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Interval-Type Fuzzy Linear Fractional Programming Problem in Neutrosophic Environment: A Fuzzy Mathematical Programming Approach

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Abstract: This article proposes an interval-valued fuzzy linear fractional programming (LFP) problem, where the coefficients in the objective functions are assumed to be single-valued trapezoidal fuzzy neutrosophic numbers. In addition to this, the coefficients in the constraints are represented by interval-valued fuzzy numbers. Auxiliary models according to different criteria are developed. Fuzzy mathematical programming approach is applied for solving each model by defining membership function. In this work, a linear membership function is used to determine the optimal compromise solutions of the auxiliary models. A numerical example is solved for the illustration and clear explanation of the proposed approach.

Keywords: Interval-type; Fractional programming; Fuzzy numbers; Neutrosophic numbers; Interval- valued fuzzy numbers; Trapezoidal fuzzy numbers; Auxiliary models.

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1. Introduction:

Linear fractional programming (LFP) model was initially developed to determine the two-objective linear programming problem (LPP). In many applications as cutting stock problem, shipping schedules problem, blending problem, etc., the optimization of ratios provides more insight into the situation than the optimization of numerator and denominator individually. Therefore, maximizing a ratio is seen as the simultaneous maximization of numerator and minimization of denominator, its solution considering one solution among several pareto optimal solutions of two-objective model. In daily life situations related problems, policy maker sometimes might face to examine the ratio between actual cost and standard cost, output and employee, inventory and sales, etc., with both denominator and nominator are linear. A LFP problem is only one ratio under linear constraints. For the benchmark trade-off among the simplicity and accuracy of a real-life model, the fractional programming provides more accuracy, and simultaneously wins to avoid the overload of model under considerations. On the other side, the fractional objective incorporated in standard membership function for fuzzy goal, makes them non-linear.

Charnes and Cooper (1962) solved LFP problem as two LP optimization models. They also suggested several applications to a ship routing problem. The fractional programming problem may also be nonlinear type in nature. Dinkelbach (1967) studied nonlinear fractional programming problems, and their methodology. Bitran and Novaes (1973) presented a LPP including the fractional objective function. Many researchers have studied LFP problems (Schaible, 1976; Charnes et al. 1987, and Craven, 1988). Moore (1979) investigated some methods with applications related to interval programming problem. Later on, Gupta and Chakraborty (1998) have applied fuzzy programming approach for obtaining optimal compromise solution for LFP problem under fuzziness. Ammar and Khalifa (2004) presented a parametric solution methodology to solve the multiple criteria LFP problem. Jain and Saksena (2012) have proposed a method for solving fractional programming in the case of there is no completely functional relationship between the decision variables and the objective function. Guzel (2013) developed a proposal to the solution of multiple objective function LFP problems.

Fuzzy set theory firstly introduced by Zadeh (1965). Fuzzy numerical data can be represented by means of fuzzy subsets of the real line known as fuzzy numbers. Decision making in a fuzzy environment has been an improvement and a great help in the management decision problems (Bellman and Zadeh, 1970). Zimmermann (1974) is one of the pioneer researchers in the fuzzy linear programming (FLP). Once of the difficulties occur in the application of mathematical programming is that the parameters in the formulation are not constants but uncertain. The fuzzy nature in a goal programming problem firstly has been discussed by Zimmermann (1978), and lots of others authors working in that field. The decision maker cannot always articulate the goal precisely in a spite of having his/ her decision making experience. Luhandjula (1984) introduced some fuzzy mathematical approaches for solving the multi objective LFP. Dutta et al. (1993) studied the effect of tolerance in fuzzy LFP problem. Sadjadi et al. (2005) presented a new methodology based on fuzzy concept for solving the multiple objective LFP model developed for inventory control problem. Ammar and Khalifa (2009) described the LFP problem considering the fuzzy parameter. Kumar and Dutta (2015) developed LFP with multiple objective functions as an inventory model of multiple items with price-sensitive demand in fuzzy environment. Veeramani and Sumathi (2017) proposed a solution procedure to solve LFP with triangular fuzzy numbers in the objective function cost, the resources, and the technological coefficients. Stanojevic et al. (2020) have introduced two crisp models for solving fuzzy multiple objective LFP problems.

Several researchers presented their work in stochastic LFP programming in fuzzy environment. Over the years, this area has become popular in fractional programming community by means of fuzzy and probabilistic parameters and variables. Chen (2005) presented the fractional programming approach with its application to two inventory models in stochastic environment. Iskander (2003) presented a case of fuzzy weighted objective function. They used various kinds of dominance criteria to linear multiple objective optimization in stochastic fuzzy environment.

Intuitionistic fuzzy sets were first proposed by Atanassov (1986), which have become a very interested topic of research in the area of fuzzy set. Wu and Liu (2013) presented a methodology to solve the multiple attribute group decision making models involving the interval-valued intuitionistic fuzzy numbers. Singh and Yadav (2016) proposed the fuzzy mathematical programming approach for solving LFP problem in intuitionistic fuzzy environment. Ali et al. (2018) studied the LFP with multiple objective functions in intuitionistic fuzzy environment with an application to inventory management. Dutta and Kumar (2015) presented the fuzzy goal programming approach with an application to solve the multiple objective LFP for inventory problem consisted of deteriorating items. Several authors further extended this problem to uncertain environment. Garai and Garg (2019) further extended introduced multi objective LFP problem to possibility and necessity constraints and generalized intuitionistic fuzzy parameters. Nasseri and Bavandi (2019) studied the stochastic LFP model. They further used the fuzzy based method to determine the single objective LFP problem.

Neutrosophic sets were first introduced by Smarandache (2005) as a new theory dealing with the origin, nature and scope of neutralities, as well as their interactions. Neutrosophic sets are the generalizations of the intuitionistic fuzzy sets. Dubois et al. (2005) presented the terminological type difficulties in the theory of fuzzy sets, which caused the case of intuitionistic fuzzy sets. Tian et al. (2016) presented the simplified neutrosophic linguistic normalized weighted bonferroni mean operator. They also presented some applications of neutrosophic set theory to multi-criteria decision-making problems. Thamaraiselvi and Santhi (2016) introduced a mathematical approach to real transportation model in neutrosophic environment. Rizk-Allah et al. (2018) developed a multiple objective transportation model in neutrosophic set environment. Ahmad et al. (2018) presented an algorithm for the computation of multi objective nonlinear optimization problem with single valued neutrosophic hesitant fuzzy criteria. Chakraborty et al. (2019) studied the various kinds of trapezoidal neutrosophic numbers along with the process of de-neutrosophication techniques. They also presented an application based on time and cost optimization method, in sequencing problem. Khalifa et al. (2020) presented a study on optimizing neutrosophic complex programming with the help of lexicographic order. In the application sometimes, determining the membership functions of fuzzy sets is not easy, but the degree of interval membership is easy to determine.

In this paper, linear programming problem with objective function coefficients represented as neutrosophic numbers and interval- valued fuzzy coefficients in the constraints is presented. In the meaning of different criteria, auxiliary models are obtained. Outlay of this article is as described under:

Section 2 presents the related preliminaries. Section 3 formulates neutrosophic linear programming with interval- valued coefficients. In Section 4, the procedure for the solution of the problem is described. In Section 5, we provide a numerical illustration for the efficiency of the solution approach. In the last, we conclude in Section 6**.**

2. Preliminaries:

In order to discuss the problem under consideration, let us introduce some results related to interval- valued fuzzy set and neutrosophic numbers.

Moore (1979) introduced the concept of closed interval number. Let

$$I(\mathbb{R}) = \{[a^-,\ a^+]: a^-,\ a^+ \in \mathbb{R} = (-\infty,\infty), a^- \leq\ a^+\}$$

represent the closed intervals on the real line $\mathbb{R}$. Wu and Liu (2013) defined the closed interval of

$[0,1]$ as $I = [0,1]$, $[I] = \{[a,b]: a \leq b, a, b \in I\}$.

Definition 2.1. (Gorzafczang, 1983). Let X be a non-empty crisp set. A mapping $P: X \longrightarrow [I]$ is said to be an interval- valued fuzzy set (IVFS). All IVFSs on X denoted as $IF(X)$.

Definition 2.2. (Wu and Liu, 2013). Assume that $Q \in IF(X)$, and $Q(x) = [Q^-(x), Q^+(x)]$. Ordinary fuzzy set

$Q^-(x): X \longrightarrow I, x \mapsto Q^-(x), Q^+(x): X \longrightarrow I, x \mapsto Q^+(x)$, are called lower and upper fuzzy sets on Q, respectively.

Definition 2.3. (Wu and Liu, 2013). Assume that $Q \in IF(X)$ and $[\xi_1,\ \xi_2] \in [I]$. The set

$Q[\xi_1,\ \xi_2] = \{x \in X: \xi_1 \leq Q^-(x),\ \xi_2 \leq\ Q^+(x)\}$, is called a $[\xi_1,\ \xi_2]$ −level set of Q.

Let $\mathrm{IFN}(\mathbb{R})$ denotes all interval- valued fuzzy numbers on the real number fields $\mathbb{R}$.

Lemma 2.1. (Wu and Liu, 2013). $Q \in \mathrm{IFN}(\mathbb{R})$, then for any $[\xi_1, \xi_2] \in [I]$, the level set of $Q[\xi_1, \xi_2]$ is an empty set or closed interval.

Lemma 2.2. (Wu and Liu, 2013). $Q \in \mathrm{IFN}(\mathbb{R})$ if and only if

$$Q^-(x) = \begin{cases} R^-(x), x > \psi; \\ L^-(x),\ x \leq \psi. \end{cases} \qquad Q^+(x) = \begin{cases} R^+(x), x > \lambda; \\ L^+(x), x < \mu; \\ 1,\ x \in [\mu, \lambda] \neq \emptyset, \end{cases}$$

where, $L^-(x)\ (0 \leq L^-(x) \leq 1)$ and $L^+(x)(0 \leq L^+(x) \leq 1)$ are increasing right continuous functions, and $\lim_{x \to -\infty} L^-(x) = \lim_{x \to -\infty} L^+(x) = 0$. In addition, $R^-(x)\ (0 \leq R^-(x) \leq 1)$ and $R^+(x)\ (0 \leq R^-(x) \leq 1)$ are decreasing left continuous functions, and $\lim_{x \to +\infty} R^-(x) = \lim_{x \to +\infty} R^+(x) = 0$.

Definition 2.4. (Score and Accuracy functions of single valued trapezoidal neutrosophic number, Thamaraiselvi, 2016). Suppose $\tilde{c}^N = \langle (a_1, a_2,\ a_2, a_2);\ w_c,\ \varpi_c, y_c \rangle$ is a single- valued trapezoidal fuzzy number. Therefore,

i. Score function $S(\tilde{c}^N) = \frac{1}{16}[a_1 +\ a_2 +\ a_2 + a_2] \times \begin{bmatrix} \nu_{\tilde{c}^N} + (1 - \pi_{\tilde{c}^N}) \\ + (1 - \rho_{\tilde{c}^N}) \end{bmatrix}$,

ii. Accuracy function

$$A(\tilde{c}^N) = \frac{1}{16}[a_1 +\ a_2 +\ a_2 + a_2] \times [\nu_{\tilde{c}^N} + \ (1 - \pi_{\tilde{c}^N}) + \ (1 + \rho_{\tilde{c}^N})].$$

3. Problem formulation & solution concepts:

Consider the following interval- valued fuzzy LFP problem in neutrosophic environment as follows:

$$\begin{aligned} &\mathrm{Max}\, \tilde{Z}^N(x) = \frac{\tilde{c}^{N^T} x + \tilde{\alpha}^N}{\tilde{d}^{N^T} x + \tilde{\beta}^N} \\ &\text{Subject to} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1) \\ &\quad x \in \tilde{\Gamma} = \{x : \tilde{A}x \leq \tilde{b}, x \geq 0\}, \end{aligned}$$

where, $\tilde{A} = (\tilde{a}_{ij})_{m \times n}, \tilde{b} = (\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_m), \in \mathrm{IVN}(\mathbb{R})$; $\tilde{c}^N = (\tilde{c}_1{}^N, \tilde{c}_2{}^N \ldots, \tilde{c}_n{}^N)^T$,

$\tilde{d}^N = \left(\tilde{d}_1{}^N, \dots, \tilde{d}_n{}^N\right)^T, \tilde{\alpha}^N$, and $\tilde{\beta}^N$ are single valued trapezoidal fuzzy neutrosophic numbers.

In accordance with Definition 4, problem (1) converted to problem (2) as follows:

$$\max Z(x) = \frac{c^T x+\alpha}{d^T x+\beta}$$

Subject to (2)

$$x \in \tilde{\Gamma} = \{x: \tilde{A}x \le \tilde{b}, x \ge 0\}.$$

Definition 3.1. If $a_{ij} = [a_{ij}{}^-(x), a_{ij}{}^+(x)]$, and $b_i = [b_i{}^-(x), b_i{}^+(x)]$, then the mappings

$$f_{ij}: \mathrm{IFN}(\mathbb{R}) \longrightarrow \mathbb{R}, a_{ij} \longmapsto f_{ij}(a_{ij}) = a_{ij}^0,$$

$g_i: \mathrm{IFN}(\mathbb{R}) \longrightarrow \mathbb{R}, b_i \mapsto g_i(b_i) = b_i^0$, are called fuzzy- crisp transformations, and hence problem becomes

$$\mathrm{Max}\, \tilde{Z}^N(x) = \frac{c^T x+\alpha}{d^T x+\beta} = \frac{M(x)}{N(x)}$$

Subject to (3)

$$x \in \Gamma = \{x: g(x) = A^0 x - b^0 \le 0, x \ge 0\}.$$

Here, problem (3) is the corresponding auxiliary model of problem (2).

There are numerous criterion for the values of a_{ij}^0, and b_i^0 depending on the decision maker's objective:

- The first criterion "choosing big from small".

  Suppose that $\mathrm{Q} \in \mathrm{IFN}(\mathbb{R})$, and $\mathrm{Q(x)} = [\mathrm{Q}^-(\mathrm{x}), \mathrm{Q}^+(\mathrm{x})]$. Then, we have

$$\underset{x\in\mathbb{R}}{\mathrm{Max}}\{\mathrm{Min}(\mathrm{Q}^-(\mathrm{x}), \mathrm{Q}^+(\mathrm{x}))\} = \underset{x\in\mathbb{R}}{\mathrm{Max}}\, \mathrm{Q}^-(\mathrm{x}) = \mathrm{Q}^-(\mathrm{Q}^0). \quad (4)$$

- The second criterion "choosing big from big". Then

$$\underset{x\in\mathbb{R}}{\mathrm{Max}}\{\mathrm{Max}(\mathrm{Q}^-(\mathrm{x}), \mathrm{Q}^+(\mathrm{x}))\} = \underset{x\in\mathbb{R}}{\mathrm{Max}}\, \mathrm{Q}^+(\mathrm{x}) = \mathrm{Q}^+(\mathrm{Q}^0). \quad (5)$$

  In the case that the first criterion is conservative and the second is risky, the compromise criterion is considered.

- The third criterion "Compromise criterion"

$$\mathrm{Q(x)} = \mathrm{k}\, \mathrm{Q}^+(\mathrm{x}) + (1-\mathrm{k})\mathrm{Q}^-(\mathrm{x}). \quad (6)$$

  Here, k is called the optimistic coefficient.

Lemma 3. Let $\mathrm{Q} \in \mathrm{IFN}(\mathbb{R})$. Hence

i. If Q is taken according to the first criterion, then $Q^0 \in Q[\xi_1, \xi_2]$, where

$\xi_1 = \underset{x \in \mathbb{R}}{\mathrm{Max}}\, Q^-(x)$, and ξ_2 any value of the interval $[0,1]$.

ii. If Q is taken referring to the second criterion, then $Q^0 \in Q[\xi_1, \xi_2]$, where

$\xi_1 = \underset{x \in \mathbb{R}}{\mathrm{Max}}\, Q^-(x)$, and ξ_2 any value of the interval $[0, \xi_1]$, and

iii. If Q is taken referring to the third criterion, then $Q^0 \in Q[\xi_1, \xi_2]$, where

$\xi_1 = Q^-(x^*)$, and

$$Q^+(x^*) = \underset{x \in \mathbb{R}}{\mathrm{Max}}\, Q^+(x),\ \xi_2 = Q^+(x^{**}),\ Q^-(x^{**}) = \underset{x \in \mathbb{R}}{\mathrm{Max}}\, Q^-(x)$$

Therefore, with the help of the variable transformation (Charnes and Cooper, 1962; Schaible, 1976), we have

$$y = t\,x \ (\, t \text{ is scalar})$$

It is shown that if for $x \in \Gamma, M(x) \geq 0$, problem (3) with $N(x) > 0$ is an equivalent to

$$\begin{aligned} &\mathrm{Max}\, t\, M\left(\tfrac{y}{t}\right) \\ &\text{Subject to} \\ &\quad t\, g\left(\tfrac{y}{t}\right) \leq 0; t\, N\left(\tfrac{y}{t}\right) \leq 1; y \geq 0, t \geq 0. \end{aligned} \tag{7}$$

In addition, for some $x \in \Gamma, M(x) < 0$, problem (3) is equivalent to

$$\begin{aligned} &\mathrm{Max}\, t\, N\left(\tfrac{y}{t}\right) \\ &\text{Subject to} \\ &\quad t\, g\left(\tfrac{y}{t}\right) \leq 0; -t\, M\left(\tfrac{y}{t}\right) \leq 1; y \geq 0, t \geq 0. \end{aligned} \tag{8}$$

If for $x \in \Gamma, M(x) \geq 0$, then the membership function of the objective function is expressed as follows:

$$\mu(t\,M(x)) = \begin{cases} 1, & \text{if } t\,M(x) \leq \bar{Z}, \\ \frac{t\,M(x)-0}{\bar{Z}-0}, & \text{if } 0 < t\,M(x) < \bar{Z}, \\ 0, & \text{if } t\,M(x) \geq \bar{Z} \end{cases} \tag{9}$$

If for all $x \in \Gamma, M(x) < 0$, then the membership function is expressed as follows:

$$\mu(t\,M(x)) = \begin{cases} 1, & \text{if}\ t\,M(x) \le \underline{Z}, \\ \frac{t\,M(x)-\underline{Z}}{\overline{Z}-\underline{Z}}, & \text{if}\ \underline{Z} < t\,M(x) < \overline{Z}, \\ 0, & \text{if}\ t\,M(x) \le \overline{Z} \end{cases} \tag{10}$$

In Equation (10), $\underline{Z}$, and $\overline{Z}$ are aspiration levels for the minimization and maximization of $tM(x)$, respectively. Using the relations (9) and (10), problem (7) reduced into the following linear programming problem using Zadeh's min operator as

$\text{Max}\,\nu$

Subject to (11)

$$\mu(t\,M(y/t)) \ge \nu, t\,N\left(\frac{y}{t}\right) \le 1, A\left(\frac{y}{t}\right) - b \le 0, y, t \ge 0.$$

Proposition (Chakraborty and Gupta, 2002). If $d^T > 0$, and $\beta > 0$, then we have

$$Z = \frac{c^T x+\alpha}{d^T x+\beta}, x \ge 0, \text{has}\ \overline{Z} = \text{Max}\left\{\frac{c_i}{d_i}, \frac{\alpha}{\beta}, i = 1, 2, \dots, n\right\},$$

$$\text{and}\ \underline{Z} = \text{Min}\left\{\frac{c_i}{d_i}, \frac{\alpha}{\beta}, i = 1, 2, \dots, n\right\},$$

where, $\bar{Z}$, and $\underline{Z}$ are the maximum and minimum values, respectively.

4. Solution procedure:

The steps of the solution method for solving interval- valued fuzzy LFP problem in neutrosophic environment are as follow:

Step 1: Consider problem (1),

Step 2: Convert problem (1) into problem (2), and hence (3),

Step 3: According to different criteria, obtaining problems (4), (5), and (6),

Step 4: Applying variable transformation method with membership functions as given in Equations (9) and (10) to problem (7) as in problem (8).

Step 5: Applying Zadah's min operator, problem (8) is converted into problem (11) which may be solved using any software (like LINGO 18.0 or MATLAB 2020a) for obtaining the optimal compromise solution.

5. Numerical example:

Consider a fractional programming problem as follows:

$$\text{Max}\,\tilde{Z}^N = \frac{\tilde{c}_1^N x_1 + \tilde{c}_2^N x_2}{\tilde{d}_1^N x_1 + \tilde{d}_2^N x_2 + \tilde{\beta}^N}$$

Subject to (12)

$$a_{11}x_1 + a_{12}x_2 \le b_1,$$

$$a_{21}x_1 + a_{22}x_2 \le b_2,$$

$$x_1,\ x_2 \ge 0\,.$$

In Equation (12), let us consider the coefficients as follows:

$a_{11} = [a_{11}^-(x), a_{11}^+(x)] \in IFN(\mathbb{R}),\ {a_{11}}^+ = (3,1,5),\ a_{12} = (3, 2.5, 5.5),$

$b_1 = (15,,13,16),\ a_{21} = (1,0,2),\ a_{22} = (1,1,2),\ b_2 = (1,0,3)$ are $L-R$ fuzzy numbers,

$\tilde{c}_1^N = \langle(-14,-10,-8,-5); 0.3, 0.6, 0.6\rangle,\ \tilde{c}_2^N = \langle(1,3,4,6); 0.6, 0.3\ 0.5\rangle,$

$\tilde{d}_1^N = \langle(0,1,3,6); 0.7, 0.5, 0.3\rangle = \tilde{d}_2^N$, and $\tilde{\beta}^N = \langle(5,9\ 14,19); 0.3, 0.7, 0.6\rangle$ are single-valued trapezoidal neutrosophic numbers.

$$a_{11}^-(x) = \begin{cases} \frac{x-1}{2}, 1 \le x \le 2; \\ \frac{-2x+9}{10}, 2 \le x \le \frac{5}{2}; \\ \frac{-6x+19}{10}, \frac{5}{2} \le x \le 3; \\ \frac{-x+5}{20}, 3 \le x \le 5; \\ 0, \text{Otherwise} \end{cases}$$

In accordance with the Definition 4, with the criterion in Equation (4), we formulate the model as presented in Equation (13):

$$\text{Max}\,Z = \frac{-3x_1 + 2x_2}{x_1 - x_2 + 3}$$

Subject to (13)

$$2x_1 + 3x_2 \le 15,$$

$$x_1 - x_2 \ge 1,$$

$$x_1 \ge 3,$$

$$x_1, x_2 \ge 0.$$

$$\bar{Z} = \text{Max}\left\{-\frac{3}{1}, -\frac{2}{1}, \frac{0}{3}\right\} = 0, \text{ and } \underline{Z} = \text{Min}\left\{-\frac{3}{1}, -\frac{2}{1}, \frac{0}{3}\right\} = -3.$$

Then, problem (13) is transformed to problem (14) as follows

$$\text{Max } \nu$$

$$\text{Subject to} \tag{14}$$

$$3y_1 - 2y_2 + 3\nu \leq 3,$$

$$y_1 - y_2 + 3t \leq 1,$$

$$2y_1 + 3y_2 - 15t \leq 0,$$

$$y_1 - y_2 - t \geq 0,$$

$$y_1 - 3t \geq 0,$$

$$y_1, y_2, t, \nu \geq 0.$$

The solution of problem (14) is given by

$$y_1 = 5.8887 \times e^{-12}, \; y_2 = 1.3614 \times e^{-12},$$

$$t = 1.8565 \times e^{-12}, \quad \nu = 0.999999,$$

$$x_1 = 3.1719, \quad x_2 = 0.7333,$$

$$\text{and} \quad Z = -0.00299586,$$

$$\tilde{Z}^N = \langle\left(\frac{-24}{12}, \frac{-16}{21}, \frac{-3}{37}, \frac{40}{7}\right); 0.3, 0.7, 0.6\rangle. \tag{15}$$

Problem (11) according to the third criterion can be presented as follows

$$\text{Max } \nu$$

$$\text{Subject to} \tag{16}$$

$$3y_1 - 2y_2 + 3\nu \leq 3,$$

$$y_1 - y_2 + 3t \leq 1,$$

$$2.5y_1 + 3y_2 - 15t \leq 0,$$

$$y_1 - y_2 - t \geq 0,$$

$$y_1 - 3t \geq 0,$$

and $$y_1, y_2, t, \nu \geq 0.$$

The solution of problem (16) is given by

$y_1 = 3.19387 \times e^{-12}$, $y_2 = 8.54807 \times e^{-13}$,

$t = 9.50564 \times e^{-13}$, $\nu = 0.999999$,

$x_1 = 1.0849$, $x_2 = 0.8993$, and $Z = -0.45709$,

$$\tilde{Z}^N = \langle \left(\frac{-24}{12}, \frac{-16}{21}, \frac{-3}{37}, \frac{40}{7} \right); 0.3, 0.7, 0.6 \rangle.$$

6. Conclusions:

In this paper, LFP problem with single- valued trapezoidal fuzzy neutrosophic numbers in the objective functions coefficients and interval- valued fuzzy numbers coefficients in the constraints has studied. Auxiliary models according to different criteria have introduced. The solution of the corresponding auxiliary model under its criterion has considered the solution of the original problem according to the subjective and objective factors combination. For further research scope, we can extend the proposed work by introducing the generalized trapezoidal neutrosophic fuzzy number to deal with LFP problems in Neutrosophic fuzzy environment. Second, to extend the proposed work for nonlinear case would be an interesting area of research.

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