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The Duality Approach of the Neutrosophic Linear Programming

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Abstract. The neutrosophic mathematical linear programming in its duality fashion is originally exhibited in this manuscript. In accordance with this concept, the relationship of the duality between the neutrosophic objective functions and neutrosophic constraints is given, three versions of linear programming related to $\mu_D(x)$, $\sigma_D(x)$, and $v_D(x)$ have been originated respectively, some important propositions have been discussed, two numerical examples were considered in the economic interpretation and in the hybrid renewable energy production.

Keyword: Neutrosophic Linear Programming Related to $\mu_D(x)$; Neutrosophic Linear Programming Related to $\sigma_D(x)$; Neutrosophic Linear Programming Related to $v_D(x)$; Parametric Dual Neutrosophic Linear Programming.

1. Introduction

It is well known that, when the region of the feasible solution of any mathematical problem is convex, then the optimality will be traditionally performed. For many years the dominant distinction in applied mathematics between problem types has rested upon linearity, or lack thereof. Our assignment here is to serve more than a half-century of work in convex analysis that has played a fundamental role in the development of computational in every branch of application whether the problem is in economical fields, industrial fields, or agricultural fields ... etc. Supposing that the problem is a neutrosophic linear programming problem and the established problem is taken from the economical point of view.

In any selling product, there are three assigned statuses:

S1- Profit situations.

S2- Loss situations.

S3- Indeterminate status which there are not clear criteria enables the decision maker to determine getting profit nor loss.

There are many factors that affecting on the degree of accomplishment for the above S1, S2, and S3 such as the season of the year, quality of the competition in each product, the spread of public pandemic as COVID 19… etc.

For a wise decision, the selling product should not mark with maximum profit often along time, since in such a case the competition, for instance, could obtain a profit of firm's policy be reducing their prices for the same articles. Therefore, and by the authors' opinion, it is very important to make an adaptation for the classical linear programming or the fuzzy linear programming into the neutrosophic linear programming with three versions related to their truth, indeterminate, and falsity membership functions, this kind of programming will parametric classical or fuzzy linear programming into three decision factors: -

1- The neutrosophic linear programming related to the truth membership function.

2- The neutrosophic linear programming related to the indeterminate membership function.

3- The neutrosophic linear programming related to the falsity membership functions.

Bellow two comparisons between Fuzzy Linear Programming and Neutrosophic Linear Programming:

1- In Fuzzy Linear Programming Problems (FLP) [1,15], as the optimal solution has depended on a limited number of constraints, therefore, much of the information that should be collected and having a good impact on the solution are absent, this is exactly what Neutrosophic Linear Programming (NLP) provides.
2- Given the power of LP, one could have expected even more applications. This might be because LP requires many well-defined and precise data which involves high information costs. In real-world applications certainty, reliability, and precision of data are often illusory. Being able to deal with vague and imprecise data may greatly contribute to the diffusion and application of LP. Neutrosophic Linear Programming problems have the ability to reformulate the soft linear programming problems

through three membership functions which are truth membership function, indeterminacy membership function, and falsity membership functions, while the Fuzzy Linear Programming deals with just one membership function.

This essay aims to advance a new way for analyzing linear programming containing three different and related faces in which the same problem can be approached from three various corners, it is neutrosophic linear programming of three membership functions. This wide insight can be appeared depending upon the neutrosophic logic, this kind of problem has established firstly in 1995 by Florentin Smarandache [2,3], the neutrosophic logic and theory have widespread since the NSS journal has been released in 2013. Dozens of papers were issued, and new mathematical concepts have been originated, such as, neutrosophic geometric programming has been established and modified at 2015-2020 by Huda et al [9,10,12-14], also presented another concept of geometric programming with neutrosophic less than or equal [6,8], neutrosophic (sleeves, Anti-sleeves, Neut-sleeves), and the neutrosophic convex set has been set up [11], the excluded middle law with the perspective of neutrosophic geometric programming [4,5,11].

The new type of linear programming that presented in this article will be defined in the triplet $(X = [0,1], N(X), c)$ corresponding to the case in which the expert- mathematician exactly knows his objective function, but the constraints set is of type neutrosophic linear programming, the upcoming preliminaries are necessary to build the mathematical structure of such problems.

Call the classical linear programming problems

$$\left.\begin{array}{ll} Max & f = cx \\ s.t. & Ax \le b \\ & x \ge 0 \end{array}\right\} \qquad (1)$$

Which defined by the triplet (R^n, X, c), the goal of this problem is to find the optimal solution $x^* \in X \subset R^n$ such that $\forall\, x \in X: cx^* \ge cx$ with $X = \{x \in R^n | Ax \le b: x \ge 0\}$.

The above classical linear programming can be redefined as a Neutrosophic Linear Programming (NLP) related to its truth, indeterminacy, and falsity membership functions. The following section contains some new definitions that coined and for the first time in this essay beside to some preliminaries which are necessary to build the mathematical formulas.

The upcoming sections of this paper have been organized as: section two contains three basic definitions that are necessary tools to follow up the mathematical requirements of this article, as well as, three new definitions were originally coined by the authors to extend the fuzzy linear programming to the neutrosophic linear programming. Section three was dedicated to two important propositions that regarded as the new mathematical vision for a new procedure that proves any neutrosophic linear programming related to its truth membership function can be regarded as the dual form for the neutrosophic linear programming related to its falsity membership function. In section four, two practical examples have been presented that assures the theoretical directions of the paper. Concluding section was the fifth section of this article.

2. Basic Concepts

2.1 Definition [16]

A neutrosophic set $\mathrm{D} \in \mathrm{N}(\mathrm{X})$ is defined as $\mathrm{D} = \{< \mu_\mathrm{D}(\mathrm{x}), \sigma_\mathrm{D}(\mathrm{x}), \mathrm{v}_\mathrm{D}(\mathrm{x}) >: \mathrm{x} \in \mathrm{X}\}$ where $\mu_\mathrm{D}(\mathrm{x}), \sigma_\mathrm{D}(\mathrm{x}), \mathrm{v}_\mathrm{D}(\mathrm{x})$ represent the membership function, the indeterminacy function, the non-membership function respectively.

2.2 Definition [11]

Let $D \in N(X), \forall\ (\alpha, \gamma, \beta) \in [0,1]$, written $D_{(\alpha,\gamma,\beta)} = \{x: \mu_\mathrm{D}(\mathrm{x}) \geq \alpha, \sigma_\mathrm{D}(\mathrm{x}) \geq \gamma, \mathrm{v}_\mathrm{D}(\mathrm{x}) \leq \beta\}$, $D_{(\alpha,\beta,\gamma)}$ is said to be an $(\alpha, \beta, \gamma) - cut$ set of a neutrosophic set D . Again, $D_{(\alpha,\gamma,\beta)^+} = \{\mathrm{x}: \mu_\mathrm{D}(\mathrm{x}) > \alpha, \sigma_\mathrm{D}(\mathrm{x}) > \gamma, \mathrm{v}_\mathrm{D}(\mathrm{x}) < \beta\}$, $D_{(\alpha,\gamma,\beta)^+}$ is said to be a strong $(\alpha, \gamma, \beta) - cut$ set of a neutrosophic set D, (α, γ, β) are confidence levels and $\alpha + \gamma + \beta \leq 3$.

2.3 Definition [7]

A mapping $D\colon \mathrm{X} \to [0,1], \mathrm{x} \to \mu_\mathrm{D}(\mathrm{x}), \mathrm{x} \to \sigma_\mathrm{D}(\mathrm{x})\ , \mathrm{x} \to \mathrm{v}_\mathrm{D}(\mathrm{x})$ is called a collection of neutrosophic elements, where μ_D a membership x corresponding to a neutrosophic set D, $\sigma_\mathrm{D}(\mathrm{x})$ an indeterminacy membership x corresponding to a neutrosophic set D, $\mathrm{v}_\mathrm{D}(\mathrm{x})$ a non-membership x corresponding to a neutrosophic set D.

The upcoming definitions are essentially requirements for completing the duties of this article, so the authors originally coined them as follow:

2.4 Definition

$$\left.\begin{array}{ll} & Max \quad f = cx \\ s.t. & \mu(A(x)_i, b) \geq \alpha \quad i = 1,2,\dots,m \\ & \alpha \in [0,1]\,, \quad x \geq 0 \end{array}\right\} \tag{2}$$

Here $\alpha \in [0,1]$ is the respective $\alpha - cut$ of the neutrosophic constraint set related to the truth membership function μ .

The above problem has been defined in the neutrosophic triplet $(X, N(X), c)$, here $X = [0,1], c \in N(X^n), b \in N(X^m), A_{mn}$ is a matrix of neutrosophic values, where $N(X) = \{x \in X^n; f: X^n \rightarrow N(X^n)\}$, $\mu = \{\mu_1, \mu_2, \dots, \mu_m\}$ is an m- vector of truth membership functions.

2.5 Definition

Depending upon the structure of the mathematical formula of neutrosophic linear programming (2), one can define a new concept named neutrosophic linear programming related to the falsity membership function as follow: -

$$\left.\begin{array}{ll} & Min \quad f = cx \\ s.t. & V(A(x)_i, b) \leq \beta \\ & \beta \in [0, 1], \quad x \geq 0 \end{array}\right\} \tag{3}$$

Here $\beta \in [0, 1]$ is the respective $\beta - cut$ of the neutrosophic constraint set in the case of neutrosophic linear programming regarded to its falsity membership function.

One can base on the intuitive idea to conclude that the inequality $V(A(x)_i, b) \leq \beta$ is equivalent to the inequality

$$1 - \mu(A(x)_i, b) \leq \beta \tag{4}$$

$$\Rightarrow 1 - \beta \leq \mu(A(x)_i, b)$$

So, the inequality (4) can be rewrite as

$$\mu(A(x)_i, b) \geq 1 - \beta \tag{5}$$

Comparing (2) & (5), we conclude that $\alpha \equiv 1-\beta$

Note that the difference between the two problems (2) & (3) is that the problem (2) gives the optimal solution for the neutrosophic linear programming with respect to the truth membership function, while the problem (3) gives the optimal solution for the neutrosophic linear programming with respect to the falsity membership function.

2.6 Definition

It is well known for any mathematical programmer who has the tools for reformulating any classical mathematical programming problems into neutrosophic programming problems, that the neutrosophic linear programming related to its indeterminacy membership function has well defined when it can be defined as:

$$\left.\begin{array}{ll} & Max \quad f = cx \\ s.t. & \sigma_D(x) \geq \gamma \\ & x \geq 0 \end{array}\right\} \tag{6}$$

Here $\gamma \in [0,1]$ is the respective $\gamma - cut$ of the neutrosophic constrain set in the case of neutrosophic linear programming with respect to its indeterminacy membership function.

The problem (6) is equivalent to

$$\left.\begin{array}{ll} & Max \quad f = cx \\ s.t. & \mu(A(x)_i, b) \cap V(A(x)_i, b) \geq \gamma \end{array}\right\} \tag{7}$$

3. The Duality Approach of Neutrosophic Linear Programming

3. 1 Proposition

Given a neutrosophic linear programming problem (2), there always exist a corresponding dual problem which is exactly the neutrosophic linear programming problem (3), and they have the same neutrosophic solution.

Proof

Consider the following neutrosophic linear programming

$$\left.\begin{array}{l} \qquad Max \quad f = cx \\ s.t. \quad \mu(A(x)_i, b) \geq \alpha \quad i = 1,2, \dots, m \\ \qquad \alpha \in [0,1], \quad x \geq 0 \end{array}\right\} \tag{8}$$

Where the m-vector of membership functions $\mu = \{\mu_1, \mu_2, \dots, \mu_m\}$ such that

$$\forall x \in X: \ \mu_j(x) = \begin{cases} 1 & x < b_j \\ \frac{(b_j+d_j)-x}{d_j} & b_j \leq x \leq \ b_j + d_j \\ 0 & x > b_j + d_j \end{cases} \tag{9}$$

Where the values of $d_j \in X \ \ (j = 1,2, \dots, m)$ expressing the admissible violations of the economic-expert allows in the accomplishment of the neutrosophic linear constraints of (9), it is obvious that the neutrosophic solution of (9) is found by obtaining the optimal solution of the linear neutrosophic problem

$$\left.\begin{array}{l} \qquad Max \quad f = cx \\ s.t. \quad \mu(A(x), b) \geq \alpha \\ \qquad \alpha \in [0,1], \quad x \geq 0 \end{array}\right\} \tag{10}$$

Depending upon (9) we have

$$\frac{b+d-Ax}{d} \geq \alpha \Leftrightarrow b + d - Ax \geq d\alpha \ \Leftrightarrow Ax - b - d \leq -d\alpha \ \Leftrightarrow Ax \leq b + d(1 - \alpha).$$

Therefore, we have

$$\left.\begin{array}{l} \qquad Max \quad f = cx \\ s.t. \quad Ax \leq b + d(1 - \alpha) \\ \qquad \alpha \in [0,1], \quad x \geq 0 \end{array}\right\} \tag{11}$$

As (11) is a classical parametric linear programming problem, its dual is given by

$$\left.\begin{array}{l} Min \ [b + d(1 - \alpha)]u \\ s.t. \qquad uA^T \geq c \\ \ u \geq 0, \ \ \alpha \in [0,1] \end{array}\right\} \tag{12}$$

Let $Y = \{u \in N(X^m) | uA^T \geq c, \ \ u \geq 0\}$

So, we have

$$\left.\begin{array}{l} \qquad Min \ \ au \\ s.t. \quad a = b + d(1 - \alpha) \\ \quad u \in Y, \ \alpha \in [0,1] \end{array}\right\} \tag{13}$$

Consider a as m-variable vectors and taking $\beta = 1 - \alpha$, this problem is equivalent to

$$\left.\begin{array}{c} Min\ \ au \\ s.t. \quad a \leq\ b + d\beta \\ u \in Y,\ \beta \in [0,1] \end{array}\right\} \tag{14}$$

Understanding the equivalence in the sense that any optimal solution of (13) is also an optimal solution of (14), but as $\frac{(b_j+d_j)-a_j}{d_j} \geq \alpha$ which implies that

$$\left(b_j + d_j\right) - a_j \geq d_j\alpha \Leftrightarrow -a_j \geq -\left(b_j + d_j\right) + d_j\alpha \ \Leftrightarrow a_j \leq b_j + d_j - d_j\alpha \Leftrightarrow a_j \leq b_j + d_j(1-\alpha)$$
$$\Leftrightarrow a_j \leq b_j + d_j\beta \quad for\ j = 1,2,\dots,m$$

So (14) may be rewritten as

$$\left.\begin{array}{c} Min\ \ au \\ s.t. \quad \mu_j\left(a_j\right) \geq 1 - \beta \\ u \in Y,\ \beta \in [0,1] \end{array}\right\} \tag{15}$$

Which implies to the following formula

$$\left.\begin{array}{c} Min\ \ au \\ s.t. \quad 1 - \mu_j\left(a_j\right) \leq \beta \\ u \in Y,\ \beta \in [0,1] \end{array}\right\} \tag{16}$$

Consequently

$$\left.\begin{array}{c} Min\ \ au \\ s.t. \quad V_j\left(a_j\right) \leq \beta \\ u \in Y,\ \beta \in [0,1] \end{array}\right\} \tag{17}$$

With $\mu_j(.)$ is given by (9), $V_j\left(a_j\right)$ is a non-membership function that stated in def. (2.3), programming (17) is exactly represented a neutrosophic linear programming with respect to its falsity membership function. Since, in the optimum, (11) and (12) have the same parametric solution, the problem (17) has the same neutrosophic solution as (8) by taking $\beta \equiv 1 - \alpha$.

If we had initially started from the neutrosophic linear programming (2), we would by the same development, in a parallel way, have come to a neutrosophic linear programming (3) with the same neutrosophic solution.

3.2 Proposition

Given a neutrosophic linear programming (2) or a neutrosophic linear programming (3), with continues and strictly monotone membership function for the economic restrictions (costs or benefits), there exists a dual neutrosophic linear programming (3), or a dual neutrosophic linear programming (2) respectively of the former in such a way that both have the same neutrosophic solution.

Proof

Let $\mu_j : X \longrightarrow N(X), j = 1,2,\dots,m$ be continuous and strictly increasing function for the neutrosophic linear programming problem (2).

Given a classical linear programming with a neutrosophic inequality in its constraint

$$\left.\begin{array}{lll} & Max & cx \\ s.t. & Ax \leq ₦ \; b & x \geq 0 \end{array}\right\} \tag{18}$$

Where ($\leq$ ₦) is the neutrosophic version of the (less than or equal) inequality. We shall find its neutrosophic solution with respect to $\mu(.)$, and for every $\alpha \in [0,1]$ of the neutrosophic constraint set

$\mu(Ax, b) \geq \alpha \qquad \alpha \in [0,1]$

But according to the hypotheses as μ is continuous and strictly monotone, μ^{-1} exists, and $\mu(Ax, b) \geq \alpha \Leftrightarrow Ax \leq \emptyset(\alpha) = \mu^{-1}(\alpha)$, and the proof follows as in proposition (3.1).

4 Numerical Examples:

4.1 Example 1

Suppose we have a neutrosophic linear programming problem with neutrosophic less than or equal in its constraints and as follows:

$$\left.\begin{array}{c} Max \;\; f(x_1, x_2) = x_1 + x_2 \\ 4x_1 - x_2 \leq ₦ \; 10 \\ x_1 + 2x_2 \leq ₦ \; 15 \\ 5x_1 + 2x_2 \leq ₦ \; 20 \\ x_i \geq 0 \end{array}\right\} \tag{19}$$

With membership functions as follow:

$\mu_1(4x_1 - x_2, 10) \geq \alpha$

Here $b_1 = 10$, if we take $d_1 = 5$ as an admissible violation of the first constraint.

So, $\mu_1(4x_1 - 2x_2, 10) = \frac{(15-4x_1+x_2)^2}{25} \geq \alpha$,

$$(15 - 4x_1 + x_2)^2 \geq 25\alpha \quad (20)$$

The optimal solution of the inequality (20) is equivalent to the optimal solution of

$15 - 4x_1 + x_2 = 5\sqrt{\alpha}$,

$$-4x_1 + x_2 = 5\sqrt{\alpha} - 15 \quad (21)$$

Also we have,

$\mu_2(x_1 + 2x_2, 15) \geq \alpha$

Here $b_2 = 15$, if we take $d_2 = 8$ as an admissible violation of the second constraint.

$\mu_2(x_1 + 2x_2, 15) = \frac{(23-x_1-2x_2)^2}{64} \geq \alpha$,

$23 - x_1 - 2x_2 = \sqrt{64\alpha}$,

$-x_1 - 2x_2 = 8\sqrt{\alpha} - 23$

$$x_1 + 2x_2 = 23 - 8\sqrt{\alpha} \quad (22)$$

Finally, the membership of the third constraint is

$\mu_3(5x_1 + 2x_2, 20) \geq \alpha$

It is obviously that $b_3 = 20$, and if we take the admissible violation for the third constraint as $d_3 = 10$, so we have

$$-5x_1 - 2x_2 = 10\sqrt{\alpha} - 30 \longrightarrow 5x_1 + 2x_2 = 30 - 10\sqrt{\alpha} \quad (23)$$

From (23) we have,

$$2x_2 = 30 - 10\sqrt{\alpha} - 5x_1 \quad (24)$$

Substitute (24) in (22),

$x_1 + 30 - 10\sqrt{\alpha} - 5x_1 = 23 - 8\sqrt{\alpha}$

$\therefore\ x_1 = \frac{7-2\sqrt{\alpha}}{4}$ (25)

Substitute (25) in (23) we get,

$[\frac{5}{4}\left(7 - 2\sqrt{\alpha}\right) + 2x_2 = 30 - 10\sqrt{\alpha}]$, simplify this formula by multiplying it by 4 getting the following formula:

$35 - 10\sqrt{\alpha} + 2x_2 = 120 - 40\sqrt{\alpha}$

$x_2 = \frac{85-30\sqrt{\alpha}}{8}$ (26)

Consequently, $\alpha \in [0,1], x_1^* = \frac{7-2\sqrt{\alpha}}{4}\ , x_2^* = \frac{85-30\sqrt{\alpha}}{8}$.

Thus,

$$f^*(x_1^*, x_2^*) = \frac{(99 - 34\sqrt{\alpha})}{8} \in [\frac{65}{8}, \frac{99}{8}]$$

And the neutrosophic solution for (19) with respect to its membership function μ becomes the neutrosophic set

$$\left\{f(x), \mu(x) \colon f(x) \in \left[\frac{65}{8}, \frac{99}{8}\right],\ \mu(x) = [\frac{99-8x}{34}]^2\right\} \tag{27}$$

On the other hand, if we solve (19) by means of its dual (i.e., the corresponding of its neutrosophic linear programming (3)), we should have

$$\left.\begin{array}{l} Min\ w = (10 + 5\beta)u_1 + (15 + 8\beta)u_2 + (20 + 10\beta)u_3 \\ \qquad\qquad s.t.\ \ 4u_1 + u_2 + 5u_3 \geq 1 \\ \qquad\qquad -u_1 + 2u_2 + 2u_3 \geq 1 \\ \qquad\qquad \beta \in [0,1], \quad u_i \geq 0 \end{array}\right\}$$

Which is equivalent to the following program

$$\left.\begin{array}{l} Min\ w = (15 - 5\alpha)u_1 + (23 - 8\alpha)u_2 + (30 - 10\alpha)u_3 \\ \qquad\qquad s.t.\ \ 4u_1 + u_2 + 5u_3 \geq 1 \\ \qquad\qquad -u_1 + 2u_2 + 2u_3 \geq 1 \\ \qquad\qquad \alpha \in [0,1], \quad u_i \geq 0 \end{array}\right\}$$

Which, when solved in the same way as parametric neutrosophic linear programming problem (2), has an optimal solution $u_1 = 0, u_2 = \frac{3}{8}, u_3 = \frac{1}{8}$.

Therefore,

$$w^* = (15 - 5\alpha)u_1 + (23 - 8\alpha)u_2 + (30 - 10\alpha)u_3 = \frac{(99-34\alpha)}{8} \in \left[\frac{65}{8}, \frac{99}{8}\right], \alpha \in [0,1],$$

And the corresponding neutrosophic solution with respect to its falsity membership function is the neutrosophic set

$\left\{w, \mu(x): w \in \left[\frac{65}{8}, \frac{99}{8}\right],\ \mu(x) = [\frac{99-8x}{34}]^2\right\}$ which coincides with (27).

4.2 Case Study (Hybrid Renewable Energy Systems in View of Neutrosophic Linear Programming)

As a result of the increasing demand for renewable energy, which is an inexhaustible and generally inexpensive source compared to traditional energy sources such as oil, natural gas, and coal...etc. Recently, an urgent need has emerged to integrate renewable energy sources in order to build up an economical hybrid energetic system in the case where each type of energy is only available as of specific units. The case study that we will shed the light on it was appeared in the essay [17] where the authors presented the capability of estimates of annual power production by combining photovoltaic panels with wind turbines (PV/Wind system) having specific capacities to meet energy demand in a specific site with the lowest cost. The purpose of this paper is different from the Zaatri and Allab [17] wherein they tried to estimate annual power production, also they formulated their problem as standard integer linear programming where the objective function to be minimized is the initial capital investment and where the decision variables are the numbers of units which should be pure integer numbers. Really, in this essay we aim to adapt the same problem in a different approach using neutrosophic linear programming (2), so our goal in this section is to show that the extension of the classical linear programming in uncertainty modeling gives more analytical information to be studied, the field of the power production of PV/Wind system with respect to uncertainties is unfathomable field and there are little discussions in the literature on it. It is well known that the intermittency is a part of Indeterminacy which is in between: interruption and non- interruption, so the neutrosophic theory will be a strong tool in getting and analyzing the optimal solutions.

Let $c_1 = 130\$$ represents the unit cost of a photovoltaic panel, $c_2 = 100\$$ is the reduced cost of the type of wind turbine. The investment for capital cost of the hybrid system which may involve the number of photovoltaic panels (N_1), and the number of wind turbine (N_2). This capital cost which is the objective function Z_T has to be minimized, therefore: -

$$\left.\begin{array}{l} MinZ_T = 130\,n_1 + 100\,n_2 \\ s.t.\;\; 66\,n_1 + 84n_2 \geq 3000 \\ \qquad\quad n_2 \geq 6 \\ \qquad\quad n_1, n_2 \;\geq 0 \end{array}\right\} \tag{28}$$

The following solution depends on the neutrosophic linear programming (2), the membership functions related to the two constraints of the program (28) are:

$$\mu_1(\,66\,n_1 + 84n_2, 3000) \geq \alpha$$

$$\mu_2(\,n_2, 6) \geq \alpha$$

Where, $d_1 = 1500\;and\;d_2 = 3$ are the admissible violations of these constraints,

$$\mu_1(\,66\,n_1 + 84n_2, 3000) = \frac{(4500-66n_1-84n_2)^2}{2250000} = \alpha, \tag{29}$$

$$\mu_2(\,n_2, 6) = \frac{(9-n_2)^2}{9} = \alpha, \tag{30}$$

The formula (29) implies to $4500 - 66n_1 - 84n_2 = 1500\sqrt{\alpha}$,

$$4500 - 1500\sqrt{\alpha} = 66n_1 + 84n_2 \tag{31}$$

While the formula (30) implies to

$$9 - n_2 = 3\sqrt{\alpha} \;\rightarrow n_2 = 9 - 3\sqrt{\alpha}, \tag{32}$$

As $\alpha \in [0,1] \rightarrow n_2 \in [6,9]$

Substituting (32) in (31), $4500 - 1500\sqrt{\alpha} = 66n_1 + 84\,(9 - 3\sqrt{\alpha})$, $4500 - 1500\sqrt{\alpha} = 66n_1 + 756 - 252\sqrt{\alpha} \;\Rightarrow 66n_1 = 3744 - 1248\,\sqrt{\alpha}$,

$$n_1 = \frac{3744}{66} - \frac{1248}{66}\sqrt{\alpha},\;\alpha \in [0,1] \tag{33}$$

$$n_1 \in \left[\frac{2496}{66}, \frac{3744}{66}\right] = [37.8, 56.7],$$

As n_1 *and* n_2 should be pure integer numbers, we will approximate the interval $[37.8, 56.7] \cong [38, 57]$, $n_1 \in [38,57]$.

The optimal value for the objective function Z_T

$$Z_T^* = 130\left(\frac{3744}{66} - \frac{1248}{66}\sqrt{\alpha}\right) + 100\left(9 - 3\sqrt{\alpha}\right) = 8274.5 - 2758.18\sqrt{\alpha}$$

$\alpha \in [0,1] \rightarrow Z_T^* \in [5517\$, 8275\$]$.

5 Conclusion

In this article, the classical linear programming has been redefined for the new type of neutrosophic linear programming with respect to its membership function, indeterminacy membership function, and non-membership function, with neutrosophic less than or equal in its constraint. Three new definitions have been posited, and two propositions were presented and proved. Two numerical examples were necessary to illustrate the theoretical direction practically.

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Reference

[1] D. Dubois, H. Prade. ''Fuzzy Sets and Systems. Theory and Applications''. Academic Press (1980).

[2] F. Smarandache, (1995). ''Neutrosophic Logic and Set''.

[3] F. Smarandache, (1998), Neutrosophy. Neutrosophic Probability, Set, and Logic, Amer. Res. Press, Rehoboth, USA, 105.

[4] F. Smarandache "Law of Included Multiple-Middle & Principle of Dynamic Neutrosophic Opposition". EuropaNova & Education Publisher Brussels-Columbus, 2014.

[5] F. Smarandache, "Unmatter as a Consequence of the Law of Included Multiple-Middles", Annual Meeting of the APS Four Corners Section, Vol. 60, Number 11, Friday–Saturday, October 16–17, 2015; Tempe, Arizona, http://meetings.aps.org/Meeting/4CF15/Session/F1.66

[6] F. Smarandache, H. E. Khalid, A. K. Essa, M. Ali, "The Concept of Neutrosophic Less Than or Equal To: A New Insight in Unconstrained Geometric Programming", Critical Review, Volume XII, 2016, pp. 72-80.
[7] F. Smarandache, Huda E. Khalid "Neutrosophic Precalculus and Neutrosophic Calculus". Second enlarged edition, Pons asbl 5, Quai du Batelage, Brussels, Belgium, European Union, 2018.
[8] F. Smarandache, H. E. Khalid, A. K. Essa, "Neutrosophic Logic: The Revolutionary Logic in Science and Philosophy", Proceedings of the National Symposium, EuropaNova, Brussels, 2018.
[9] H. E. Khalid, "An Original Notion to Find Maximal Solution in the Fuzzy Neutrosophic Relation Equations (FNRE) with Geometric Programming (GP)", Neutrosophic Sets and Systems, vol. 7, 2015, pp. 3-7.
[10] H. E. Khalid, "The Novel Attempt for Finding Minimum Solution in Fuzzy Neutrosophic Relational Geometric Programming (FNRGP) with (max, min) Composition", Neutrosophic Sets and Systems, vol. 11, 2016, pp. 107-111.
[11] H. E. Khalid, F. Smarandache, A. K. Essa, (2018). The Basic Notions for (over, off, under) Neutrosophic Geometric Programming Problems. Neutrosophic Sets and Systems, 22, 50-62.
[12] H. E. Khalid, (2020). Geometric Programming Dealt with a Neutrosophic Relational Equations Under the ($\boldsymbol{max - min}$) Operation. Neutrosophic Sets in Decision Analysis and Operations Research, chapter four. IGI Global Publishing House.
[13] H. E. Khalid, "Neutrosophic Geometric Programming (NGP) with (max-product) Operator, An Innovative Model", Neutrosophic Sets and Systems, vol. 32, 2020.
[14] H. E. Khalid, F. Smarandache, A. K. Essa, (2016). A Neutrosophic Binomial Factorial Theorem with their Refrains. Neutrosophic Sets and Systems, 14, 50-62.
[15] H. Tanaka, K. Asai,. ''Fuzzy linear programming problems with fuzzy numbers'', Fuzzy sets and systems 13, (1984) , 108-110.
[16] V. Kandasamy, F. Smarandache, "Fuzzy Relational Maps and Neutrosophic Relational Maps", American Research Press, Rehoboth,2004.
[17] Zaatri A., Allab K., (2012), Analysis of PV/Wind systems by integer linear programming. Revue des Energies Renouvelables Vol. 15 N°1 (2012) 111 – 120.

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