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NN-TOPSIS strategy for MADM in neutrosophic number setting

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Abstract: Selecting an appropriate alternative among the feasible selection options is a difficult activity for decision makers. Because of the imprecise information and the complexity of decision problem, it is not easy to evaluate the attributes in terms of crisp numbers. To deal with the problem, neutrosophic numbers can be used during the decision process. Neutrosophic numbers can easily describe cognitive information. In this paper, we use neutrosophic numbers to state evaluation information. We define unit neutrosophic number as an effective tool to express cognitive information. We propose novel NN-TOPSIS strategy in neutrosophic number environment. Moreover, we define Euclidean distance in neutrosophic numbers environment, and propose a tangent function to determine unknown attribute weights. We propose linguistic variables in neutrosophic number environment. We present a numerical example dealing with a decision-making problem based on the proposed NN-TOPSIS strategy We present a sensitivity analysis for reflecting the influence of indeterminacy values. We then conduct a comparison analysis between the proposed NN-TOPSIS and other existing decision- making strategies.

Keywords: Neutrosophic number; Unit neutrosophic number; TOPSIS, NN-TOPSIS; MADM

1. Introduction

Decision-making, in general, refers to a cognitive process that is continuously performed by a human being or a group of human beings. Generally, cognitive process is done consciously or unconsciously [1–3]. Some studies have been appeared in the literature dealing with the utilization of cognitive information for decision making process [4–8]. Zhang et al. [9] opined that specific numerical rating values provided by the decision makers do not always accurately present the behaviors and opinions of the decision makers, especially in the fields of decision-making [10–40] in

general, linear programming problems [41], cloud computing [42, 43], supplier chain management [44–46] image processing [47-48], medical diagnosis [49–52], etc.
Basset et al. [53] proposed an included neutrosophic and SWOT and AHP process for strategic setting up methodology choice. Basset et al. [54] presented a group MADM algorithm based on triangular neutrosophic sets. Chang et al. [55] developed a recycle strategic decision outline framework from theories to practical. Basset et al. [56] proposed a group decision structure based on VIKOR (neutrosophic field) method for e-governance website evaluation. Basset and Mohamed [57] proposed the role of rough sets and SVNS in smart city under defective and incomplete data. Mondal et al. [58] developed similarity measure (with tangent function) based model for interval neutrosophic sets. Pramanik et al. [59] developed NC-VIKOR technique for neutrosophic cubic sets. Mondal et al. [60] proposed hybrid similarity measure (based on logarithm function) under SVNSs assessments. Dalapati et al. [61] developed IN-cross entropy technique for INSs. Mondal et al. [62] introduced sine similarity measures based on hyperbolic function for MADM in SVNS According to Zeleny [63], human decision making involves multi-attributes. So, MADM problems are common in human life. The attribute values in the MADM problems often involve indeterminacy. Therefore, it is difficult to describe the attribute values by the crisp numbers.

Smarandache [64, 65] introduced the neutrosophic number (NN) to deal with partial and indeterminate information. An NN comprises of two components namely, determinate component and indeterminate component. An NN is presented in the form: $N = r + sI$, where r stands for the determinate part and sI stands for the indeterminate part. If $N = sI$, we obtain the worst situation. If $N = r$, we obtain the best situation. Thus, it seems that NNs are capable to deal with the imprecise information in realistic decision-making situation.

Ye [66] initiated to study of linear programming for neutrosophic numbers. Ye et al. [67] developed a nonlinear programming in NN environment and provided general solution. Banerjee and Pramanik [68] proposed a linear goal programming with a single-objective in NN environment. Pramanik and Banerjee [69] developed a goal programming for multiple objective linear programming in neutrosophic number setting. Pramanik and Dey [70] developed a Bi-Level Linear Programming (BLP) model in NN environment. Maiti et al. [[71] extended BLP to Bi-Level Decentralized Programming (BLDP)in NN environment. Pramanik and Dey [72] extended BLP to Multi-Level Programming (MLP) in NN environment. Maiti et al. [73] extended MLP To Multi-Level Multi-Objective Linear Programming (MLMOLP) in NN environment.

Ye [74] studied possibility degree based ranking method and proposed an MADM technique in NN environment. Ye [75] proposed an MAGDM approach based on bidirectional measure in NN environment. Kong et al. [76] defined a cosine similarity for NNs to solve the misfire error finding gasoline engines. Ye [77] defined exponential similarity measure for NNs and used it to the fault analysis of vapor turbine. Ye et al. [78] proposed a joint roughness coefficient using NN functions. Liu and Liu [79] defined weighted power averaging operator and proposed an MAGDM technique in NN environment. Zheng et al. [80] developed an MAGDM policy based on NN fusion weighted averaging operator. Employing aggregation operators of NN-Harmonic mean, Mondal et al. [81] proposed an MAGDM technique in NN environment. Pramanik et al. [82] presented a teacher selection strategy in NN environment. Shi and Ye [83] presented a linguistic NN and presented an

MAGDM technique in NN environment based on cosine measures. Ye [84] defined the hesitant neutrosophic linguistic number and developed an MADM technique based on the probable value and similarity measure in an HNLN environment. The study for MAGDM in NN environment is its infancy. New research is necessary to handle the MAGDM problems in NN environment.

TOPSIS is a well-liked technique to deal with MAGDM. TOPSIS [85] selects the best option, which is the nearest to the solution (ideal) and the farthest from the solution (negative ideal). The TOPSIS technique is based on information of attributes from decision maker/makers. In SVNS setting, Biswas et al. [86] proposed TOPSIS strategy for MAGDM in Single Valued Neutrosophic Set (SVNS) environment. Ye [87] extended TOPSIS approach for MAGDM based on SVNS linguistic numbers. Pramanik et al. [88] presented a TOPSIS technique for neutrosophic cubic set environment. Mondal et al. [89] developed TOPSIS for MAGDM in rough neutrosophic setting. Pramanik et al. [90] presented a TOPSIS technique for MADM in single valued neutrosophic soft expert set. Biswas et al. [91] studied a TOPSIS approach for MADM with trapezoidal neutrosophic numbers. García-Cascales and Lamata [92] proposed an improved version of TOPSIS based on rank reversal technique.

TOPSIS is yet to come into view NN environment. For the research gap, we develop an MAGDM technique based on TOPSIS in NN environment namely, NN-TOPSIS method for solving MAGDM problems.

Contribution of the paper:

- We develop an NN-TOPSIS technique to solve MAGDM problem in NN environment.
- We define an UNN and establish its basic properties.
- We define NN weighted arithmetic aggregation operator (NNWANO) to aggregate NN decision matrices.
- We propose a tangent function to decide unknown weights of attributes in NN environment.
- We propose a linguistic variable to present NN.
- We present sensitivity analysis for different values of I to reflect the influence of I on ranking order of selection options.
- The proposed NN-TOPSIS is comprehensive because when $I = 0$, NN-TOPSIS reduces to classical TOPSIS.

The paper is structured as follows. Section 2 presents several basic ideas of NNs, operations on NNs, unit neutrosophic number (UNN), Euclidean distance between two NNs, tangent function for NNs, NN relative ideal solution (positive) and NN relative ideal solution (negative). Section 3 defines 'NN weighted arithmetic aggregation operator (NNWANO) to aggregate NN decision matrix and develops a novel NN-TOPSIS technique for solving MAGDM problem in NN environment. Section 4 provides an example based on proposed NN-TOPSIS technique. Section 5 conducts sensitivity study to show the impact of ranking for different indeterminacy values. Section 6 presents a comparison analysis with other existing strategies. Section 7 presents conclusion and future scope of research.

2. Preliminaries

In this section, the idea of NN, operations on NNs, unit neutrosophic numbers and Euclidean distance between two NNs, tangent function for NNs, NN relative ideal solution (positive) and NN relative ideal solution (negative) are outlined.

2.1. Neutrosophic numbers (NNs)

An NN [64, 65] is expressed as $z = p + qI$ for $p, q \in R$, where I denotes indeterminacy and R denotes the set of real numbers. An NN z is expressed as a interval number in the form: $z = [p+ qI^L, p+ qI^U]$ for $z \in Z$ and $I \in [I^L, I^U]$. The interval $I \in [I^L, I^U]$ is regarded as an indeterminate interval.

Here, Z = set of all neutrosophic numbers.

- If $qI = 0$, then $z = p$ *i.e.*, real number or crisp number.
- If $p = 0$, then $z = qI$ *i.e.*, indeterminate number
- If $I^L = I^U$, then z is a crisp number.

Assume that $z_1 = p_1 + q_1I$ and $z_2 = p_2 + q_2I$ for $z_1, z_2 \in Z$ and $I \in [I^L, I^U]$ are two NNs. Some basic operational laws [66] for z_1 and z_2 are presented as follows:

(1) $I^2 = I$

(2) $I.0 = 0$

(3) I/I = Undefined

(4) $z_1 + z_2 = p_1 + p_2 + (q_1 + q_2)I = [p_1 + p_2 + (q_1 + q_2)I^L, p_1 + p_2 + (q_1 + q_2)I^U]$

(5) $z_1 - z_2 = p_1 - p_2 + (q_1 - q_2)I = [p_1 - p_2 + (q_1 - q_2)I^L, p_1 - p_2 + (q_1 - q_2)I^U]$

(6) $z_1 \times z_2 = p_1p_2 + (p_1q_2 + p_2q_1)I + q_1q_2I^2 = p_1p_2 + (p_1q_2 + p_2q_1 + q_1q_2)I$

(7) $\frac{z_1}{z_2} = \frac{p_1 + q_1 I}{p_2 + q_2 I} = \frac{p_1}{p_2} + \frac{p_2 q_1 - p_1 q_2}{p_2(p_2 + q_2)} I;\ p_2 \neq 0, p_2 \neq -q_2$

(8) $z_1^2 = p_1^2 + (2p_1q_1 + q_1^2)I$

(9) $\lambda z_1 = \lambda p_1 + \lambda q_1 I$

2.2. Unit neutrosophic numbers (UNNs)

In this subsection, we define unit neutrosophic number (UNN).

Definition 1 Let $A=\{\langle a_1+b_1I\rangle,\langle a_2+b_2I\rangle,\cdots,\langle a_n+b_nI\rangle\}$ (i = 1, 2, . . . , n) be a set of neutrosophic numbers. Then $A^\circ=\left\{\left\langle\frac{a_1+b_1I}{2\sqrt{a_1^2+b_1^2}}\right\rangle,\left\langle\frac{a_2+b_2I}{2\sqrt{a_2^2+b_2^2}}\right\rangle,\cdots,\left\langle\frac{a_n+b_nI}{2\sqrt{a_n^2+b_n^2}}\right\rangle\right\}$, (i = 1, 2, . . . , n) is the set of UNNs.

Example 1 Let A = {3+4I, 4–3I} be a set of NNs. Then, we obtain $A^\circ=\left\{\frac{3+4I}{10},\frac{4-3I}{10}\right\}$.

Theorem 1 Each element of the set of UNNs lies in the interval [−1, 1].

Proof Let a_i, $b_i \in R$ (set of real numbers), and $I\in[0, 1]$.

$$\Rightarrow -1\le\frac{a_i}{\sqrt{a_i^2+b_i^2}}\le 1;\quad -1\le\frac{b_iI}{\sqrt{a_i^2+b_i^2}}\le 1.$$

$$\Rightarrow -1\le\frac{a_i+b_iI}{2\sqrt{a_i^2+b_i^2}}\le 1;\ \text{i} = 1, 2, \dots, \text{n}.$$

2.3. Euclidean distance of two Sets of NNs and UNNs

Definition 2 Let $A=\{\langle a_1+b_1I\rangle,\langle a_2+b_2I\rangle,\cdots,\langle a_n+b_nI\rangle\}$ and $X=\{\langle x_1+y_1I\rangle,\langle x_2+y_2I\rangle,\cdots,\langle x_n+y_nI\rangle\}$ (i = 1, 2, . . . , n.) be any two sets of NNs and $I\in[0, 1]$. Then the Euclidean distance of A and X is defined as:

$$D_{Eucl}(A,\ X)=\sqrt{\sum_{i=1}^{n}\left\langle a_i-x_i)+(b_i-y_i)I^*\right\rangle^2} \tag{1}$$

Here, $I^*=\lambda I^L+(1-\lambda)I^U$, and $0\le\lambda\le 1$. Let,

$$A^\circ=\left\{\left\langle\frac{a_1+b_1I}{2\sqrt{a_1^2+b_1^2}}\right\rangle,\left\langle\frac{a_2+b_2I}{2\sqrt{a_2^2+b_2^2}}\right\rangle,\cdots,\left\langle\frac{a_n+b_nI}{2\sqrt{a_n^2+b_n^2}}\right\rangle\right\},\ X^\circ=\left\{\left\langle\frac{x_1+y_1I}{2\sqrt{x_1^2+y_1^2}}\right\rangle,\left\langle\frac{x_2+y_2I}{2\sqrt{x_2^2+y_2^2}}\right\rangle,\cdots,\left\langle\frac{x_n+y_nI}{2\sqrt{x_n^2+y_n^2}}\right\rangle\right\}$$

be the sets of UNNs. Then the Euclidean distance of A° and X° is defined as:

$$D_{Eucl}(A^\circ,\ X^\circ)=\sqrt{\sum_{i=1}^{n}\left\langle\frac{(a_i+b_iI^*)}{2\sqrt{a_i^2+b_i^2}}-\frac{(x_i+y_iI^*)}{2\sqrt{x_i^2+y_i^2}}\right\rangle^2} \tag{2}$$

Definition 3 The normalized Euclidean distance of two sets of UNNs A° and X° is defined as:

$$D_{Eucl}^{N}(A^{\circ}, X^{\circ}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left\langle \frac{(a_i+b_iI^*)}{2\sqrt{a_i^2+b_i^2}} - \frac{(x_i+y_iI^*)}{2\sqrt{x_i^2+y_i^2}} \right\rangle^2} \quad (3)$$

Note 1 In decision making situation we use expectation value (mean value) of λ i.e., $\lambda = 0.5$.

2.4. Tangent function for NNs

Definition 4 The tangent function of a neutrosophic number $P= x_{ij}+ y_{ij}I = [x_{ij}+ y_{ij}I^L, x_{ij}+ y_{ij}I^U]$, where x_{ij} and y_{ij} are not simultaneously zeroes, (i = 1, 2, ..., m; j = 1, 2, ..., n) is defined as:

$$T_j(P) = 1 - \frac{1}{n}\sum_{i=1}^{n} \tan\frac{\pi}{4}\left(\left|\frac{y_{ij}}{\sqrt{x_{ij}^2+y_{ij}^2}}\right|\right) \quad (4)$$

The weight structure is defined as:

$$\omega_j = \frac{T_j(P)}{\sum_{j=1}^{n} T_j(P)}\text{; j = 1, 2, ..., n} \quad (5)$$

Here, $\sum_{j=1}^{n}\omega_j = 1$.

Theorem 2 The function $T_j(P)$ is bounded.

Proof Since x_{ij}, $y_{ij} \in$ R and x_{ij}, y_{ij} are not both zero, we have.

$$0 \le \frac{|y_{ij}|}{\left|\sqrt{x_{ij}^2+y_{ij}^2}\right|} \le 1 \Rightarrow 0 \le \tan\frac{\pi}{4}\left(\frac{|y_{ij}|}{\left|\sqrt{x_{ij}^2+y_{ij}^2}\right|}\right) \le 1 \Rightarrow 0 \le 1 - \tan\frac{\pi}{4}\left(\frac{|y_{ij}|}{\left|\sqrt{x_{ij}^2+y_{ij}^2}\right|}\right) \le 1 \Rightarrow 0 \le T_j(P) \le 1.$$

Hence, the function $T_j(P)$ is bounded.

Theorem 3 The function $T_j(P)$ is monotone decreasing.

Proof Here, $\frac{1}{n}\sum_{i=1}^{n} \tan\frac{\pi}{4}\left(\left|\frac{y_{ij}}{\sqrt{x_{ij}^2+y_{ij}^2}}\right|\right)$ is monotone increasing in the interval $[0, \pi/4]$ and $0 \le \frac{|y_{ij}|}{\left|\sqrt{x_{ij}^2+y_{ij}^2}\right|} \le 1$.

$\Rightarrow\ 1-\frac{1}{n}\sum_{i=1}^{n}\tan\frac{\pi}{4}\left(\left|\frac{y_{ij}}{\sqrt{x_{ij}^2+y_{ij}^2}}\right|\right)$ is a monotone decreasing function in the interval $[0,\pi/4]$.

$\Rightarrow\ T_j(P)$ is monotone decreasing in the interval $[0,\pi/4]$.

Example 2 Assume that $P_1 = 3 + 2I$, and $P_2 = 3 + 5I$. Then, we obtain $T(P_1) = 0.5345,\ T(P_2) = 0.2020$.

Example 3 Assume that $P_1 = 3 + I$, and $P_2 = 7 + I$. Then, we obtain $T(P_1) = 0.7464,\ T(P_2) = 0.8883$.

Example 4 Assume that $P_1 = 10 + 2I$, and $P_2 = 2 + 10I$. Then, we obtain $T(P_1) = 0.8447,\ T(P_2) = 0.0299$.

2.5. NN relative positive ideal solution and NN relative negative ideal solution

Definition 5 Assume that C^+ and C^- denote respectively two the type modifiers, namely, the benefit attribute and cost attribute. Assume that G_N^+ denotes the NN relative positive ideal solution (NNRPIS) and G_N^- denotes the NN relative negative ideal solution (NNRNIS).

Then G_N^+ is defined as:

$$G_N^+ = \left\langle d_1^{\omega+}, d_2^{\omega+}, \cdots, d_n^{\omega+} \right\rangle \tag{6}$$

Here $d_j^{\omega+} = \left\langle x_j^{\omega+} + y_j^{\omega+} I \right\rangle$ for j = 1, 2, . . . , n.

$$x_j^{\omega+} = \{(\max_i\{x_{ij}^{\omega j}\} / j \in C^+),\ (\min_i(x_{ij}^{\omega j}) / j \in C^-)\}$$

$$y_j^{\omega+} = \{(\min_i\{y_{ij}^{\omega j}\} / j \in C^+),\ (\max_i(y_{ij}^{\omega j}) / j \in C^-)\}$$

Then G_N^- is defined as follows:

$$G_N^- = \left\langle d_1^{\omega-}, d_2^{\omega-}, \cdots, d_n^{\omega-} \right\rangle \tag{7}$$

Here $d_j^{\omega-} = \left\langle x_j^{\omega-} + y_j^{\omega-} I \right\rangle$ for j = 1, 2, . . . , n.

$$x_j^{\omega-} = \{(\min_i\{x_{ij}^{\omega j}\} / j \in C^+),\ (\max_i(x_{ij}^{\omega j}) / j \in C^-)\}$$

$$y_j^{\omega-} = \{(\max_i\{y_{ij}^{\omega j}\} / j \in C^+),\ (\min_i(y_{ij}^{\omega j}) / j \in C^-)\}$$

3. NN-TOPSIS technique for MADM

Assume that an MADM problem is characterized by m selection options and n attributes. Also let $D = (D_1, D_2, \ldots, D_r)$ be the set of decision makers, $K = (K_1, K_2, \ldots, K_m)$ be the set of selection options, and $C = (C_1, C_2, \ldots, C_n)$ be the set of attributes. The ratings offered by the decision maker describe the performance of the selection option K_i against the attribute C_j. Let $\{\omega_1, \omega_2, \cdots, \omega_n\}$ be the weight vector assigned for the attributes $C_1, C_2, \ldots, C_n$. The rating values associated with the selection options with respect to the attributes is presented in the following NN based decision matrix (for r^{th} decision maker).

$$D_r[K \mid C_1, C_2, \ldots, C_n] = \begin{matrix} \\ K_1 \\ K_2 \\ \vdots \\ K_m \end{matrix} \begin{pmatrix} C_1 & C_2 & \cdots & C_n \\ \langle x_{11}+y_{11}I \rangle_r & \langle x_{12}+y_{12}I \rangle_r & \cdots & \langle x_{1n}+y_{1n}I \rangle_r \\ \langle x_{21}+y_{21}I \rangle_r & \langle x_{22}+y_{22}I \rangle_r & \cdots & \langle x_{2n}+y_{2n}I \rangle_r \\ \vdots & \vdots & \ddots & \vdots \\ \langle x_{m1}+y_{m1}I \rangle_r & \langle x_{m2}+y_{m2}I \rangle_r & \cdots & \langle x_{mn}+y_{mn}I \rangle_r \end{pmatrix} \tag{8}$$

Here, $\langle x_{ij}+y_{ij}I \rangle_r$ is the rating value (in terms of NN) for ij^{th} element of the decision matrix of r-th decision maker.

Now the steps (Figure 1) of NN-TOPSIS technique in NN environment are described below:

Step 1: Determine the weights of the decision makers.

Assume that $D = \{D_1, D_2, \ldots, D_r\}$ be a group of decision makers. In decision making situation, the decision maker's weight may be different as their importance is not identical. The importance of each decision maker is expressed in terms of linguistic variables. The linguistic variables are then transformed into NNs (see Table 1).

Table 1 Transformation of linguistic variable into NN

Linguistic terms (LTs)	NNs
Very Important (VI)	5
Important (I)	4+*I*
Medium (M)	3+2*I*
Unimportant (UI)	2+3*I*

Very unimportant (VUI)	1+4*I*

Assume that $a_r + b_r I$ presents the rating of r[th] decision maker. Then, the weight (ξ_r) of the r[th] decision maker is presented as:

$$\xi_r = \frac{[2a_r + b_r(I^L + I^U)]}{\sum_{t=1}^{r}[2a_t + b_t(I^L + I^U)]} \tag{9}$$

and $\sum_{t=1}^{r}\xi_t = 1$.

Step 2: Calculate the aggregated NN decision matrix based on decision makers' assessments.

Assume that $D^r = \langle a_k + b_k I \rangle_{m\times n}$ denotes the NN decision matrix of the r[th] decision maker and $\xi = (\xi_1, \xi_2, ..., \xi_r)^T$ be the weight format of decision makers such that each $\xi_r \in (0, 1)$. The aggregated matrix is obtained using NN weighted arithmetic mean aggregation operator (NNWAMAO) as:

$$D_{aggr} = (d_{ij})_{m\times n} = \text{NNWAMAO}\left(d_{ij}^1, d_{ij}^2, \cdots, d_{ij}^r\right) = \xi_1 d_{ij}^1 \oplus \xi_2 d_{ij}^2 \oplus \cdots \oplus \xi_r d_{ij}^r = \left\langle \sum \xi_{ij} a_{ij} + I \sum \xi_{ij} b_{ij} \right\rangle \tag{10}$$

Now the aggregated NN decision matrix is defined as:

$$D_{aggr}[K \mid C_1, C_2, ..., C_n] = \begin{matrix} \\ K_1 \\ K_2 \\ \vdots \\ K_m \end{matrix} \begin{pmatrix} C_1 & C_2 & \cdots & C_n \\ \langle x_{11} + y_{11} I \rangle_{aggr} & \langle x_{12} + y_{12} I \rangle_{aggr} & \cdots & \langle x_{1n} + y_{1n} I \rangle_{aggr} \\ \langle x_{21} + y_{21} I \rangle_{aggr} & \langle x_{22} + y_{22} I \rangle_{aggr} & \cdots & \langle x_{2n} + y_{2n} I \rangle_{aggr} \\ \vdots & \vdots & \ddots & \vdots \\ \langle x_{m1} + y_{m1} I \rangle_{aggr} & \langle x_{m2} + y_{m2} I \rangle_{aggr} & \cdots & \langle x_{mn} + y_{mn} I \rangle_{aggr} \end{pmatrix} \tag{11}$$

Here, $\langle x_{ij} + y_{ij} I \rangle_{aggr}$ is the rating value for ij[th] element of aggregated decision

matrix $D_{aggr}[K \mid C_1, C_2, ..., C_n]$

(i = 1, 2, . . . m; j = 1, 2, . .. n).

Step 3: Calculate the attribute weights.

When weights of attributes are completely unknown to decision makers, the entropy measure [93] is used to calculate attribute weights. Entropy method [94] is used to find out completely unknown attribute weights of single valued neutrosophic sets. Method to determine unidentified attribute weights in NN environment is yet to come into view in literature. We define a function for measuring unknown attribute weights (see definition 4).

Step 4: Aggregate the weighted NN decision matrix.

The calculated weights of the attributes and aggregated NN decision matrix are fused to construct the aggregated weighted NN decision matrix. The aggregated weighted NN decision matrix is defined by utilizing the multiplication rules between attribute weights and corresponding rating values of attributes as:

$$D\otimes\omega=\left\langle d^{\omega j}_{ij}\right\rangle_{m\times n}=\left\langle a^{\omega j}_{ij}+b^{\omega j}_{ij}I\right\rangle_{m\times n}=\begin{matrix} \\ K_1 \\ K_2 \\ \vdots \\ K_m \end{matrix}\begin{pmatrix} C_1 & C_2 & \cdots & C_n \\ \langle x_{11}+y_{11}I\rangle_{\omega j} & \langle x_{12}+y_{12}I\rangle_{\omega j} & \cdots & \langle x_{1n}+y_{1n}I\rangle_{\omega j} \\ \langle x_{21}+y_{21}I\rangle_{\omega j} & \langle x_{22}+y_{22}I\rangle_{\omega j} & \cdots & \langle x_{2n}+y_{2n}I\rangle_{\omega j} \\ \vdots & \vdots & \ddots & \vdots \\ \langle x_{m1}+y_{m1}I\rangle_{\omega j} & \langle x_{m2}+y_{m2}I\rangle_{\omega j} & \cdots & \langle x_{mn}+y_{mn}I\rangle_{\omega j} \end{pmatrix} \quad (12)$$

Here, $d^{\omega j}_{ij}=\left\langle T^{\omega j}_{ij}, I^{\omega j}_{ij}, F^{\omega j}_{ij}\right\rangle$ denotes the rating value for $(ij)^{th}$ element of the aggregated weighted NN decision matrix $D\otimes\omega$ (i = 1, 2, . . . , m and j = 1, 2, . . . , n).

Step 5: Determine the NNRPIS and the NNRNIS.

Assume that $\left\langle d^{\omega}_{ij}\right\rangle_{m\times n}=\left\langle x_{ij}+y_{ij}I\right\rangle_{m\times n}$ is an NN decision matrix, where, x_{ij} and $y_{ij}I$ are respectively the determinant part and indeterminate part of the evaluation for the attribute C_j with respect to the selection option K_i.

Step 6: Determine the distance measures of each selection option from the NNRPIS and the NNRNIS.

The normalized Euclidean distance measure of all selection option $\left\langle x^{\omega j}_{ij}+y^{\omega j}_{ij}I^{*}\right\rangle$ from the NNRPIS $\left\langle d^{\omega+}_1, d^{\omega+}_2,\ldots, d^{\omega+}_n\right\rangle$ for i = 1, 2, …, m and j = 1, 2, …, n is written as:

$$\Delta^{i+}_{Eucl}(d^{\omega j}_{ij}, d^{\omega+}_{j})=\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left\langle\frac{(a^{\omega j}_{ij}+b^{\omega j}_{j}I^{*})}{(a^{\omega j}_{ij}+b^{\omega j}_{j})}-\frac{(a^{\omega+}_{ij}+b^{\omega+}_{j}I^{*})}{(a^{\omega+}_{ij}+b^{\omega+}_{j})}\right\rangle^{2}} \quad (13)$$

The normalized Euclidean distance measure of all selection option $\left\langle x_{ij}^{\omega j} + y_{ij}^{\omega j} I^* \right\rangle$ from the NNRNIS $\left\langle d_1^{\omega -}, d_2^{\omega -}, \ldots, d_n^{\omega -} \right\rangle$ for i = 1, 2, …, m and j = 1, 2, …, n is presented as:

$$\Delta_{Eucl}^{i-}(d_{ij}^{\omega j}, d_j^{\omega -}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left\langle \frac{(a_{ij}^{\omega j}+b_j^{\omega j} I^*)}{(a_{ij}^{\omega j}+b_j^{\omega j})} - \frac{(a_{ij}^{\omega -}+b_j^{\omega -} I^*)}{(a_{ij}^{\omega -}+b_j^{\omega -})}\right\rangle^2} \tag{14}$$

Step 7: compute the relative closeness co-efficient (RCC) to the NN ideal solution.

RCC of each selection option K_i with respect to the NN positive ideal solution G_N^+ is defined as:

$$\text{RCC}(K_\text{i}) = \frac{\left\langle \Delta_{Eucl}^{-}(d_{ij}^{\omega j}, d_j^{\omega -}) \right\rangle}{\left\langle \Delta_{Eucl}^{-}(d_{ij}^{\omega j}, d_j^{\omega -}) + \Delta_{Eucl}^{+}(d_{ij}^{\omega j}, d_j^{\omega +}) \right\rangle} \tag{15}$$

Here, $0 \le \text{RCC}(K_\text{i}) \le 1$.

Step 8: Rank the priority

All the RCC values are arranged in descending order. A set of alternatives is then preference ranked order. We select the alternative corresponding to the highest value of $\text{RCC}(K_\text{i})$ as the best choice K_i for i = 1, 2, . . . , m.

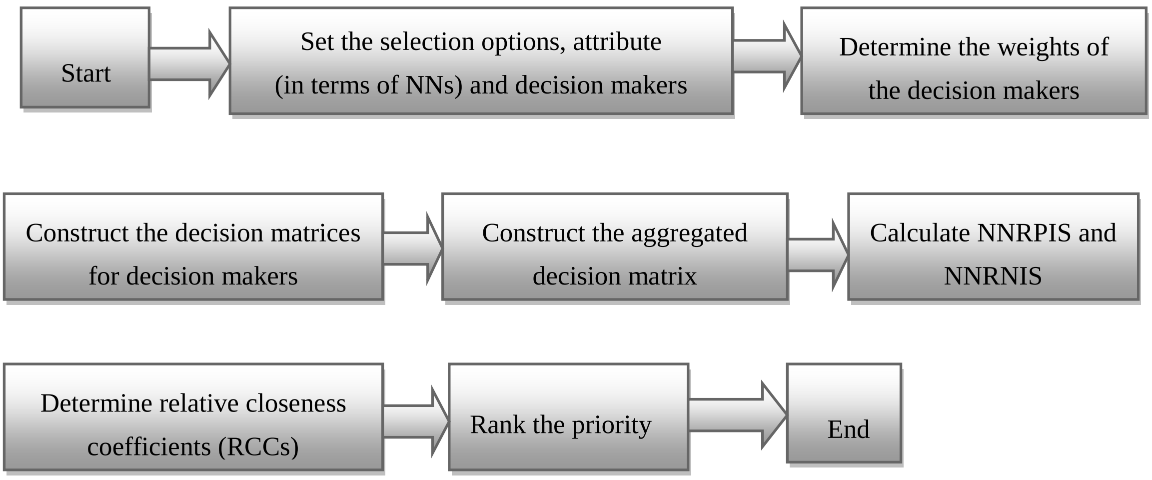

Figure 1: Steps of NN-TOPSIS technique

Step 9: End.

4. Illustrative example

Let a multi-national company wants to recruit managing director for their company. An interview board with three members D_1, D_2, D_3 is formed to select the managing director. The selection options (candidates) are K_1, K_2, K_3, and K_4. Decision makers must take their decisions based on the following attributes (C_1): academic qualification, (C_2): interview performance (C_3): management experience and (C_4): risk factor. Assume that ω_1, ω_2, ω_3 and ω_4 be the weights assigned to the attributes C_1, C_2, C_3, and C_4 respectively. The rating values of the selection options for the MAGDM problem with respect to the attribute are presented in NN based decision matrices (Eqs. (16), (17), and (18)).

Each decision maker uses five-point scale (see Table 1) to express his/her rating values. Each decision maker forms an NN based decision matrix to express rating values. The decision matrices corresponding to decision makers D_1, D_2, and D_3 are shown in (16), (17), and (18) respectively.

$$D_1[K|C_1,C_2,C_3] = \begin{matrix} \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{matrix}\begin{pmatrix} C_1 & C_2 & C_3 & C_4 \\ 1+2I & 5 & 4+I & 2+I \\ 2+I & 1+I & 2+2I & 1+2I \\ 1+3I & 2+2I & 5+I & 2I \\ 3+I & 2+3I & 4+I & I \end{pmatrix} \tag{16}$$

$$D_2[K|C_1,C_2,C_3] = \begin{matrix} \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{matrix}\begin{pmatrix} C_1 & C_2 & C_3 & C_4 \\ 2+3I & 4 & 3+I & I \\ 2+I & 2+2I & 2+I & 2I \\ 3I & 2+2I & 5 & 1+I \\ 1+I & 2+I & 4 & I \end{pmatrix} \tag{17}$$

$$D_3[K|C_1,C_2,C_3] = \begin{matrix} \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{matrix}\begin{pmatrix} C_1 & C_2 & C_3 & C_4 \\ 3+2I & 3 & 4+I & I \\ 1+3I & 2+I & 3 & I \\ 2I & 1+I & 4 & 2I \\ 2+I & 2+I & 3+I & 1+I \end{pmatrix} \tag{18}$$

The problem is solved using the following steps of the proposed NN-TOPSIS technique.

Step 1: Determine the weights of decision makers.

Linguistic variables are employed to represent the weights of decision makers and their corresponding neutrosophic numbers are shown in Table 2.

Table 2 Transformation of linguistic variable into NN

	D_1	D_2	D_3
LTs	Important	Medium	Important
NNs	4+I	3+2I	4+I

Using Eq. (9), we obtain the weights of the decision makers as:

$\xi_1 = 0.346$, $\xi_2 = 0.308$, $\xi_3 = 0.346$.

Step 2: Construct the aggregated NN decision matrix based on the decision makers' assessments.

Using Eq. (10), we calculate aggregated NN decision matrix as:

$$D_{aggr}[K \mid C] = \begin{matrix} \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{matrix} \begin{pmatrix} C_1 & C_2 & C_3 & C_4 \\ 1.997+2.337I & 4.026 & 3.683+I & 0.696+I \\ 1.683+1.673I & 1.667+1.337I & 2.343+1.039I & 0.356+1.683I \\ 0.343+3I & 1.683+1.680I & 4.680+0.356I & 0.337+1.683I \\ 2.026+I & 2+1.712I & 3.683+0.683I & 0.330+I \end{pmatrix} \quad (19)$$

Step 3: Determine the weights of the attributes.

Using Eqs. (4) and (5), we calculate the weights of the attributes as follows:

$\omega_1 = 0.1875$, $\omega_2 = 0.3180$, $\omega_3 = 0.4535$, $\omega_4 = 0.0410$.

Step 4: Aggregate the weighted NN decision matrices

Using Eq. (12), we calculate the aggregated weighted NN decision matrix as follows:

$$\left\langle a^{\omega_j}_{ij} + b^{\omega_j}_{ij} I \right\rangle_{4\times 4} = \begin{matrix} \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{matrix} \begin{pmatrix} C_1 & C_2 & C_3 & C_4 \\ 0.374+0.438I & 1.276 & 1.663+0.4533I & 0.028+0.041I \\ 0.315+0.313I & 0.527+0.422I & 1.054+0.464I & 0.014+0.068I \\ 0.066+0.564I & 0.531+0.531I & 2.115+0.153I & 0.013+0.069I \\ 0.379+0.1875I & 0.636+0.541I & 1.662+0.302I & 0.014+0.041I \end{pmatrix} \quad (20)$$

Step 5: Determine the NNRPIS and the NNRNIS.

Here, C_1, C_2 and C_3 are benefit attributes and C_4 is the cost attribute. Then we obtain NNRPIS and NNRNIS as follows:

$$\text{NNRPIS} = \{(0.379+0.1875I), (1.276+0.422I), (1.054+0.464I), (0.013+0.069I)\}$$

$\text{NNRNIS} = \{(0.066+0.564I), (0.527+0.541I), (2.115+0.153I), (0.028+0.041I)$

Step 6: Determine the distance measures of each selection option from the NNRPIS and the NNRNIS.

Using Eq. (13), the normalized Euclidean distance measures of all selection options from the NNRPIS are calculated and shown in Table 3. Using Eq. (14), the normalized Euclidean distance measures of all selection options from the NNRNIS are calculated and shown in Table 3.

Step 7: Determine the RCC to the NN ideal solution.

The RCC of each selection option K_i with respect to the NN positive ideal solution is calculated as follows:

$\text{RCC}(K_1) = 0.3791$, $\text{RCC}(K_2) = 0.3159$, $\text{RCC}(K_3) = 0.0723$, $\text{RCC}(K_4) = 0.4788$.

Step 8: Ranking the priority.

According to the RCC values, we have,

$\text{RCC}(K_4) \succ \text{RCC}(K_1) \succ \text{RCC}(K_2) \succ \text{RCC}(K_3)$.

Hence, the candidate K_4 is the best selection option.

Table 3 Distance measures and RCC values of selection options

Selection options	Δ_{Eucl}^{i+}	Δ_{Eucl}^{i-}	$\text{RCC}(K_i)$
K_1	0.5562	0.3397	0.3791
K_2	0.5271	0.2434	0.3159
K_3	0.3582	0.0279	0.0723
K_4	0.9395	0.8629	0.4788

Step 9: End.

5. Sensivity study

In this section, we present sensitivity analysis to demonstrate the impact of different values of I on ranking order of selection options (see Figure 2). The ranking order for different intervals of I, is shown in Table 4. Table 4 reflects that the ranking order of selection options are same for selected values of *I*.

Table 4 Ranking order of the selection options for different *I*

I	RCC(K_i)	Ranking order
$I = 0$	RCC(K_1) = 0.3889, RCC(K_2) = 0.3291, RCC(K_3) = 0.1179, RCC(K_4) = 0.4899	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.1]$	RCC(K_1) = 0.3876, RCC(K_2) = 0.3274, RCC(K_3) = 0.1077, RCC(K_4) = 0.4881	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.2]$	RCC(K_1) = 0.3865, RCC(K_2) = 0.3264, RCC(K_3) = 0.1075, RCC(K_4) = 0.4866	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.3]$	RCC(K_1) = 0.3857, RCC(K_2) = 0.3251, RCC(K_3) = 0.1031, RCC(K_4) = 0.4850	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.4]$	RCC(K_1) = 0.3842, RCC(K_2) = 0.3241, RCC(K_3) = 0.0992, RCC(K_4) = 0.4838	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.5]$	RCC(K_1) = 0.3831, RCC(K_2) = 0.3225, RCC(K_3) = 0.0945, RCC(K_4) = 0.4823	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.6]$	RCC(K_1) = 0.3821, RCC(K_2) = 0.3212, RCC(K_3) = 0.0899, RCC(K_4) = 0.4815	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.7]$	RCC(K_1) = 0.3812, RCC(K_2) = 0.3196, RCC(K_3) = 0.0952, RCC(K_4) = 0.4807	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.8]$	RCC(K_1) = 0.3805, RCC(K_2) = 0.3183, RCC(K_3) = 0.0807, RCC(K_4) = 0.4797	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.9]$	RCC(K_1) = 0.3798, RCC(K_2) = 0.3170, RCC(K_3) = 0.0765, RCC(K_4) = 0.4776	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 1]$	RCC(K_1) = 0.3791, RCC(K_2) = 0.3159, RCC(K_3) = 0.0723, RCC(K_4) = 0.4758	$K_4 \succ K_1 \succ K_2 \succ K_3$

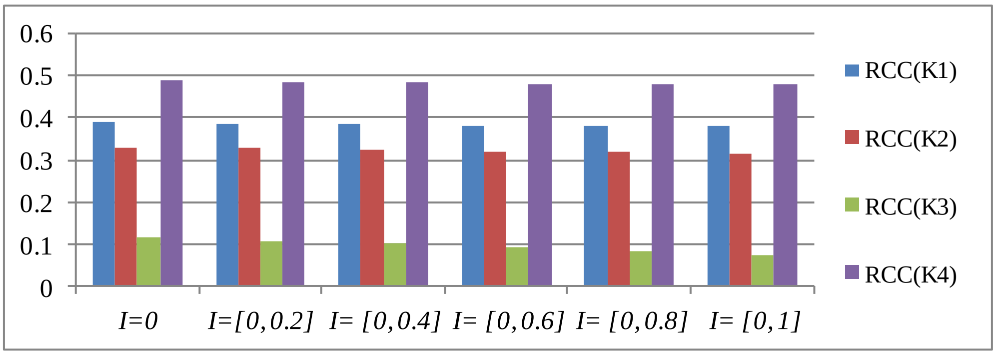

Figure 2: Ranking order of the alternatives with different values of *I*

6. Comparison analysis

In this section, a comparison analysis is presented between the proposed NN-TOPSIS technique and other existing decision-making strategies in NN environment. The ranking results obtained from the existing strategies [71, 76, 77] are furnished in Table 5. From the second column (Ye [71]) of Table 5, we see that K_4 is the best selection option for $I = 0$, $I \in [0, 0.2]$, and $I \in [0, 0.4]$. K_1 is the best selection option for other selected indeterminacy intervals. From the third column (Liu and Liu [76]) of Table 5, we observe that K_4 is the best selection option for $I = 0$, $I \in [0, 0.2]$, $I \in [0, 0.4]$, and $I \in [0, 0.6]$. K_1 is the best selection option for $I \in [0, 0.8]$, and $I \in [0, 1]$. From the fourth column (Zheng et al. [77]) of Table 5, we state that, K_4 is the best selection option for every selected indeterminacy interval. In the proposed technique, ranking order of selection option is unaltered for every selected indeterminacy interval. The comparison of ranking order of selection options between the proposed NN-TOPSIS technique and existing MADM strategies is shown in Table 5.

Table 5 The ranking order of existing strategies with different values of 'I'

I	Ye [71]	Liu and Liu [76]	Zheng et al. [77]	NN-TOPSIS
$I = 0$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.1]$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.2]$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.3]$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.4]$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_3 \succ K_2$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.5]$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.6]$	$K_1 \succ K_4 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_3 \succ K_2$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.7]$	$K_1 \succ K_4 \succ K_2 \succ K_3$	$K_1 \succ K_4 \succ K_3 \succ K_2$	$K_4 \succ K_1 \succ K_2 \succ K_3$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.8]$	$K_1 \succ K_4 \succ K_2 \succ K_3$	$K_1 \succ K_4 \succ K_3 \succ K_2$	$K_4 \succ K_1 \succ K_3 \succ K_2$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 0.9]$	$K_1 \succ K_4 \succ K_2 \succ K_3$	$K_1 \succ K_4 \succ K_3 \succ K_2$	$K_4 \succ K_1 \succ K_3 \succ K_2$	$K_4 \succ K_1 \succ K_2 \succ K_3$
$I \in [0, 1]$	$K_1 \succ K_4 \succ K_2 \succ K_3$	$K_1 \succ K_4 \succ K_3 \succ K_2$	$K_4 \succ K_1 \succ K_3 \succ K_2$	$K_4 \succ K_1 \succ K_2 \succ K_3$

7. Conclusion

In real decision making, indeterminacy plays a very important role. In this article, the selection process is studied based on proposed NN-TOPSIS technique. To develop the NN-TOPSIS technique, we have defined an UNN and proved the basic properties. The defined UNN is an effective mathematical tool to express cognitive information considering the reliability of the information. We have defined Euclidean distance between two sets of NNs. We have defined NN weighted arithmetic aggregation operator (NNWANO) to aggregate NN decision matrices. We have also proposed a tangent function to determine unknown weights of attributes in NN environment. We have proposed a linguistic variable to present NN. We have performed sensitivity analysis for different values of I to show the influence of I on ranking order of selection options. The proposed technique simply and reliably represents human cognition by considering the interactivity of attribute and the cognition towards indeterminacy involved in the problem. The developed NN-TOPSIS technique combines the advantages of NN and TOPSIS. NN-TOPSIS is more comprehensive because when $I = 0$, NN-TOPSIS reduces to classical TOPSIS. Finally, we have addressed a problem of selecting the managing director of a multi-national company based on the proposed NN-TOPSIS technique. Future studies may consider the following problems: (i) the case when I varies for different NNs, (ii) more than 5 point-scale can be employed for rating purpose, (iii) Rank reversal in TOPSIS technique in NN environment, etc.

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