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Multi-Objective Inventory Model with Deterioration under Space Constraint: Neutrosophic Hesitant Fuzzy Programming Approach

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Abstract: We have considered a deterministic inventory model with time-dependent demand and holding cost and time varying deterioration where shortages are allowed and partially backlogged. To reduce deterioration we have considered here a preservation condition. In the presence uncertainty we have taken cost parameters as generalized trapezoidal fuzzy number. The proposed model has been solved by neutrosophic hesitant fuzzy programming approach, fuzzy nonlinear programming approach and fuzzy additive goal programming technique. The model is illustrated with numerical example and we presented sensitivity analysis finally.

Keywords: Inventory model, Multi-item, Preservation condition, demand, holding cost, deterioration, generalized trapezoidal fuzzy number, Neutrosophic Hesitant Fuzzy programming approach.

1. Introduction: In inventory models the effect of deterioration plays a very important role but in traditional inventory model the most popular assumptions was that the products kept their characteristic and physical structure while they were stored in the storage of inventory. But this types of assumptions are not be true always for all types of products. Deterioration is defined as erosion, change or spoilage that prevents the item from being used for its original purpose. Food products, drugs, radioactive substances, photograph, electronic components are a few examples of products in which decay may occur during the normal storage period of the units, and that is why this loss we have to consider into account while developing the inventory model. But with the help of some electronic devices like freeze, Micro-oven, etc. or some other type of devices it is possible to reduce the effect of deterioration.

Formulating an inventory model most of the researchers take deteriorating cost, inventory set-up cost ,holding cost as constant but in reality they may not constant always ,so if we take those cost parameters in fuzzy variable then it will be most realistic and interesting.

F. Harris (1915) developed first inventory model. Lotfi A. Zadeh in 1965 introduced the concept of fuzzy set theory in inventory modeling. L. A. Zadeh and R. E. Bellman in 1970 considered an inventory model on decision making in fuzzy environment. In 1934, Wilson gave a formula to obtain EOQ. In 1996, M. Vujosevic, D. Petrovic and R. Petrovic developed an EOQ formula by assuming inventory cost as a fuzzy number. M. Lee developed a fuzzy inventory model by considering backorder as a trapezoidal fuzzy number in 1999. In 1999, Chang and Dye developed an inventory model with time –varying demand and partial backlogging.

Deterioration of a product is the most realistic thing to be considered. Ghare and Schrader developed an inventory model where they took demand rate and deterioration rate as a constant. T.K Roy and M.Maity presented an inventory fuzzy EOQ model with demand-dependent unit cost under limited storage capacity. Vinod Kumar Mishra, Lal Sahab Singh and Rakesh Kumar developed an inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging. Zaid T, Balkhi, Lakdere Benkherouf represent an inventory model for deterioration items with stock dependent and time-varying demand rates. In 2001, Horng-Jinh Chang and Chung-Yuan Dye developed an inventory model for deterioration items with partial backlogging and permissible delay in payment. Misra and Singh gave an inventory model for ramp –type demand, time-dependent deterioration items with salvage value and shortages and deteriorating inventory model for time-dependent demand and holding cost with partial backlogging. Sumana Saha and Tripti Chakrabarty in 2012 considered a fuzzy EOQ model with time varying demand and shortages. D. Dutta and Pawan Kumar studied a fuzzy inventory model without shortages using a trapezoidal fuzzy number. In 2013, D. Dutta and Pawan Kumar considered an optimal replenishment for an inventory model without shortages by assuming fuzziness in demand, holding cost and ordering cost.

Smarandache. F introduced the neutrosophic set. Smarandache. F developed a generalization of the intuitionistic fuzzy set and gave geometric interpretation of the Neutrosophic set which is the generalization of the intuitionistic fuzzy set. Ye. J studied on multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment. Abdel-Basset et al. proposed a novel approach to solving fully neutrosophic linear programming problem and applied to production planning problem. Ye et.al. formulated neutrosophic number nonlinear programming problem (NN-NPP) and proposed an effective method to solve the problem under neutrosophic number environments. Firoz Ahmad, Ahmad Yusuf Adhami, Florentin Smarandache developed Single valued Neutrosophic Hesitant Fuzzy Computational Algorithm for Multi objective Nonlinear Optimization Problem. C. kar, B. Mondal and T.K Roy developed an Inventory Model under Space Constraint in Neutrosophic Environment by Geometric Programming Approach. T. Garai and T.K. Roy studied on optimization of EOQ Model with Limited Storage Capacity by neutrosophic geometric programming.P. Biswas, S.Pramanik and B.C. Giri presented multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. In 2018, I. Ali, S.

Gupta, A. Ahmed represented Multi-objective bi-level supply chain network order allocation problem under fuzziness. V. Charles, G. Gupta and I. Ali developed A Supply Chain Network model in Fuzzy Goal Programming Approach with Pareto-Distributed Random Variables in 2019. In 2021, P. Gautam; S. Maheshwari, A. Kausar and C.K. Jaggi described an inventory Models for Imperfect Quality Items a Two-Decade Review. S. Gupta, A. Haq, I. Ali. And B. Sarkar described the Significance of multi-objective optimization in logistics problem for multi-product supply chain network under the intuitionistic fuzzy environment in 2021.

Research Motivation: In the present scenario in every situation of inventory storage it is very much obvious that there must be some kind of deterioration occur in every product sometimes it could be less or sometimes it could be more. Looking at this types of situation many authors takes many types of deteriorating function while developing their inventory models, sometimes they took constant deterioration, sometimes time depending linear type deterioration or sometimes stock dependent linear or quadratic type deteriorating function ect.But a few authors think about how to control these type of problems that is what should be the necessary step to be taken to reduce the very much effect of deterioration in inventory. In this paper we developed a method to reduce the effect of deterioration, here we introduced a constant "k" in the differential equation which gives some reduction on deterioration. In real life it is very often where many companies use freezer, Micro-oven or some other type of device to reduce the effect of deterioration. After getting the total average cost we have discussed several methodology to find the minimum average cost and what should be the timing to get minimum average cost.

AUTHORS CONTRIBUTION:

Names of authors	Papers title	Contribution
Ghare and Schrader	An EOQ Model for Deteriorating Items with Linear Demand, Variable Deterioration and Partial Backlogging.	Demand rate deterioration rate as a constant rate
T.K Roy and M.Maity	A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity	Demand dependent unit cost under limited storage capacity
Vinod Kumar Mishra, Lal Sahab Singh and Rakesh Kumar	An Inventory Model for Deteriorating Items with Time Dependent Demand and Holding Cost under Partial Backlogging	Inventory model for deteriorating items with time dependent demand and time varying holding cost under partial backlogging

D. Dutta and Pawan Kumar	Fuzzy Inventory Model without Shortage Using Trapezoidal Fuzzy Number with Sensitivity Analysis	Fuzzy inventory model without shortages using trapezoidal fuzzy number
P. Gautam, S.Maheshwari;A.Kausar,C.K.Jaggi	Inventory Models for Imperfect Quality Items: A Two-Decade Review.	an inventory Models for Imperfect Quality Items
F. Smarandache	A Geometric Interpretation of the Neutrosophic Set - A Generalization of the Intuitionistic Fuzzy Set	developed a generalization of the intuitionistic fuzzy set and gave geometric interpretation of the Neutrosophic set
Jun Ye, Surapati Pramanik	Multiple-attribute Decision-Making Method under a Single-Valued Neutrosophic Hesitant Fuzzy Environment	multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment
Firoz Ahmad, Ahmad Yusuf Adhami, Florentin Smarandache	Single valued Neutrosophic Hesitant Fuzzy Computational Algorithm for Multi objective Nonlinear Optimization Problem	developed Single valued Neutrosophic Hesitant Fuzzy Computational Algorithm for Multi objective Nonlinear Optimization Problem
I. Ali, S. Gupta, A. Ahmed	Multi-objective linear fractional inventory problem under intuitionistic fuzzy environment.	represent Multi-objective bi-level supply chain network order allocation problem under fuzziness
Sahidul Islam and Kausik Das	Multi-objective inventory model with deterioration under space constrain:Neutrosophic fuzzy programming approach	Inventory model with time dependent demand and holding cost with preservation condition under neutrosophic hesitant

		fuzzy programming approach.

In this present paper we have taken a deterministic inventory model with time-dependent demand and holding cost and time varying deterioration. Shortages are allowed and partially backlogged. In this proposed model we have taken deterioration of item as linearity increasing function of time. We have considered here a preservation condition. In the presence of uncertainty the cost parameters are considered as generalized trapezoidal fuzzy number. This model has been solved by neutrosophic hesitant fuzzy programming approach, fuzzy nonlinear programming approach and fuzzy additive goal programming technique. Numerical examples have been finally.

2. Mathematical Model:

The following notations and assumptions are considered to formulate the model for i'th item.

2.1 Notations For i'th item (i=1, 2, 3,…., n):

A_i : Ordering cost per unit item for i'th item..

H_i: Holding cost per unit time for i'th item..

θ_i : Deterioration rate is time proportional for i'th item.

k_i:The preservation constant for i'th item

D_i : Demand rate is time proportional for i'th item.

C_{1i} : the purchase cost per unit time for i'th item.

C_{2i} : Deterioration cost per unit time for i'th item.

C_{3i} : Shortage cost per unit time for i'th item.

C_{4i} : Opportunity cost(lost sale cost) per unit time for i'th item.

B_i : The backlogging rate , $0 \leq B_i \leq 1$ for i'th item.

T_i : The length of cycle time for i'th item, $T_i \geq 0$.

t_{0i} : The time when the inventory level starts to reduce due to demand only ,$t_{0i} \geq 0$.

t_{1i} : The time when the inventory level starts to reduce due to both demand and deterioration for i'th item, $t_{1i} \geq 0$.

I_{1i} : the level of positive inventory for i'th item when deterioration is not occurring in time $[0, t_{0i}]$.

I_{2i} : the level of positive inventory for i'th item when deterioration starts to occur in time $[t_{0i}, t_{1i}]$.

I_{3i} : the level of negative inventory due to shortages for i'th item in the time interval $[t_{1i}, T_i]$.

I_{max}: the maximum inventory at time t=0 for i'th item.

S: the maximum back order quantity during stock-out period for i'th item.

$Q_i(I_{max} + S)$: the order quantity for the duration of a cycle of length T_i for i'th item.

$TAC_i(t_{0i}.t_{1i},T_i)$: total average cost per unit for i'th item.

w_i: storage space per unit time for i'th item.

W: total area space.

$\widetilde{A_i}$: Fuzzy ordering cost for i'th item.

$\widetilde{C_{1i}}$ Fuzzy purchase cost for i'th item.

$\widetilde{C_{2i}}$: Fuzzy Deterioration cost per unit time for i'th item.

$\widetilde{C_{3\iota}}$: Fuzzy Shortage cost per unit time for i'th item.

$\widetilde{C_{4\iota}}$:Fuzzy Opportunity cost (lost sale cost) per unit time for i'th item.

$\widetilde{H_\iota}$:Fuzzy Holding cost per unit time for i'th item.

$\widetilde{TAC_\iota}$:Total average cost per unit for i'th item.

$\widetilde{w_\iota}$:Fuzzy storage space per unit time for i'th item.

2.2 Assumptions:

i. The inventory model involves multi-item.

ii. The replenishment occurs instantaneously at infinite rate.

iii. The lead time is negligible.

iv. Demand rate is constant for $t \in [0, t_{0i}]$ and demand rate is time dependent for $t \in [t_{0i}, t_{1i}]$.That is

$$Demand(D) = \begin{cases} a_i & if\ t \in [0, t_{0i}] \\ b_i e^{-\alpha_i t} & if\ t \in [t_{0i}, t_{1i}] \end{cases}$$

v. The deterioration rate for i'th item is $\theta_i(t) = \theta_i t$. $0 < \theta_i < 1$.

vi. The backlogging rate is $B_i(t) = \frac{1}{1+\delta_i(T_i - t)}$, where $t_{1i} \leq t \leq T_i$.δ_i is the positive back logging parameter.

vii. The constant preservation constant is k_i .

viii. The inventory holding cost $H_i(t)=h_i.t$ is a linear function.

2.3. Mathematical formulation

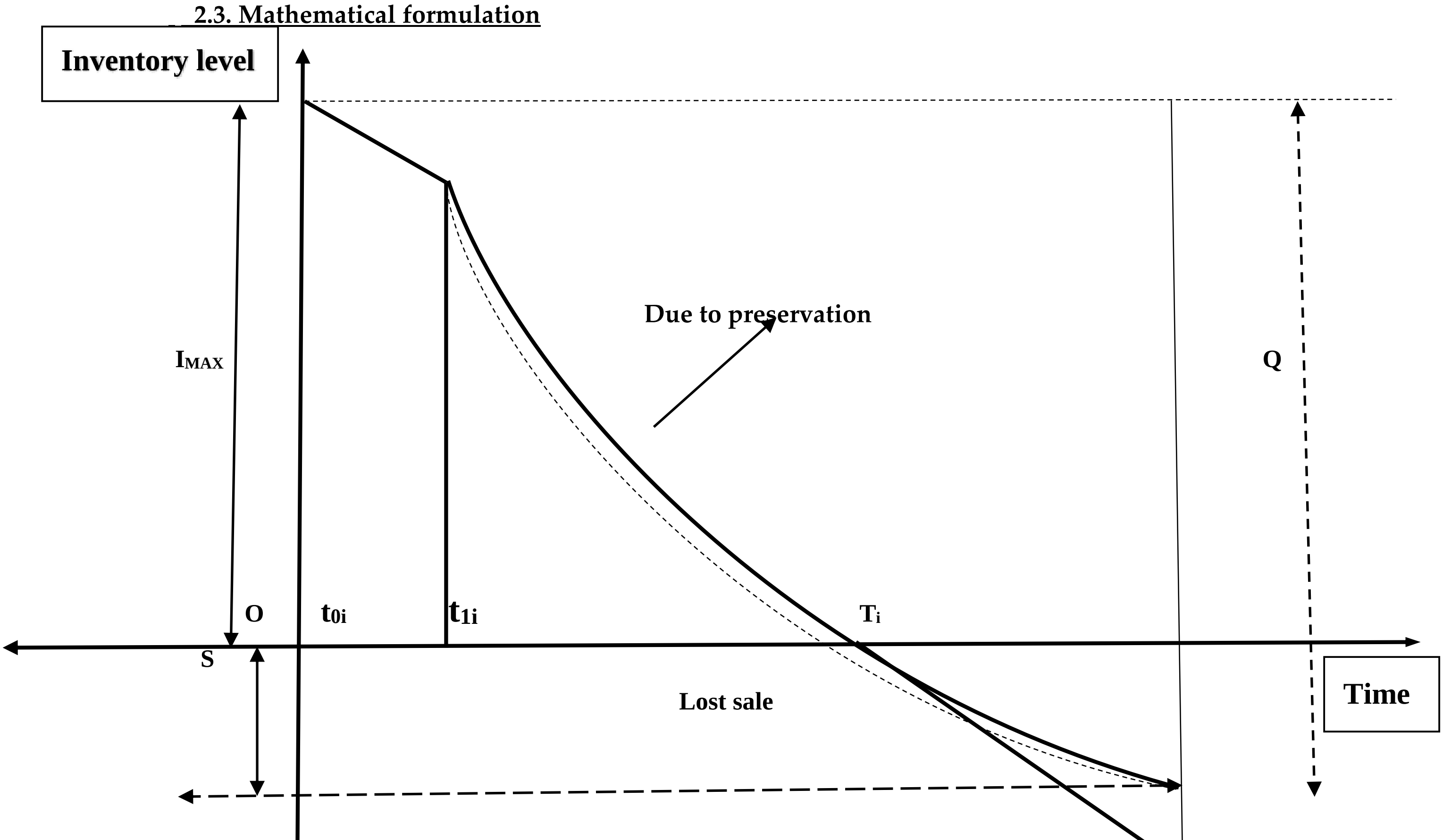


Figure-1: Representation of Inventory model

The initial inventory level is I_{max} unit at time t=0.From t=0 to $t=t_{0i}$, the inventory reduces just for demand. In time $t=t_{0i}$ to $t=t_{1i}$ the inventory level reduces for both demand as well as deterioration. At this time, shortage is accrued which is partially backlogged at rate $B_i(t)$ for the time $[t_{1i},T_i]$.At the end of the cycle, the inventory reaches a maximum shortage level so as to clear the backlogged and again the inventory level to I_{max} (Figure 1).

The rate of change of inventory during the positive stock period $[0, t_{0i}]$, $[t_{0i}, t_{1i}]$and negative stock period (i.e shortage period) $[t_{1i}, T_i]$ is governed by the following differential equation for i'th item.

$$\frac{dI_{1i}}{dt} = -D_i = -a_i\,, \qquad 0 \le t \le t_{0i} \tag{2.1}$$

$$\frac{dI_{2i}}{dt} = -D_i(t) - (\theta_i(t) - k_i)I_{2i}(t) = -b_i e^{-\alpha_i t} - (\theta_i t - k)I_{2i}(t), \quad t_{0i} \le t \le t_{1i} \tag{2.2}$$

$$\frac{dI_{3i}}{dt} = -a_i B_i(t) = -\frac{a_i}{1+\delta_i(T_i - t)}\,, \quad t_{1i} \le t \le T_i \tag{2.3}$$

Thus the boundary condition are as follows:

$I_{1i}(0)=I_{max}$; $I_{2i}(t_1)=0$; $I_{3i}(t_1)=0$;

From (2.1) we get, using the initial condition (2.4)

$I_{1i}(t)= I_{max} - a_i t$, $0 \le t \le t_{0i}$

From (2.2) we get using initial condition and neglecting the higher power of θ_i,multiplication of θ_i with k_i and $\alpha_i$also neglecting higher power $of\ k_i$ we get.

$$I_{2i}(t)=b_i[(t_{1i} - t) - \frac{\theta_i}{3}(t_{1i}^3 - t^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t^2)]\,,\ t_{0i} \le t \le t_{1i} \tag{2.5}$$

Since, $I_{1i}(t_0)=$, $I_{2i}(t_0)$,so we have

$$I_{max} = a_i t_{0i} + b_i[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)] \tag{2.6}$$

Hence we have from (2.4)

$$I_{1i}(t)= a_i(t_{0i} - t) + b_i[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)]\,,\ 0 \le t \le t_{0i} \tag{2.7}$$

Now solving (2.3) with the boundary condition we get

$$I_{3i} = -\frac{a_i}{\delta_i}[\log\{1 + \delta_i(T_i - t_{1i})\}]\ ,\ t_{1i} \le t \le T_i \tag{2.8}$$

The maximum backordered unit is

$$S= -I_{3i}(T_i) = \frac{a_i}{\delta_i}[\log\{1 + \delta_i(T_i - t_{1i})\}] \tag{2.9}$$

Hence, the order size during $[0,T_i]$ is Q_i for i'th item is given by,

$Q_i= I_{max}+S$.

$$= a_i t_{0i} + b_i[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{oi}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)] + \frac{a_i}{\delta_i}[\log\{1 + \delta_i(T_i - t_{1i})\}] \tag{2.10}$$

Now,

1. The ordering cost per cycle for i'th item.

$OC_i = A_i$ (2.11)

2. The inventory holding cost per cycle for i'th item.

$IHC_i = \int_0^{t_{0i}} h_i.t.I_{1i}(t)\,dt + \int_{t_{0i}}^{t_{1i}} h_i.t.I_{2i}(t)dt$

$= \frac{a_i h_i t_{0i}^3}{6} + \frac{b_i h_i t_{0i}^2}{2}\left[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)\right] +$

$b_i h_i[\frac{t_{1i}}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{3}(t_{1i}^3 - t_{0i}^3) - \frac{\theta_i}{3}\{\frac{t_{1i}^3}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{5}(t_{1i}^5 - t_{0i}^5)\} + \frac{k_i - \alpha_i}{2}\{\frac{t_{1i}^2}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{4}(t_{1i}^4 - t_{0i}^4)\}]$

(2.12)

3. The deterioration cost for the time interval $[t_{0i}, t_{1i}]$.

$DC_i = c_{2i}[\frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2) + \frac{b_i \alpha_i}{2}(t_{1i}^2 - t_{0i}^2) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) - k_i(t_{1i} - t_{0i})]$ (2.13)

4. Shortage due to accumulation in the system during the time interval $[t_{1i}, T_i]$

The optimum level of the shortage is present at t = T_i ; Therefore the total shortage cost during this time period is as follows:

$SC_i = \frac{C_{3i} a_i}{\delta_i}(T_i - t_{1i})\ \log\{1 + \delta_i(T_i - t_{1i})\}$ (2.14)

5. Due to stock out during $[t_{1i}, T_i]$ shortage is accumulated, but not all customers are willing to wait for the next lot size arrive. Hence, this results in some loss of sale which accounts to loss in profit.

Opportunity cost (Lost sale cost) calculated for the i'th item as follows

$OC_i = C_{4i} a_i\ [T_i - t_{1i} - \frac{\log\{1 + \delta_i(T_i - t_{1i})\}}{\delta_i}]$ (2.15)

6. Purchase cost for i'th item is as follows:

$PC_i = C_{1i}[a_i t_{0i} + b_i\{(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{oi}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)\}$

$+ \frac{a_i}{\delta_i}[\log\{1 + \delta_i(T_i - t_{1i})\}]]$ (2.16)

So, the total average cost for i'th item is as follows

$TAC_i(t_{0i}.t_{1i}, T_i) = \frac{1}{T_i}[A_i + \frac{a_i h_i t_{0i}^3}{6} + \frac{b_i h_i t_{0i}^2}{2}\left[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)\right] + b_i h_i\{\frac{t_{1i}}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{3}(t_{1i}^3 - t_{0i}^3) - \frac{\theta_i}{3}\{\frac{t_{1i}^3}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{5}(t_{1i}^5 - t_{0i}^5)\} + \frac{k_i - \alpha_i}{2}\{\frac{t_{1i}^2}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{4}(t_{1i}^4 - t_{0i}^4)\}\ \} + c_{2i}\{\frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2) + \frac{b_i \alpha_i}{2}(t_{1i}^2 - t_{0i}^2) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) - k_i(t_{1i} - t_{0i})\} + \frac{C_{3i} a_i}{\delta_i}(T_i - t_{1i}).\log\{1 + \delta_i(T_i - t_{1i})\} + C_{4i} a_i\left\{T_i - t_{1i} - \frac{\log\{1 + \delta_i(T_i - t_{1i})\}}{\delta_i}\right\} + C_{1i}\ [a_i t_{0i} + b_i\{(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{oi}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)\} + \frac{a_i}{\delta_i}\{\log\{1 + \delta_i(T_i - t_{1i})\}\}]]$ (2.17)

From above we have a multi-objective (MOIM) inventory model given below

Minimize { $TAC_1(t_{01}.t_{11}, T_1)$, $TAC_2(t_{02}.t_{12}, T_2)$,…………………., $TAC_n(t_{0n}.t_{1n}, T_n)$}

Subject to: $\sum_{i=i}^{n} w_i Q_i \le W$ for i=1,2,3,4,……….,n

Where $TAC_i(t_{0i}.t_{1i},T_i)=\frac{1}{T_i}[A_i+\frac{a_ih_it_{0i}^3}{6}+\frac{b_ih_it_{0i}^2}{2}\left[(t_{1i}-t_{0i})-\frac{\theta_i}{3}(t_{1i}^3-t_{0i}^3)+\frac{k_i-\alpha_i}{2}(t_{1i}^2-t_{0i}^2)\right]+$

$b_ih_i\{\frac{t_{1i}}{2}(t_{1i}^2-t_{0i}^2)-\frac{1}{3}(t_{1i}^3-t_{0i}^3)-\frac{\theta_i}{3}\{\frac{t_{1i}^3}{2}(t_{1i}^2-t_{0i}^2)-\frac{1}{5}(t_{1i}^5-t_{0i}^5)\}+\frac{k_i-\alpha_i}{2}\{\frac{t_{1i}^2}{2}(t_{1i}^2-t_{0i}^2)-\frac{1}{4}(t_{1i}^4-t_{0i}^4)\}\}$

$+c_{2i}\{\frac{k_i-\alpha_i}{2}(t_{1i}^2-t_{0i}^2)+\frac{b_i\alpha_i}{2}(t_{1i}^2-t_{0i}^2)-\frac{\theta_i}{3}(t_{1i}^3-t_{0i}^3)-k_i(t_{1i}-t_{0i})\}+\frac{c_{3i}\alpha_i}{\delta_i}(T_i-t_{1i}).\log\{1+\delta_i(T_i-t_{1i})\}$

$+C_{4i}a_i\left\{T_i-t_{1i}-\frac{\log\{1+\delta_i(T_i-t_{1i})\}}{\delta_i}\right\}+C_{1i}[a_it_{0i}+b_i\{(t_{1i}-t_{0i})-\frac{\theta_i}{3}(t_{1i}^3-t_{oi}^3)+\frac{k_i-\alpha_i}{2}(t_{1i}^2-t_{0i}^2)\}$

$+\frac{a_i}{\delta_i}\{\log\{1+\delta_i(T_i-t_{1i})\}\}]]$

And $Q_i=a_it_{0i}+b_i\{(t_{1i}-t_{0i})-\frac{\theta_i}{3}(t_{1i}^3-t_{oi}^3)+\frac{k_i-\alpha_i}{2}(t_{1i}^2-t_{0i}^2)\}+\frac{a_i}{\delta_i}\{\log\{1+\delta_i(T_i-t_{1i})\}\}$ (2.18)

3. Prerequisite mathematics:

3.1 Fuzzy set:

Let S be the collection of substance called universe of discourse. A fuzzy set is a subset of S denoted by Ã and is defined by a set of ordered pairs Ã= $\{(x,\mu_{\tilde{A}}(x)): x\in S\}$.

Where $\mu_{\tilde{A}}: S\rightarrow[0,1]$ is a membership function of the fuzzy set Ã and $\mu_{\tilde{A}}(x)$ is called the grade of membership of x∈ S in the fuzzy set Ã.

3.2 Equality of fuzzy two sets: Two fuzzy sets Ã and $\tilde{B}$ are called equal iff $\mu_{\tilde{A}}(x)=\mu_{\tilde{B}}(x)$,∀ x∈S.It is usually denoted by Ã=$\tilde{B}$.

3.3 Union of two fuzzy sets: Let Ã and $\tilde{B}$ be two fuzzy sets. The union of Ã and $\tilde{B}$ is a fuzzy set in S is denoted by Ã ∪ $\tilde{B}$ and is defined by

$$\mu_{\tilde{A}\cup\tilde{B}}(x)=\mu_{\tilde{A}}(x)\vee\mu_{\tilde{B}}(x)=\max\{\mu_{\tilde{A}}(x),\mu_{\tilde{B}}(x)\}\quad\forall\, x\in S.$$

3.4 Intersection of two fuzzy sets: Let Ã and $\tilde{B}$ be two fuzzy sets. The intersection of Ã and $\tilde{B}$ is a fuzzy set in S is denoted by Ã ∩ $\tilde{B}$ and is defined by

$$\mu_{\tilde{A}\cap\tilde{B}}(x)=\mu_{\tilde{A}}(x)\wedge\mu_{\tilde{B}}(x)=\min\{\mu_{\tilde{A}}(x),\mu_{\tilde{B}}(x)\}\quad\forall\, x\in S.$$

3.5 Trapezoidal fuzzy number (TrFN): Let F ($\mathcal{R}$) be the set of all TrFN in the line $\mathcal{R}$ .A trapezoidal fuzzy number Ã∈ F ($\mathcal{R}$) is parameterized by (a, b, c, d) with the membership function $\mu_{\tilde{A}}:\mathcal{R}\rightarrow[0,1]$ which is defined as

$$\mu_{\tilde{A}}(x)=\begin{cases}\frac{x-a}{b-a} & for\ a\le x\le b\\ 1 & for\ b\le x\le c\\ \frac{d-x}{d-c} & for\ c\le x\le d\\ 0 & otherwise\end{cases}$$

Where $a<b<c<d$.Here a and d represent the lower and upper limits of sustains of a fuzzy set Ã.

3.6 Generalized Trapezoidal fuzzy number (GTrFN):A generalized TrFN Ã can be represented as Ã=(a, b, c, d; w) 0≤w≤ 1 and a,b,c,d∈ $\mathcal{R}$.Here Ã is a fuzzy subset of $\mathcal{R}$,with the membership function $\mu_{\tilde{A}}:\mathcal{R}\rightarrow[0,w]$ is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} w\frac{x-a}{b-a} & for\ a \le x \le b \\ w & for\ b \le x \le c \\ w\frac{d-x}{d-c} & for\ c \le x \le d \\ 0 & otherwise \end{cases}$$

Where a$< b < c < d$.Here a and d represent the lower and upper limits of sustains of a fuzzy set Ã and w ∈[0, 1].If w=1 then GTrFN Ã is called TrFN.

3.7 Neutrosophic set: Let S be a universe of discourse. A neutrosophic set $\widetilde{A^n}$ in S is defined by

$\widetilde{A^n}$ = {(x,T_A(x), I_A(x), F_A(x))∣ x∈S }

Where T_A(x) is the truth membership function, I_A(x) is the indeterminacy membership function, F_A(x) is falsity membership function.

Where,

$T_A(x):S \to]0^-,1^+[$

$I_A(x):S \to]0^-,1^+[$

$F_A(x):S \to]0^-,1^+[$

So we can say $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$

3.7.1 Single valued neutrosophic set: Let S be a universe of discourse. A single valued neutrosophic set $\widetilde{A^n}$ in S is defined by

$\widetilde{A^n}$= {(x,T_A(x), I_A(x), F_A(x))∣ x∈S }

Where T_A(x) is the truth membership function, I_A(x) is the indeterminacy membership function, F_A(x) is falsity membership function.

Together with,

$T_A(x): S \to [0, 1]$

$I_A(x): S \to [0, 1]$

$F_A(x): S \to [0, 1]$

So we can say $0 \le T_A(x) + I_A(x) + F_A(x) \le 3\ \forall\ x \in S$.

3.8 Hesitant Fuzzy set (HFS): Let S be a non-empty reference set, a hesitant fuzzy set A^h of S is defined by a function A_h(x) which is applied to S returns a finite subset of [0,1]. Whose mathematical representation I given by

A^h= {(x, A_h(x) ∣ x∈ S}

Here h_A(x)∈ [0, 1], representing the possible membership degree of the elements x∈ S to A^h.The set h_A(x) is known as hesitant fuzzy element (HFE).

e.g.Let S ={x_1, x_2, x_3} be a reference set . A_h (x_1)=(0.1,0.2,0.3), A_h (x_2)=(0.2,0.3,0.4), A_h (x_3)=(0.1,0.2,0.5) be hesitant elements of x_1, x_2, x_3 to a set A^h.Therefore the hesitant fuzzy set A^h is given by

A^h={(x_1,{ 0.1,0.2,0.3}),(x_2,{0.2,0.3,0.4}).(x_3,{ 0.1,0.2,0.5})}

3.9 Single valued neutrosophic Hesitant Fuzzy Set (NVNHFS):

We take S be non-empty reference set, then a single valued neutrosophic hesitant fuzzy set A on S is defined as

A^h= {(x,$\widetilde{T_A}$(x), $\widetilde{I_A}$(x), $\widetilde{F_A}$(x))∣ x∈S}

Here $\widetilde{T_A}$(x)={μ∣ $\mu \in \widetilde{T_A}$(x) : x∈S } is known as possible truth membership hesitant degree , $\widetilde{I_A}$(x)={ρ∣{$\rho \in \widetilde{I_A}$(x) : x∈S } is known as possible indeterminacy membership hesitant degree and $\widetilde{F_A}$(x)={σ | σ ∈ $\widetilde{F_A}$(x) : x∈S } is known as possible falsity membership hesitant degree. These sets takes the different values in [0, 1].Which satisfies the following conditions

μ,ρ,σ ⊆[0,1] and 0 ≤supμ^++ sup ρ^+ + supσ^+ ≤3

Where μ^+=$\cup_{\mu\in\widetilde{T_A}(x)}$max {$\mu$}, ρ^+=$\cup_{\rho\in\widetilde{T_A}(x)}$max {$\rho$}, σ^+=$\cup_{\sigma\in\widetilde{T_A}(x)}$max { σ} for x∈S.

3.10 Union of two SVNS sets: Let S be a universe of discourse and $\widetilde{A^n}$ and $\widetilde{B^n}$ are any two subsets of S.

Where $T_A(x)$ is the truth membership function, $I_A(x)$ is the indeterminacy membership function, $F_A(x)$ is falsity membership function of respectively. The union of $\widetilde{A^n}$ and $\widetilde{B^n}$ is denoted by $\widetilde{A^n} \cup \widetilde{B^n}$ and defined by

$\widetilde{A^n} \cup \widetilde{B^n}$ = {(x, max ($T_A(x)$, $T_B(x)$), max ($I_A(x)$, $I_B(x)$), min ($F_A(x)$, $F_B(x)$))| x∈S}

3.11 Intersection of two SVNs sets: Let S be a universe of discourse and $\widetilde{A^n}$ and $\widetilde{B^n}$ are any two subsets of S.

Where $T_A(x)$ is the truth membership function, $I_A(x)$ is the indeterminacy membership function, $F_A(x)$ is falsity membership function of respectively. The union of $\widetilde{A^n}$ and $\widetilde{B^n}$ is denoted by $\widetilde{A^n} \cap \widetilde{B^n}$ and defined by

$\widetilde{A^n} \cap \widetilde{B^n}$ = {(x, min ($T_A(x)$, $T_B(x)$),min ($I_A(x)$, $I_B(x)$),max($F_A(x)$, $F_B(x)$))| x∈S}

4. Fuzzy model for i'th item

For the presence of uncertainty we take some cost parameters as generalized trapezoidal fuzzy number which are already mentioned above.

Let us take the following fuzzy cost parameters

$\widetilde{a_i} = (a_i^1, a_i^2, a_i^3, a_i^4; w_{a_i})$,$0 \le w_{a_i} \le 1$;

$\widetilde{b_i} = (b_i^1, b_i^2, b_i^3, b_i^4; w_{b_i})$,$0 \le w_{b_i} \le 1$;

$\widetilde{A_i} = (A_i^1, A_i^2, A_i^3, A_i^4; w_{A_i})$,$0 \le w_{A_i} \le 1$;

$\widetilde{C_{1i}} = (C_{1i}^1, C_{1i}^2, C_{1i}^3, C_{1i}^4; w_{C_{1i}})$,$0 \le w_{C_{1i}} \le 1$;

$\widetilde{C_{2i}} = (C_{2i}^1, C_{2i}^2, C_{2i}^3, C_{2i}^4; w_{C_{2i}})$,$0 \le w_{C_{2i}} \le 1$;

$\widetilde{C_{3i}} = (C_{3i}^1, C_{3i}^2, C_{3i}^3, C_{3i}^4; w_{C_{3i}})$,$0 \le w_{C_{3i}} \le 1$;

$\widetilde{C_{4i}} = (C_{4i}^1, C_{4i}^2, C_{4i}^3, C_{4i}^4; w_{C_{4i}})$,$0 \le w_{C_{4i}} \le 1$;

$\widetilde{H_i} = (H_i^1, H_i^2, H_i^3, H_i^4; w_{H_i})$,$0 \le w_{H_i} \le 1$;

$\widetilde{w_i} = (w_i^1, w_i^2, w_i^3, w_i^4; w_{w_i})$,$0 \le w_{w_i} \le 1$;

The fuzzy total average cost is given by:

$$\widetilde{TAC_i}(t_{0i}, t_{1i}, T_i) = \frac{1}{T_i}[\widetilde{A_i} + \frac{\widetilde{a_i}\widetilde{h_i}t_{0i}^3}{6} + \frac{\widetilde{b_i}\widetilde{h_i}t_{0i}^2}{2}\Big[(t_{1i}-t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) + \frac{k_i-\alpha_i}{2}(t_{1i}^2 - t_{0i}^2)\Big] +$$

$$\widetilde{b_i}\widetilde{h_i}\{\frac{t_{1i}}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{3}(t_{1i}^3 - t_{0i}^3) - \frac{\theta_i}{3}\{\frac{t_{1i}^3}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{5}(t_{1i}^5 - t_{0i}^5)\} + \frac{k_i-\alpha_i}{2}\{\frac{t_{1i}^2}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{4}(t_{1i}^4 - t_{0i}^4)\}\}$$

$$+\widetilde{C_{2i}}\{\frac{k_i-\alpha_i}{2}(t_{1i}^2 - t_{0i}^2) + \frac{\widetilde{b_i}\alpha_i}{2}(t_{1i}^2 - t_{0i}^2) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) - k_i(t_{1i} - t_{0i})\} + \widetilde{C_{3i}}\frac{\widetilde{a_i}}{\delta_i}(T_i - t_{1i}).\log\{1 +$$

$$\delta_i(T_i - t_{1i})\} + \widetilde{C_{4i}}\widetilde{a_i}\{T_i - t_{1i} - \frac{\log\{1+\delta_i(T_i-t_{1i})\}}{\delta_i}\} + \widetilde{C_{1i}}\{\widetilde{a_i}t_{0i} + \widetilde{b_i}[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{oi}^3) +$$

$$\frac{k_i-\alpha_i}{2}(t_{1i}^2 - t_{0i}^2)] + \frac{\widetilde{a_i}}{\delta_i}\{\log\{1 + \delta_i(T_i - t_{1i})\}\}\}] \quad (4.1)$$

And our Multi-objective inventory model (MOIM) become fuzzy model as
Min { $\widetilde{TAC_1}(t_{01}, t_{11}, T_1), \widetilde{TAC_2}(t_{02}, t_{12}, T_2), \ldots\ldots\ldots\ldots\ldots\ldots, \widetilde{TAC_n}(t_{0n}, t_{1n}, T_n)$}
Subject to: $\sum_{i=i}^{n} w_i Q_i \le W$ for i=1, 2, 3, 4,……….,n
Where, $\widetilde{TAC_\iota}(t_{0i}, t_{1i}, T_i) = \frac{1}{T_i}[\widetilde{A_\iota} + \frac{\widetilde{a_\iota}\widetilde{h_\iota}t_{0i}^3}{6} + \frac{\widetilde{b_\iota}\widetilde{h_\iota}t_{0i}^2}{2}\left[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)\right] +$

$\widetilde{b_\iota}\widetilde{h_\iota}\{\frac{t_{1i}}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{3}(t_{1i}^3 - t_{0i}^3) - \frac{\theta_i}{3}\{\frac{t_{1i}^3}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{5}(t_{1i}^5 - t_{0i}^5)\} + \frac{k_i - \alpha_i}{2}\{\frac{t_{1i}^2}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{4}(t_{1i}^4 - t_{0i}^4)\}\}$

$+\widetilde{C_{2\iota}}\{\frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2) + \frac{\widetilde{b_\iota}\alpha_i}{2}(t_{1i}^2 - t_{0i}^2) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) - k_i(t_{1i} - t_{0i})\} + \widetilde{C_{3\iota}}\frac{\widetilde{a_\iota}}{\delta_i}(T_i - t_{1i}).\log\{1 +$

$\delta_i(T_i - t_{1i})\} + \widetilde{C_{4\iota}}\widetilde{a_\iota}\{T_i - t_{1i} - \frac{\log\{1+\delta_i(T_i - t_{1i})\}}{\delta_i}\} + \widetilde{C_{1\iota}}\{\widetilde{a_\iota}t_{0i} + \widetilde{b_\iota}[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{oi}^3) +$

$\frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)] + \frac{\widetilde{a_\iota}}{\delta_i}\{\log\{1 + \delta_i(T_i - t_{1i})\}\}\}]$

And $Q_i = \widetilde{a_\iota}t_{0i} + \widetilde{b_\iota}[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{oi}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)] + \frac{\widetilde{a_\iota}}{\delta_i}\{\log\{1 + \delta_i(T_i - t_{1i})\}\}$ (4.2)

Now we have to defuzzify all the fuzzy number we have considered earlier. In this model we have considered GTrFN. Now for defuzzification we have considered here the following technique.

Let $\tilde{A} = (a, b, c, d; w)$ be any GTrFN fuzzy number, then the total λ-integer value of $\tilde{A}$ is

$$I_\lambda^w(\tilde{A}) = \lambda\omega\left(\frac{c+d}{2}\right) + (1-\lambda)\omega\left(\frac{a+b}{2}\right)$$

For simplification we have considered $\lambda = \frac{1}{2}$, therefore we get the approximate value of $\tilde{A} = (a, b, c, d, \omega)$ as $\omega\left(\frac{a+b+c+d}{4}\right)$

On the basis of this method all the GTrFN fuzzy number $(\widetilde{a_\iota}, \widetilde{b_\iota}, \widetilde{C_{1\iota}}, \widetilde{C_{2\iota}}, \widetilde{C_{3\iota}}, \widetilde{C_{4\iota}}, \widetilde{H_\iota}, \widetilde{A_\iota}, \widetilde{w_\iota})$ converted to the crisp value as $(\widehat{a_\iota}, \widehat{b_\iota}, \widehat{C_{1\iota}}, \widehat{C_{2\iota}}, \widehat{C_{3\iota}}, \widehat{C_{4\iota}}, \widehat{H_\iota}, \widehat{A_\iota}, \widehat{w_\iota})$.

So the above model (4.2) becomes a crisp model as
Min {$\widehat{TAC_1}(t_{01}, t_{11}, T_1), \widehat{TAC_2}(t_{02}, t_{12}, T_2), \ldots\ldots\ldots\ldots\ldots\ldots, \widehat{TAC_n}(t_{0n}, t_{1n}, T_n)$}
(4.3)
Subject to, Subject to: $\sum_{i=i}^{n} w_i Q_i \le W$ for i=1, 2, 3, 4,……….,n
Where,

$$\widehat{TAC_\iota}(t_{0i}, t_{1i}, T_i) = \frac{1}{T_i}[\widehat{A_\iota} + \frac{\widehat{a_\iota}\widehat{h_\iota}t_{0i}^3}{6} + \frac{\widehat{b_\iota}\widehat{h_\iota}t_{0i}^2}{2}\left[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)\right] +$$

$\widehat{b_\iota}\widehat{h_\iota}\{\frac{t_{1i}}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{3}(t_{1i}^3 - t_{0i}^3) - \frac{\theta_i}{3}\{\frac{t_{1i}^3}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{5}(t_{1i}^5 - t_{0i}^5)\} + \frac{k_i - \alpha_i}{2}\{\frac{t_{1i}^2}{2}(t_{1i}^2 - t_{0i}^2) - \frac{1}{4}(t_{1i}^4 - t_{0i}^4)\}\}$

$+\widehat{C_{2\iota}}\{\frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2) + \frac{\widehat{b_\iota}\alpha_i}{2}(t_{1i}^2 - t_{0i}^2) - \frac{\theta_i}{3}(t_{1i}^3 - t_{0i}^3) - k_i(t_{1i} - t_{0i})\} + \widehat{C_{3\iota}}.\frac{\widehat{a_\iota}}{\delta_i}(T_i - t_{1i}).\log\{1 +$

$\delta_i(T_i - t_{1i})\} + \widehat{C_{4\iota}}\widehat{a_\iota}\{T_i - t_{1i} - \frac{\log\{1+\delta_i(T_i - t_{1i})\}}{\delta_i}\} + \widehat{C_{1\iota}}\{\widehat{a_\iota}t_{0i} + \widehat{b_\iota}[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{oi}^3) +$

$\frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)] + \frac{\widehat{a_\iota}}{\delta_i}\{\log\{1 + \delta_i(T_i - t_{1i})\}\}\}]$

And $Q_i = \widehat{a_\iota}t_{0i} + \widehat{b_\iota}[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{oi}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)] ++ \frac{\widehat{a_\iota}}{\delta_i}\{\log\{1 + \delta_i(T_i - t_{1i})\}\}$ for i=1, 2, ..., n.

5. Fuzzy programming technique (FNLP AND FAGP):

To solve the above MOIM (4.3) problem we take one objective at a time and we ignore the others. Using this we find out the value of every objective function and from this we formulate the pay-of-matrix as follows.

$$\begin{array}{c} \\ (t_{01}^1, t_{11}^1, T_1^1) \\ (t_{02}^2, t_{12}^2, T_2^2) \\ \dots\dots\dots\dots \\ (t_{0n}^n, t_{1n}^n, T_n^n) \end{array} \begin{array}{cccc} TAC_1(t_{01}, t_{11}, T_1) & TAC_2(t_{02}, t_{12}, T_2) & \dots\dots\dots & TAC_n(t_{0n}, t_{1n}, T_n) \\ \left[\begin{array}{c} TAC_1^*(t_{01}^1, t_{11}^1, T_1^1) \\ TAC_1(t_{02}^2, t_{12}^2, T_2^2) \\ \dots\dots\dots \\ TAC_1(t_{0n}^n, t_{1n}^n, T_n^n) \end{array}\right. & \begin{array}{c} TAC_2(t_{01}^1, t_{11}^1, T_1^1) \\ TAC_1^*(t_{02}^2, t_{12}^2, T_2^2) \\ \dots\dots\dots\dots \\ TAC_2(t_{0n}^n, t_{1n}^n, T_n^n) \end{array} & \begin{array}{c} \dots\dots.. \\ \dots\dots\dots \\ \dots\dots\dots\dots.. \\ \dots\dots\dots\dots \end{array} & \left.\begin{array}{c} TAC_n(t_{01}^1, t_{11}^1, T_1^1) \\ TAC_n(t_{02}^2, t_{12}^2, T_2^2) \\ \dots\dots\dots\dots\dots \\ TAC_n^*(t_{0n}^n, t_{1n}^n, T_n^n) \end{array}\right] \end{array}$$

Now we assume that $U^k = \max\{TAC_k(t_{0i}^i, t_{1i}^i, T_i^i), i = 1,2,3, \dots\dots, n\}$ for k= 1, 2, 3, ….., n

And $L^k = \{TAC_k^*(t_{0k}^k, t_{1k}^k, T_k^k), k = 1,2,3, \dots\dots, n\}$

Here $L^k \leq TAC_k(t_{0i}^i, t_{2i}^i, T_i^i) \leq U^k$ for $i = 1,2,3, \dots\dots, n$; and k= 1, 2, 3, ….., n; (5.1)

Now we take the linear fuzzy membership function $\mu_{TAC_k}(TAC_k(t_{0k}, t_{1k}, T_k))$ for the k'th objective function $TAC_k(t_{0k}, t_{1k}, T_k)$ as follows.

$$\mu_{TAC_k}(TAC_k(t_{0k}, t_{1k}, T_k)) = \begin{cases} 1 & for \quad TAC_k(t_{0k}, t_{1k}, T_k) \leq L^k \\ \frac{U^k - TAC_k(t_{0k}, t_{1k}, T_k)}{U^k - L^k} & for \ L^k \leq TAC_k(t_{0k}, t_{1k}, T_k) \leq U^k \\ 0 & for \quad TAC_k(t_{0k}, t_{1k}, T_k) \geq U^k \end{cases} \quad (5.2)$$

For k=1, 2,3,….., n;

After getting the membership function we formulate the fuzzy non-linear programming problems (FNLP) based on max-min operator as follow:

Max p

Subject to,

$$p(U^k - L^k) + TAC_k(t_{0k}, t_{1k}, T_k) \leq U^k \quad \text{For k=1, 2, 3,……, n}$$

$$0 \leq p \leq 1, \qquad t_{0k} \geq 0, t_{1k} \geq 0, T_k \geq 0; \qquad (5.3)$$

And the same constraints and restriction as the problem (4.3)

Now based on max-additive operator we formulate Fuzzy additive goal programming (FAGP) as follows:

$$\text{Max} \sum_{k=1}^{n} \frac{U^k - TAC_k(t_{0k}, t_{1k}, T_k)}{U^k - L^k} \qquad (5.4)$$

Subject to, $0 \leq \mu_{TAC_k}(TAC_k(t_{0k}, t_{1k}, T_k)) \leq 1$, for k=1,2,3,…….,n

And the same constraints and restrictions as in the problem (4.3)

Solving the above reduced problem (5.3) and (5.4) by using the above FNLP and FAGP method we shall find the optimal solutions.

6. Weighted Fuzzy programming technique (WFNLP AND WFAGP):

Here we take a positive weight for ω_k for each of the objective $\left(TAC_k(t_{0k}, t_{1k}, T_k)\right)$

(Where k=1, 2, 3,….., n) and $\sum_{k=1}^{n} \omega_k = 1$.

Having the membership function (5.2) WFNLP technique becomes

Max p

Subject to,

$$\omega_k . \mu_{TAC_k}\left(TAC_k(t_{0k}, t_{1k}, T_k)\right) \geq \text{p} \quad \text{For k=1, 2, 3,……, n}$$

$$0 \leq p \leq 1, \qquad t_{0k} \geq 0, t_{1k} \geq 0, T_k \geq 0 \text{ and } \sum_{k=1}^{n} \omega_k = 1 \qquad (6.1)$$

And the same constraints and restriction as the problem (4.3)

Having the membership function (5.2) WFAGP technique becomes

$$\text{Max } \sum_{k=1}^{n} \omega_k . \mu_{TAC_k}\left(TAC_k(t_{0k}, t_{1k}, T_k)\right)$$

$$\text{Subject to,} 0 \leq \mu_{TAC_k}\left(TAC_k(t_{0k}, t_{1k}, T_k)\right) \leq 1, \text{ for k=1,2,3,…….,n and } \sum_{k=1}^{n} \omega_k = 1 \qquad (6.2)$$

And the same constraints and restrictions as in the problem (4.3)

Solving the problem by using the above WFNLP and WFAGP method we shall find the optimal solution.

7. Neutrosophic Hesitant Fuzzy Non-Linear Programming (NHFNLP) technique:

Using (5.1) we can defined now the different hesitant membership function under neutrosophic hesitant fuzzy environment as follows.

For minimization type objective function

The truth hesitant-membership function:

$$T_{h^-}^{E_1}(TAC_k(t_{0k}, t_{1k}, T_k)) = \begin{cases} 1 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) < \mathrm{L^k} \\ \zeta_1 \frac{(U^k)^t - (TAC_k(t_{0k}, t_{1k}, T_k))^t}{(U^k)^t - (L^k)^t} & if \ L^k \leq TAC_k(t_{0k}, t_{1k}, T_k) \leq \mathrm{U^k} \\ 0 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) > \mathrm{U^k} \end{cases}$$

$$T_{h^-}^{E_2}(TAC_k(t_{0k}, t_{1k}, T_k)) = \begin{cases} 1 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) < \mathrm{L^k} \\ \zeta_2 \frac{(U^k)^t - (TAC_k(t_{0k}, t_{1k}, T_k))^t}{(U^k)^t - (L^k)^t} & if \ L^k \leq TAC_k(t_{0k}, t_{1k}, T_k) \leq \mathrm{U^k} \\ 0 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) > \mathrm{U^k} \end{cases}$$

………………………………….

$$T_{h^-}^{E_n}(TAC_k(t_{0k}, t_{1k}, T_k)) = \begin{cases} 1 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) < \mathrm{L^k} \\ \zeta_n \frac{(U^k)^t - (TAC_k(t_{0k}, t_{1k}, T_k))^t}{(U^k)^t - (L^k)^t} & if \ L^k \leq TAC_k(t_{0k}, t_{1k}, T_k) \leq \mathrm{U^k} \\ 0 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) > \mathrm{U^k} \end{cases}$$

The indeterminacy hesitant-membership function:

$$I_{h^-}^{E_1}(TAC_k(t_{0k}, t_{1k}, T_k)) = \begin{cases} 1 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) < \mathrm{L^k} - s^k \\ \eta_1 \frac{(U^k)^t - (TAC_k(t_{0k}, t_{1k}, T_k))^t}{(s^k)^t} & if \ U^k - s^k \leq TAC_k(t_{0k}, t_{1k}, T_k) \leq \mathrm{U^k} \\ 0 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) > \mathrm{U^k} \end{cases}$$

$$I_{h^-}^{E_2}(TAC_k(t_{0k}, t_{1k}, T_k)) = \begin{cases} 1 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) < \mathrm{L^k} - s^k \\ \eta_2 \frac{(U^k)^t - (TAC_k(t_{0k}, t_{1k}, T_k))^t}{(s^k)^t} & if \ U^k - s^k \leq TAC_k(t_{0k}, t_{1k}, T_k) \leq \mathrm{U^k} \\ 0 & if \quad TAC_k(t_{0k}, t_{1k}, T_k) > \mathrm{U^k} \end{cases}$$

……………………………………………………………………………………….

$$I_{h^-}^{E_n}(TAC_k(t_{0k},t_{1k},T_k)) = \begin{cases} 1 & if \quad TAC_k(t_{0k},t_{1k},T_k) < \mathrm{L^k} - s^k \\ \eta_n \frac{(U^k)^t - (TAC_k(t_{0k},t_{1k},T_k))^t}{(S^k)^t} & if \ U^k - s^k \le TAC_k(t_{0k},t_{1k},T_k) \le \mathrm{U^k} \\ 0 & if \quad TAC_k(t_{0k},t_{1k},T_k) > \mathrm{U^k} \end{cases}$$

The falsity hesitant-membership function:

$$F_{h^-}^{E_1}(TAC_k(t_{0k},t_{1k},T_k)) = \begin{cases} 0 & if \ TAC_k(t_{0k},t_{1k},T_k) < \mathrm{L^k} + r^k \\ \xi_1 \frac{(TAC_k(t_{0k},t_{1k},T_k))^t - (L^k)^t - (r^k)^t}{(U^k)^t - (L^k)^t - (r^k)^t} & if \ L^k + r^k \le TAC_k(t_{0k},t_{1k},T_k) \le \mathrm{U^k} \\ 1 & if \quad TAC_k(t_{0k},t_{1k},T_k) > \mathrm{U^k} \end{cases}$$

$$F_{h^-}^{E_2}(TAC_k(t_{0k},t_{1k},T_k)) = \begin{cases} 0 & if \ TAC_k(t_{0k},t_{1k},T_k) < \mathrm{L^k} + r^k \\ \xi_2 \frac{(TAC_k(t_{0k},t_{1k},T_k))^t - (L^k)^t - (r^k)^t}{(U^k)^t - (L^k)^t - (r^k)^t} & if \ L^k + r^k \le TAC_k(t_{0k},t_{1k},T_k) \le \mathrm{U^k} \\ 1 & if \quad TAC_k(t_{0k},t_{1k},T_k) > \mathrm{U^k} \end{cases}$$

……………………………………..

$$F_{h^-}^{E_n}(TAC_k(t_{0k},t_{1k},T_k)) = \begin{cases} 0 & if \ TAC_k(t_{0k},t_{1k},T_k) < \mathrm{L^k} + r^k \\ \xi_n \frac{(TAC_k(t_{0k},t_{1k},T_k))^t - (L^k)^t - (r^k)^t}{(U^k)^t - (L^k)^t - (r^k)^t} & if \ L^k + r^k \le TAC_k(t_{0k},t_{1k},T_k) \le \mathrm{U^k} \\ 1 & if \quad TAC_k(t_{0k},t_{1k},T_k) > \mathrm{U^k} \end{cases}$$

Here t>0 is a parameter and $s^k, r^k \in (0,1) \forall \ k = 1,2,3, \dots\dots, n$ are the indeterminacy and falsity tolerance values. They are assigned by decision making and h^- represents the minimization type hesitant objective function.

$T_{h^-}^{E_1}(TAC_k(t_{0k},t_{1k},T_k))$, $I_{h^-}^{E_1}(TAC_k(t_{0k},t_{1k},T_k))$, $F_{h^-}^{E_1}(TAC_k(t_{0k},t_{1k},T_k))$ are the truth, indeterminacy and falsity hesitant membership degree assigned by 1^{st} expert.

$T_{h^-}^{E_2}(TAC_k(t_{0k},t_{1k},T_k))$, $I_{h^-}^{E_2}(TAC_k(t_{0k},t_{1k},T_k))$, $F_{h^-}^{E_2}(TAC_k(t_{0k},t_{1k},T_k))$ are the truth, indeterminacy and falsity hesitant membership degree assigned by 2^{st} expert.

……………………………

$T_{h^-}^{E_n}(TAC_k(t_{0k},t_{1k},T_k))$, $I_{h^-}^{E_n}(TAC_k(t_{0k},t_{1k},T_k))$, $F_{h^-}^{E_n}(TAC_k(t_{0k},t_{1k},T_k))$ are the truth, indeterminacy and falsity hesitant membership degree assigned by n^{st} expert.

With the help of above membership function our multi-objective inventory model transforms into the following form

$$\text{Max} \frac{\sum_1^n \zeta_i}{n}$$

$$\text{Max} \frac{\sum_1^n \eta_i}{n}$$

$$\text{Max} \frac{\sum_1^n \xi_i}{n}$$

s.t,

$$T_{h^-}^{E_i}\left(TAC_k(t_{0k},t_{1k},T_k)\right) \ge \zeta_i \ ; I_{h^-}^{E_i}\left(TAC_k(t_{0k},t_{1k},T_k)\right) \ge \eta_i \ ; F_{h^-}^{E_i}\left(TAC_k(t_{0k},t_{1k},T_k)\right) \le \xi_i$$

$$\sum_{i=i}^n w_i Q_i \le W \qquad \text{For i=1, 2, 3, 4,………., n}$$

$$\text{Where } Q_i = \widetilde{a_i} \mathrm{t_{0i}} + \widetilde{b_i}[(t_{1i} - t_{0i}) - \frac{\theta_i}{3}(t_{1i}^3 - t_{oi}^3) + \frac{k_i - \alpha_i}{2}(t_{1i}^2 - t_{0i}^2)] + \frac{\widetilde{a_i}}{\delta_i}\{\log\{1 + \delta_i(T_i - t_{1i})\}\}$$

$$t_{0k}, t_{1k}, T_k \ge 0; \zeta_i, \ \eta_i, \ \xi_i \in (0,1); \ \zeta_i + \eta_i + \xi_i \le 3; \zeta_i \ge \eta_i; \ \zeta_i \ge \xi_i; \ \forall \ i = 1,2,3, \dots\dots, n \tag{7.1}$$

Using the linear membership function our multi-item inventory model can be written as follows:

Max $\frac{\zeta_1+\zeta_2+\ldots\ldots\ldots+\zeta_n}{n}+\frac{\eta_1+\eta_2+\ldots\ldots\ldots+\eta_n}{n}+\frac{\xi_1+\xi_2+\ldots\ldots\ldots+\xi_n}{n}$

Subject to,

$T_{h^-}^{E_1}\left(TAC_k(t_{0k},t_{1k},T_k)\right)\geq\zeta_1,T_{h^-}^{E_2}\left(TAC_k(t_{0k},t_{1k},T_k)\right)\geq\zeta_2,\ldots,T_{h^-}^{E_n}\left(TAC_k(t_{0k},t_{1k},T_k)\right)\geq\zeta_n;$

$I_{h^-}^{E_1}\left(TAC_k(t_{0k},t_{1k},T_k)\right)\geq\eta_1,I_{h^-}^{E_2}\left(TAC_k(t_{0k},t_{1k},T_k)\right)\geq\eta_2,\ldots\ldots,I_{h^-}^{E_n}\left(TAC_k(t_{0k},t_{1k},T_k)\right)\geq\eta_n;$

$F_{h^-}^{E_1}\left(TAC_k(t_{0k},t_{1k},T_k)\right)\leq\xi_1,F_{h^-}^{E_2}\left(TAC_k(t_{0k},t_{1k},T_k)\right)\leq\xi_2,\ldots,F_{h^-}^{E_n}\left(TAC_k(t_{0k},t_{1k},T_k)\right)\leq\xi_n;$

$\sum_{i=i}^{n}w_iQ_i\leq W$; For i=1, 2, 3, 4, ………, n

Where $Q_i=\widetilde{a_i}t_{0i}+\widetilde{b_i}[(t_{1i}-t_{0i})-\frac{\theta_i}{3}(t_{1i}^3-t_{oi}^3)+\frac{k_i-\alpha_i}{2}(t_{1i}^2-t_{0i}^2)]+\frac{\widetilde{a_i}}{\delta_i}\{\log\{1+\delta_i(T_i-t_{1i})\}\}$

$0\leq\zeta_1,\zeta_2,\ldots\ldots\ldots..,\zeta_n\leq1;0\leq\eta_1,\eta_2,\ldots\ldots\ldots..,\eta_n\leq1;0\leq\xi_1,\xi_2,\ldots\ldots\ldots..,\xi_n\leq1;$

$t_{0k},t_{1k},T_k\geq0;\ \zeta_i,\eta_i,\xi_i\in(0,1);\ \zeta_i+\eta_i+\xi_i\leq3;\zeta_i\geq\eta_i;\ \zeta_i\geq\xi_i;\ \forall\ i=1,2,3,\ldots\ldots,n$ (7.2)

From this we get the optimal solution for our multi-item inventory model.

8. NUMERICAL EXAMPLE:

Let us take our inventory model which consists of two items only. And the several parameter's values are taken at form of generalized fuzzy number and some values are in crisp. Here we take total storage area W=3500m[2].

Minimize { $\widetilde{TAC_1}(t_{01},t_{11},T_1),\widetilde{TAC_2}(t_{02},t_{12},T_2)$}

Subject to: $\sum_{i=i}^{n}w_iQ_i\leq W$ for i=1, 2

Where, $\widetilde{TAC_i}(t_{0i},t_{1i},T_i)=\frac{1}{T_i}[\widetilde{A_i}+\frac{\widetilde{a_i}\widetilde{h_i}t_{0i}^3}{6}+\frac{\widetilde{b_i}\widetilde{h_i}t_{0i}^2}{2}\left[(t_{1i}-t_{0i})-\frac{\theta_i}{3}(t_{1i}^3-t_{0i}^3)+\frac{k_i-\alpha_i}{2}(t_{1i}^2-t_{0i}^2)\right]+$

$\widetilde{b_i}\widetilde{h_i}\{\frac{t_{1i}}{2}(t_{1i}^2-t_{0i}^2)-\frac{1}{3}(t_{1i}^3-t_{0i}^3)-\frac{\theta_i}{3}\{\frac{t_{1i}^3}{2}(t_{1i}^2-t_{0i}^2)-\frac{1}{5}\left(t_{1i}^5-t_{0i}^5\right)\}+\frac{k_i-\alpha_i}{2}\{\frac{t_{1i}^2}{2}(t_{1i}^2-t_{0i}^2)-\frac{1}{4}(t_{1i}^4-t_{0i}^4)\}\quad\}$

$+\widetilde{C_{2i}}\{\frac{k_i-\alpha_i}{2}(t_{1i}^2-t_{0i}^2)+\frac{\widetilde{b_i}\alpha_i}{2}(t_{1i}^2-t_{0i}^2)-\frac{\theta_i}{3}(t_{1i}^3-t_{0i}^3)-k_i(t_{1i}-t_{0i})\}\quad+\widetilde{C_{3i}}\quad\frac{\widetilde{a_i}}{\delta_i}(T_i-t_{1i}).\log\{1+\delta_i(T_i-$

$t_{1i})\}\quad+\widetilde{C_{4i}}\widetilde{a_i}\{T_i-t_{1i}-\frac{\log\{1+\delta_i(T_i-t_{1i})\}}{\delta_i}\}\quad+\widetilde{C_{1i}}\ \{\widetilde{a_i}t_{0i}+\widetilde{b_i}[(t_{1i}-t_{0i})-\frac{\theta_i}{3}(t_{1i}^3-t_{oi}^3)+\frac{k_i-\alpha_i}{2}(t_{1i}^2-t_{0i}^2)]+$

$\frac{\widetilde{a_i}}{\delta_i}\{\log\{1+\delta_i(T_i-t_{1i})\}\}\}]$

And $Q_i=\widetilde{a_i}t_{0i}+\widetilde{b_i}[(t_{1i}-t_{0i})-\frac{\theta_i}{3}(t_{1i}^3-t_{oi}^3)+\frac{k_i-\alpha_i}{2}(t_{1i}^2-t_{0i}^2)]+\frac{\widetilde{a_i}}{\delta_i}\{\log\{1+\delta_i(T_i-t_{1i})\}\}$ for i=1, 2

(8.1)

The values of those parameters which are taken as generalized trapezoidal fuzzy number are given in table-1.

Table-1

Input data

Parameters	Item

	Item1	Item2
$\widetilde{a_i}$	(45,43,39,33 ;0.9)	(39,41,44,46 ;0.8)
$\widetilde{b_i}$	(50,45,41,39 ;0.8)	(49,53,57,61 ;0.6)
$\widetilde{C_{1i}}$	(17,15,13,10 ;0.8)	(16,19,22,23 ;0.6)
$\widetilde{C_{2i}}$	(15,16,18,19 ;0.5)	(9.3,11.2,13.1,13.9 ;0.8)
$\widetilde{C_{3i}}$	(7,6,4,3 ;0.8)	(7,9,11,13 ;0.5)
$\widetilde{C_{4i}}$	(2.8,2.6,2.5,2.1 ;0.8)	(2,3,4,6 ;0.8)
$\widetilde{H_i}$	(3.5,4.3,5.5,6.7 ;0.5)	(3.2,3.3,4.7,5.8 ;0.6)
$\widetilde{A_i}$	(113,101,97,89 ;0.5)	(99,105,101,111 ;0.5)
$\widetilde{w_i}$	(2.7,2.5,2.6,2.2 ;0.8)	(2.5,2.6,2.8,2.1 ;0.8)

The values of those parameters which are taken as crisp are given below.

$$\alpha_1 = .017, \delta_1 = .216, k_1 = .001, \theta_1 = .004, t_{01} = 0.12$$

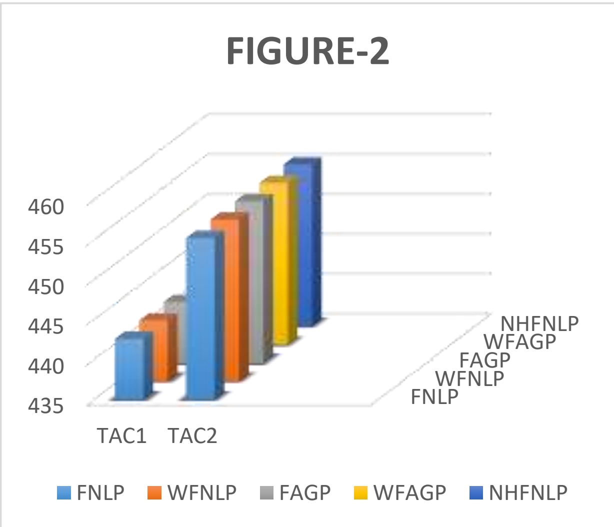
$$\alpha_2 = .018, \delta_2 = .216, k_2 = .001, \theta_2 = .004, t_{02} = 0.12$$

We take the weights w_1=.5, w_2=.5 for WFNLP and weights w_1=.5, w_2=.5 for WFAGP.

Table-2

Optimal solution by FNLP, WFNLP, FAGP, WFAGP, NHFNLP

Methods	t_{11}	T_1	$TAC_1(t_{01}^*, t_{11}^*, T_1^*)$	t_{12}	T_2	$TAC_2(t_{02}^*, t_{12}^*, T_2^*)$
FNLP	1.199857	1.420192	442.6481	1.244292	1.421455	455.2850
WFNLP	1.199205	1.419957	422.6481	1.244292	1.421455	455.2850
FAGP	1.200245	1.420618	442.6481	1.244230	1.421975	455.2850
WFAGP	1.199774	1.419792	442.6481	1.244322	1.421549	455.2850
NHFNLP	1.99023	1.420511	442.6482	1.244454	1.421594	455.2850



We can see from the figure-2 that total average cost of two items is almost same. FNLP, WFNLP, FAGP, WFAGP all are giving the same average cost but NHFNLP gives slide different value of total average cost. So we can say all the methodology applied here are almost identical for this proposed model.

Figure-2: GRAPH OF TOTAL AVERAGE COST WITH DIFERENT METHODS.

9. Sensitivity analysis:

Here we have discussed the optimal solution of multi-item (two-item) inventory model with different weights by WFNLP technique.

TABLE-3

Different weight	TAC1($t_{01}^*, t_{11}^*, T_1^*$)	TAC2($t_{02}^*, t_{12}^*, T_2^*$)
w_1=.1, w_2=.9	442.6481	455.4970
w_1=.2, w_2=.8	442.4681	455.4639
w_1=.3, w_2=.7	442.4681	455.4213
w_1=.4, w_2=.6	442.4681	455.3645
w_1=.5, w_2=.5	442.4681	455.2850
w_1=.6, w_2=.4	442.7164	455.2850
w_1=.7, w_2=.3	442.7652	455.2850
w_1=.8, w_2=.2	442.8018	455.2850
w_1=.9, w_2=.1	442.8302	455.2850

Here we can see that if w_1 increases total average cost TAC1 ($t_{01}^*, t_{11}^*, T_1^*$) increases and total average cost TAC2 ($t_{02}^*, t_{12}^*, T_2^*$) decreases. If w_2 increases total average cost TAC1 ($t_{01}^*, t_{11}^*, T_1^*$) decreases and total average cost TAC2 ($t_{02}^*, t_{12}^*, T_2^*$) increases.

Here we have discussed the optimal solution of multi-item (two-item) inventory model with different weight by WFAGP technique.

TABLE-4

Different weight	TAC1($t_{01}^*, t_{11}^*, T_1^*$)	TAC2($t_{02}^*, t_{12}^*, T_2^*$)
w_1=.1, w_2=.9	442.6481	455.2850
w_1=.2, w_2=.8	442.4681	455.2850
w_1=.3, w_2=.7	442.4681	455.2850
w_1=.4, w_2=.6	442.4681	455.2850
w_1=.5, w_2=.5	442.4681	455.2850
w_1=.6, w_2=.4	442.6481	455.2850
w_1=.7, w_2=.3	442.6481	455.2850
w_1=.8, w_2=.2	442.6481	455.2850
w_1=.9, w_2=.1	442.6481	455.2850

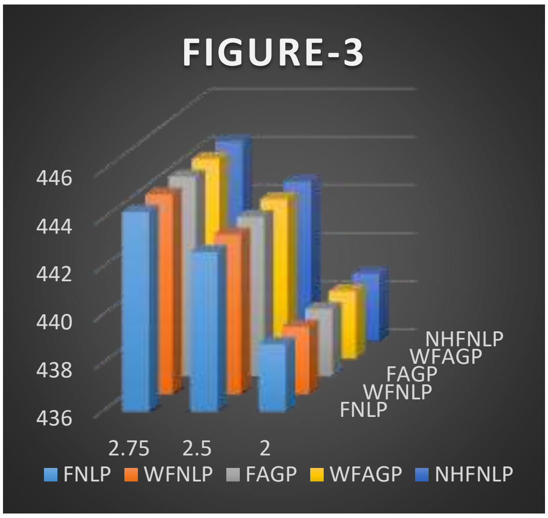
Here we can see that total average cost of TAC1 ($t_{01}^*, t_{11}^*, T_1^*$) and TAC2 ($t_{02}^*, t_{12}^*, T_2^*$) are constant.it does not depend on different values of weight w_1 and w_2.

Now we focus on the change of total average cost depending on various holding cost by FNLP, WFNLP, FAGP, WFAGP and NHFNLP techniques are represented in table -5. Here we take the weights (w_1=w_2=.5) for both WFNLP and WFAGP technique.

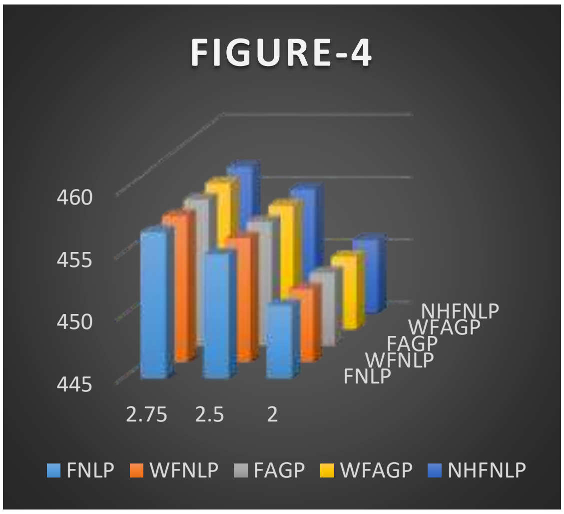
TABLE-5

Methods	HOLDING	TAC1($t_{01}^*, t_{11}^*, T_1^*$)	t_{11}^*	T_1^*	TAC2($t_{02}^*, t_{12}^*, T_2^*$)	t_{12}^*	T_2^*

	COST[h_1 (1st item)=h_2 (2nd item=h]						
FNLP	2.75	444.3175	1.156815	1.385719	456.7003	1.210349	1.392935
	2.5	442.6481	1.200486	1.421524	454.9165	1.253672	1.429418
	2.0	438.8166	1.307034	1.509127	450.8424	1.365131	1.524196
WFNLP	2.75	444.3175	1.156815	1.385719	456.7003	1.208066	1.390538
	2.5	442.6481	1.199541	1.420801	454.9165	1.253672	1.429418
	2.0	438.8166	1.307034	1.509127	450.8424	1.366006	1.525305
FAGP	2.75	444.3175	1.156815	1.385719	456.7003	1.209225	1.392011
	2.5	442.6481	1.200144	1.420725	454.9125	1.253672	1.429418
	2.0	438.8166	1.307012	1.508977	450.8423	1.364764	1.525177
WFAGP	2.75	444.3175	1.156815	1.385719	456.7003	1.209045	1.391500
	2.5	442.6481	1.199810	1.420557	454.9165	1.253672	1.429418
	2.0	438.8166	1.307034	1.509127	450.8423	1.364750	1.525606
NHFNLP	2.75	444.3177	1.154959	1.384276	456.7004	1.210351	1.393492
	2.5	442.6481	1.199795	1.420212	454.9165	1.253362	1.429104
	2.0	438.8166	1.307033	1.509126	450.8423	1.364590	1.523901



Total average cost of first item in different technique and different values of holding cost.



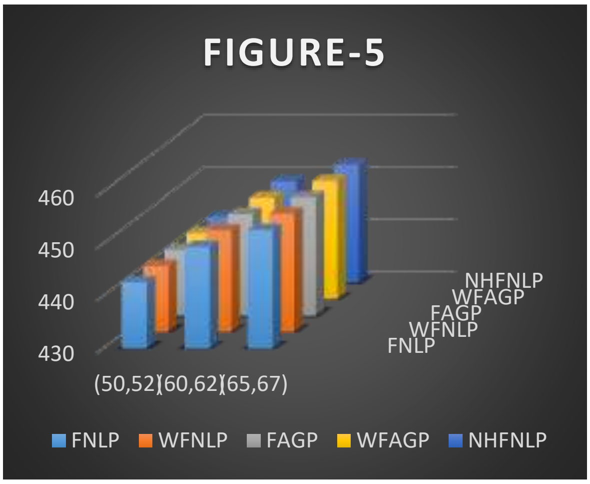
Total average cost of second item in different technique and different values of holding cost.

In above figures (figue-3 and figure-4) we can see that when we increase the value of holding cost (h) corresponding average cost value also increases. And it is true for all methodology we are dealing with.

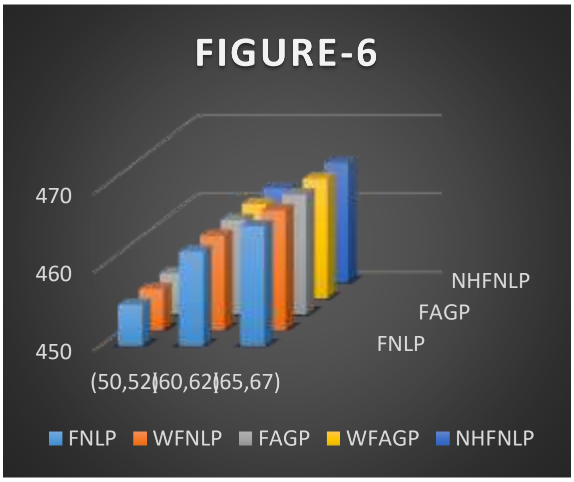
Now we focused on the change of total average cost depending on various ordering cost by FNLP, WFNLP, FAGP, WFAGP and NHFNLP techniques are represented in table-6. Here we take the weights (w_1=w_2=.5) for both WFNLP and WFAGP technique.

Table-6

Methods	Ordering cost for first item(A_1)	TAC$_1$($t_{01}^*, t_{11}^*, T_1^*$)	t_{11}^*	T_1^*	Ordering cost for second item(A_2)	TAC$_2$($t_{02}^*, t_{12}^*, T_2^*$)	t_{12}^*	T_2^*
FNLP	50	442.6481	1.199857	1.420192	52	455.2850	1.244292	1.421455
	60	449.4415	1.268755	1.522320	62	462.0886	1.314198	1.517613
	65	452.6740	1.299211	1.569278	67	465.3352	1.346345	1.562357
WFNLP	50	442.6481	1.199205	1.419957	52	455.2850	1.244292	1.421455
	60	449.4415	1.269526	1.523659	62	462.0886	1.314198	1.517613
	65	452.6740	1.300564	1.569681	67	465.3352	1.346345	1.562357
FAGP	50	442.6481	1.200245	1.420618	52	455.2850	1.244230	1.421975
	60	449.4415	1.268980	1.521919	62	462.0886	1.313272	1.516051
	65	452.6740	1.301278	1.571637	67	465.3352	1.346345	1.562357
WFAGP	50	442.6481	1.199774	1.419792	52	455.2850	1.244322	1.421549
	60	449.4415	1.270147	1.524057	62	462.0886	1.313251	1.516190
	65	452.6740	1.300205	1.569592	67	465.3352	1.346344	1.562291
NHFNLP	50	442.6482	1.99023	1.420511	52	455.2850	1.244454	1.421594
	60	449.4415	1.267611	1.522379	62	462.0886	1.314243	1.5185587
	65	452.6740	1.300222	1.569239	67	465.3353	1.348005	1.563531



Total average cost of first item in different technique and different values of ordering Cost.



Total average cost of second item in different technique and different values of ordering Cost.

In above figures (figue-5 and figure-6) we can see that when we increase the value of ordering cost (A) corresponding average cost value also increases. And it is true for all methodology we are dealing with.

9. CONCLUSION:

This paper presents a multi-objective inventory model in fuzzy environment. Here the model is directly connected with the real life business enterprises which consider the fact that storage space is limited and storage item is deteriorated during storage period and here the demand rate, deterioration and holding cost depend upon time. The model is developed in deterministic manner in which we use a preservation constant to reduce deterioration. After getting the result we have seen that the effect of deterioration can be controlled slightly by using a preservation condition. The model allows for shortages and in the shortage period the demand is partially backlogging. For the presence of uncertainty some cost parameters are considered as generalized trapezoidal fuzzy number. Finally the proposed model has been verified in several techniques as FNLP, WFNLP, FAGP, WFAGP and NHFNLP Techniques. The acquired results assure us the stability and validity of this model.

The proposed model can be implemented by taking more realistic assumptions such as probabilistic demand, power demand, finite replenishment etc. Instead of taking generalized trapezoidal fuzzy number we may also take triangular fuzzy number, pentagonal fuzzy number, parabolic flat fuzzy number etc. for all cost parameters we have considered to develop our multi-item inventory model. It will be also very interesting if we can develop a model where deterioration is depending with preservation condition w.r.t time. Instead of taking preservation condition as a constant function it would be more realistic and interesting if it would be taken as some function of time that means preservation power should be decrease with the increasing of time.

Highlights of the manuscript:

i) An inventory model with time dependent deterioration and time-dependent holding cost.

ii) Storage space is limited for this proposed model

iii) Preservation constant is used for controlling the effect of deterioration.

iv) Different methodologies are used for solving this model that is for finding the minimum total average cost.

v) All the methods gives us more or less same optimum values of total average cost.

vi) Neutrosophic hesitant technique gives us almost same optimum value as FNLP and FAGP methods give.

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Author's Contribution:

Dr Sahidul Islam gave the idea in developing the Inventory model and described how the preservation condition could be used for this model and also discussed various methodologies for solving this model. Kausik Das implemented this concept in this model and describes various methodology for solving the inventory model mainly focused on the technique neutrosophic hesitant fuzzy programming. Both authors participate in developing the manuscript.

References:

1. Roy, T.K.; Maity, M. A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity. European journal of Operation Research 1997, 99, 425-432.

2. Goswami, A.; Chaudhuri, K.S. An EOQ Model for Deterioration Items with Shortage and a Linear Trend in Demand. Journal of the Operational Research Society 1991, 42, 1105-1110.

3. Balkhi, Z.T.; Benkherouf, L. On an inventory model for deterioration items with stock dependent and time-varying demand rates. Computers and Operation Research 2004, 31, 223-240.

4. Hariga, M. An EOQ Model for deterioration Items with Shortages and Time-varying Demand. Journal of operational research Society 1995, 46, 398-404.

5. Chang, H.J.; Dye, C.Y. An inventory model for deterioration items with partial backlogging and permissible delay in payment. International journal of System of science 2001, 32, 345-352.

6. Kaur. J.; Sharma, R. Inventory Model: Deterioration Items with price and Time Dependent Demand Rate. International Journal of Modern Engineering Research (IJMER) 2012, 2, 3650-3652.

7. Chang, H.J.; Dye, C.Y. An EOQ model for deteriorating items with time varying demand and partial backlogging. Journal of the Operational Research Society 1991, 50, 1176-1182.

8. Ahmad, F.; Adhami, Y.A.; Smarandache, F. Single valued Neutrosophic Hesitant Fuzzy Computational Algorithm for Multi objective Nonlinear Optimization Problem. Neutrosophic Sets and System 2018, 22, 76-86.

9. Bharati, S. K. Hesitant fuzzy computational algorithm for multiobjective optimization problems. International Journal of Dynamics and Control 2018, 6, 1799-1806

10. Singh, S. K.; Yadav, S. P. Modeling and optimization of multi objective non-linear programming problem in intuitionistic fuzzy environment. Applied Mathematical Modelling 2015, 39, 4617–4629.

11. Islam, S.; Mandal, W. A. A Fuzzy Inventory Model (EOQ MODEL) with Unit Production Cost, Time Depended Holding Cost, with-out Shortages under A Space Constraint: A Parametric Geometric Programming (FPGP) Approach. Independent Journal of Management & Production 2017, 8, 299-318.

12. Zadeh, L. A. Fuzzy sets Information and Control 1965, 8, 338-353.

13. Nabeeh, N. A.; Basset, M.A.; El-Ghareeb, H. A.; Aboelfetouh. A. Neutrosophic multi-criteria decision making approach for Io T-based enterprises. IEEE Access 2019, 7, 59559 - 59574.

14. Kar, C.; Mondal, B.; Roy, T.K. An Inventory Model under Space Constraint in Neutrosophic Environment: A Neutrosophic Geometric Programming Approach. Neutrosophic Sets and Systems 2018, 21, 93-109.

15. Pramanik, S.; Mallick, R.; Dasgupta, A. Contributions of selected Indian researchers to multi attribute decision making in neutrosophic environment: an over view. Neutrosophic Sets and Systems 2018, 20, 108-131.

16. Charles, V.; Gupta, S.; Ali, I. A Fuzzy Goal Programming Approach for Solving Multi-Objective Supply Chain Network Problems with Pareto-Distributed Random Variables. International journal of Uncertainty, Fuzziness and Knowledge-Based system 2019, 27, 559-593.

17. Ali, I..; Fugenschuh, A.; Gupta, S.; Modibbo, U.M. The LR-Type Fuzzy Multi-Objective Vendor Selection Problem in Supply Chain Management. Mathematics 2020, 8, 1-25.

18. Gupta, S.; Haq, A.; Ali, I; Sarkar, Significance of multi-objective optimization in logistics problem for multi-product supply chain network under the intuitionistic fuzzy environment. Complex & intelligent system 2021, 7, 2119–2139.

19. Janssen, L.; Claus, T.; Sauer, J. Literature review of deteriorating inventory models by key topics from 2012 to 2015. International Journal of Production Economics 2016, 182, 86-112.

20. Gautam, P.; Maheshwari, S.; Kausar, A.; Jaggi, C.K. Inventory Models for Imperfect Quality Items: A Two-Decade Review.Advances in Interdisciplinary Research in Engineering and Business Management 2021, 185-215.

21. Ali, I.; Gupta, S.; Ahmed, A. Multi-objective linear fractional inventory problem under intuitionistic fuzzy environment. International Journal of System Assurance Engineering and Management 2018, 10, 173–189.

22. Masae, M.; Glock, C.H.; Grosse, E.H. Order picker routing in warehouses: A systematic literature review. International Journal of Production Economics 2020, 224, 107564.

23. Ali, I.; Gupta, S.; Ahmed, A. Multi-objective bi-level supply chain network order allocation problem under fuzziness. Springer 2018, 55, 721–748.

24. Charles, V.; Gupta, G.; Ali, I. A Fuzzy Goal Programming Approach for Solving Multi-Objective Supply Chain Network Problems with Pareto-Distributed Random Variables. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 2019, 27, 559-593.

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