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Complex Bipolar- Valued Neutrosophic Soft Set and its Decision Making Method

Ashraf Al-Quran[1], Shawkat Alkhazaleh[2]*, and Lazim Abdullah[3]

[1] Preparatory Year Deanship, King Faisal University, Hofuf 31982, Al-Ahsa, Saudi Arabia
Email: aalquran@kfu.edu.sa
[2] Department of Mathematics, Faculty of Science and Information Technology, Jadara University, Irbid, Jordan
Email: shmk79@gmail.com
[3] School of Informatics and Applied Mathematics, University Malaysia Terengganu, Kuala Nerus 21030, Terengganu, Malaysia; Email: lazim_m@umt.edu.my

*Correspondence: shmk79@gmail.com

Abstract: We establish the hybrid concept of complex bipolar- valued neutrosophic soft set (CBVNSS) as a hybrid model of bipolar neutrosophic soft set (BNSS)and complex fuzzy set (CFS). A CBVNSS is highly suitable for use in real life situations where the decision makers are interested to deal with bi-polarity as well as truth membership, indeterminacy membership and falsity membership grades to the alternatives in an extended range with complex numbers. Certain operations on CBVNSS like complement, subset, union and intersection operations are defined. Some related examples are also given to enhance the understanding of the proposed concept. The basic properties are also verified. We then provide a decision-making method on the CBVNSS. Finally, a numerical example has been presented to verify validity and feasibility of the developed method.

Keywords: neutrosophic set; complex neutrosophic set; neutrosophic soft set; complex neutrosophic soft set; bipolar neutrosophic soft set; decision making

1. Introduction

Smarandache [1, 2] introduced the notation of neutrosophic set (NS) to examine and process the truth, indeterminate and false information simultaneously to help with making decisions. NS is one of the most effective techniques for presenting uncertainty and vagueness in decision making, which is the more generality of fuzzy set (FS) [3] and intuitionistic fuzzy set (IFS) [4]. Yet, in order to adapt these uncertainty sets with more real complex cases, complex fuzzy sets (CFSs) [5], complex intuitionistic fuzzy sets (CIFSs) [6] and complex neutrosophic sets(CNSs) [7] have been proposed accordingly. In CFS, the degree of membership is expressed by a complex-valued function where the amplitude term and the phase term are both real-valued functions. The CFSs are utilized to represent data with uncertainty and periodicity in the form of amplitude term which handles uncertainty and the phase term to represent periodicity. As a generalization of CFS, CIFS is traded by a complex-valued truth membership function which handles the uncertainty with periodicity and a complex-valued false membership function which handles the falsity with periodicity. In other words, CIFS is used to handle the intuitionistic fuzzy information that happen periodically. Some problems have imprecise, indeterminate, inconsistent, and incomplete information which appear in a periodic manner in our real life. These types of information cannot be handled by CFS and CIFS. To overcome this problem CNS has been introduced as a generalization of CFS and CIFS. A CNS is defined by a complex-valued truth membership function which represents uncertainty with periodicity, complex-valued indeterminacy membership function which represents indeterminacy with periodicity, and a complex-valued falsity membership function which represents falsity with periodicity.

A soft set (SS) is a set-valued map defined by Molodtsov [8], to approximately describe objects using several parameters. Both CNS [7] and neutrosophic soft set (NSS) [9] are improved and generalized models of the neutrosophic set but in different spaces. Complex neutrosophic set handles the neutrosophic data which has the periodic manner, while neutrosophic soft set provides a parameterization tool to hanle the neutrosophic data. Subsequently, these uncertainty sets have been actively applied in various decision making problems to address the uncertainty [10-20].

A wide variety of human decision making is based on double-sided or bipolar judgmental thinking on a positive side and a negative side. A great deal of research have been conducted to integrate the idea of bipolarity in decision making techniques by virtue of the uncertainty sets like fuzzy, intuitionistic fuzzy, complex fuzzy and complex neutrosophic sets [21- 31]. Bipolar complex neutrosophic set (BCNS) [22] and BNSS [28] are the most advanced methods which have the advantages of both bipolarity and neutrosophy which make them superior to all of the aforementioned uncertainty sets. However, the BCNS lacks the adequate parameterization tool to facilitate the representation of parameters. On the other hand, bipolar neutrosophic soft set lacks the phase terms of the complex numbers which have the ability to represent two-dimensional neutrosophic information side by side with the amplitude terms. Motivated by these results and as per our knowledge there is no work available on CBVNSS and its application. Accordingly, we introduce CBVNSS and its operations. CBVNSS is equipped with adequate parameterization. It also has the ability to handle the imprecise, indeterminate, inconsistent and incomplete information that is captured by the amplitude terms and phase terms of the complex numbers, simultaneously. The results of this paper also have been applied to solve a decision-making problem.

2. Preliminaries

We provide a brief overview of some concepts on NSs and CNSs. We begin by defining the concepts of NS [2], NSS [9] and BNSS [28].

Definition 1. [2] Let M be a universe. A NS S in M is defined as $S = \{< m;\ T_S(m),\ I_S(m)\ , F_S(m) >: m \in M\}$, where $T_S(m), I_S(m)$ and $F_S(m)$ are the truth, the indeterminacy and the falsity membership functions, such that $T, I, F: U \to]^{-}0, 1^{+}[$, and $0^{-} \leq T + I + F \leq 3^{+}$.

Definition 2. [9] If M is the universe and A is a set of parameters set. A pair (S, A) is called a NSS over M, where S is a mapping given by $S: A \to \rho(S)$, where $\rho(S\ )$ denotes the power neutrosophic set of M.

Definition 3. [28] Let M be a universe and E be a set of parameters. A BNSS B in M is defined as $B = \{< e, \{T^+(m), I^+(m), F^+(m), T^-(m), I^-(m), F^-(m)\} >: e \in E,\ m \in M\}$, where $T^+, I^+, F^+: M \to [0,1]$ and $T^-, I^-, F^-: M \to [-1,0]$. The membership degrees T^+, I^+, F^+ denote the truth membership, indeterminate membership and false membership of an element corresponding to a bipolar neutrosophic soft set B and the membership degrees T^-, I^-, F^- denote the truth membership,indeterminate membership and false membership of an element $m \in M$ to some implicit counter-property corresponding to a bipolar neutrosophic soft set B.

Now, we define the concepts of CNS and BCNS as follows.

Definition 4. [7] Let R be the universe. A complex neutrosophic set S in R is defined as $S = \{< r;\ T_s(r),\ I_s(r),\ F_s(r) >: r \in R\}$, where $T_s(r),\ I_s(r)$ and $F_s(r)$ are complex-valued truth, indeterminate and false membership functions and are of the form $T_s(r) = P_s(r), e^{j\mu_s(r)},\ I_s(r) = q_s(r), e^{ju_s(r)}$ and $F_s(r) = v_s(r), e^{j\omega_s(r)}$. By definition, $P_s(r), q_s(r), v_s(r)$ and $\mu_s(r),\ u(r),\ \omega_s(r)$ are, respectively, real valued and $P_s(r), q_s(r), v_s(r) \in [0,\ 1]$, such that $0^- \le P_s(r) + q_s(r) + v_s(r) \le 3^+$.

Definition 5. [22] A BCNS S in U is defined as:

$S = \{< u;\ p^+e^{i\mu^+},\ q^+e^{iv^+}, r^+e^{i\omega^+},\ p^-e^{i\mu^-},\ q^-e^{iv^-}, r^-e^{i\omega^-} >: u \in U\}$, where $p^+, q^+, r^+: U \to [0,1]$ and $p^-, q^-, r^-: U \to [-1,0]$.

A bipolar complex neutrosophic number can be represented as follows.

$S =< p^+e^{i\mu^+}, q^+e^{iv^+}, r^+e^{i\omega^+}, p^-e^{i\mu^-}, q^-e^{iv^-}, r^-e^{i\omega^-} >.$

3. Complex bipolar- valued neutrosophic soft set

Definition 6. Let X be a universe and A be a set of parameters. A complex bipolar- valued neutrosophic soft set (CBVNSS) (B, A) is defined as:

$(B, A) = \{< a, \{T^+_{B(a)}(x), I^+_{B(a)}(x), F^+_{B(a)}(x), T^-_{B(a)}(x), I^-_{B(a)}(x), F^-_{B(a)}(x)\} >: a \in A,\ x \in X\}$, where $\forall\, a \in A, \forall\, x \in X, T^+_{B(a)}(x) = P^+_{B(a)}(x)e^{2\pi i\mu^+_{B(a)}(x)}, I^+_{B(a)}(x) = q^+_{B(a)}(x)e^{2\pi i v^+_{B(a)}(x)}, F^+_{B(a)}(x) = r^+_{B(a)}(x)e^{2\pi i\omega^+_{B(a)}(x)}, T^-_{B(a)}(x) = P^-_{B(a)}(x)e^{2\pi i\mu^-_{B(a)}(x)}, I^-_{B(a)}(x) = q^-_{B(a)}(x)e^{2\pi i v^-_{B(a)}(x)}$, and $F^-_{B(a)}(x) = r^-_{B(a)}(x)e^{2\pi i\omega^-_{B(a)}(x)}$, such that :

$p^+, q^+, r^+, \mu^+, v^+, \omega^+: X \to [0,1]$ and $p^-, q^-, r^-, \mu^-, v^-, \omega^-: X \to [-1,0]$. The positive membership degrees T^+, I^+, F^+ denote, respectively the complex valued truth, indeterminacy, and falsity membership degrees of an element $x \in X$ to the property corresponding to a CBVNSS (B, A), and the negative membership degrees T^-, I^-, F^- are to denote the complex valued truth, indeterminacy, and falsity membership degrees of an element $x \in X$ to some implicit counter-property corresponding to a CBVNSS (B, A).

To illustrate the above definition, we provide the following example.

Example 1. Suppose $X = \{x_1, x_2\}$ is the universe and $A = \{a_1, a_2\}$ is the parameters set. Then the CBVNSS (B, A) is defined as below:

$(B,A) =$

$$\{< a_1, \left\{\frac{x_1}{<0{,}2\,e^{2\pi i(0{,}5)},\ 0{,}1\,e^{2\pi i(0{,}4)},\ 0{,}3\,e^{2\pi i(0{,}8)},-0{,}2\,e^{2\pi i(-0{,}5)},-0{,}8\,e^{2\pi i(-0{,}7)},\ -0{,}1\,e^{2\pi i(-0{,}2)}>}\right\},$$

$$\left\{\frac{x_2}{<0{,}9\,e^{2\pi i(0{,}7)},\ 0{,}2\,e^{2\pi i(0{,}5)},\ 0{,}4\,e^{2\pi i(0{,}1)},-0{,}3\,e^{2\pi i(-0{,}6)},-0{,}1\,e^{2\pi i(-0{,}5)},\ -0{,}4\,e^{2\pi i(-0{,}5)}>}\right\}>,$$

$$< a_2, \left\{\frac{x_1}{<0{,}5\,e^{2\pi i(0{,}6)},\ 0{,}4\,e^{2\pi i(0{,}3)},\ 0{,}1\,e^{2\pi i(0{,}5)},-0{,}2\,e^{2\pi i(-0{,}7)},-0{,}3\,e^{2\pi i(-0{,}4)},\ -0{,}2\,e^{2\pi i(-0{,}6)}>}\right\},$$

$$\left\{\frac{x_2}{<0{,}8\,e^{2\pi i(0{,}4)},\ 0{,}2\,e^{2\pi i(0{,}4)},\ 0{,}7\,e^{2\pi i(0{,}9)},-0{,}9\,e^{2\pi i(-0{,}4)},-0{,}8\,e^{2\pi i(-0{,}2)},\ -0{,}7\,e^{2\pi i(-0{,}5)}>}\right\}>\}.$$

In the following we define the empty CBVNSS and the the absolute CBVNSS.

Definition 7. Let (B,A) be a CBVNSS over M. Then (B,A) is said to be empty CBVNSS denoted by $B_\emptyset$, if $T^+_{B(a)}(m) = 0, I^+_{B(a)}(m) = 1, F^+_{B(a)}(m) = 1$ and $T^-_{B(a)}(m) = 0, I^-_{B(a)}(m) = -1, F^-_{B(a)}(m) = -1, \forall a \in A, \forall m \in M$ and defined as:

$$(B_\emptyset, A) = \{< a, \{0, 1, 1, 0, -1, -1\} >: a \in A, m \in M\}.$$

Definition 8. Let (B,A) be a CBVNSS over M. Then (B,A) is said to be absolute CBVNSS denoted by B_M, if $T^+_{B(a)}(m) = 1, I^+_{B(a)}(m) = 0, F^+_{B(a)}(m) = 0$ and $T^-_{B(a)}(m) = -1, I^-_{B(a)}(m) = 0, F^-_{B(a)}(m) = 0, \forall a \in A, \forall m \in M$ and defined as:

$$(B_M, A) = \{< a, \{1, 0, 0, -1,0,0\} >: a \in A, m \in M\}.$$

Now, we define the concept of the complement of the CBVNSS.

Definition 9. Let M be a universe of discourse and (B,A) be a CBVNSS on M, which is defined below:

$$(B,A) = \left\{< a, \left\{T^+_{B(a)}(m), I^+_{B(a)}(m), F^+_{B(a)}(m), T^-_{B(a)}(m), I^-_{B(a)}(m), F^-_{B(a)}(m)\right\} >: a \in A, \quad m \in M\right\}.$$

The complement of (B,A) is denoted by $(B,A)^c = (B^c, A)$ and is defined as:

$(B,A)^c = \{< a, \{T^+_{B^c(a)}(m), I^+_{B^c(a)}(m), F^+_{B^c(a)}(m), T^-_{B^c(a)}(m), I^-_{B^c(a)}(m), F^-_{B^c(a)}(m)\} >: a \in A, m \in M\}$, where

$T^+_{B^c(a)}(m) = P^+_{B^c(a)}(m)e^{2\pi i \mu^+_{B^c(a)}(m)} = r^+_{B(a)}(m)e^{2\pi i \omega^+_{B(a)}(m)}$, $I^+_{B^c(a)}(m) = q^+_{B^c(a)}(m)e^{2\pi i \nu^+_{B^c(a)}(m)} =$

$\left(1 - q^+_{B(a)}(m)\right) e^{2\pi i(1-\nu^+_{B(a)}(m))}$, $F^+_{B^c(a)}(m) = r^+_{B^c(a)}(m)e^{2\pi i \omega^+_{B^c(a)}(m)} = P^+_{B(a)}(m)e^{2\pi i \mu^+_{B(a)}(m)}$,

$T^-_{B^c(a)}(m) = P^-_{B^c(a)}(m)e^{2\pi i \mu^-_{B^c(a)}(m)} = r^-_{B(a)}(m)e^{2\pi i \omega^-_{B(a)}(m)}$, $I^-_{B^c(a)}(m) = q^-_{B^c(a)}(m)e^{2\pi i \nu^-_{B^c(a)}(m)} =$

$\left(-1 - q^-_{B(a)}(m)\right) e^{2\pi i(-1-\nu^-_{B(a)}(m))}$, $F^-_{B^c(a)}(m) = r^-_{B^c(a)}(m)e^{2\pi i \omega^-_{B^c(a)}(m)} = P^-_{B(a)}(m)e^{2\pi i \mu^-_{B(a)}(m)}$.

Example 2. Consider Example 1. By Definition 9, we get the complement of the CBVNSS (B,A) as:

$(B, A)^{c} =$

$$\{< a_1, \left\{\frac{x_1}{<0{,}3\, e^{2\pi i(0{,}8)},\ 0{,}9\, e^{2\pi i(0{,}6)},\ 0{,}2\, e^{2\pi i(0{,}5)}, -0{,}1\, e^{2\pi i(-0{,}2)}, -0{,}2\, e^{2\pi i(-0{,}3)},\ -0{,}2\, e^{2\pi i(-0{,}5)} >}\right\},$$
$$\left\{\frac{x_2}{<0{,}4\, e^{2\pi i(0{,}1)},\ 0{,}8\, e^{2\pi i(0{,}5)},\ 0{,}9\, e^{2\pi i(0{,}7)}, -0{,}4\, e^{2\pi i(-0{,}5)}, -0{,}9\, e^{2\pi i(-0{,}5)},\ -0{,}3\, e^{2\pi i(-0{,}6)} >}\right\},$$
$$a_2, \left\{\frac{x_1}{<0{,}1\, e^{2\pi i(0{,}5)},\ 0{,}6\, e^{2\pi i(0{,}2)},\ 0{,}5\, e^{2\pi i(0{,}6)}, -0{,}2\, e^{2\pi i(-0{,}6)}, -0{,}7\, e^{2\pi i(-0{,}6)},\ -0{,}2\, e^{2\pi i(-0{,}7)} >}\right\},$$
$$\left\{\frac{x_2}{<0{,}7\, e^{2\pi i(0{,}9)},\ 0{,}8\, e^{2\pi i(0{,}6)},\ 0{,}8\, e^{2\pi i(0{,}4)}, -0{,}7\, e^{2\pi i(-0{,}5)}, -0{,}2\, e^{2\pi i(-0{,}8)},\ -0{,}9\, e^{2\pi i(-0{,}4)} >}\right\} >\}.$$

Proposition 1. If (B, A) is a CBVNSS over the universe X, then $((B,\ A)^c)^c = (B,\ A)$.

Proof. The proof is straightforward from Definition 9. □

Now, we put forward the definition of the subset of two CBVNSSs.

Definition 10. For two CBVNSSs (B, A) and $(B',\ A')$ over a universe U, CBVNSS (B, A) is contained in CBVNSS $(B',\ A')$, denoted as $(B, A) \sqsubseteq (B',\ A')$ if:

(1) $A \subseteq A'$, and (2) $\forall a \in A,\ \forall m \in M,\ P^+_{B(a)}(m) \le P^+_{B'(a)}(m),\ q^+_{B(a)}(m) \ge q^+_{B'(a)}(m),\ r^+_{B(a)}(m) \ge r^+_{B'(a)}(m),\ \mu^+_{B(a)}(m) \le \mu^+_{B'(a)}(m),\ v^+_{B(a)}(m) \ge v^+_{B'(a)}(m),\ \omega^+_{B(a)}(m) \ge \omega^+_{B'(a)}(m)$ and $P^-_{B(a)}(m) \ge P^-_{B'(a)}(m), q^-_{B(a)}(m) \le q^-_{B'(a)}(m),\ r^-_{B(a)}(m) \le r^-_{B'(a)}(m), \mu^-_{B(a)}(m) \ge \mu^-_{B'(a)}(m),\ v^-_{B(a)}(m)\ v^-_{B'(a)}(m),\ \omega^-_{B(a)}(m) \le \omega^-_{B'(a)}(m)$.

Definition 11. For two CBVNSSs (B, A) and $(B',\ A')$ over a universe M, (B, A) is equal to $(B',\ A')$ and it is denoted as $(B, A)=(B',\ A')$ if and only if $(B, A) \sqsubseteq (B',\ A')$ and $(B',\ A') \sqsubseteq (B, A)$.

We establish the definitions of the union and intersection of two CBVNSSs below.

Definition 12. Lex X be a universe. The union of two CBVNSSs (B, A) and $(B',\ A')$ denoted as

$(B, A) \sqcup (B',\ A')$ is a CBVNSS (C, D), where $D = A \cup A'$ and $\forall \epsilon \in D,\ \forall m \in M$,

$$T^+_{C(\epsilon)} = \begin{cases} P^+_{B(\epsilon)}(m) e^{2\pi i \mu^+_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ P^+_{B'(\epsilon)}(m) e^{2\pi i \mu^+_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(P^+_{B(\epsilon)}(m) \vee P^+_{B'(\epsilon)}(m) \right). e^{2\pi i (\mu^+_{B(\epsilon)}(m) \vee \mu^+_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$I^+_{C(\epsilon)} = \begin{cases} q^+_{B(\epsilon)}(m) e^{2\pi i v^+_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ q^+_{B'(\epsilon)}(m) e^{2\pi i v^+_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(q^+_{B(\epsilon)}(m) \wedge q^+_{B'(\epsilon)}(m) \right). e^{2\pi i (v^+_{B(\epsilon)}(m) \wedge v^+_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$F^+_{C(\epsilon)} = \begin{cases} r^+_{B(\epsilon)}(m) e^{2\pi i \omega^+_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ r^+_{B'(\epsilon)}(m) e^{2\pi i \omega^+_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(r^+_{B(\epsilon)}(m) \wedge r^+_{B'(\epsilon)}(m) \right). e^{2\pi i (\omega^+_{B(\epsilon)}(m) \wedge \omega^+_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$T^{-}_{C(\epsilon)} = \begin{cases} P^{-}_{B(\epsilon)}(m)e^{2\pi i\mu^{-}_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ P^{-}_{B'(\epsilon)}(m)e^{2\pi i\mu^{-}_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(P^{-}_{B(\epsilon)}(m) \wedge P^{-}_{B'(\epsilon)}(m)\right).e^{2\pi i(\mu^{-}_{B(\epsilon)}(m) \wedge \mu^{-}_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$I^{-}_{C(\epsilon)} = \begin{cases} q^{-}_{B(\epsilon)}(m)e^{2\pi i\nu^{-}_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ q^{-}_{B'(\epsilon)}(m)e^{2\pi i\nu^{-}_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(q^{-}_{B(\epsilon)}(m) \vee q^{-}_{B'(\epsilon)}(m)\right).e^{2\pi i(\nu^{-}_{B(\epsilon)}(m) \vee \nu^{-}_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$F^{-}_{C(\epsilon)} = \begin{cases} r^{-}_{B(\epsilon)}(m)e^{2\pi i\omega^{-}_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ r^{-}_{B'(\epsilon)}(m)e^{2\pi i\omega^{-}_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(r^{-}_{B(\epsilon)}(m) \vee r^{-}_{B'(\epsilon)}(m)\right).e^{2\pi i(\omega^{-}_{B(\epsilon)}(m) \vee \omega^{-}_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

Definition 13. Lex M be a universe. The intersection of two CBVNSSs (B, A) and $(B',\ A')$ denoted as $(B, A) \sqcap (B',\ A')$ is a CBVNSS (C, D), where $D = A\ \cup\ A'$ and $\forall\epsilon \in D,\ \forall m\ \in M,$

$$T^{+}_{C(\epsilon)} = \begin{cases} P^{+}_{B(\epsilon)}(m)e^{2\pi i\mu^{+}_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ P^{+}_{B'(\epsilon)}(m)e^{2\pi i\mu^{+}_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(P^{+}_{B(\epsilon)}(m) \wedge P^{+}_{B'(\epsilon)}(m)\right).e^{2\pi i(\mu^{+}_{B(\epsilon)}(m) \wedge \mu^{+}_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$I^{+}_{C(\epsilon)} = \begin{cases} q^{+}_{B(\epsilon)}(m)e^{2\pi i\nu^{+}_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ q^{+}_{B'(\epsilon)}(m)e^{2\pi i\nu^{+}_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(q^{+}_{B(\epsilon)}(m) \vee q^{+}_{B'(\epsilon)}(m)\right).e^{2\pi i(\nu^{+}_{B(\epsilon)}(m) \vee \nu^{+}_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$F^{+}_{C(\epsilon)} = \begin{cases} r^{+}_{B(\epsilon)}(m)e^{2\pi i\omega^{+}_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ r^{+}_{B'(\epsilon)}(m)e^{2\pi i\omega^{+}_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(r^{+}_{B(\epsilon)}(m) \vee r^{+}_{B'(\epsilon)}(m)\right).e^{2\pi i(\omega^{+}_{B(\epsilon)}(m) \vee \omega^{+}_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$T^{-}_{C(\epsilon)} = \begin{cases} P^{-}_{B(\epsilon)}(m)e^{2\pi i\mu^{-}_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ P^{-}_{B'(\epsilon)}(m)e^{2\pi i\mu^{-}_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(P^{-}_{B(\epsilon)}(m) \vee P^{-}_{B'(\epsilon)}(m)\right).e^{2\pi i(\mu^{-}_{B(\epsilon)}(m) \vee \mu^{-}_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$I^{-}_{C(\epsilon)} = \begin{cases} q^{-}_{B(\epsilon)}(m)e^{2\pi i\nu^{-}_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ q^{-}_{B'(\epsilon)}(m)e^{2\pi i\nu^{-}_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(q^{-}_{B(\epsilon)}(m) \wedge q^{-}_{B'(\epsilon)}(m)\right).e^{2\pi i(\nu^{-}_{B(\epsilon)}(m) \wedge \nu^{-}_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

$$F^{-}_{C(\epsilon)} = \begin{cases} r^{-}_{B(\epsilon)}(m)e^{2\pi i\omega^{-}_{B(\epsilon)}(m)} & if\ \epsilon \in A - A' \\ r^{-}_{B'(\epsilon)}(m)e^{2\pi i\omega^{-}_{B'(\epsilon)}(m)} & if\ \epsilon \in A' - A \\ \left(r^{-}_{B(\epsilon)}(m) \wedge r^{-}_{B'(\epsilon)}(m)\right).e^{2\pi i(\omega^{-}_{B(\epsilon)}(m) \wedge \omega^{-}_{B'(\epsilon)}(m))} & if\ \epsilon \in A \cap A' \end{cases}$$

Theorem 1. If (B, A) and (B', A') are two CBVNSSs over the universe $\boldsymbol{X}$, then the union $(B, A) \sqcup (B', A')$ is the smallest CBVNSS which contains both these two sets.

Proof. The proof can be easily stated according to Definitions 10 and 12.

Theorem 2. If (B, A) and (B', A') are two CBVNSSs over the universe X, then the intersection $(B, A) \sqcap (B', A')$ is the largest CBVNSS which is contained in both of these two sets.

Proof. The proof can be easily stated according to Definitions 10 and 13.

Proposition 2. The following properties hold for the CBVNSSs (B, A) , (B', A') and (B'', A'').

1. $(B_{\emptyset}, A)^c = (B_X, A)$,
2. $(B_X, A)^c = (B_{\emptyset}, A)$,
3. $(B, A) \sqcup (B_{\emptyset}, A) = (B, A)$,
4. $(B, A) \sqcup (B_X, A) = (B_X, A)$,
5. $(B, A) \sqcap (B_{\emptyset}, A) = (B_{\emptyset}, A)$,
6. $(B, A) \sqcap (B_X, A) = (B, A)$,
7. $(B, A) \sqcup (B', A') = (B', A') \sqcup (B, A)$,
8. $(B, A) \sqcap (B', A') = (B', A') \sqcap (B, A)$,
9. $(B, A) \sqcup ((B', A') \sqcup (B'', A'')) = ((B, A) \sqcup (B', A')) \sqcup (B'', A'')$,
10. $(B, A) \sqcap ((B', A') \sqcap (B'', A'')) = ((B, A) \sqcap (B', A')) \sqcap (B'', A'')$,
11. $(B, A) \sqcup ((B', A') \sqcap (B'', A'')) = ((B, A) \sqcup (B', A')) \sqcap \big((B, A) \sqcup (B'', A'')\big)$,
12. $(B, A) \sqcap ((B', A') \sqcup (B'', A'')) = ((B, A) \sqcap (B', A')) \sqcup \big((B, A) \sqcap (B'', A'')\big)$,
13. $((B, A) \sqcup ((B', A'))^c = (B, A)^c \sqcap (B', A')^c$,
14. $((B, A) \sqcap ((B', A'))^c = (B, A)^c \sqcup (B', A')^c$.

Proof. The proof is straightforward by Definitions 9, 12 and 13.

4. Application of the CBVNSS in the decision making.

In this section, we establish an approach to decision making problem based on the CBVNSS model proposed in this paper.

In fact, all the existing approaches to decision making based on NSS and its extensions theory have solved kinds of decision problem effectively. In 2013, Maji [9] first give the decision method based on NSS theory by using the comparison matrix of the NSS to compute the scores of a set of alternatives. In the same year Broumi and Smarandache [32] also applied the intuitionistic neutrosophic soft set to solve the blouse purchase problem by computing the comparison matrix of the intuitionistic neutrosophic soft set. As an adaptation of the algorithm proposed in [32], Broumi et al. [33] developed an algorithm used the score function and the comparison matrix of CNSS to determine the country with the strongest economic indicators among a set of selected countries. On the other hand, Ali et al. [28] proposed an aggregation bipolar neutrosophic soft operator of a bipolar neutrosophic soft set and developed a decision making algorithm based on bipolar neutrosophic soft sets.

In the following, we establish a new approach to decision making based on the CBVNSS theory. In this approach, a modified algorithm and an accompanying score function is presented as an adaptation of the method proposed in [33], which was then made compatible with the structure of the CBVNSS model. To achieve this, we present the definitions of the comparison matrix of the CBVNSS and the score function as follows.

Definition 14. Assume that $(B, A) = \{< a, \{T^+_{B(a)}(m),\ I^+_{B(a)}(m),\ F^+_{B(a)}(m),\ T^-_{B(a)}(m),\ I^-_{B(a)}(m), F^-_{B(a)}(m)\ \} >: a \in A,\ m\ \in M\}$ is a CBVNSS, where $\forall\ a \in A,\ \forall\ m \in M$,

$$T^+_{B(a)}(m) = P^+_{B(a)}(m)e^{2\pi i\mu^+_{B(a)}(m)},\ I^+_{B(a)}(m) = q^+_{B(a)}(m)e^{2\pi i v^+_{B(a)}(m)},\ F^+_{B(a)}(m) = r^+_{B(a)}(m)e^{2\pi i\omega^+_{B(a)}(m)},$$

$$T^-_{B(a)}(m) = P^-_{B(a)}(m)e^{2\pi i\mu^-_{B(a)}(m)},\ I^-_{B(a)}(m) = q^-_{B(a)}(m)e^{2\pi i v^-_{B(a)}(m)},\ F^-_{B(a)}(m) = r^-_{B(a)}(m)\ e^{2\pi i\omega^-_{B(a)}(m)}.$$

Then the comparison matrix of (B, A) is a matrix whose rows comprise the alternatives variables m_1, m_2, $x_3, \dots m_k$ and the columns represent the parameters variables $a_1, a_2, a_3 \dots, a_n$. The entries μ_{ij} of this matrix are calculated as $\mu_{ij} = \left(\theta^+_{amp} + \sigma^+_{amp} - \tau^+_{amp}\right) + \left(\theta^+_{phase} + \sigma^+_{phase} - \tau^+_{phase}\right) -$

$\left(\theta^-_{amp} + \sigma^-_{amp} - \tau^-_{amp}\right) - \left(\theta^-_{phase} + \sigma^-_{phase} - \tau^-_{phase}\right)$, where the components of this formula are as defined below for all $m_i, m_k\ \in M$, such that $i \neq k$.

θ^+_{amp} = The number of times $P^+_{B(a_j)}(m_i)$ exceeds or is equal to $P^+_{B(a_j)}(m_k)$,
σ^+_{amp}= The number of times $q^+_{B(a_j)}(m_i)$ exceeds or is equal to $q^+_{B(a_j)}(m_k)$,
τ^+_{amp} = The number of times $r^+_{B(a_j)}(m_i)$ exceeds or is equal to $r^+_{B(a_j)}(m_k)$,

θ^+_{phase} = The number of times $\mu^+_{B(a_j)}(m_i)$ exceeds or is equal to $\mu^+_{B(a_j)}(m_k)$,
σ^+_{phase}= The number of times $v^+_{B(a_j)}(m_i)$ exceeds or is equal to $v^+_{B(a_j)}(m_k)$,
τ^+_{phase} = The number of times $\omega^+_{B(a_j)}(m_i)$ exceeds or is equal to $\omega^+_{B(a_j)}(m_k)$,

θ^-_{amp} = The number of times $P^-_{B(a_j)}(m_i)$ exceeds or is equal to $P^-_{B(a_j)}(m_k)$,
σ^-_{amp}= The number of times $q^-_{B(a_j)}(m_i)$ exceeds or is equal to $q^-_{B(a_j)}(m_k)$,
τ^-_{amp} = The number of times $r^-_{B(a_j)}(m_i)$ exceeds or is equal to $r^-_{B(a_j)}(m_k)$,
and
θ^-_{phase} = The number of times $\mu^-_{B(a_j)}(m_i)$ exceeds or is equal to $\mu^-_{B(a_j)}(m_k)$,
σ^-_{phase}= The number of times $v^-_{B(a_j)}(m_i)$ exceeds or is equal to $v^-_{B(a_j)}(m_k)$,
τ^-_{phase} = The number of times $\omega^-_{B(a_j)}(m_i)$ exceeds or is equal to $\omega^-_{B(a_j)}(m_k)$.

Definition 15. Score of the complex bipolar-valued neutrosophic element x_i in the universe X is calculated using the score function $\mathcal{R}_i$ as $\mathcal{R}_i = \sum_j \mu_{ij}$.

Example 3. Consider that an automobile manufacturer produces three models of cars, where $X = \{ x_1, x_2, x_3 \}$ represents the set of alternative models. Suppose that the manufacturer wants to examine these three models of cars two times, once before and again after trying a sample of each model of these cars. Then decide to make a decision about the most desirable model of these cars. Suppose that there are three attributes have been approved in this decision making process, where a_1 stands for comfortability, a_2 stands for reliability and a_3 stands for durability.

In this context, the CBVNSS has been applied such that the amplitude terms represent the decision information in first stage (before testing the cars), whereas the phase terms represent the decision information in second stage (after testing the cars. On the other hand, the positive decision information denote the complex valued membership degrees of an element $x \in X$ to the attribute corresponding to a CBVNSS, and the negative decision information denote the complex valued membership degrees of an element $x \in X$ to some implicit counter-attribute corresponding to a CBVNSS. All of these different types of valuable information can be expressed using the CBVNSS (B, A) as follows.

$$(B, A) =$$

$$\{< a_1, \left\{\frac{x_1}{<0,5\, e^{2\pi i(0,7)},\ 0,1\, e^{2\pi i(0,2)},\ 0,1\, e^{2\pi i(0,2)}, -0,4\, e^{2\pi i(-0,5)}, -0,8\, e^{2\pi i(-0,7)}, -0,2\, e^{2\pi i(-0,2)}>}\right\},$$

$$\left\{\frac{x_2}{<0,8\, e^{2\pi i(0,7)},\ 0,4\, e^{2\pi i(0,5)},\ 0,1\, e^{2\pi i(0,1)}, -0,1\, e^{2\pi i(-0,3)}, -0,1\, e^{2\pi i(-0,5)}, -0,8\, e^{2\pi i(-0,7)}>}\right\},$$

$$\left\{\frac{x_3}{<0,1\, e^{2\pi i(0,3)},\ 0,5\, e^{2\pi i(0,4)},\ 0,8\, e^{2\pi i(0,8)}, -0,9\, e^{2\pi i(-0,8)}, -0,3\, e^{2\pi i(-0,4)}, -0,1\, e^{2\pi i(-0,2)}>}\right\}>,$$

$$< a_2, \left\{\frac{x_1}{<0,3\, e^{2\pi i(0,4)},\ 0,3\, e^{2\pi i(0,4)},\ 0,7\, e^{2\pi i(0,8)}, -0,7\, e^{2\pi i(-0,5)}, -0,5\, e^{2\pi i(-0,7)}, -0,6\, e^{2\pi i(-0,7)}>}\right\},$$

$$\left\{\frac{x_2}{<0,9\, e^{2\pi i(0,8)},\ 0,1\, e^{2\pi i(0,3)},\ 0,2\, e^{2\pi i(0,4)}, -0,1\, e^{2\pi i(-0,4)}, -0,2\, e^{2\pi i(-0,3)}, -0,1\, e^{2\pi i(-0,2)}>}\right\},$$

$$\left\{\frac{x_3}{<0,2\, e^{2\pi i(0,3)},\ 0,4\, e^{2\pi i(0,3)},\ 0,5\, e^{2\pi i(0,6)}, -0,4\, e^{2\pi i(-0,5)}, -0,5\, e^{2\pi i(-0,6)}, -0,6\, e^{2\pi i(-0,7)}>}\right\}>,$$

$$< a_3, \left\{\frac{x_1}{<0,4\, e^{2\pi i(0,3)},\ 0,1\, e^{2\pi i(0,2)},\ 0,9\, e^{2\pi i(0,7)}, -0,5\, e^{2\pi i(-0,5)}, -0,2\, e^{2\pi i(-0,5)}, -0,7\, e^{2\pi i(-0,6)}>}\right\},$$

$$\left\{\frac{x_2}{<0,7\, e^{2\pi i(0,6)},\ 0,3\, e^{2\pi i(0,5)},\ 0,4\, e^{2\pi i(0,3)}, -0,3\, e^{2\pi i(-0,4)}, -0,1\, e^{2\pi i(-0,3)}, -0,5\, e^{2\pi i(-0,6)}>}\right\},$$

$$\left\{\frac{x_3}{<0,3\, e^{2\pi i(0,5)},\ 0,8\, e^{2\pi i(0,5)},\ 0,7\, e^{2\pi i(0,6)}, -0,7\, e^{2\pi i(-0,5)}, -0,8\, e^{2\pi i(-0,7)}, -0,4\, e^{2\pi i(-0,2)}>}\right\}>\}.$$

Now, our problem is to determine the most desirable model of cars to the manufacturer.
To solve this decision making problem, we use the above CBVNSS along with a modified algorithm adopted from [33] as follows.

Step 1 : Input the CBVNSS(B, A) .

Step 2: Compute the CBVNSS comparison matrix using the formula given in Definition 14.

Step 3: Compute the score $\mathcal{R}_i$ for each alternative x_i in the universe X using Definition 15.

Step 4: Choose the alternative with the maximum score as the optimal alternative, If there are more than one alternative have the maximum score, anyone can be chosen as the optimal alternative.

Now, we apply the above algorithm to solve our decision making problem.

The comparison matrix of the CBVNSS (B, A) is constructed as in Table 1.

Table 1. Comparison matrix of the CBVNSS (B, A)

Attributes	Alternatives		
	x_1	x_2	x_3
a_1	2	-1	0
a_2	1	1	-1
a_3	-6	1	6

Next, we compute the score values $\mathcal{R}_i$ for each alternative x_i as shown in Table 2.

Table 2. Score values of the alternatives x_i

x_i	x_1	x_2	x_3
$\mathcal{R}_i$	-3	1	5

Clearly the maximum score is 5. Thus, the decision is to choose the alternative x_3 as the optimal Solution. Therefore, we conclude that model x_3 is the most desirable model of cars, followed by model x_2 and model x_1.

Conclusion

We established the notion of CBVNSS as an extension of BNSS by extending its range from the real space to the complex space. The formal definition of the CBVNSS is stated based on the definitions of the bipolar complex neutrosophic set and soft set. Some essential operations such as complement, subset, union and intersection with their properties are defined and verified. A modified algorithm has been presented and its decision steps constructed. It was shown to be workable and successful in producing a desired result as illustrated by the application in decision making. CBVNSS seems to be a promising new concept, open the way toward various future research. We intend to study this model further to conduct some real applications which involve bipolarity, uncertainty and periodicity simultaneously in the field of physics, signal processing, stock marketing, and so on.

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