under some conditions. It is a good remedy to calculate the properties of the unknown direction through the properties of rock joints obtained in the direction with better measurement conditions. The similarity measurement method is an important mathematical tool to determine the similarity between two objects [5,25-26]. The Dice similarity measure can be effectively used in the evaluation of the degree of similarity between the studied objects. But this similarity measure has not been applied to the JRC anisotropy assessment. Therefore, this paper extends the generalized Dice similarity measure by NIP, which can solve the problem of not being able to accurately obtain the roughness along the potential sliding direction. The main advantage of the similarity measure method is that it can effectively handle indeterminate information in anisotropic environments.

This paper is formed by the following parts. First, we will introduce the generalized Dice similarity measures between NISNs. Second, insight will be gained into the statistical measurement results of the similarity of joint roughness coefficients in different orientations by the generalized Dice similarity methods. Lastly, it gives conclusions and future study of this paper. These findings are of great significance for solving the problem that the joint roughness along the potential sliding direction cannot be obtained accurately on site.

2. Neutrosophic interval probability and Neutrosophic interval statistical number

Chen et al. (2017) defined the NIP in an interesting range $[x^L, x^U]$ of all individuals in the sample. The form of a NIP was expressed as $P = <[x^L, x^U], (P_T, P_I, P_F)>$, where P_T, P_I, P_F are the true, indeterminate, and false probabilities belonging to the determinate, indeterminate, and failure ranges, respectively. For each trial data, the neutrosophic interval probability by the following equations:

$$\begin{cases} P_T = n_T / n \\ P_I = n_I / n \\ P_F = n_F / n \end{cases}, \quad (1)$$

where n is the total number of the individuals; n_T is the number of samples that fall in the interval $[x_m - \sigma, x_m + \sigma]$; n_I is the number of samples that fall in the interval $[x_m - 3\sigma, x_m - \sigma]$ and $(x_m + \sigma, x_m + 3\sigma)$; n_F is the number of the rest samples. Here, x_m denotes the statistical mean value and σ for standard deviation. The sum of true, indeterminate, and false probabilities is equal to 1.

A NISN R, which is combined NN with the expected value of NIP (confidence level λ), could be expressed as follows:

$$\begin{cases} R = x_m + (1-\lambda)I = x_m + (1-\frac{P_T}{\sqrt{P_T^2+P_I^2+P_F^2}})I, \quad \text{for } I \in [\inf I, \sup I] \\ \lambda = \frac{P_T}{\sqrt{P_T^2+P_I^2+P_F^2}}, \quad \text{for } \lambda \in [0,1] \end{cases} \tag{2}$$

where I denote the indeterminacy and it ranges in the robust interval $[-\sigma, \sigma]$.

The NISN is very suitable to express the interval value under indeterminate environments.

3. Generalized Dice similarity measures between NISNs

Definition 1. Let $R_A = a_A + b_A I$ and $R_B = a_B + b_B I$ be two neutrosophic numbers, where a_A, b_A, a_B, $b_B \geq 0$. A generalized Dice similarity measure between R_A and R_B is defined as follows:

$$\begin{aligned} D(R_A,R_B) &= \frac{2R_A \cdot R_B}{|R_A|^2+|R_B|^2} \\ &= 2\times\frac{(a_A+\inf(b_A I))(a_B+\inf(b_B I))+(a_A+\sup(b_A I))(a_B+\sup(b_B I))}{(a_A+\inf(b_A I))^2+(a_A+\sup(b_A I))^2+(a_B+\inf(b_B I))^2+(a_B+\sup(b_B I))^2} \end{aligned} \tag{3}$$

The generalized Dice similarity measure should satisfy the following properties (P1-P3):

(P1) $0 \leq D(R_A,R_B) \leq 1$;

(P2) $D(R_A,R_B)=1$ if $R_A=R_B$;

(P3) $D(R_A,R_B)=D(R_B,R_A)$.

Definition 2. Let $A=\{R_{A1},R_{A2},\cdots,R_{An}\}$ and $B=\{R_{B1},R_{B2},\cdots,R_{Bn}\}$ be two sets of neutrosophic numbers, where $R_{Ak}=a_{Ak}+b_{Ak}I$ and $R_{Bk}=a_{Bk}+b_{Bk}I$, and (k=1, 2, ⋯, n) a_{Aj}, b_{Aj}, a_{Bj}, $b_{Bj} \geq 0$. Then, the generalized Dice similarity measure between A and B can be calculated by

$$\begin{aligned} D(A,B) &= \sum_{j=1}^{n} w_j \frac{2R_{Aj}\cdot R_{Bj}}{|R_{Aj}|^2+|R_{Bj}|^2} \\ &= 2\times\sum_{j=1}^{n} w_j \frac{(a_{Aj}+\inf(b_{Aj}I))(a_{Bj}+\inf(b_{Bj}I))+(a_{Aj}+\sup(b_{Aj}I))(a_{Bj}+\sup(b_{Bj}I))}{(a_{Aj}+\inf(b_{Aj}I))^2+(a_{Aj}+\sup(b_{Aj}I))^2+(a_{Bj}+\inf(b_{Bj}I))^2+(a_{Bj}+\sup(b_{Bj}I))^2} \end{aligned} \tag{4}$$

The generalized Dice similarity measure of two sets of neutrosophic numbers is satisfied the properties (P4-P5):

(P4) $0 \leq D(A,B) \leq 1$;

(P5) $D(A,B)=1$ if $A=B$;

(P6) $D(A,B)=D(B,A)$.

According to **Definition 1**, the generalized Dice similarity measure between two neutrosophic intervals statistical number R_A and R_B can be expressed as

$$\begin{aligned} D(R_A,R_B) &= \frac{2R_A\cdot R_B}{|R_A|^2+|R_B|^2} \\ &= 2\times\frac{(x_{mA}-\sigma_A(1-\lambda_A))(x_{mB}-\sigma_B(1-\lambda_B))+(x_{mA}+\sigma_A(1-\lambda_A))(x_{mB}+\sigma_B(1-\lambda_B))}{(x_{mA}-\sigma_A(1-\lambda_A))^2+(x_{mA}+\sigma_A(1-\lambda_A))^2+(x_{mB}-\sigma_B(1-\lambda_B))^2+(x_{mB}-\sigma_B(1-\lambda_B))^2} \end{aligned} \tag{5}$$

4. Applications

Anisotropy is one of the basic characteristics of rock joints. The degree of anisotropy of JRC depends on the contact condition between the upper and lower joint surfaces, which controls the shear strength of the rock joint in different directions and the internal hydraulic transmission mechanism. In this study, for the evaluation of the anisotropic characteristics of JRC, statistical analysis is carried out based on classical mathematical methods. In this section, we use the NISNs proposed by Chen et al. (2017) to express the indeterminacy of JRC and then adopt the generalized

Dice similarity measure method to illustrate the similarity of the statistical results of JRC in all directions. To verify the validity and rationality of the proposed statistical similarity analysis based on the NIP, we selected the calcareous slate rock joint collected from Changshan County, Zhejiang Province. The joint surface is hard and complete, the joint wall is dense and slightly weathered, and the joint surface is smooth to rough, fully meets the requirements of this statistical analysis.

In this study, the generalized Dice similarity measure method is developed to estimate the similarity between the potential slip direction and the remaining directions of the structural plane, as described below.

Step1. According to the NIP statistical method, NISNs are utilized to represent the JRC values in each direction.

Table 1. Related value of JRC and NISNs.

Orientation θ (°)	x_m	σ	λ	R
0	10.5425	2.2385	0.921	[10.3651,10.7199]
15	10.0131	2.8392	0.857	[9.6084,10.4178]
30	10.5944	2.3528	0.918	[10.4014,10.7874]
45	9.9244	2.5120	0.884	[9.6321,10.2166]
60	9.0253	2.4592	0.919	[8.8250,9.2255]
75	7.9352	2.1063	0.901	[7.7260,8.1444]
90	7.0467	2.4054	0.728	[6.3935,.6998]
105	7.7766	2.4105	0.930	[7.6080,7.9452]
120	9.1324	2.3250	0.937	[8.9858,9.2790]
135	9.2258	1.9104	0.927	[9.0857,9.3659]
150	10.4673	2.4365	0.866	[9.9325,10.6021]
165	10.6035	2.2090	0.912	[9.9005,10.3066]
180	9.8501	2.1439	0.906	[9.6486,10.0515]
195	9.9383	2.2254	0.916	[9.7522,10.1245]
210	9.5903	1.9444	0.896	[9.3872,9.7933]
225	8.9167	1.9764	0.906	[8.7299,9.1034]
240	7.8582	1.8456	0.921	[7.7130,8.0034]
255	7.2166	1.9341	0.907	[7.0362,7.3970]
270	6.8025	2.1165	0.671	[6.1059,7.4990]
285	7.0061	1.5474	0.916	[6.8767,7.1355]
300	8.4720	1.7448	0.923	[8.3373,8.6068]
315	10.1428	2.4790	0.883	[9.9868,10.2988]
330	9.8295	2.2844	0.910	[9.6230,10.0360]
345	9.6831	2.0192	0.918	[9.5183,9.8479]

First, we conduct a statistical analysis of the JRC values, giving the mean value x_m, the standard deviation σ and the confidence level λ of each orientation θ, and then using equations (1) and (2) to obtain the results of NISNs in each direction, which are shown in Table 1.

Step 2. According to the generalized Dice similarity measure approach, determine the similariti**es** between the potential slip direction and other directions**,** respectively.

Here, we take an azimuth angle of 0° as the reference object and assume that it is the potential slip direction of this case. Based on the obtained NISNs data, we calculate the similarity between 0° orientation and other orientations by equation (5).

Step 3. For determining the similarity between the potential slip direction and other directions, the range of the obtained similarity value is normalized from [0,1] to [−1,1].

To obtain the normalized correlation coefficient χ_k, the results of the similarity measure need to be changed as follows:

$$\chi_k = \frac{2\phi_k - \phi_{\min} - \phi_{\max}}{\phi_{\max} - \phi_{\min}} \tag{6}$$

where $\phi_{\max}$ and $\phi_{\min}$ represent the maximum and minimum values of similarity measure results, respectively.

If the correlation coefficient χ_k is negative 1, it means that this direction does not have the same JRC statistical characteristics as the potential slip direction. However, if the correlation coefficient χ_k of positive 1, the direction has the same JRC statistical characteristics as the potential slip direction.

For example, taking the azimuth angle $\theta = 0^o$ and $\theta = 30^o$ as a set of data to be measured, and the calculation process is as follows.

First, according to Equation (5), the generalized Dice similarity measure value is calculated as

$$\begin{aligned} D(R_A, R_B) &= \frac{2R_A \cdot R_B}{|R_A|^2 + |R_B|^2} \\ &= 2\times\frac{\left(x_{mA}-\sigma_A(1-\lambda_A)\right)\left(x_{mB}-\sigma_B(1-\lambda_B)\right)+\left(x_{mA}+\sigma_A(1-\lambda_A)\right)\left(x_{mB}+\sigma_B(1-\lambda_B)\right)}{\left(x_{mA}-\sigma_A(1-\lambda_A)\right)^2+\left(x_{mA}+\sigma_A(1-\lambda_A)\right)^2+\left(x_{mB}-\sigma_B(1-\lambda_B)\right)^2+\left(x_{mB}-\sigma_B(1-\lambda_B)\right)^2} \\ &= 1.0000 \end{aligned}$$

Then, obtaining the normalized correlation coefficient by using equation (6):

$$\begin{aligned} \chi_k &= \frac{2\phi_k - \phi_{\min} - \phi_{\max}}{\phi_{\max} - \phi_{\min}} \\ &= 0.9997 \end{aligned}$$

For other orientations of the data, we also calculated according to the above steps, the results are shown in Table 2.

It can be seen in Figure 1 that when the sample length is constant, as the orientation θ changes, the generalized similarity measure of JRC changes but without any special rules to follow. When the measurement direction is 30°, the generalized Dice similarity measure value reaches the highest value. While, when the measurement direction is 270°, it shows the smallest similarity. By calculating the variation of similarity in different directions, it can be concluded that the generalized Dice similarity measure value varies due to different measurement directions. In this case, for the orientations range within 60° to 135° and 225°to 300°, the similarity of JRC is small. When the measurement orientations are in the ranges of 15° to 45° and 150° to 195°, the similarity is large.

Through the above analysis, it can be seen that JRC has a significant anisotropy, and the evaluation results of similarity measures obtained in different directions are quite different. Therefore, in practice, according to the similarity measures of JRC, the roughness along the potential slip direction that is difficult to measure can be predicted based on the obtained roughness in the direction with better exposure conditions.

Table 2. Table of normalized values corresponding to each orientation

Orientation (°)	$D(R_A, R_B)$	χ_k
0	1.0000	1.0000
15	0.9984	0.9652
30	1.0000	0.9997
45	0.9981	0.9582
60	0.9880	0.7352
75	0.9610	0.1352
90	0.9228	–0.7101
105	0.9554	0.0128
120	0.9898	0.7735
135	0.9912	0.8042
150	0.9999	0.9897
165	0.9991	0.9799
180	0.9977	0.9489
195	0.9983	0.9615
210	0.9955	0.9011
225	0.9861	0.6929
240	0.9583	0.0768
255	0.9323	–0.5008
270	0.9097	–1.0000
285	0.9220	–0.7290
300	0.9766	0.4807
315	0.9992	0.9834
330	0.9975	0.9457
345	0.9964	0.9201

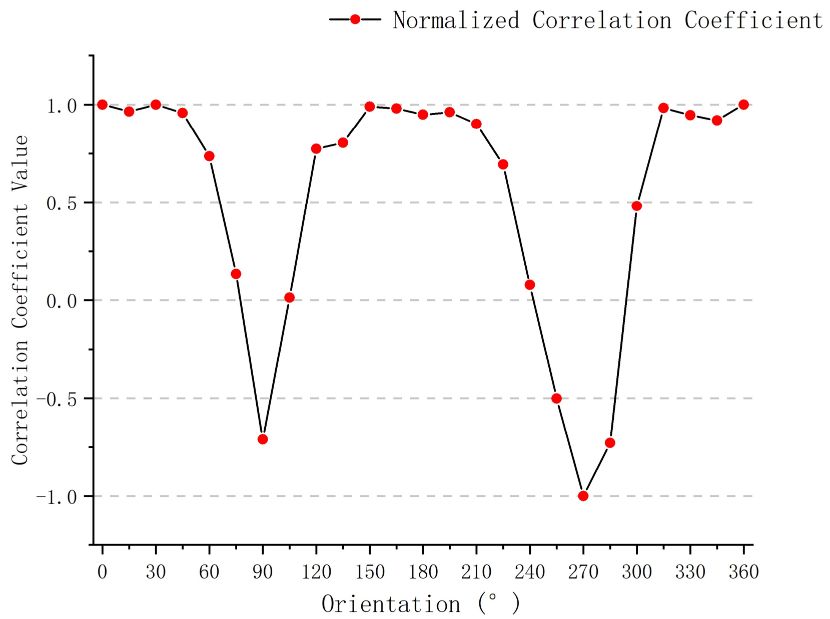

Figure 1. The relationship between orientation and normalized correlation coefficient value

5. Conclusion

In this study, the generalized Dice measures of NISNs are presented to estimate the similarity of the JRC values in different orientations. A quantitative evaluation of the similarity between the potential slip direction and other measurement directions was made. It could be applied to predict the properties of the potential slip direction based on the obtained roughness in the direction with better exposure conditions. Compared to classic statistical methods, for the processing of JRC data, not only are the average value and standard deviation considered, but also the confidence level is introduced, so that some uncertain information can be displayed more specifically. The JRC value of the potential slip direction is taken as the characteristic interval of the reference scale, and the JRC values in the remaining directions are used as the estimated characteristic intervals. Statistical similarity analysis based on neutrosophic interval probability overcomes the deficiency of the existing exposed rock joint area is small, and the required effectiveness information cannot be obtained on the potential slip surface, indicating its necessity in the JRC evaluation.

For future work, we can do more research on the anisotropy of joint surface roughness. For instance, we can propose more statistical analysis models and can also apply the proposed method to different aspects, such as extending the similarity measure to the scale effect of rock joints.

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