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On Refined Neutrosophic Canonical Hypergroups

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Abstract. Refinement of neutrosophic algebraic structure or hyperstructure allows for the splitting of the indeterminate factor into different sub-indeterminate and gives a detailed information about the neutrosophic structure/hyperstructure considered. This paper is concerned with the development of a refined neutrosophic canonical hypergroup from a canonical hypergroup R and sub-indeterminate I_1 and I_2. Several interesting results and examples are presented. The paper also studies refined neutrosophic homomorphisms and establishes the existence of a good homomorphism between a refined neutrosophic canonical hypergroup $R(I_1, I_2)$ and a neutrosophic canonical hypergroup $R(I)$.

Keywords: Neutrosophic, neutrosophic canonical hypergroup, neutrosophic subcanonical hypergroup, refined neutrosophic canonical hypergroup, refined neutrosophic subcanonical hypergroup, refined neutrosophic canonical hypergroup homomorphism.

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1. **Introduction**

The refinement of neutrosophic components of the form $< T_1, T_2, \cdots, T_p; I_1, I_2, \cdots, I_r; F_1, F_2, \cdots, F_s >$ was introduced by Florentine Smarandache in [17] . The birth of this refinement led to the extension of neutrosophic numbers $a + bI$ into refined neutrosophic numbers of the form $(a + b_1I_1 + b_2I_2 + \cdots + b_nI_n)$ where $a, b_1, b_2, \cdots, b_n$ are real or complex numbers. The concept of refined neutrosophic set was later studied using refined neutrosophic number and this paved way for the introduction of refined neutrosophic algebraic structures by Agboola in [5]. Ever since he studied and introduced this structure, several researchers in this field have studied this concept and a great deal of results have been published. For example recently, Ibrahim et al., published in [11–14] their results on refined neutrosophic vector spaces, refined neutrosophic hypergroup and refined neutrosophic hypervector spaces. Also, Adeleke et al., in [1,2] studied refined neutrosophic rings, refined neutrosophic subrings, refined neutrosophic ideals and refined neutrosophic ring homomorphisms. And in [8] Agboola et al., refined neutrosophic algebraic hyperstructures and presented some of its elementary properties. For details about algebraic hyperstructure, neutrosophic structures/hyperstructure and new trends in neutrosophic theory the readers should see [3, 6, 7, 9, 10, 15, 16].

2. Preliminaries

In this section, we will give some definitions, examples and results that will be used in the sequel.

Definition 2.1. [4] If $* : X(I_1, I_2) \times X(I_1, I_2) \mapsto X(I_1, I_2)$ is a binary operation defined on $X(I_1, I_2)$, then the couple $(X(I_1, I_2), *)$ is called a refined neutrosophic algebraic structure and it is named according to the laws (axioms) satisfied by $*$.

For the purposes of this paper, it will be assumed that I splits into two indeterminacies I_1 [contradiction (true (T) and false (F))] and I_2 [ignorance (true (T) or false (F))]. It then follows logically that:

$$\begin{aligned} I_1I_1 &= I_1^2 &&= I_1, \\ I_2I_2 &= I_2^2 &&= I_2 \text{ and} \\ I_1I_2 &= I_2I_1 &&= I_1. \end{aligned}$$

Definition 2.2. [4] Let $(X(I_1, I_2), +, \cdot)$ be any refined neutrosophic algebraic structure where $+$ and . are ordinary addition and multiplication respectively.
For any two elements $(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$, we define

$$(a, bI_1, cI_2) + (d, eI_1, fI_2) = (a + d, (b + e)I_1, (c + f)I_2),$$

$$(a, bI_1, cI_2) \cdot (d, eI_1, fI_2) = (ad, (ae + bd + be + bf + ce)I_1, (af + cd + cf)I_2).$$

Definition 2.3. [4] If $+$ and . are ordinary addition and multiplication, I_k with $k = 1, 2$ have the following properties:

(1) $I_k + I_k + \cdots + I_k = nI_k$.
(2) $I_k + (-I_k) = 0$.
(3) $I_k \cdot I_k \cdot \cdots \cdot I_k = I_k^n = I_k$ for all positive integers $n > 1$.
(4) $0 \cdot I_k = 0$.
(5) I_k^{-1} is undefined and therefore does not exist.

Definition 2.4. [4] Let $(G, *)$ be any group. The couple $(G(I_1, I_2), *)$ is called a refined neutrosophic group generated by G, I_1 and I_2. $(G(I_1, I_2), *)$ is said to be commutative if for all $x, y \in G(I_1, I_2)$, we have $x * y = y * x$. Otherwise, we call $(G(I_1, I_2), *)$ a non -commutative refined neutrosophic group.

Definition 2.5. [4] If $(X(I_1, I_2), *)$ and $(Y(I_1, I_2), *')$ are two refined neutrosophic algebraic structures, the mapping

$$\phi : (X(I_1, I_2), *) \longrightarrow (Y(I_1, I_2), *')$$

is called a neutrosophic homomorphism if the following conditions hold:

(1) $\phi((a, bI_1, cI_2) * (d, eI_1, fI_2)) = \phi((a, bI_1, cI_2)) *' \phi((d, eI_1, fI_2))$.
(2) $\phi(I_k) = I_k$ for all $(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$ and $k = 1, 2$.

Example 2.6. [4] Let $\mathbb{Z}_2(I_1, I_2) = \{(0, 0, 0), (1, 0, 0), (0, I_1, 0), (0, 0, I_2), (0, I_1, I_2), (1, I_1, 0), (1, 0, I_2), (1, I_1, I_2)\}$. Then $(\mathbb{Z}_2(I_1, I_2), +)$ is a commutative refined neutrosophic group of integers modulo 2. Generally for a positive integer $n \geq 2$, $(\mathbb{Z}_n(I_1, I_2), +)$ is a finite commutative refined neutrosophic group of integers modulo n.

Example 2.7. [4] Let $(G(I_1, I_2), *)$ and and $(H(I_1, I_2), *')$ be two refined neutrosophic groups.
Let $\phi : G(I_1, I_2) \times H(I_1, I_2) \to G(I_1, I_2)$ be a mapping defined by $\phi(x, y) = x$ and let
$\psi : G(I_1, I_2) \times H(I_1, I_2) \to H(I_1, I_2)$ be a mapping defined by $\psi(x, y) = y$. Then ϕ and ψ are refined neutrosophic group homomorphisms.

Definition 2.8. [10] Let H be a non-empty set and $\circ : H \times H \longrightarrow P^*(H)$ be a hyperoperation. The couple $(H, \circ)$ is called a hypergroupoid. For any two non-empty subsets A and B of H and $x \in H$, we define

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \quad A \circ x = A \circ \{x\} \quad \text{and} \quad x \circ B = \{x\} \circ B.$$

Definition 2.9. [10] Let H be a non-empty set and let $+$ be a hyperoperation on H. The couple $(H, +)$ is called a canonical hypergroup if the following conditions hold:

(1) $x + y = y + x$, for all $x, y \in H$,
(2) $x + (y + z) = (x + y) + z$, for all $x, y, z \in H$,
(3) there exists a neutral element $0 \in H$ such that $x + 0 = \{x\} = 0 + x$, for all $x \in H$,
(4) for every $x \in H$, there exists a unique element $-x \in H$ such that $0 \in x + (-x) \cap (-x) + x$,
(5) $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$, for all $x, y, z \in H$. A nonempty subset A of H is called a subcanonical hypergroup if A is a canonical hypergroup under the same hyperaddition as that of H that is, for every $a, b \in A$, $a - b \in A$. If in addition $a + A - a \subseteq A$ for all $a \in H$, A is said to be normal.

Definition 2.10. [6] Let $(H, +)$ be any canonical hypergroup and let I be an indeterminate.
Let $H(I) =< H \cup I >= \{(a, bI) : a, b \in H\}$ be a set generated by H and I. The hyperstructure $(H(I), +)$ is called a neutrosophic canonical hypergroup . For all $(a, bI), (c, dI) \in H(I)$ with $b \neq 0$ or $d \neq 0$, we define

$$(a, bI) + (c, dI) = \{(x, yI) : x \in a + c, \ y \in a + d \cup b + c \cup b + d\}.$$

An element $I \in H(I)$ is represented by $(0, I)$ in $H(I)$ and any element $x \in H$ is represented by $(x, 0)$ in $H(I)$. For any nonempty subset $A(I)$ of $H(I)$, we define $-A(I) = \{-(a, bI) = (-a, -bI) : a, b \in H\}$.

Definition 2.11. [6] Let $(H(I), +)$ be a neutrosophic canonical hypergroup .

(1) A nonempty subset $A(I)$ of $H(I)$ is called a neutrosophic subcanonical hypergroup of $H(I)$ if $(A(I), +)$ is itself a neutrosophic canonical hypergroup . It is essential that $A(I)$ must contain a proper subset which is a subcanonical hypergroup of H.
If $A(I)$ does not contain a proper subset which is a subcanonical hypergroup of H, then it is called a pseudo neutrosophic subcanonical hypergroup of $H(I)$.

(2) If $A(I)$ is a neutrosophic subcanonical hypergroup (pseudo neutrosophic subcanonical hypergroup), $A(I)$ is said to be normal in $H(I)$ if for all $(a, bI) \in H(I)$, $(a, bI) + A(I) - (a, bI) \subseteq A(I)$.

Definition 2.12. [6] Let $(H_1(I), +)$ and $(H_2(I), +)$ be two neutrosophic canonical hyper- groups and let
$\phi : H_1(I) \longrightarrow H_2(I)$ be a mapping from $H_1(I)$ into $H_2(I)$.

(1) ϕ is called a homomorphism if :
  (a) ϕ is a canonical hypergroup homomorphism,
  (b) $\phi((0, I)) = (0, I)$.
(2) ϕ is called a good or strong homomorphism if:
  (a) ϕ is a good or strong canonical hypergroup homomorphism,
  (b) $\phi((0, I)) = (0, I)$.
(3) ϕ is called an isomorphism (strong isomorphism) if ϕ is a bijective homomorphism (strong homomorphism).

3. Development of a refined neutrosophic canonical hypergroup

In this section, we study and present the development of refined neutrosophic canonical hypergroup and some of their basic properties.

Definition 3.1. Let $(R, +)$ be any canonical hypergroup. The couple $(R(I_1, I_2), +)$ is a neutrosophic canonical hypergroup generated by R, I_1 and I_2, where $+$ hyperoperations. i.e.,

$$+ : R(I_1, I_2) \times R(I_1, I_2) \longrightarrow 2^{R(I_1,I_2)}.$$

For all $(a, bI_1, cI_2), (d, eI_1, fI_2) \in R(I_1, I_2)$ with $a, b, c, d, e, f \in R$, we define

$$(a, bI_1, cI_2) + (d, eI_1, fI_2) = \{(p, qI_1, rI_2) : p \in a + d, q \in (b + e), r \in (c + f)\}.$$

Lemma 3.2. *Let $(R(I_1, I_2), +)$ be any neutrosophic canonical hypergroup. Let $h_1 = (u, vI_1, tI_2)$, $h_2 = (m, nI_1, kI_2) \in R(I_1, I_2)$ with $u, v, t, m, n, k \in R$. For all $h_1, h_2 \in R(I_1, I_2)$ we have*

(1) $-(-h_1) = -(-u, -vI_1, -tI_2) = (-(-u), -(-v)I_1, -(-t)I_2) = (u, vI_1, tI_2)$.

(2) $(0,0I_1,0I_2)$ *is the unique element such that for every* $h_1 \in R(I_1,I_2)$, *there is an element* $-h_1 \in R(I_1,I_2)$ *with the property* $(0,0I_1,0I_2) \in h_1 - h_2$.

(3) $-(0,0I_1,0I_2) = (0,0I_1,0I_2)$.

(4) $-(h_1+h_2) = -h_1 - h_2$.

Proof. The proof is similar to the proof in classical case. □

Definition 3.3. Let $R(I_1,I_2)$ be a refined neutrosophic canonical hypergroup and let $K(I_1,I_2)$ be a proper subset of $R(I_1,I_2)$. Then

(1) $K(I_1,I_2)$ is said to be a refined neutrosophic subcanonical hypergroup of $R(I_1,I_2)$ if $K(I_1,I_2)$ is a refined neutrosophic canonical hypergroup. It is essential that $K(I_1,I_2)$ contains a proper subset which is a canonical hypergroup.

(2) $K(I_1,I_2)$ is said to be a refined pseudo neutrosophic subcanonical hypergroup of $R(I_1,I_2)$ if $K(I_1,I_2)$ is a refined neutrosophic canonical hypergroup which contains no proper subset which is a canonical hypergroup.

Proposition 3.4. *Every refined neutrosophic canonical hypergroup is a canonical hypergroup.*

Proof. Let $(R(I_1,I_2),+)$ be a refined neutrosophic canonical hypergroup and let $x=(a,bI_1,cI_2), y=(d,eI_1,fI_2), z=(g,hI_1,kI_2) \in R(I_1,I_2)$. Then :

$$
\begin{aligned}
(i)\ x+y &= (a,bI_1,cI_2)+(d,eI_1,fI_2)\\
&= \{(p,qI_1,sI_2): p\in a+d, q\in b+e, s\in c+f\}\\
&= \{(p,qI_1,sI_2): p\in d+a, q\in e+b, s\in f+c\}\\
&= (d,eI_1,fI_2)+(a,bI_1,cI_2)\\
&= y+x.
\end{aligned}
$$

$$
\begin{aligned}
(ii)\ (x+y)+z &= ((a,bI_1,cI_2)+(d,eI_1,fI_2))+(g,hI_1,kI_2)\\
&= \{(p,qI_1,sI_2): p\in a+d, q\in b+e, s\in c+f\}+(g,hI_1,kI_2)\\
&= \{(p',q'I_1,s'I_2): p'\in p+g, q'\in q+h, s'\in s+k\}\\
&= \{(p',q'I_1,s'I_2): p'\in (a+d)+g,\ q'\in (b+e)+h,\ s'\in (c+f)+k\}\\
&= \{(p',q'I_1,s'I_2): p'\in a+(d+g),\ q'\in b+(e+h),\ s'\in c+(f+k)\}\\
&= (a,bI_1,cI_2)+((d,eI_1,fI_2)+(g,hI_1,kI_2))\\
&= x+(y+z).
\end{aligned}
$$

$$
\begin{aligned}
(iii)\ (0,0I_1,0I_2)+(a,bI_1,cI_2) &= \{(p,qI_1,tI_2): p\in 0+a, q\in 0+b, t\in 0+c\}\\
&= \{(p,qI_1,tI_2): p\in \{a\}, q\in \{b\}, t\in \{c\}\}\\
&= \{(a,bI_1,cI_2)\}.
\end{aligned}
$$

Also, it can be shown that $(a, bI_1, cI_2) + (0, 0I_1, 0I_1) = \{(a, bI_1, cI_2)\}$. Hence, there exists a neutral element $(0, 0I_1, 0I_2) \in R(I_1, I_2)$.

$$\begin{aligned}(iv)\ ((a, bI_1, cI_2) + (-a, -bI_1, cI_2)) \quad \cap \quad & ((-a, -bI_1, cI_2) + (a, bI_1, cI_2)) \\ = \quad & \{(p, qI_1, tI_2) : p \in a + (-a), q \in b + (-b), t \in c + (-c)\} \\ & \cap \{(m, nI_1, uI_2) : m \in (-a) + a, n \in (-b) + b, u \in (-c) + c\} \\ = \quad & \{(p, qI_1, tI_2) : p \in \{0\},\ q \in \{0\},\ t \in \{0\}\} \\ & \cap \{(m, nI_1tI_2) : m \in \{0\},\ n \in \{0\},\ t \in \{0\}\}.\end{aligned}$$

Then we can say that $(0, 0I_1, 0I_2) \in ((a, bI_1, cI_2) + (-a, -bI_1, -cI_2)) \cap ((-a, -bI_1, -cI_2) + (a, bI_1, cI_2))$ and therefore, $-(a, bI_1, cI_2)$ is the unique inverse of any $(a, bI_1, cI_2) \in R(I_1, I_2)$.

$$\begin{aligned}(v)\ \text{Let } z \in x + y, \quad i.e \quad & (g, hI_1, kI_2) \in (a, bI_1, cI_2) + (d, eI_1, fI_2).\ Then \\ (g, hI_1, kI_2) \quad \in \quad & \{(p, qI_1, tI_2) : p \in a + d, q \in b + e, t \in c + f\} \\ = \quad & \{(p, qI_1, tI_2) : d \in -a + p, e \in -b + q, f \in -c + t\} \\ = \quad & \{(d, eI_1, fI_2) : d \in -a + p,\ e \in -b + q,\ f \in -c + t\}.\end{aligned}$$

So, we have $(d, eI_1, fI_2) \in -(a, bI_1, cI_2) + (g, hI_1, kI_2)$.

Also we can show that $(a, bI_1, cI_2) \in (g, hI_1, kI_2) - (d, eI_1, fI_2)$. Hence, $z \in x + y$ implies that $x \in z - y$ and $y \in -x + z$. Accordingly, $R(I_1, I_2)$ is a canonical hypergroup. □

Example 3.5. Let $R(I_1, I_2) = \{a_1 = (s, sI_1, sI_2), a_2 = (s, sI_1, tI_2), a_3 = (s, tI_1, sI_2),$ $a_4 = (s, tI_1, tI_2),\ b_1 = (t, tI_1, tI_2), b_2 = (t, tI_1, sI_2), b_3 = (t, sI_1, tI_2), b_4 = (t, sI_1, sI_2)\}$ be a refined neutrosophic set and let $+$ be the hyperoperation on $R(I_1, I_2)$ defined as in the tables below. Let $a = \{a_1, a_2, a_3, a_4\}$ and $b = \{b_1, b_2, b_3, b_4\}$.

TABLE 1. Cayley table for the binary operation " + "

$+$	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
a_1	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
a_2	a_2	$\{a_1, a_2\}$	a_4	$\{a_3, a_4\}$	$\{b_1, b_2\}$	b_1	$\{b_3, b_4\}$	b_3
a_3	a_3	a_4	$\{a_1, a_3\}$	$\{a_2, a_4\}$	$\{b_1, b_3\}$	$\{b_2, b_4\}$	b_1	b_2
a_4	a_4	$\{a_3, a_4\}$	$\{a_2, a_4\}$	a	b	$\{b_1, b_3\}$	$\{b_1, b_2\}$	b_1
b_1	b_1	$\{b_1, b_2\}$	$\{b_1, b_3\}$	b	$R(I_1, I_2)$	$\{a_2, a_4, b_1, b_3\}$	$\{a_3, a_4, b_1, b_2\}$	$\{a_4, b_1\}$
b_2	b_2	b_1	$\{b_2, b_4\}$	$\{b_1, b_3\}$	$\{a_2, a_4, b_1, b_3\}$	$\{a_1, a_3, b_2, b_4\}$	$\{a_4, b_1\}$	$\{a_3, b_2\}$
b_3	b_3	$\{b_3, b_4\}$	b_1	$\{b_1, b_2\}$	$\{a_3, a_4, b_1, b_2\}$	$\{a_4, b_1\}$	$\{a_1, a_2, b_3, b_4\}$	$\{a_2, b_3\}$
b_4	b_4	b_3	b_2	b_1	$\{a_4, b_1\}$	$\{a_3, b_2\}$	$\{a_2, b_3\}$	$\{a_1, b_4\}$

It is clear from the table that $(R(I_1, I_2), +)$ is a refined neutrosophic canonical hypergroups.

Example 3.6. Let $R = \{0, u, v\}$ and define " + " on R as follows

TABLE 2. Cayley table for the hyper operation " + "

$+$	0	u	v
0	0	u	v
u	u	$\{0, u\}$	v
v	v	v	$\{0, u, v\}$

Let $R(I_1, I_2) = \{a_1 = (0, 0I_1, 0I_2), a_2 = (0, 0I_1, uI_2), a_3 = (0, 0I_1, vI_2), a_4 = (0, uI_1, 0I_2),$ $a_5 = (0, vI_1, 0I_2), a_6 = (0, uI_1, vI_2), a_7 = (0, vI_1, uI_2), a_8 = (0, uI_1, uI_2), a_9 = (0, vI_1, vI_2),$ $b_1 = (u, uI_1, uI_2), b_2 = (u, uI_1, 0I_2), b_3 = (u, uI_1, vI_2), b_4 = (u, 0I_1, uI_2), b_5 = (u, vI_1, uI_2),$ $b_6 = (u, 0I_1, vI_2), b_7 = (u, vI_1, 0I_2), b_8 = (u, 0I_1, 0I_2), b_9 = (u, vI_1, vI_2), c_1 = (v, vI_1, vI_2),$ $c_2 = (v, vI_1, 0I_2), c_3 = (v, vI_1, uI_2), c_4 = (v, 0I_1, vI_2), c_5 = (v, uI_1, vI_2), c_6 = (v, 0I_1, uI_2),$ $c_7 = (v, uI_1, 0I_2), c_8 = (v, 0I_1, 0I_2), c_9 = (v, uI_1, uI_2)\}$ be a refined neutrosophic set.

For any $(x, yI_1, zI_1), (p, qI_1, rI_2) \in R(I_1, I_2)$ define the hyperoperation $+'$ by

$$(x, yI_1, zI_1) +' (p, qI_1, rI_2) = \{(m, nI_1, sI_2) : m \in x + p, n \in y + q, s \in z + r\}.$$

Then $(R(I_1, I_2), +')$ is a refined neutrosophic canonical hypergroup.

Proposition 3.7. *Let $(R(I_1, I_2), +_1)$ be a refined neutrosophic canonical hypergroup and let $(K, +_2)$ be a canonical hypergroup. Define for all $(x_1, k_1), (x_2, k_2) \in R(I_1, I_2) \times K$ the hyperoperation $''+''$ by*

$$(x_1, k_1) + (x_2, k_2) = \{(x_3, k_3) : x_3 \in x_1 +_1 x_2,\ k_3 \in k_1 +_2 k_2\}.$$

Where $x_i = (a_i, b_iI_1, c_iI_2)$ for $i = 1, 2 \cdots n$. Then $(R(I_1, I_2) \times K, +)$ is a refined neutrosophic canonical hypergroup.

Proof. Let $(x_1, k_1), (x_2, k_2), (x_3, k_3) \in R(I_1, I_2) \times K$ for $x_1, x_2, x_3 \in R(I_1, I_2)$ and $k_1, k_2, k_3 \in K$. Then :

(1) For commutativity;

$$\begin{aligned}
(x_1, k_1) + (x_2, k_2) &= ((a_1, b_1I_1, c_1I_2), k_1) + ((a_2, b_2I_1, c_2I_2), k_2) \\
&= \{((p, qI_1, sI_2), k) : p \in a_1 +_1 a_2,\ q \in b_1 +_1 b_2,\ s \in c_1 +_1 f_2,\ k \in k_1 +_2 k_2\} \\
&= \{((p, qI_1, sI_2), k) : p \in a_2 +_1 a_1,\ q \in b_2 +_1 b_1,\ s \in c_2 +_1 c_1,\ k \in k_2 +_2 k_1\} \\
&= ((a_2, b_2I_1, c_2I_2), k_2) + ((a_1, b_1I_1, c_1I_2), k_1) \\
&= (x_2, k_2) + (x_1, k_2).
\end{aligned}$$

(2) For associativity;

$$\begin{aligned}
[(x_1, k_1) + (x_2, k_2)] + (x_3, k_3) &= [((a_1, b_1I_1, c_1I_2), k_1) + ((a_2, b_2I_1, c_2I_2), k_2)] + ((a_3, b_3I_1, c_3I_2), k_3) \\
&= \{((p, qI_1, sI_2), k) : p \in a_1 +_1 a_2,\ q \in b_1 +_1 b_2,\ s \in c_1 +_1 c_2, \\
&\quad k \in k_1 +_2 k_2\} + ((a_3, b_3I_1, c_3I_2), k_3) \\
&= \{((p', q'I_1, s'I_2), k') : p' \in p +_1 a_3,\ q' \in q +_1 b_3,\ s' \in s +_1 c_3, \\
&\quad k' \in k +_2 k_3\} \\
&= \{((p', q'I_1, s'I_2), k') : p' \in (a_1 +_1 a_2) +_1 a_3,\ q' \in (b_1 +_1 b_2) +_1 b_3, \\
&\quad s' \in (c_1 +_1 c_2) +_1 c_3,\ k' \in (k_1 +_2 k_2) +_2 k_3\} \\
&= \{((p', q'I_1, s'I_2), k') : p' \in a_1 +_1 (a_2 +_1 a_3),\ q' \in b_1 +_1 (b_2 +_1 b_3), \\
&\quad s' \in c_1 +_1 (c_2 +_1 c_3),\ k' \in k_1 +_2 (k_2 +_2 k_3)\} \\
&= ((a_1, bI_1, c_1I_2), k_1) + [((a_2, b_2I_1, c_2I_2), k_2) + ((a_3, b_3I_1, c_3I_2), k_3)] \\
&= (x_1, k_1) + [(x_2, k_2) + (x_3, k_3)].
\end{aligned}$$

(3) Existence of inverse element:

We want to show that the element $((0, 0I_1, 0I_2), 0_k)$ is the neutral element in $R(I_1, I_2) \times K$.

Where 0_k is the neutral element in K. Now, consider

$$\begin{aligned}
((0, 0I_1, 0I_2), 0_K) + ((a_1, b_1I_1, c_1I_2), k_1) &= \{((p, qI_1, tI_2), k) : p \in 0 +_1 a_1, q \in 0 +_1 b_1,\ t \in 0 +_1 c_1, \\
&\quad k \in 0_K +_2 k_1\} \\
&= \{((p, qI_1, tI_2), k)) : p \in \{a_1\}, q \in \{b_1\}, t \in \{c_1\},\ k \in \{k_1\} \\
&= \{((a_1, b_1I_1, c_1I_2), k_1\}.
\end{aligned}$$

Similarly, we can show that

$$((a_1, b_1I_1, c_1I_2), k_1) + ((0, 0I_1, 0I_1), 0_K) = \{((a_1, b_1I_1, c_1I_2), k_1)\}.$$

Hence, we can conclude that there exists a neutral element $((0, 0I_1, 0I_2), 0_K) \in R(I_1, I_2) \times K$.

(4) Existence of unique inverse:

We want to show that there exist a unique inverse for any $((a_1, b_1I_1, c_1I_2), k_1) \in R(I_1, I_2) \times K$.

Now, consider

$$\begin{aligned}
[((a_1, b_1I_1, c_1I_2), k_1) &+ ((-a_1, -b_1I_1, c_1I_2), -k_1)] \cap [((-a, -bI_1, -cI_2), -k_1) + ((a_1, bI_1, c_1I_2), k_1)] \\
&= \{((p, qI_1, tI_2), k) : p \in a_1 +_1 (-a_1), q \in b_1 +_1 (-b_1), t \in c_1 +_1 (-c_1), \\
&\quad k \in k_1 +_2 (-k_1)\} \\
&\quad \cap \{((m, nI_1, uI_2), k') : m \in -a_1 +_1 a_1, n \in -b_1 +_1 b_1, u \in -c +_1 c, \\
&\quad k' \in -k_1 +_2 k_1\} \\
&= \{((p, qI_1, tI_2), k) : p \in \{0\},\ q \in \{0\},\ t \in \{0\},\ k \in \{0_K\} \\
&\quad \cap \{((m, nI_1uI_2), k') : m \in \{0\},\ n \in \{0\},\ u \in \{0\},\ k' \in \{0_K\}.
\end{aligned}$$

Then we can say that

$((0, 0I_1, 0I_2), 0_K) \in (((a_1, b_1I_1, c_1I_2), k_1) + ((-a_1, -b_1I_1, -c_1I_2), -k_1) \cap (((-a_1, -b_1I_1, -c_1I_2), -k_1) + ((a_1, b_1I_1, c_1I_2), k_1)$ and therefore, $-((a_1, b_1I_1, c_1I_2), k_1)$ is the unique inverse of any $((a_1, b_1I_1, c_1I_2), k_1) \in R(I_1, I_2) \times K$.

(5) Let $(x_3, k_3) \in (x_1, k_1) + (x_2, k_2)$, *i.e.*, $((a_3, b_3I_1, c_3I_2), k_3) \in ((a_1, b_1I_1, c_1I_2), k_1) + ((a_2, b_2I_1, c_2I_2), k_2)$. Then

$$\begin{aligned}
((a_3, b_3I_1, c_3I_2), k_3) &\in \{((p, qI_1, tI_2), k) : p \in a_1 +_1 a_2,\ q \in b_1 +_1 b_2,\ t \in c_1 +_1 c_2,\ k \in k_1 +_2 k_2\} \\
&= \{((p, qI_1, tI_2), k) : a_2 \in -a_1 +_1 p, b_2 \in -b_1 +_1 q,\ c_2 \in -c_1 +_1 t, \\
&\quad k_2 \in -k_1 +_2 k\} \\
&= \{((a_2, b_2I_1, c_2I_2), k_2) : a_2 \in -a_1 +_1 p,\ b_2 \in -b_1 +_1 q,\ c_2 \in -c_1 +_1 t, \\
&\quad k_2 \in -k_1 +_2 k\}.
\end{aligned}$$

So, we have $((a_2, b_2I_1, c_2I_2), k_2) \in -((a_1, b_1I_1, c_1I_2), k_1) + ((a_3, b_3I_1, c_3I_2), k_3)$.

Also, we can show that $((a_1, b_1I_1, c_1I_2), k_1) \in ((a_3, b_3I_1, c_3I_2), k_3) - ((a_2, b_2I_1, c_2I_2), k_2)$. Hence, $(x_3, k_3) \in (x_1, k_1) + (x_2, k_2)$ implies that $(x_1, k_1) \in (x_3, k_3) - (x_2, k_2)$ and $(x_2, k_2) \in -(x_1, k_1) + (x_3, k_3)$.

Accordingly, $R(I_1, I_2)$ is a refined neutrosophic canonical hypergroup. □

Proposition 3.8. *Let $(R(I_1, I_2), +_1)$ and $(K(I_1, I_2), +_2)$ be two refined neutrosophic canonical hypergroup. Define for all $(x_1, k_1), (x_2, k_2) \in R(I_1, I_2) \times K(I_1, I_2)$ the hyperoperations $''+''$ by*

$$(x_1, k_1) + (x_2, k_2) = \{(x_3, k_3) : x_3 \in x_1 +_1 x_2,\ k_3 \in k_1 +_2 k_2\}.$$

Where $x_i = (a_i, b_iI_1, c_iI_2)$ and $k_i = (u_i, v_iI_1, s_iI_2)$ for $i = 1, 2 \cdots n$.
Then $(R(I_1, I_2) \times K(I_1, I_2), +)$ is a refined neutrosophic canonical hypergroup.

Proof. The proof is similar to the prof of Proposition 3.7 . □

Lemma 3.9. *Let $R(I_1, I_2)$ be a refined neutrosophic canonical hypergroup. A non-empty subset $K(I_1, I_2)$ of $R(I_1, I_2)$ is a refined neutrosophic subcanonical hypergroup if and only if for $k_1 = (p_1, q_1 I_1, s_1 I_1), k_2 = (p_2, q_2 I_1, s_2 I_1) \in K(I_1, I_2)$ the following conditions hold:*

(1) $k_1 - k_2 \subseteq K(I_1, I_2)$,

(2) $K(I_1, I_2)$ *contains a proper subset which is a canonical hypergroup.*

Proposition 3.10. *Let $M(I_1, I_2)$ and $N(I_1, I_2)$ be any two refined neutrosophic subcanonical hypergroups of a refined neutrosophic canonical hypergroup $R(I_1, I_2)$ and let K be a subcanonical hypergroup of R. Then,*

(1) $M(I_1, I_2) + N(I_1, I_2)$ *is a refined neutrosophic subcanonical hypergroup of* $R(I_1, I_2)$.

(2) $M(I_1, I_2) \cap N(I_1, I_2)$ *is a refined neutrosophic subcanonical hypergroup of* $R(I_1, I_2)$.

(3) $M(I_1, I_2) + K$ *is a refined neutrosophic subcanonical hypergroup of* $R(I_1, I_2)$.

Proof. (1) It is clear that $(0, 0I_1, 0I_2) \in M(I_1, I_2) + N(I_1, I_2)$ since $M(I_1, I_2)$ and $N(I_1, I_2)$ are refined neutrosophic subcanonical hypergroup.

Let $(x, yI_1, zI_2), (u, vI_1 wI_2) \in M(I_1, I_2) + N(I_1, I_2)$. Where $x = x_1 + x_2, y = y_1 + y_2$, $z = z_1 + z_2, u = u_1 + u_2, v = v_1 + v_2$ and $w = w_1 + w_2$. With $x_1, y_1, z_1, u_1, v_1, w_1 \in M$ and $x_2, y_2, z_2, u_2, v_2, w_2 \in N$. Then

$$\begin{aligned}
(x, yI_1, zI_2) - (u, vI_1 wI_2) &= ((x_1 + x_2), (y_1 + y_2)I_1, (z_1 + z_2)I_2) - \\
&\quad ((u_1 + u_2), (v_1 + v_2)I_1, (w_1 + w_2)I_2) \\
&= ((x_1 + x_2) - (u_1 + u_2), ((y_1 + y_2) - (v_1 + v_2))I_1, \\
&\quad ((z_1 + z_2) - (w_1 + w_2)I_2) \\
&= \{(p, qI_1, rI_2) : p \in (x_1 - u_1) + (x_2 - u_2), q \in (y_1 - v_1) + (y_2 - v_2), \\
&\quad r \in (z_1 - w_1) + (z_2 - w_2)\} \\
&\subseteq M(I_1, I_2) + N(I_1, I_2).
\end{aligned}$$

Now, since the refined neutrosophic subcanonical hypergroups $M(I_1, I_2)$ and $N(I_1, I_2)$ contain a proper subset M and N respectively, which are canonical hypergroups. Then, $M + N$ is a canonical hypergroup which is contained in $M(I_1, I_2) + N(I_1, I_2)$. Hence $M(I_1, I_2) + N(I_1, I_2)$ is a refined neutrosophic subcanonical hypergroup of $R(I_1, I_2)$.

(2) The proof is similar to the proof in classical case.

(3) The proof follows the same approach as the proof of 1. □

Remark 3.11. It should be noted that if $M(I_1, I_2)$ is a refined pseudo neutrosophic subcanonical hypergroup of a refined neutrosophic canonical hypergroup $R(I_1, I_2)$ and K is a subcanonical hypergroup of a canonical hypergroup R. Then $M(I_1, I_2) + K$ is a refined neutrosophic subcanonical hypergroup of $R(I_1, I_2)$.

Definition 3.12. Let $R(I_1, I_2)$ be a refined neutrosophic canonical hypergroup. The refined neutrosophic subcanonical hypergroup $M(I_1, I_2)$ is said to be normal in $R(I_1, I_2)$ if

$$(a, bI_1, cI_2) + M(I_1, I_2) - (a, bI_1, cI_2) \subseteq M(I_1, I_2) \text{ for all } (a, bI_1, cI_2) \in R(I_1, I_2).$$

Definition 3.13. Let $M(I_1, I_2)$ be a normal refined neutrosophic subcanonical hypergroup of a refined neutrosophic canonical hypergroup $R(I_1, I_2)$. The quotient $R(I_1, I_2)/M(I_1, I_2)$ is defined by the set

$$\{r + M(I_1, I_2) : r = (x, yI_1, zI_2) \in R(I_1, I_2)\}.$$

Proposition 3.14. *Let $R(I_1, I_2)/M(I_1, I_2) = \{r + M(I_1, I_2) : r = (x, yI_1, zI_2) \in R(I_1, I_2)\}$. For $r_1 + M(I_1, I_2), r_2 + M(I_1, I_2) \in R(I_1, I_2)/M(I_1, I_2)$, if $r_1 + M(I_1, I_2) \cap r_2 + M(I_1, I_2) \neq \emptyset$ then $r_1 + M(I_1, I_2) = r_2 + M(I_1, I_2)$.*

Proof. Let $r_3 \in r_1 + M(I_1, I_2) \cap r_2 + M(I_1, I_2)$ i.e.,

$$(x_3, y_3I_1, z_3I_2) \in (x_1, y_1I_1, z_1I_2) + M(I_1, I_2) \cap (x_2, y_2I_1, z_2I_2) + M(I_1, I_2).$$

Obviously,

$$(x_3, y_3I_1, z_3I_2) \in (x_1, y_1I_1, z_1I_2) + M(I_1, I_2) \text{ and } (x_3, y_3I_1, z_3I_2) \in (x_2, y_2I_1, z_2I_2) + M(I_1, I_2).$$

So, for $m_1 = (u_1, v_1I_1, t_1I_2), m_2 = (u_2, v_2I_1, t_2I_2) \in M(I_1, I_2)$, with $u_1, u_2, u_3, v_1, v_2, v_3, t_1, t_2, t_3 \in M$, we have

$$(x_3, y_3I_1, z_3I_2) \in (x_1, y_1I_1, z_1I_2) + (u_1, v_1I_1, t_1I_2) \text{ and } (x_3, y_3I_1, z_3I_2) \in (x_2, y_2I_1, z_2I_2) + (u_2, v_2I_1, t_2I_2).$$

$$(x_3, y_3I_1, z_3I_2) \in (x_1 + u_1, (y_1 + v_1)I_1, (z_1 + t_1)I_2) \text{ and } (x_3, y_3I_1, z_3I_2) \in (x_2 + u_2, (y_2 + v_2)I_1, (z_2 + t_2)I_2),$$

$$\implies x_3 \in x_1 + u_1, y_3 \in y_1 + v_1, z_3 \in z_1 + t_1 \text{ and } x_3 \in x_2 + u_2, y_3 \in y_2 + v_2, z_3 \in z_2 + t_2.$$

Since $x_3 \in x_1 + u_1, y_3 \in y_1 + v_1, z_3 \in z_1 + t_1$ implies $x_1 \in x_3 - u_1, y_1 \in y_3 - v_1, z_1 \in z_3 - t_1$.
Then we have

$$x_1 \in x_3 - u_1 \subseteq (x_2 + u_2) - u_1 = x_2 + (u_2 - u_1) \subseteq x_2 + M,$$

$$y_1 \in y_3 - v_1 \subseteq (y_2 + v_2) - v_1 = y_2 + (v_2 - v_1) \subseteq y_2 + M,$$

$$z_1 \in z_3 - t_1 \subseteq (z_2 + t_2) - t_1 = z_2 + (t_2 - t_1) \subseteq z_2 + M,$$

$$\implies (x_1, y_1I_1, z_1I_2) \subseteq (x_2, y_2I_1, z_2I_2) + M(I_1, I_2).$$

$$\therefore (x_1, y_1I_1, z_1I_2) + M(I_1, I_2) \subseteq (x_2, y_2I_1, z_2I_2) + M(I_1, I_2) + M(I_1, I_2) = (x_2, y_2I_1, z_2I_2) + M(I_1, I_2).$$

□

Similarly it can be shown that $(x_2, y_2I_1, z_2I_2) + M(I_1, I_2) \subseteq (x_2, y_2I_1, z_2I_2) + M(I_1, I_2)$. Hence the proof.

Proposition 3.15. *Let $M(I_1, I_2)$ be a normal refined neutrosophic subcanonical hypergroup of a refined neutrosophic canonical hypergroup $R(I_1, I_2)$. Let $R(I_1, I_2)/M(I_1, I_2)$ be as defined in Proposition 3.14. For all $r_1 + M(I_1, I_2), r_2 + M(I_1, I_2) \in R(I_1, I_2)/M(I_1, I_2)$ define the hyperoperation $+'$ by*

$$r_1 + M(I_1, I_2) +' r_2 + M(I_1, I_2) = (r_1 +' r_2) + M(I_1, I_2).$$

Then, $(R(I_1, I_2)/M(I_1, I_2), +')$ is a neutrosophic canonical hypergroup if R/M is a canonical hypergroup.

Definition 3.16. Let $(R(I_1, I_2), +_1)$ and $(M(I_1, I_2), +_2)$ be any two refined neutrosophic canonical hypergroups and let

$$\phi : R(I_1, I_2) \longrightarrow M(I_1, I_2)$$

be a mapping from $R(I_1, I_2)$ into $M(I_1, I_2)$.

(1) ϕ is called a refined neutrosophic canonical hypergroup homomorphism if:
   (a) for all x, y of $R(I_1, I_2)$, $\phi(x +_1 y) \subseteq \phi(x) +_2 \phi(y)$,
   (b) $\phi(0, 0I_1, 0I_2) = (0, 0I_1, 0I_2)$,
   (c) $\phi(I_k) = I_k$ for $\quad k = 1, 2$.
(2) ϕ is called a good refined neutrosophic canonical hypergroup homomorphism if:
   (a) for all x, y of $R(I_1, I_2)$, $\phi(x +_1 y) = \phi(x) +_2 \phi(y)$,
   (b) $\phi(0, 0I_1, 0I_2) = (0, 0I_1, 0I_2)$,
   (c) $\phi(I_k) = I_k$ for $\quad k = 1, 2$.
(3) ϕ is called a refined neutrosophic isomorphism if ϕ is a refined neutrosophic homomorphism and ϕ^{-1} is also a refined neutrosophic homomorphism.

Definition 3.17. Let $\phi : R(I_1, I_2) \longrightarrow M(I_1, I_2)$ be a refined neutrosophic canonical hypergroup homomorphism from a refined neutrosophic canonical hypergroup $R(I_1, I_2)$ into a refined neutrosophic canonical hypergroup $M(I_1, I_2)$.

(1) The kernel of ϕ denoted by $Ker\phi$ is the set $\{(u, vI_1, wI_2) \in R(I_1, I_2) : \phi((u, vI_1, wI_2)) = (0, 0I_1, 0I_2)\}$.
(2) The image of ϕ denoted by $Im\phi$ is the set $\{\phi((u, vI_1, wI_2)) : (u, vI_1, wI_2) \in R(I_1, I_2)\}$.

Proposition 3.18. *Let $\phi : R(I_1, I_2) \longrightarrow M(I_1, I_2)$ be a refined neutrosophic canonical hypergroup homomorphism from a refined neutrosophic canonical hypergroup $R(I_1, I_2)$ into a refined neutrosophic canonical hypergroup $M(I_1, I_2)$.*

(1) *The kernel of ϕ is not a neutrosophic subcanonical hypergroup of $R(I_1, I_2)$.*
(2) *The image of ϕ is a neutrosophic subcanonical hypergroup of $M(I_1, I_2)$.*

Proof. (1) It can be seen from the definition of Kernel that $Ker\phi$ is a subcanonical hypergroup and not a neutrosophic subcanonical hypergroup.
(2) The proof is similar to the proof in classical case. □

Remark 3.19. If ϕ in Proposition 3.18 is a good refined neutrosophic canonical hypergroup homomorphism and $P(I_1, I_2)$ is a normal refined neutrosophic subcanonical hypergroup of $R(I_1, I_2)$ then $\phi(P(I_1, I_2))$ is normal in $M(I_1, I_2)$. Also, if $Q(I_1, I_2)$ is a normal refined neutrosophic subcanonical hypergroup of $M(I_1, I_2)$, then $\phi^{-1}(Q(I_1, I_2))$ is normal in $R(I_1, I_2)$.

In what follows we shall establish the relationship between the refined neutrosophic canonical hypergroups and the parent or any neutrosophic canonical hypergroups. Since every neutrosophic (refined neutrosophic) canonical hypergroup is a canonical hypergroup. Then, our task will be to find a classical map ψ say, such that

$$\psi : R(I_1, I_2) \longrightarrow R(I).$$

And for all $(u, vI_1, wI_2) \in R(I_1, I_2)$ we define ψ by

$$\psi((u, vI_1, wI_2)) = (u, (v + w)I).$$

Proposition 3.20. *Let $(R(I_1, I_2), +')$ be a refined neutrosophic canonical hypergroup and let $(R(I), +)$ be a neutrosophic canonical hypergroup. The mapping ψ defined above is a good homomorphism.*

Proof. It can be easily shown that ψ is well defined.
Now, for $(u, vI_1, wI_2), (p, qI_1, tI_2) \in R(I_1, I_2)$ then

$$\begin{aligned} \psi((u, vI_1, wI_2) +' (p, qI_1, tI_2)) &= \psi((u + p), (v + q)I_1, (w + t)I_2) \\ &= ((u + p), (v + q + w + t)I) \\ &= ((u + p), (v + w)I + (q + t)I) \\ &= (u, (v + w)I) + (p, (q + t)I) \\ &= \psi((u, vI_1, wI_2)) +' \psi((p, qI_1, tI_2)). \end{aligned}$$

Hence ψ is a good homomorphism. □

Remark 3.21. The kernel of this map is given by

$$\begin{aligned} ker\psi &= \{(u, vI_1, wI_2) : \psi((u, vI_1, wI_2)) = (0, 0I_1, 0I_2)\} \\ &= \{(0, vI_1, (-v)I_2)\}. \end{aligned}$$

It can be shown that $ker\psi$ is a subcanonical hypergroup of $R(I_1, I_2)$.

4. Conclusions

This paper studied refinement of neutrosophic canonical hypergroup and presented some of its basic properties. Also, the existence of a good homomorphism between a refined neutrosophic canonical hypergroup $R(I_1, I_2)$ and a neutrosophic canonical hypergroup $R(I)$ was established. We hope to present and study more advance properties of refined neutrosophic canonical hypergroup and its substructures in our future papers.

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