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Neutrosophic Set Approach to Study the Characteristic Behavior of Left Almost Rings

Inayatur Rehman[1], Asima Razzaque[2,*] and Muhammad Iftikhar Faraz[3]

[1]Department of Mathematics and Sciences, College of Arts and Applied Sciences, Dhofar University, Salalah, Oman

[1]irehman@du.edu.om

[2]Department of Basic Science, Deanship of Preparatory year, King Faisal University, Hofuf 31982, Al Hasa, Saudi Arabia

[2]arazzaque@kfu.edu.sa

[3]Department of Mechanical Engineering, College of Engineering, King Faisal University, Hofuf 31982, Al Hasa, Saudi Arabia

[3]mfaraz@kfu.edu.sa

* Correspondence: arazzaque@kfu.edu.sa; Tel.: (+966562744993)

Abstract: In this paper, the concept of Neutrosophic LA-rings is introduced. Furthermore, we investigate their algebraic structures. We discuss various types of ideals and establish a number of results to better understand the characteristic behavior of Neutrosophic LA-rings. In addition, we investigate the properties of the Neutrosophic M-system, Neutrosophic P-system, and Neutrosophic I-system in order to characterize the Neutrosophic LA-ring.

Keywords: Neutrosophic Sets; Neutrosophic LA-rings; Neutrosophic ideals; Neutrosophic M-systems; Neutrosophic P-systems and Neutrosophic I-systems.

1. Introduction

Different researchers have defined algebraic structures which were based on the crisp set. But the real-life problems could not be solved by crisp set theory. The crisp set deals with yes or no only and it never tells about in between yes and no. In 1965, Zadeh [1] introduced a fuzzy set theory to address the vagueness of various real-life problems. The fuzzy sets deals with membership in between 0 and 1. Later, Atanassov [2] in 1986, initiated intuitionistic fuzzy set. However, these theories have remained unsuccessful in finding a solution to many real-life mathematical challenges.

In 1999, Smarandache [3] gave the notion of Neutrosophic set. Nowadays, Neutrosophic set attains more attention of researchers due to its characteristic behavior to solve the indeterminate situations in the different fields of life. In 2006, Smarandache et al., [4] were the first ones who applied the concept of Neutrosophic sets on some algebraic structure and in their work, they introduced the Neutrosophic rings. Later, in 2011 Agboola et al., [5], discussed Neutrosophic rings-I. Neutrosophic groups and Neutrosophic sub-groups were introduced in 2012 by Agboola et al., [6]. Ali et al., [7-10] have used Neutrosophic set approach for different algebraic structures. In 2016, Khan et al.,[11] briefly discussed the characterization of Neutrosophic left almost semigroups.

The tremendous application of Neutrosophic sets is the main motivator for us to work in this field. Intuitively, Neutrosophic sets are gaining popularity among researchers. To investigate the application aspect of Neutrosophic sets, readers are directed to the most recent research work of Abdel Basset et al., [15-19], as well as [12-14].

LA-rings is generalized form of the commutative rings. LA-rings is non-commutative and non-associative algebraic structure. In recent times, a lot of research work has been done by different researches on this area of study. No doubt, LA-rings has a remarkable contribution in the development of non-associative theory in the current decade. Shah, Rehman, Asima and many other researchers have done noteworthy work in this ring structure. And they have published articles. The readers are referred to study [20-29] for comprehensive study of LA-rings.

We used Neutrosophic set approach to give the notion of Neutrosophic LA-rings. This may be a useful contribution to the non-associative field of mathematics. It may provide a new direction for future researchers to extend the non-associative area of mathematics. We discussed characteristic properties of substructures of Neutrosophic LA-rings. We gave the concept of different type of Neutrosophic ideals. We defined Neutrosophic prime ideals, Neutrosophic quasi ideals and Neutrosophic bi-ideals and established some results. One of the main results is: If $\boldsymbol{e}$ the left identity in Neutrosophic LA-ring $\boldsymbol{N(LR)}$, then $\boldsymbol{N(LR)}$ is fully Neutrosophic prime iff set ideal$(\boldsymbol{N(LR)})$ becomes totally ordered under inclusion and every ideal becomes idempotent. In last section, we discussed the characterizations of neutrosophic LA-ring by exploring the Neutrosophic $\boldsymbol{M}$-system, Neutrosophic $\boldsymbol{P}$-system and Neutrosophic $\boldsymbol{I}$-system. It is shown that: If $\boldsymbol{e}$ the left identity in $\boldsymbol{N(LR)}$, then a neutrosophic left ideal $\boldsymbol{N(LI)}$ is neutrosophic quasi-prime iff $\boldsymbol{N(LR)}\setminus \boldsymbol{N(LI)}$ is neutrosophic $\boldsymbol{M}$-system. Also, a relation is developed between neutrosophic $\boldsymbol{M}$-system and neutrosophic $\boldsymbol{P}$-system i.e., In a neutrosophic LA-ring $\boldsymbol{N(LR)}$, every neutrosophic $\boldsymbol{M}$-system is a neutrosophic $\boldsymbol{P}$-system.

2. Neutrosophic LA-rings

As preliminary, we recall the following definitions from refrences [3], [24] and [25].

Definition 2.1. [24] Let R be a set with at least two elements and two binary operations `+' and `·' defined on R. Suppose $(R,+)$ is an LA-group and $(R,.)$ is an LA-semigroup satisfying both left and right distributive laws: $a(b+c)=ab+ac$ and $(a+b)c=ac+bc$ for all $a,b,c\in R$. Then $(R,+,.)$ is called an LA-ring.

Definition 2.2. [24] Let $(R,+,.)$ be an LA-ring. If S is a non-empty subset of R and S is itself an LA-ring under the binary operation induced by R, then S is called an LA-subring of R.

Definition 2.3. [25] If A is an LA-subring of an LA-ring $(R,+,.)$, then A is called a left ideal if $RA\subseteq A$. Right ideal and two sided ideal are defined in the usual manner.

Definition 2.4. [25] A nonempty subset S of an LA-ring R is called an M-system if for $a,b\in S$, there exists r in R such that $a(rb)\in S$.

Definition 2.5. [25] A nonempty subset Q of an LA-ring R with left identity e is called P-system if for all $a\in Q$, there exists $r\in R$ such that $a(ra)\in Q$.

Definition 2.6. [25] A nonempty subset S of an LA-ring R with left identity e is called an I-system if for all $a,b\in S$, $(\langle a\rangle\cap\langle b\rangle)\cap S\neq\varphi$.

Definition 2.7. [3] A Neurosophics set is define as $A=\{\langle x,T(x),I(x),F(x)\rangle : x\in X\}$, where X is a universe of discoveries and A is characterized by a truth-membership function $T: X\to \left]0^-,1^+\right[$, an indeterminacy-membership function $I: X\to \left]0^-,1^+\right[$ and a falsity-membership function $F: X\to \left]0^-,1^+\right[$ and $0\leq T(x)+I(x)+F(x)\leq 3$.

We initaiate our work with the following definition.

Definition 2.8. If R is a LA-ring, I is a neutrosophic element with the property $I^2 = I$. Then a non-empty set $\langle R \cup I \rangle = \{r + sI : r, s \in R\}$ under the "$\boxplus$" and "$\boxdot$" is a Neutrosophic LA- ring if:

i) $(\langle R \cup I \rangle, \boxplus)$ is Left Almost group

ii) $(\langle R \cup I \rangle, \boxdot)$ is Left Almost semigroup

iii) $\boxdot$ is distributive over $\boxplus$ from both sides

Throughout this paper we denote Neutrosophic Left Almost ring by $N(LR)$.

Example 2.9. Following are the Cayley tables (1 and 2) for an LA-ring $R = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under the binary operations '$+$' and '.'

Cayley Table 1

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	2	0	3	1	6	4	7	5
2	1	3	0	2	5	7	4	6
3	3	2	1	0	7	6	5	4
4	4	5	6	7	0	1	2	3
5	6	4	7	5	2	0	3	1
6	5	7	4	6	1	3	0	2
7	7	6	5	4	3	2	1	0

Cayley Table 2

·	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	4	4	0	0	4	4	0
2	0	4	4	0	0	4	4	0
3	0	0	0	0	0	0	0	0
4	0	3	3	0	0	3	3	0
5	0	7	7	0	0	7	7	0
6	0	7	7	0	0	7	7	0
7	0	3	3	0	0	3	3	0

Then $N(LR) = \langle R \cup I \rangle = \{0, 1, 2, 3, 4, 5, 6, 7, 0I, 1I, 2I, 3I, 4I, 5I, 6I, 7I\}$ becomes neutrosophic LA-ring under "$\boxplus$" and "$\boxdot$" as defined in Cayley tables (3 and 4):

Cayley Table 3

⊞	0	1	2	3	4	5	6	7	0I	1I	2I	3I	4I	5I	6I	7I
0	0	1	2	3	4	5	6	7	0I	1I	2I	3I	4I	5I	6I	7I
1	2	0	3	1	6	4	7	5	2I	0I	3I	1I	6I	4I	7I	5I
2	1	3	0	2	5	7	4	6	1I	3I	0I	2I	5I	7I	4I	6I
3	3	2	1	0	7	6	5	4	3I	2I	1I	0I	7I	6I	5I	4I
4	4	5	6	7	0	1	2	3	4I	5I	6I	7I	0I	1I	2I	3I
5	6	4	7	5	2	0	3	1	6I	4I	7I	5I	2I	0I	3I	1I
6	5	7	4	6	1	3	0	2	5I	7I	4I	6I	1I	3I	0I	2I
7	7	6	5	4	3	2	1	0	7I	6I	5I	4I	3I	2I	1I	0I
0I	0I	1I	2I	3I	4I	5I	6I	7I	0I	1I	2I	3I	4I	5I	6I	7I
1I	2I	0I	3I	1I	6I	4I	7I	5I	2I	0I	3I	1I	6I	4I	7I	5I
2I	1I	3I	0I	2I	5I	7I	4I	6I	1I	3I	0I	2I	5I	7I	4I	6I
3I	3I	2I	1I	0I	7I	6I	5I	4I	3I	2I	1I	0I	7I	6I	5I	4I
4I	4I	5I	6I	7I	0I	1I	2I	3I	4I	5I	6I	7I	0I	1I	2I	3I
5I	6I	4I	7I	5I	2I	0I	3I	1I	6I	4I	7I	5I	2I	0I	3I	1I
6I	5I	7I	4I	6I	1I	3I	0I	2I	5I	7I	4I	6I	1I	3I	0I	2I
7I	7I	6I	5I	4I	3I	2I	1I	0I	7I	6I	5I	4I	3I	2I	1I	0I

.

Cayley Table 4

⊡	0	1	2	3	4	5	6	7	0I	1I	2I	3I	4I	5I	6I	7I
0	0	0	0	0	0	0	0	0	0I	0I	0I	0I	0I	0I	0I	0I
1	0	4	4	0	0	4	4	0	0I	4I	4I	0I	0I	4I	4I	0I
2	0	4	4	0	0	4	4	0	0I	4I	4I	0I	0I	4I	4I	0I
3	0	0	0	0	0	0	0	0	0I	0I	0I	0I	0I	0I	0I	0I
4	0	3	3	0	0	3	3	0	0I	3I	3I	0I	0I	3I	3I	0I
5	0	7	7	0	0	7	7	0	0I	7I	7I	0I	0I	7I	7I	0I
6	0	7	7	0	0	7	7	0	0I	7I	7I	0I	0I	7I	7I	0I
7	0	3	3	0	0	3	3	0	0I	3I	3I	0I	0I	3I	3I	0I
0I	0I	0I	0I	0I	0I	0I	0I	0I	0I	0I	0I	0I	0I	0I	0I	0I
1I	0I	4I	4I	0I	0I	4I	4I	0I	0I	4I	4I	0I	0I	4I	4I	0I
2I	0I	4I	4I	0I	0I	4I	4I	0I	0I	4I	4I	0I	0I	4I	4I	0I

$3I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$	$0I$
$4I$	$0I$	$3I$	$3I$	$0I$	$0I$	$3I$	$3I$	$0I$	$0I$	$3I$	$3I$	$0I$	$0I$	$3I$	$3I$	$0I$
$5I$	$0I$	$7I$	$7I$	$0I$	$0I$	$7I$	$7I$	$0I$	$0I$	$7I$	$7I$	$0I$	$0I$	$7I$	$7I$	$0I$
$6I$	$0I$	$7I$	$7I$	$0I$	$0I$	$7I$	$7I$	$0I$	$0I$	$7I$	$7I$	$0I$	$0I$	$7I$	$7I$	$0I$
$7I$	$0I$	$3I$	$3I$	$0I$	$0I$	$3I$	$3I$	$0I$	$0I$	$3I$	$3I$	$0I$	$0I$	$3I$	$3I$	$0I$

Definition 2.10. Let $N(LR)$ be a neutrosophic LA-ring under the binary operations $\boxplus$ and $\boxdot$. A non-empty proper subset $N(SLR)$ of $N(LR)$ is said to be a neutrosophic subLA-ring if $N(SLR)$ is itself a neutrosophic LA-ring under " $\boxplus$ " and " $\boxdot$ " defined in $N(LR)$.

Lemma 2.11. Let $N(LR)$ be a Neutrosophic LA-ring. Then the proper subset $N(SLR)$ of $N(LR)$ is a Neutrosophic subLA-ring iff, every $(r' + s'I)$, $(p' + q'I) \in N(SLR)$ satisfies the following conditions:

(i) $(r' + s'I) \boxplus (p' + q'I)$ belongs $N(SLR)$
(ii) $(r' + s'I) \boxdot (p' + q'I)$ belongs $N(SLR)$

Proof. If $N(SLR)$ is neutrosophic subLA-ring, then it is clear from definition that $(N(SLR), \boxplus))$ becomes LA-group as well as $(N(SLR), \boxdot)$ becomes LA-semigroup. Consequently, the closure property holds for $N(SLR)$. Hence (i) and (ii) hold.

Conversely, suppose that (i) and (ii) is true for all $(r' + s'I)$, $(p' + q'I) \in N(SLR)$. Since the binary operations " $\boxplus$ " and " $\boxdot$ " are closed, so $(N(SLR), \boxplus))$ being the subset of $N(LR)$ will be LA-group, likewise $(N(SLR), \boxdot))$ will be LA-semigroup. Moreover, inheritably $\boxdot$ is distributive over $\boxplus$ from both sides. Hence, $N(SLR)$ is a neutrosophic subLA-ring.

Lemma 2.12. If $\{(N(SLR))_i, i \in J\}$ is the collection of neutrosophic subLA-rings of $N(LR)$. Then the intersection of this collection is either empty or again a neutrosophic subLA-ring.

Proof. Let $\{(N(SLR))_i, i \in J\}$ be a collection of neutrosophic subLA-rings of $N(LR)$. Assume that $\cap (N(SLR))_i$ is not empty. Let $(r' + s'I)$, $(p' + q'I) \in \cap(N(SLR))_i$. This implies $(r' + s'I) \in (N(SLR))_i$ and $(p' + q'I) \in (N(SLR))_i$ where $i \in J$. Since $(N(SLR))_i$ is the collection of neutrosophic subLA-rings. Therefore, each $(N(SLR))_i, \boxplus)$ will be LA-group, $(N(SLR))_i, \boxdot)$ will be LA-semigroup. Also $\boxdot$ is distributive over $\boxplus$ from both sides. Consequently, $(r' + s'I) \boxplus (p' + q'I) \in (N(SLR))_i$ for all $i \in J$ and likewise $(r' + s'I) \boxdot (p' + q'I) \in (N(SLR))_i$ for all $i \in J$. Therefore, $(r' + s'I) \boxplus (p' + q'I)$ and $(r' + s'I) \boxdot (p' + q'I) \in \cap(N(SLR))_i$ for all $i \in J$.

Definition 2.13. If $e \in N(LR)$, then e is left identity if $e \boxdot N(LR) = N(LR)$.

Example 2.14. The following Cayley tables (5 and 6) form a neutrosophic LA-ring $N(LR) = \langle R \cup I \rangle = \{0,1,2,3,4,5,6,7,8,0I,1I,2I,3I,4I,5I,6I,7I,8I\}$ and it can be easily observed that the element $7 \in N(LR)$ is the left identity.

Cayley Table 5

$\boxplus$	0	1	2	3	4	5	6	7	8	$0I$	$1I$	$2I$	$3I$	$4I$	$5I$	$6I$	$7I$	$8I$
0	3	4	6	8	7	2	5	1	0	$3I$	$4I$	$6I$	$8I$	$7I$	$2I$	$5I$	$1I$	$0I$
1	2	3	7	6	8	4	1	0	5	$2I$	$3I$	$7I$	$6I$	$8I$	$4I$	$1I$	$0I$	$5I$
2	1	5	3	4	2	0	8	6	7	$1I$	$5I$	$3I$	$4I$	$2I$	$0I$	$8I$	$6I$	$7I$
3	0	1	2	3	4	5	6	7	8	$0I$	$1I$	$2I$	$3I$	$4I$	$5I$	$6I$	$7I$	$8I$
4	5	0	4	2	3	1	7	8	6	$5I$	$0I$	$4I$	$2I$	$3I$	$1I$	$7I$	$8I$	$6I$
5	4	2	8	7	6	3	0	5	1	$4I$	$2I$	$8I$	$7I$	$6I$	$3I$	$0I$	$5I$	$1I$
6	7	6	0	1	5	8	3	2	4	$7I$	$6I$	$0I$	$1I$	$5I$	$8I$	$3I$	$2I$	$4I$
7	6	8	1	5	0	7	4	3	2	$6I$	$8I$	$1I$	$5I$	$0I$	$7I$	$4I$	$3I$	$2I$
8	8	7	5	0	1	6	2	4	3	$8I$	$7I$	$5I$	$0I$	$1I$	$6I$	$2I$	$4I$	$3I$

0I	3I	4I	6I	8I	7I	2I	5I	1I	0I	3I	4I	6I	8I	7I	2I	5I	1I	0I
1I	2I	3I	7I	6I	8I	4I	1I	0I	5I	2I	3I	7I	6I	8I	4I	1I	0I	5I
2I	1I	5I	3I	4I	2I	0I	8I	6I	7I	1I	5I	3I	4I	2I	0I	8I	6I	7I
3I	0I	1I	2I	3I	4I	5I	6I	7I	8I	0I	1I	2I	3I	4I	5I	6I	7I	8I
4I	5I	0I	4I	2I	3I	1I	7I	8I	6I	5I	0I	4I	2I	3I	1I	7I	8I	6I
5I	4I	2I	8I	7I	6I	3I	0I	5I	1I	4I	2I	8I	7I	6I	3I	0I	5I	1I
6I	7I	6I	0I	1I	5I	8I	3I	2I	4I	7I	6I	0I	1I	5I	8I	3I	2I	4I
7I	6I	8I	1I	5I	0I	7I	4I	3I	2I	6I	8I	1I	5I	0I	7I	4I	3I	2I
8I	8I	7I	5I	0I	1I	6I	2I	4I	3I	8I	7I	5I	0I	1I	6I	2I	4I	3I

Cayley Table 6

$\boxdot$	0	1	2	3	4	5	6	7	8	0I	1I	2I	3I	4I	5I	6I	7I	8I
0	3	1	6	3	1	6	6	1	3	3I	1I	6I	3I	1I	6I	6I	1I	3I
1	0	3	0	3	8	8	3	0	8	0I	3I	0I	3I	8I	8I	3I	0I	8I
2	8	1	5	3	7	2	6	4	0	8I	1I	5I	3I	7I	2I	6I	4I	0I
3	3	3	3	3	3	3	3	3	3	3I	3I	3I	3I	3I	3I	3I	3I	3I
4	0	6	7	3	5	4	1	2	8	0I	6I	7I	3I	5I	4I	1I	2I	8I
5	8	6	4	3	2	7	1	5	0	8I	6I	4I	3I	2I	7I	1I	5I	0I
6	8	3	8	3	0	0	3	8	0	8I	3I	8I	3I	0I	0I	3I	8I	0I
7	0	1	2	3	4	5	6	7	8	0I	1I	2I	3I	4I	5I	6I	7I	8I
8	3	6	1	3	6	1	1	6	3	3I	6I	1I	3I	6I	1I	1I	6I	3I
0I	3I	1I	6I	3I	1I	6I	6I	1I	3I	3I	1I	6I	3I	1I	6I	6I	1I	3I
1I	0I	3I	0I	3I	8I	8I	3I	0I	8I	0I	3I	0I	3I	8I	8I	3I	0I	8I
2I	8I	1I	5I	3I	7I	2I	6I	4I	0I	8I	1I	5I	3I	7I	2I	6I	4I	0I
3I	3I	3I	3I	3I	3I	3I	3I	3I	3I	3I	3I	3I	3I	3I	3I	3I	3I	3I
4I	0I	6I	7I	3I	5I	4I	1I	2I	8I	0I	6I	7I	3I	5I	4I	1I	2I	8I
5I	8I	6I	4I	3I	2I	7I	1I	5I	0I	8I	6I	4I	3I	2I	7I	1I	5I	0I
6I	8I	3I	8I	3I	0I	0I	3I	8I	0I	8I	3I	8I	3I	0I	0I	3I	8I	0I
7I	0I	1I	2I	3I	4I	5I	6I	7I	8I	0I	1I	2I	3I	4I	5I	6I	7I	8I
8I	3I	6I	1I	3I	6I	1I	1I	6I	3I	3I	6I	1I	3I	6I	1I	1I	6I	3I

3. Neutrosophic Ideals

Definition 3.1. A neutrosophic subLA-ring $N(SLR)$ of $N(LR)$ is known as a neutrosophic left ideal if $N(LR) \boxdot N(SLR) \subseteq N(SLR)$. Likewise, the right ideal and two sided ideal of $N(LR)$ can be easily defined.

We denote neutrosophic left ideal by $N(LI)$, neutrosophic right ideal by $N(RI)$ and two sided neutrosophic ideal will be denoted by $N(I)$.

Lemma 3.2. If e is the left identity in neutrosophic LA-ring $N(LR)$, then $N(RI)$ will be neutrosophic left ideal.

Proof. Suppose $r' + s'I \in N(LR)$, $m' + n'I \in N(RI)$. Then $r' + s'I \boxdot (m' + n'I) = (e \boxdot (r' + s'I)) \boxdot (m' + n'I)$=$((m' + n'I) \boxdot (r' + s'I)) \boxdot e \in N(RI)$. Therefore, $N(RI)$ becomes a neutrosophic left ideal also.

Remark 3.3. From **Lemma 3.2**, it is concluded a neutrosophic LA-rings having e the left identity, the neutrosophic ideal means the neutrosophic right ideal.

Proposition 3.4. Let $N(LR)$ a neutrosophic LA-ring having left identity. Then:

(i) $N(LR) \boxdot N(LI) = N(LI)$, $N(LI)$ neutrosophic left ideal of $N(LR)$.

(ii) $N(RI) \boxdot N(LR) = N(RI)$, $N(RI)$ neutrosophic right ideal of $N(LR)$.

Proof. (i) By definition, if $N(LI)$ is neutrosophic left ideal of $N(LR)$, then $N(LR) \boxdot N(LI) \subseteq N(LI)$. Let $p' + q'I \in N(LI)$. Then $p' + q'I = e \boxdot (p' + q'I) \in N(LR) \boxdot N(LI)$. Consequently, $N(LI) \subseteq N(LR) \boxdot N(LI)$ and hence $N(LR) \boxdot N(LI) = N(LI)$.

(ii) By definition, if $N(RI)$ is neutrosophic right ideal of $N(LR)$, then $N(RI) \boxdot N(LR) \subseteq N(RI)$. Let $m' + n'I \in N(RI)$. Then

$$\begin{aligned} m' + n'I &= e \boxdot (m' + n'I) \\ &= (e \boxdot e) \boxdot (m' + n'I) \\ &= ((m' + n'I) \boxdot e) \boxdot e \\ &\in (N(RI) \boxdot N(LR)) \boxdot N(LR) \\ &\subseteq N(RI) \boxdot N(LR). \end{aligned}$$

This implies $N(RI) \subseteq N(RI) \boxdot N(LR)$. Thus, $N(RI) \boxdot N(LR) = N(RI)$.

Lemma 3.5. If e is a left identity and $N(RI)$ is the neutrosophic right ideal of $N(LR)$, then $(N(RI))^2$ is a neutrosophic ideal of $N(LR)$.

Proof. If an element $l' + k'I \in \left(N(RI)\right)^2$, then $l' + k'I = (m' + n'I) \boxdot (p' + q'I)$, where $(m' + n'I)$, $(p' + q'I) \in N(RI)$. Let $(r' + s'I)$ be any element of $N(LR)$.

Now consider

$$\begin{aligned} (l' + k'I) \boxdot (r' + s'I) &= \left((m' + n'I) \boxdot (p' + q'I)\right) \boxdot (r' + s'I) \\ &= \left((r' + s'I) \boxdot (p' + q'I)\right) \boxdot (m' + n'I) \in N(RI) \boxdot N(RI) \\ &= \left(N(RI)\right)^2. \end{aligned}$$

This means $\left(N(RI)\right)^2$ is neutrosophic right ideal. Therefore, from **Lemma 3.2**, $\left(N(RI)\right)^2$becomes neutrosophic left ideal. Thus $\left(N(RI)\right)^2$ is neutrosophic ideal.

Remark 3.6. It is interesting to note that in a neutrosophic LA-ring $N(LR)$ having left identity, $\left(N(LI)\right)^2$ becomes neutrosophic ideal, where $N(LI)$ is neutrosophic ideal.

Lemma 3.7. If $N(LR)$ is a neutrosophic LA-rings having left identity. Let $N(I')$ is proper neutrosophic ideal of $N(LR)$. Then the left identity e does not belong to $N(I')$.

Proof. Contrarily, let $e \in N(I')$ and $r' + s'I \in N(LR)$. Now consider

$$\begin{aligned} r' + s'I &= e \boxdot (r' + s'I) \\ &\in N(I') \boxdot N(LR) \\ &\subseteq N(I'). \end{aligned}$$

This implies $N(LR) \subseteq N(I')$. But $N(I') \subseteq N(LR)$. This means $N(I') = N(LR)$. Hence a contradiction. Thus $e \notin N(I')$.

Definition 3.8. $N(PI)$ is neutrosophic ideal of $N(LR)$. $N(PI)$ is called neutrosophic prime ideal iff for any neutrosophic ideals $N(AI)$ and $N(BI)$, $N(AI) \boxdot N(BI) \subseteq N(PI)$ then either $N(AI) \subseteq N(PI)$ or $N(BI) \subseteq N(PI)$. $N(P)$ is called neutrosophic semi-prime if $N(I')^2 \subseteq N(PI)$ implies that $N(I') \subseteq N(PI)$, where $N(I')$ is any neutrosophic ideal of $N(LR)$.

If each neutrosophic ideal of $N(LR)$ is neutrosophic prime ideal, then $N(LR)$ is called fully neutrosophic prime and if all the neutrosophic ideals are neutrosophic semi-prime ideals than $N(LR)$ is called fully neutrosophic semi-prime.

Definition 3.9. If for all neutrosophic ideals $N(AI)$, $N(BI)$, either $N(AI) \subseteq N(BI)$ or $N(BI) \subseteq N(AI)$, then $N(LR)$ is called totally ordered under inclusion. It is symbolized by a set ideal$(N(LR))$.

Theorem 3.10. If e left identity in neutrosophic LA-rings $N(LR)$, then $N(LR)$ is fully neutrosophic prime iff set ideal$(N(LR))$ becomes totally ordered under inclusion and every ideal becomes idempotent.

Proof. Suppose $N(LR)$ is fully neutrosophic prime and $N(AI)$, $N(BI)$ be any neutrosophic ideals in $N(LR)$. Since $N(AI) \boxdot N(BI) \subseteq N(AI)$ and $N(AI) \boxdot N(BI) \subseteq N(BI)$, therefore $N(AI) \boxdot N(BI) \subseteq$

$N(AI) \cap N(BI)$. Since the intersection of neutrosophic prime ideals is prime. This implies that $N(AI) \cap N(BI)$ is prime and hence by definition, $N(AI) \subseteq N(AI) \cap N(BI)$ or $N(BI) \subseteq N(AI) \cap N(BI)$. This further implies that either $N(AI) \subseteq N(BI)$ or $N(BI) \subseteq N(AI)$. Thus set ideal$(N(LR))$ is totally ordered under the inclusion. Assume $N(I^{'})$ a neutrosophic ideal of $N(LR)$, where $N(LR)$ is fully neutrosophic prime. Then from **Lemma 3.5**, it is proved that $(N(I))^2$ is neutrosophic ideal in $N(LR)$, therefore $(N(I'))^2 \subseteq N(I')$. Also, $N(I') \subseteq (N(I'))^2$. Consequently, $(N(I'))^2 = N(I')$ this implies $N(I')$ is idempotent. Conversely, assume that set ideal$(N(LR))$ is totally ordered under the inclusion and each ideal becomes idempotent. Consider $N(UI)$, $N(VI)$ and $N(WI)$ be neutrosophic ideals in $N(LR)$. Let $N(UI) \boxdot N(VI) \subseteq N(WI)$ where $N(UI) \subseteq N(VI)$. As $N(UI)$ is an idempotent neutrosophic ideal in $N(LR)$, so $N(UI) = (N(UI))^2 = N(UI) \boxdot N(UI) \subseteq N(UI) \boxdot N(VI) \subseteq N(WI)$. Hence $N(VI) \subseteq N(WI)$. This $N(WI)$ is neutrosophic prime ideal. Similarly, on the same lines it can be proved that $N(UI)$ and $N(VI)$ are prime ideals in $N(LR)$. Hence $N(LR)$ is fully neutrosophic prime.

Definition 3.11. Let $N(LR)$ be a neutrosophic LA-ring. $N(QI)$ a non-empty subset is called neutrosophic quasi ideal if $N(QI) \boxdot N(LR) \cap N(LR) \boxdot N(QI) \subseteq N(QI)$.

Lemma 3.12. If $N(LR)$ is neutrosophic LA-ring. Let $N(RI)$, $N(LI)$ be the neutrosophic right and left ideal respectively. Then the intersection of $N(RI)$ and $N(LI)$ is a neutrosophic quasi ideal in $N(LR)$.

Proof. From the properties of neutrosophic right and left ideals it can be written that $N(LI) \cap N(RI) \subseteq N(RI)$ and $N(LI) \cap N(RI) \subseteq N(LI)$. Also $N(LR) \boxdot N(LI) \subseteq N(LI)$ and $N(RI) \boxdot N(LR) \subseteq N(RI)$. Now consider,

$$\begin{aligned}(N(LI) \cap N(RI)) \boxdot N(LR) \cap N(LR) \boxdot (N(LI) \cap N(RI)) &\subseteq N(RI) \boxdot N(LR) \cap N(LR) \boxdot N(LI)\\ &\subseteq N(RI) \cap N(LI)\\ &= N(LI) \cap N(RI).\end{aligned}$$

Result proved.

Definition 3.13. Let $N(LR)$ be neutrosophic LA-rings. $N(BI)$ is neutrosophic generalized bi-ideal, if $(N(BI) \boxdot N(LR)) \boxdot N(BI) \subseteq N(BI)$. It is important to note that if the non-empty subset of $N(LR)$ is a neutrosophic subLA-ring then $N(B)$ is called neutrosophic bi-ideal of $N(LR)$.

Proposition 3.14. Let e be left identity in $N(LR)$. Then each idempotent neutrosophic quasi ideal $N(QI)$ becomes a neutrosophic bi-ideal in $N(LR)$.

Proof. $N(QI)$ being a neutrosophic quasi ideal is a neutrosophic subLA-ring. Consider,

$$\begin{aligned}(N(QI) \boxdot N(LR)) \boxdot N(QI) &\subseteq (N(QI) \boxdot N(LR)) \boxdot N(LR)\\ &= \big(N(LR) \boxdot N(LR)\big) \boxdot N(QI)\\ &= N(LR) \boxdot N(QI)\end{aligned}$$

Again consider,

$$\begin{aligned}\big(N(QI) \boxdot N(LR)\big) \boxdot N(QI) &\subseteq \big(N(LR) \boxdot N(LR)\big) \boxdot N(QI)\\ &= \big(N(LR) \boxdot N(LR)\big) \boxdot \big(N(QI) \boxdot N(QI)\big)\\ &= \big(N(QI) \boxdot N(QI)\big) \boxdot \big(N(LR) \boxdot N(QLR)\big)\\ &= N(QI) \boxdot N(LR).\end{aligned}$$

Therefore, $\big(N(QI) \boxdot N(LR)\big) \boxdot N(QI) \subseteq (N(QI) \boxdot N(LR)) \cap (N(LR) \boxdot N(QI)) \subseteq N(QI)$. Hence proved.

Theorem 3.15. $N(QI^{'})$, $N(QI^{''})$ be neutrosophic quasi ideals in neutrosophic LA-rings $N(LR)$. Then the intersection of $N(Q^{'}) \cap N(Q^{''})$ is empty or neutrosophic quasi ideal of $N(LR)$.

Proof. Consider $N(LR) \boxdot [N(QI^{'}) \cap N(QI^{''})] \cap [N(QI^{'}) \cap N(QI^{''})] \boxdot N(LR)$

$$= [N(LR) \boxdot N(QI^{'}) \cap N(LR) \boxdot N(QI^{''})] \cap [N(QI^{'}) \boxdot N(LR) \cap N(QI^{''}) \boxdot N(LR)]$$
$$= [N(LR) \boxdot N(QI^{'}) \cap N(QI^{'}) \boxdot N(LR)] \cap [N(LR) \boxdot N(QI^{''}) \cap N(QI^{''}) \boxdot N(LR)]$$
$$\subseteq N(QI^{'}) \cap N(QI^{''}).$$

This complete the result.

Remark 3.16. It can be concluded from **Theorem 3.15**, that the intersection of neutrosophic quasi ideals in $N(LR)$ is empty or neutrosophic quasi ideal.

4. Neutrosophic Systems in Neutrosophic LA-ring

In this section of paper, we explore Neutrosophic M-system, Neutrosophic P-system and Neutrosophic I-system in neutrosophic LA-rings.

Definition 4.1. For $m' + n'I, p' + q'I \in M'$, an element $r' + s'I \in N(LR)$ and if $m' + n'I \boxdot (r' + s'I \boxdot p' + q'I) \subseteq M'$, then M' is called a Neutrosophic M-system, where M' is non-empty subset of $N(LR)$.

Example 4.2. One can easily check that in a neutrosophic LA-rings having left identity, $(N(LR),\boxdot)$ being a neutrosophic LA-semigroup becomes a neutrosophic M-system.

Definition 4.3. A neutrosophic left ideal $N(LI)$ of neutrosophic LA-rings is neutrosophic quasi-prime if for any neutrosophic left ideals $S(LI)$ and $T(LI)$, $S(LI) \boxdot T(LI) \subseteq N(LI)$ gives either $S(LI) \subseteq N(LI)$ or $T(LI) \subseteq N(LI)$. While $N(LI)$ is said to be a neutrosophic quasi-semiprime if for any neutrosophic left ideal $S(LI)$, $(S(LI))^2 \subseteq N(LI) \Rightarrow S(LI) \subseteq N(LI)$.

Proposition 4.4. In a neutrosophic LA-ring $N(LR)$ having left identity, following claims are equivalent:

(i) $N(LI)$ is a neutrosophic quasi-prime.

(ii) $S(LI) \boxdot T(LI) = \langle S(LI) \boxdot T(LI)\rangle \subseteq N(LI)$ means either $S(LI) \subseteq N(LI)$ or $T(LI) \subseteq N(LI)$.

(iii) If $S(LI) \nsubseteq N(LI)$ and $T(LI) \nsubseteq N(LI)$, then $S(LI) \boxdot T(LI) \nsubseteq N(LI)$.

(iv) If $r' + s'I$, $l' + m'I \in N(LR)$ but $r' + s'I$, $l' + m'I \notin N(LI)$ then $\langle r' + s'I\rangle \boxdot \langle l' + m'I\rangle \nsubseteq N(LI)$, then either $r' + s'I \in N(LI)$ or $l' + m'I \in N(LI)$.

(v) If $r' + s'I$, $l' + m'I \in N(LR)$ such that $r' + s'I \boxdot (N(LR) \boxdot l' + m'I) \subseteq N(LI)$, then either $r' + s'I \in N(LI)$ or $l' + m'I \in N(LI)$.

Proof. (i)⟺(ii) Suppose $N(LI)$ is a neutrosophic quasi-prime. Then it is quite clear from definition that if $S(LI) \boxdot T(LI) = \langle S(LI) \boxdot T(LI)\rangle \subseteq N(LI)$, then either $S(LI) \subseteq N(LI)$or $T(LI) \subseteq N(LI)$. Converse can be proved directly.

(ii)⟺(iii) obvious from given information.

(i) ⟹ (iv) Assume $\langle r' + s'I\rangle \boxdot \langle l' + m'I\rangle \subseteq N(LI)$, this means $\langle r' + s'I\rangle \subseteq N(LI)$ or $\langle l' + m'I\rangle \subseteq N(LI)$, which further means $r' + s'I \in N(LI)$ or $l' + m'I \in N(LI)$.

(iv)⟹(ii) Assume $S(LI) \boxdot T(LI) \subseteq N(LI)$. If $r' + s'I \in S(LI)$ and $l' + m'I \in T(LI)$, then $\langle r' + s'I\rangle \boxdot \langle l' + m'I\rangle \subseteq N(LI)$. Thus from hypothesis $r' + s'I \in N(LI)$ or $l' + m'I \in N(LI)$. Hence either

$S(LI) \subseteq N(LI)$ or $T(LI) \subseteq N(LI)$.

(i)⟺(iv) Assume $r' + s'I \boxdot (N(LR) \boxdot l' + m''I) \subseteq N(LI)$, then $N(LR) \boxdot [r' + s'I \boxdot (N(LR) \boxdot l' + m'I)] \subseteq N(LR) \boxdot N(LI) \subseteq N(LI)$. Now applying medial law and paramedical law, we conclude that $N(LR) \boxdot [r' + s'I \boxdot (N(LR) \boxdot l' + m'I)] = (N(LR) \boxdot r' + s'I) \boxdot (N(LR) \boxdot l' + m'I) \subseteq N(LI)$. As $N(LR) \boxdot r' + s'I$ and $N(LR) \boxdot l' + m'I$ are neutrosophic left ideals, this means $r' + s'I \in N(LI)$ or $l' + m'I \in N(LI)$. Converse is trivial.

Theorem 4.5. If e is a left identity in $N(LR)$, then a neutrosophic left ideal $N(LI)$ is neutrosophic quasi-prime iff $N(LR)\backslash N(LI)$ is neutrosophic M-system.

Proof. Suppose $N(LI)$ is neutrosophic quasi-prime. Assume $r' + s'I,\ l' + m'I \in N(LR)\backslash N(LI)$. Which means $r' + s'I \notin N(LI)$ and $l' + m'I \notin N(LI)$. Therefore by **Proposition 4.4,** $r' + s'I \boxdot (N(LR) \boxdot l' + m'I) \nsubseteq N(LI)$. It means there is some element $(p' + q'I) \in N(LR)$ such that $r' + s'I \boxdot (p' + q'I \boxdot l' + m'I) \nsubseteq N(LI)$ which further implies $r' + s'I \boxdot (p' + q'I \boxdot l' + m'I) \subseteq N(LR)\backslash N(LI)$. Therefore $N(LR)\backslash N(LI)$ is a neutrosophic M-system. Conversely assume that $N(LR)\backslash N(LI)$ is a neutrosophic M-system. Let $r' + s'I \boxdot (N(LR) \boxdot l' + m'I) \subseteq N(LI)$ and $r' + s'I \notin N(LI)$ and $l' + m'I \notin N(LI)$. This means $r' + s'I,\ l' + m'I \in N(LR)\backslash N(LI)$. As by hypothesis $N(LR)\backslash N(LI)$ is a neutrosophic M-system, so $(p' + q'I) \in N(LR)$ and $r' + s'I \boxdot (p' + q'I \boxdot l' + m'I) \subseteq N(LR)\backslash N(LI)$. This implies $r' + s'I \boxdot (p' + q'I \boxdot l' + m'I) \nsubseteq N(LI)$, which is a contradiction. Thus $r' + s'I \in N(LI)$ or $l' + m'I \in N(LI)$. Hence $N(LI)$ is neutrosophic quasi-prime.

Definition 4.6. A subset $P'$of $N(LR)$ is a neutrosophic P-system, if for any $p' + q'I \in P'$, there is $r' + s'I \in N(LR)$ such that $p' + q'I \boxdot (r' + s'I \boxdot p' + q'I) \subseteq P'$.

Proposition 4.7. In a neutrosophic LA-ring $N(LR)$ having left identity, given claims are equivalent:

(i) $N(LI)$ is neutrosophic quasi-semiprime.

(ii) $(S(LI))^2 = \langle (S(LI))^2 \rangle \subseteq N(LI)$ implies that $S(LI) \subseteq N(LI)$, where $S(LI)$ is a neutrosophic left ideal.

(iii) For any neutrosophic left ideal $S(LI)$, if $S(LI) \nsubseteq N(LI)$, then $(S(LI))^2 \nsubseteq N(LI)$.

(iv) For any element $r' + s'I \in N(LR)$, if $\langle r' + s'I \rangle^2 \subseteq N(LI)$, then $r' + s'I \in N(LI)$.

(v) For any element $r' + s'I \in N(LR)$, $p' + q'I \boxdot (r' + s'I \boxdot p' + q'I) \subseteq N(LI) \Rightarrow r' + s'I \in N(LI)$.

Proof. (i)⟺(ii) ⟺(iii) obviously by definition true.

(i)⟹(iv). Let $\langle r' + s'I \rangle^2 \subseteq N(LI)$. From hypothesis as $N(LI)$ is quasi-semiprime, so $r' + s'I \subseteq N(LI)$ and it means $r' + s'I \in N(LI)$.

(i)⟹(iv). Assume $(S(LI))^2 = \langle (S(LI))^2 \rangle \subseteq N(LI)$. Let $r' + s'I \in N(LI)$. Then by given condition $\langle r' + s'I \rangle^2 \subseteq N(LI)$ and it means $r' + s'I \in N(LI)$. Hence $S(LI) \subseteq N(LI)$.

(i)⟺(v) Obvious.

Theorem 4.8. If e is a left identity in $N(LR)$, then a neutrosophic left ideal $N(LI)$ is neutrosophic quasi-semiprime iff $N(LR)\backslash N(LI)$ is neutrosophic P-system.

Proof. Suppose $N(LI)$ is neutrosophic quasi-semiprime. Let $p' + q'I \in N(LR)\backslash N(LI)$. On contrary, assume that $r' + s'I \in N(LR)$ and $p' + q'I \boxdot (r' + s'I \boxdot p' + q'I) \subseteq N(LR)\backslash N(LI)$. This means $p' + q'I \boxdot (r' + s'I \boxdot p' + q'I) \subseteq N(LI)$. But as $N(LI)$ is neutrosophic quasi-semiprime, so from **Proposition 4.4,** $p' + q'I \in N(LI)$ a contradiction arise. Therefore, $r' + s'I \in N(LR)$ and $p' + q'I \boxdot (r' + s'I \boxdot p' + q'I) \subseteq N(LR)\backslash N(LI)$. Thus $N(LR)\backslash N(LI)$ is a neutrosophic P-system. Now let for any element $p' + q'I \in N(LR)\backslash N(LI)$, there is $r' + s'I \in N(LR)$ and $p' + q'I \boxdot (r' + s'I \boxdot p' + q'I) \subseteq N(LR)\backslash N(LI)$. Suppose $p' + q'I \boxdot (N(LR) \boxdot p' + q'I) \subseteq N(LI)$. This means $r' + s'I \in N(LR)$ and $p' + q'I \boxdot (r' + s'I \boxdot p' + q'I) \subseteq N(LR)\backslash N(LI)$. This further means that $p' + q'I \in N(LI)$. Thus by **Proposition 4.7** $N(LI)$ is a neutrosophic quasi-semiprime.

Lemma 4.9. In a neutrosophic LA-ring $N(LR)$, every neutrosophic M-system is a neutrosophic P-system.

Proof. Obvious from definition.

Definition 4.10. In a neutrosophic LA-ring $N(LR)$, a neutrosophic ideal $N(I')$ is called strongly irreducible iff for any neutrosophic ideals $S(I')$ and $T(I')$, if $S(I') \cap T(I') \subseteq N(I')$ implies that either $S(I') \subseteq N(I')$ or $T(I') \subseteq N(I')$.

Definition 4.11. For $m' + n'I,\ p' + q'I \in I'$, if $(\langle m + nI\rangle \cap \langle p' + q'I\rangle) \cap I' \neq \emptyset$, then I' is called a neutrosophic I-system, where I' is a subset of $N(LR)$.

Proposition 4.12. For neutrosophic ideal $N(I')$ of neutrosophic LA-ring $N(LR)$, the below statements are equivalent:

(i) $N(I')$ is strongly irreducible.

(ii) For any elements $m' + n'I,\ p' + q'I \in N(LR)$ such that $\langle m' + n'I\rangle \cap \langle p' + q'I\rangle \subseteq N(I')$ implies that either $m' + n'I \in N(I')$ or $p' + q'I \in N(I')$.

(iii) $N(LR)\backslash N(I')$ is a neutrosophic I-system.

Proof. (i)⟹(ii). Clear from definition.

(ii)⟹(iii). Assume that $m' + n'I,\ p' + q'I \in N(LR)\backslash N(I')$ and let $(\langle m' + n'I\rangle \cap \langle p' + q'I\rangle) \cap N(LR)\backslash N(I') = \emptyset$. This means $\langle m' + n'I\rangle \cap \langle p' + q'I\rangle \subseteq N(I')$, hence either $m' + n'I \in N(I')$ or $p' + q'I \in N(I')$ which is a contradiction. Thus $(\langle m' + n'I\rangle \cap \langle p' + q'I\rangle) \cap N(LR)\backslash N(I') \neq \emptyset$.

(iii)⟹(i). Let $S(I')$ and $T(I')$ be neutrosophic ideals such that $S(I') \cap T(I') \subseteq N(I')$. Assume that Let $S(I')$ and $T(I')$ do not contained in $N(I')$. This means there will be elements $m' + n'I \in S(I')\backslash N(I')$ and $p' + q'I \in T(I')\backslash N(I')$. This further implies that $m' + n'I,\ p' + q'I \in N(LR)\backslash N(I')$. Hence by hypothesis $(\langle m' + n'I\rangle \cap \langle p' + q'I\rangle) \cap N(LR)\backslash N(I') \neq \emptyset$, which implies that there will be an element $r' + s'I \in \langle m' + n'I\rangle \cap \langle p' + q'I\rangle$ such that $r' + s'I \in N(LR)\backslash N(I')$. It means $r' + s'I \in \langle m' + n'I\rangle \cap \langle p' + q'I\rangle \subseteq S(I') \cap T(I') \subseteq N(I')$. Which further means $S(I') \cap T(I') \nsubseteq N(I')$. Here arise a contradiction, hence either $S(I') \subseteq N(I')$ or $T(I') \subseteq N(I')$. Thus $N(I')$ is strongly irreducible.

5. Conclusions

We initiated Neutrosophic LA-rings in this research work. Which will be a first attempt to enhance and develop non-associative area of mathematical sciences. It will open a new gateway for the upcoming researchers to extend this non-associative field of mathematics. In order to look at the algebraic characteristics of Neutrosophic LA-rings, we studied their ideals (Neutrosophic prime ideals, Neutrosophic semi-prime, Neutrosophic quasi ideals and Neutrosophic bi-ideals). We established number of results to study the characteristic properties of Neutrosophic LA-rings. We explored the characterizations of Neutrosophic LA-ring by the properties of Neutrosophic M-system, Neutrosophic P-system and Neutrosophic I-system and established number of results. Also, a relation is developed between neutrosophic M-system and neutrosophic P-system. In the light of our findings, we may conclude that our work is going to be a good and helpful contribution to the study of algebraic structures based on Neutrosophic sets. Further, we are planning to work out the structural study of Neutrosophic LA-rings by extending it to some theoretical applications in Neutrosophic fuzzy algebraic structures. Particularly, Neutrosophic soft LA-rings, Neutrosophic LA-semirings, Neutrosophic soft LA-semirings and related structures.

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References

1. Zadeh, L. Fuzzy sets. *Information and Control* **1965**, 338-353.
2. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **1986**, 20, 87-96.
3. Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. *American Reserch Press, Rehoboth, NM,* **1999.**
4. Kandasamy, W. B. V.; Smarandache, F. Neutrosophic rings. *Phoenix, Arizona **2006.***
5. Agboola, A. A. A.; Akinola, A. D.; Oyebola, O. Y. Neutrosophic rings I. *International Journal of Mathematical Combinatorics* **2011,** 4, 1-14.
6. Agboola, A. A. A.; Akinola, A. D.; Oyebola, O. Y. Neutrosophic Groups and Neutrosophic Subgroups. *International Journal of Mathematical Combinatorics* **2012,** 3, 1-9.
7. Ali, M.; Smarandache, F.; Shabir, M.; Vladareanu, L. Generalization of Neutrosophic rings and Neutrosophic fields. *Neutrosophic Sets and Systems* **2014**, 5, 9-14.
8. Ali, M.; Shabir, M.; Naz, M.; Smarandache, F. Neutrosophic left almost semigroup. , *Neutrosophic Sets and Systems* **2014**, 3, 18-28.
9. Ali, M.; Smarandache, F.; Shabir, M.; Naz, M.. Neutrosophic bi-LA-semi group and Neutosophic N-LA-semi group. *Neutrosophic Sets and Systems* **2014**, 4, 19-24.
10. Ali, M.; Shabir, M.; Smarandache, F.; Vladareanu, L. Neutrosophic LA-semigroup rings. *Neutrosophic Sets and Systems* **2015**, 7, 81-88.
11. Khan, M.; Smarandache, F.; Afzal, S. Neutrosophic set approach for characterizations of left almost semi groups. *Neutrosophic Sets and Systems* **2016**, 11, 79-94.
12. Hashim, R. M.; Gulistan, M.; Rehman, I.; Hassan, N.; Nasruddin, A. M. Neutrosophic Bipolar Fuzzy Set and its Application in Medicines Preparation. *Neutrosophic Sets and Systems* **2020**, DOI. 10.5281/zenodo.3639213.
13. Hashim, R. M.; Gulistan, M.; Smarandache, F. Applications of Neutrosophic Bipolar fuzzy Sets in HOPE Foundation for Planning to Build a children Hospital with Different Types of similarity measures. Symmetry **2018**, 10-31.
14. Abdel-Basset, M. A novel and powerful framework based on Neutrosophic sets to aid patients with cancer. *Future Generation Computer Systems* **2019**, 98, 144-153.
15. Abdel-Basset, M.; Manogaran, G.; Gamal, A.; Smarandache, F. A group decision making framework based on Neutrosophic TOPSIS approach for smart medical device selection. *Journal of Medical Systems* **2019**, 43, 1-13.
16. Abdel-Basset, M.; Atef, A.; & Smarandache, F. A hybrid Neutrosophic multiple criteria group decision making approach for project selection. *Cognitive Systems Research* **2019**, *57*, 216-227.
17. Abdel-Basset, M.; Manogaran, G.; Mohamed, M; & Chilamkurti, N. Three-way decisions based on neutrosophic sets and AHP-QFD framework for supplier selection problem. *Future Generation Computer Systems* **2018**, *89*, 19-30.
18. Abdel-Basset, M., Gunasekaran, M., Mohamed, M., & Chilamkurti, N. A framework for risk assessment, management and evaluation: Economic tool for quantifying risks in supply chain. *Future Generation Computer Systems* **2019**, *90*, 489-502.

19. Abdel-Basset, M., & Mohamed, R. A novel plithogenic TOPSIS-CRITIC model for sustainable supply chain risk management. *Journal of Cleaner Production* **2020**, *247*, 119586.
20. Rehman, I.; Shah, M.; Shah, T.; Asima, R. On existence of non-associative LA-rings*. Analele Stiintifice ale Universitatii ovidius Constanta* **2013**, 21, 223-228.
21. Asima, R.; Rehman, I.; Shum, K. P. On Soft LA-modules and Exact Sequences. Italian Journal of Pure and Applied Mathematics 2017, 38, 797-814.
22. Shah, T.; Kausar, N.; Rehman, I. Intuitionistics fuzzy normal subring over a non-associative ring. *Analele Stiintifice ale Universitatii ovidius Constanta* **2012,** 20, 369-386.
23. Shah, T.; Raees, M.; Ali, G. On LA-Modules. *Int. J. Contemp. Math. Sciences* **2011**, 6, 999-1006.
24. Shah, T.; Rehman, I. On LA-rings of finitely non-zero functions*. Int. J. Contemp. Math. Sciences* **2010**, 5, 209-222.
25. Shah, T.; Rehman, I. On characterizations of LA-rings through some properties of their ideals. *Southeast Asian Bull. Math.* **2012**, 36, 695-705.
26. Shah, M.; Shah, T. Some basic properties of LA-rings. *Int. Math. Forum* **2011**, 6, 2195-2199.
27. Shah, T.; Asima, R.; Rehman, I. Application of soft sets to non-associative rings*. Journal of Intelligent & Fuzzy Systems* **2016**, 30, 1537-1546.
28. Shah, T.; Asima, R. Soft M-systems in a class of soft Non-associative rings*. U.P.B. Sci. Bull., Series A* **2015**, 77, 131-142.
29. Shah, T.; Asima, R.; Rehman, I.; Gondal, M. A; Faraz, M. I.; Shum, K. P. Literature Survey on Non-associative Rings and Developments. *European Journal of Pure and Applied Mathematics* **2019,** *12, 370-408.*

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