Neutrosophic Sets and Systems, Vol. 47, 2021
University of New Mexico

Similarity Measure for m-Polar Interval Valued Neutrosophic Soft Set with Application for Medical Diagnoses

Rana Muhammad Zulqarnain[1*], Imran Siddique[2], Muhammad Asif[1], Shahzad Ahmad[1], Said Broumi[3], Sehrish Ayaz[2]

[1] Department of Mathematics, University of Management and Technology, Lahore, Sialkot Campus, Pakistan. E-mail: muhammad.asif@math.qau.edu.pk; shahzad.ahmed@skt.umt.edu.pk
[2] School of Science, Department of Mathematics, University of Management and Technology, Lahore, Pakistan. E-mail: imransiddique@umt.edu.pk;
[3] Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca. E-mail: broumisaid78@gmail.com.
[4] College of Urban and Environmental Sciences, Northwest University, Xian, China.
[1*] Correspondence: E-mail: ranazulqarnain7777@gmail.com

Abstract:

The similarity measure is used to tackle many issues that include indistinct as well as blurred information excluding is not in a position to deal with the general fuzziness along with obscurity of the problems that have various information. The main purpose of this research is to propose a multi-polar interval-valued neutrosophic soft set (mPIVNSS) with operations and basic properties. We also develop Hamming distance and Euclidean distance by using mPIVNSS and numerical examples and use the developed distances to introduce similarity measures. By using the developed similarity measures a decision-making approach is presented for mPIVNSS. Finally, we used the developed decision-making approach for medical diagnosis.

Keywords: Multipolar interval-valued neutrosophic set; multipolar interval-valued neutrosophic soft set; similarity measures.

1. Introduction

Uncertainty plays a dynamic role in many areas of life (such as modeling, medicine, engineering, etc.). However, researchers raised a general question, that is, how do we express and use the concept of uncertainty in mathematical modeling. Many researchers in the world have proposed and recommended different methods of using uncertainty. First of all, Zadeh proposed the concept of fuzzy sets [1] to solve those problems containing uncertainty and ambiguity. It can be seen that in some cases, fuzzy sets cannot handle situations. To overcome such situations, Turksen [2] proposed the idea of interval-valued fuzzy sets (IVFS). In some cases, we must consider the unbiased value of the appropriate representation of the object under the conditions of uncertainty and vagueness, as the non-membership values of the appropriate representation of the object, these fuzzy sets or IVFS cannot handle. To overcome these difficulties, Atanassov proposed the concept of an Intuitionistic Fuzzy Set (IFS) [3]. Zulqarnain et. [4] introduced the correlation coefficient for interval-valued intuitionistic fuzzy soft sets and established the TOPSIS technique based on their developed correlation measures to solve decision-making complications. The theory proposed by Atanassov only deals with under-considered data and membership and non-membership values. However, the

IFS theory cannot deal with overall incompatibility and imprecise information. To solve this incompatible and imprecise information, Smarandache [5] proposed the idea of NS.

Molodtsov [6] proposed a general mathematical tool to deal with uncertain, ambiguous, and undefined substances, called soft sets (SS). Maji et al. [7] Extended the work of SS and defined some operations and their features. They also used the SS theory to make decisions [8]. Ali et. al. [9] Modified the Maji method of SS and developed some new operations with its properties. Sezgin and Atagun [10] proved De Morgan's SS theory and law by using different operators. Cagman and Enginoglu [11] proposed the concept of soft matrices with operations and discussed their properties. They also introduced a decision-making method to solve problems that contain uncertainty. In [12], they modified the actions proposed by Molottsov's SS. In [13], the author plans to perform some new operations on soft matrices, such as soft differential product, soft limited differential product, soft extended differential product, and weak extended differential product. Zulqarnain et al. [14, 15] discussed the Pythagorean fuzzy soft sets and established the aggregation operator and TOPSIS technique to solve the MCDM problem.

Maji [16] put forward the idea of NSS with necessary operations and characteristics. The idea of NSS possibility was put forward by Karaaslan [17] and introduced a neutrosophic soft decision method to solve those uncertain problems based on And-product. Broumi [18] developed a generalized NSS with certain operations and properties and used the proposed concept for decision-making. To solve the MCDM problem with single-valued neutrosophic numbers (SVNN) proposed by Deli and Subas [19], they constructed the concept of SVNN cut sets. Based on the correlation of IFS, the CC term of SVNS was introduced [20]. In [21], the idea of simplifying NS introduced some operational laws and aggregation operators, such as weighted arithmetic and weighted geometric average operator. They constructed the MCDM method based on the proposed aggregation operator. Mukherjee and Das [22] neutrosophic bipolar vague soft sets and some of its operations using. It is the combination of neutrosophic bipolar vague sets and soft sets neutrosophic bipolar vague soft sets and some of its operations. It is the combination of neutrosophic bipolar vague sets and soft sets. Zulqarnain et al. [23, 24] utilized the neutrosophic TOPSIS model to solve the MCDM problem and for the selection of suppliers in the production industry. Masooma et al. [25] by combining multi-polar fuzzy sets and neutrosophic sets, developed a new concept called multi-polar neutrosophic sets. They also established various representations and instance arithmetic.

In the past few years, many mathematicians have developed various similarity measures, correlation coefficients, aggregation operators, and decision-making applications. These structures are based on different sets and provide better solutions to decision-making problems. It has multiple applications in different fields such as pattern recognition, medical diagnosis, artificial intelligence, social science, business, and multi-attribute decision-making problems. Garg [26] developed the MCDM method based on weighted cosine similarity measures under an intuitionistic fuzzy environment and used the proposed technique for pattern recognition and medical diagnoses. To measure the relative strength of IFS Garg and Kumar [27] presented some new similarity measures, they also formulated a connection number for set pair analysis (SPA) and developed some new similarity measures and weighted similarity measures based on defined SPA. Nguyen et al. [28] defined some similarity measures for PFS by using the exponential function for the membership and non-membership degrees with its several properties and relations. Peng and Garg [29] presented some diverse types of similarity measures for PFS with multiple parameters. In [30] the authors established the concept of mPNSS with its properties and operators, they also developed the distance-based similarity measures and used the proposed similarity measures for decision making and medical diagnoses. Recently, Smarandache [31] extended the concept of the SS to hypersoft set (HSS) by replacing the single-parameter function F with a multi-parameter (sub-attribute) function defined on Cartesian products of n different attributes. The established HSS is more flexible than SS and is more suitable for the decision-making environment. He also introduced the further extension of HSS, such as crisp HSS, fuzzy HSS, intuitionistic fuzzy HSS, neutrosophic HSS, and plithogenic HSS. Nowadays, HSS theory and its extensions are developing rapidly. Many researchers have developed different operators and properties based on HSS and its extensions [32-44].

In this era, professionals believe that real life is moving in the direction of multi-polarization. Therefore, there is no doubt that the multi-polarization of information has played an important role in the prosperity of many fields of science and technology. In neurobiology, multipolar neurons in the brain collect a lot of information from other neurons. In information technology, multi-polar technology can be used to control a wide range of structures. In the full text, the motivation for the expansion and mixed work of this research is gradually given. We proved that under any appropriate circumstances, different hybrid structures containing fuzzy sets will be converted into special privileges of mPIVNSS. The concept of a neutrosophic environment to a multipolar interval-valued neutrosophic soft set is novel. We tend to discuss the effectiveness, flexibility, quality, and advantages of planning work and algorithms. This research will be the most versatile form and will combine data to a considerable extent, as well as appropriate medicine, engineering, artificial intelligence, agriculture, and other daily life complications. In the future, the current work may be competent for other methods and different types of mixed structures.

The following research is organized as follows: In section 2, we recollected some basic definitions which are used in the following sequel such as NS, SS, NSS, and multipolar neutrosophic set. In section 3, we proposed the mPIVNSS with its properties and operations. In section 4, distance-based similarity measures have been developed by using Hamming distance and Euclidean distance between two mPIVNSS. In section 5, we use the developed distance-based similarity measures for medical diagnoses. Finally, the conclusion and future directions are presented in section 6.

2. Preliminaries

In this section, some basic concepts have been recalled such as NS, SS, NSS, and IVNSS, etc. which are used in the following sequel.

Definition 2.1 [7]

Let $\mathcal{U}$ be a universe and $\mathcal{A}$ be an NS on $\mathcal{U}$ is defined as $\mathcal{A} = \{< u, \mathcal{u}_{\mathcal{A}}(u), \mathcal{v}_{\mathcal{A}}(u), \mathcal{w}_{\mathcal{A}}(u) > : u \in \mathcal{U}\}$, where $\mathcal{u}$, $\mathcal{v}$, $\mathcal{w}$: $\mathcal{U} \to]0^-, 1^+[$ and $0^- \le \mathcal{u}_{\mathcal{A}}(u) + \mathcal{v}_{\mathcal{A}}(u) + \mathcal{w}_{\mathcal{A}}(u) \le 3^+$.

Definition 2.2 [25]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$, then $\mathcal{F}_{\mathcal{E}}$ is said to multipolar neutrosophic set if

$\mathcal{F}_{\mathcal{E}} = \{< u, (s_i \bullet \mathcal{u}_e(u),\ s_i \bullet \mathcal{v}_e(u), s_i \bullet \mathcal{w}_e(u)) > : u \in \mathcal{U},\ e \in \mathcal{E}, i = 1, 2, 3, \dots . m\}$, where $s_i \bullet \mathcal{u}_{\mathcal{E}}$, $s_i \bullet \mathcal{v}_{\mathcal{E}}, s_i \bullet \mathcal{w}_{\mathcal{E}}$: $\mathcal{U} \to [0, 1]$, and $0 \le s_i \bullet \mathcal{u}_{\mathcal{E}}(u) + s_i \bullet \mathcal{v}_{\mathcal{E}}(u) + s_i \bullet \mathcal{w}_{\mathcal{E}}(u) \le 3$; $i = 1, 2, 3, \dots . m$. $\mathcal{u}_e$, $\mathcal{v}_e$, and $\mathcal{w}_e$ represent the truth, indeterminacy, and falsity of the considered alternative.

Definition 2.3 [3]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called a soft set over $\mathcal{U}$ and its mapping is given as

$$\mathcal{F}: \mathcal{A} \to \mathcal{P}(\mathcal{U})$$

It is also defined as:

$$(\mathcal{F}, \mathcal{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}): e \in \mathcal{E}, \mathcal{F}(e) = \emptyset \ if \ e \neq \mathcal{A}\}$$

Definition 2.4 [16]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the set of Neutrosophic values of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called a Neutrosophic soft set over $\mathcal{U}$ and its mapping is given as

$$\mathcal{F}: \mathcal{A} \rightarrow \mathcal{P}(\mathcal{U})$$

Definition 2.5 [46]

Let $\boldsymbol{\mathcal{U}}$ be a universal set, then interval valued neutrosophic set can be expressed by the set $\boldsymbol{\mathcal{A}} = \{< \boldsymbol{u}, \boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u}) >: \boldsymbol{u} \in \boldsymbol{\mathcal{U}}\}$, where $\boldsymbol{u}_{\mathcal{A}}$, $\boldsymbol{v}_{\mathcal{A}}$, and $\boldsymbol{w}_{\mathcal{A}}$ are truth, indeterminacy, and falsity membership functions for $\boldsymbol{\mathcal{A}}$ respectively, $\boldsymbol{u}_{\mathcal{A}}$, $\boldsymbol{v}_{\mathcal{A}}$, and $\boldsymbol{w}_{\mathcal{A}} \subseteq [0, 1]$ for each $\boldsymbol{u} \in \boldsymbol{\mathcal{U}}$. Where

$\boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}) = [\boldsymbol{u}^{L}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{u}^{U}_{\mathcal{A}}(\boldsymbol{u})]$

$\boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}) = [\boldsymbol{v}^{L}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{v}^{U}_{\mathcal{A}}(\boldsymbol{u})]$

$\boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u}) = [\boldsymbol{w}^{L}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{w}^{U}_{\mathcal{A}}(\boldsymbol{u})]$

For each point $\boldsymbol{u} \in \boldsymbol{\mathcal{U}}$, $0 \leq \boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}) + \boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}) + \boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u}) \leq 3$ and IVN($\boldsymbol{\mathcal{U}}$) represent the family of all interval valued neutrosophic sets.

Definition 2.6 [45]

Let U be an initial universe set, IVN(U) denotes the set of all interval valued neutrosophic sets of U and $\boldsymbol{\mathcal{E}}$ be a set of parameters that describe the elements of U. An interval-valued neutrosophic soft sets(ivn-soft sets) over U is a set defined by a set-valued function $\boldsymbol{\Upsilon_K}$ representing a mapping $\boldsymbol{v_K}$: $\boldsymbol{\mathcal{E}} \rightarrow$ IVN(U) It can be written as a set of ordered pairs

$\boldsymbol{\Upsilon_K} = \{(x, \boldsymbol{v_K}(x)): x \in \boldsymbol{\mathcal{E}}\}$

Here, $\boldsymbol{v_K}$, which is interval-valued neutrosophic sets, is called the approximate function of the ivn-soft sets $\boldsymbol{\Upsilon_K}$ and $\boldsymbol{v_K}$(x) is called the x-approximate value of x $\in \boldsymbol{\mathcal{E}}$. The subscript K in the $\boldsymbol{v_K}$ indicates that $\boldsymbol{v_K}$ is the approximate function of $\boldsymbol{\Upsilon_K}$. Generally if $\boldsymbol{v_K}$, $\boldsymbol{v_L}$, $\boldsymbol{v_M}$,… will be used as an approximate function of $\boldsymbol{\Upsilon_K}$, $\boldsymbol{\Upsilon_L}$, $\boldsymbol{\Upsilon_M}$,…, respectively. Note that the sets of all ivn-soft sets over U will be denoted by IVNSS.

3. Multi-Polar Interval Valued Neutrosophic Soft Set with Aggregate Operators and Properties

In this section, we develop the concept of mPIVNSS and introduce some basic operations on mPIVNSS with their properties.

Definition 3.1

Let $\mathcal{U}$ and E are universal and set of attributes respectively, and $\mathcal{A} \subseteq$ E, if there exists a mapping Φ such as

Φ: $\mathcal{A} \rightarrow mPIVNSS^{\mathcal{U}}$

Then (Φ, $\mathcal{A}$) is called mPIVNSS over $\mathcal{U}$ defined as follows

$\Upsilon_K = (\Phi, \mathcal{A}) = \left\{\left(u, \Phi_{\mathcal{A}(e)}(u)\right): e \in E, u \in \mathcal{U}\right\}$, where

$\Phi_{\mathcal{A}}(e) = \{(e, < u, [s_i \bullet inf\ u_{\mathcal{A}(e)}(u), s_i \bullet sup\ u_{\mathcal{A}(e)}(u)], [s_i \bullet inf\ v_{\mathcal{A}(e)}(u), s_i \bullet sup\ v_{\mathcal{A}(e)}(u)], [s_i \bullet inf\ w_{\mathcal{A}(e)}(u), s_i \bullet sup\ w_{\mathcal{A}(e)}(u)] >: u \in \mathcal{U}, e \in E)\}$, and

$0 \leq s_i \bullet sup\ u_{\mathcal{A}(e)}(u) + s_i \bullet sup\ v_{\mathcal{A}(e)}(u) + s_i \bullet sup\ w_{\mathcal{A}(e)}(u) \leq 3$ for all $i \in 1, 2, 3, \ldots, m$; $e \in E$ and $u \in \mathcal{U}$.

Definition 3.2

Let Υ_A and $\Upsilon_B \in$ mPIVNSS over $\mathcal{U}$, then Υ_A is called a multi-polar interval-valued neutrosophic soft subset of Υ_B. If

$s_i \bullet inf\ u_{A(e)}(u) \leq s_i \bullet inf\ u_{B(e)}(u),\ s_i \bullet sup\ u_{A(e)}(u) \leq s_i \bullet sup\ u_{B(e)}(u)$

$s_i \bullet inf\ v_{A(e)}(u) \leq s_i \bullet inf\ v_{B(e)}(u),\ s_i \bullet sup\ v_{A(e)}(u) \leq s_i \bullet sup\ v_{B(e)}(u)$

$s_i \bullet inf\ w_{A(e)}(u) \geq s_i \bullet inf\ w_{B(e)}(u),\ s_i \bullet sup\ w_{A(e)}(u) \geq s_i \bullet sup\ w_{B(e)}(u)$

for all $i \in 1, 2, 3, \ldots, m;\ e \in E$ and $u \in \mathcal{U}$.

Example 1 Assume $\mathcal{U} = \{u_1,\ u_2\}$ be a universe of discourse and $E = \{x_1,\ x_2,\ x_3,\ x_4\}$ be a set of attribuites and $A = B = \{x_1,\ x_2\} \subseteq E$. Consider F_A and $G_B \in$ 3-PIVNSS over $\mathcal{U}$ can be represented as follows

$$F_A = \left\{\begin{matrix} (x_1, \{\langle u_1, ([.5, .8], [.2, .5], [.1, .6]), ([.3, .5], [.1, .3], [.3, .7]), ([.4, .6], [.3, .7], [.8, 1]), \\ (u_2, ([.2, .4], [.3, 0.4], [.2, .5]), ([.2, .5], [.1, .6], [.3, .8]), ([.3, .8], [.4, .9], [.6, .7]))\rangle, \\ (x_2, \{\langle u_1, ([.3, .6], [.1, .6], [.4, .7]), ([0, .2], [.1, .4], [.5, .9]), ([.3, .6], [.1, .4], [.5, .8])), \\ (u_2, ([.2, .5], [.2, .3], [.5, .6]), ([.3, .5], [.1, .5], [.5, .8]), ([.4, .6], [.3, .5], [.6, .9]))\rangle \end{matrix}\right\}$$

and

$$G_B = \left\{\begin{matrix} (x_1, \{\langle u_1, ([.6, .8], [.4, .6], [.1, .4]), ([.4, .7], [.3, .4], [.2, .6]), ([.5, .7], [.4, .7], [.5, 1]), \\ (u_2, ([.3, .6], [.5, 0.7], [.1, .5]), ([.3, .8], [.2, .6], [.1, .5]), ([.4, 1], [.5, .9], [.4, .6]))\rangle, \\ (x_2, \{\langle u_1, ([.4, .7], [.3, .7], [.3, .5]), ([0, .3], [.2, .5], [.3, .7]), ([.4, .9], [.2, .6], [.5, .7])), \\ (u_2, ([.2, .9], [.1, .5], [.3, .6]), ([.6, .9], [.3, .5], [1, 1]), ([.5, .7], [.3, .7], [.1, .8]))\rangle \end{matrix}\right\}.$$

Thus

$F_A \subseteq G_B$.

Definition 3.3

Let Υ_A and $\Upsilon_B \in$ mPIVNSS over $\mathcal{U}$, then $\Upsilon_A = \Upsilon_B$, if

$s_i \bullet inf\ u_{A(e)}(u) \leq s_i \bullet inf\ u_{B(e)}(u),\ s_i \bullet inf\ u_{B(e)}(u) \leq s_i \bullet inf\ u_{A(e)}(u)$

$s_i \bullet sup\ u_{A(e)}(u) \leq s_i \bullet sup\ u_{B(e)}(u),\ s_i \bullet sup\ u_{B(e)}(u) \leq s_i \bullet sup\ u_{A(e)}(u)$

$s_i \bullet inf\ v_{A(e)}(u) \leq s_i \bullet inf\ v_{B(e)}(u),\ s_i \bullet inf\ v_{B(e)}(u) \leq s_i \bullet inf\ v_{A(e)}(u)$

$s_i \bullet sup\ v_{A(e)}(u) \leq s_i \bullet sup\ v_{B(e)}(u),\ s_i \bullet sup\ v_{B(e)}(u) \leq s_i \bullet sup\ v_{A(e)}(u)$

$s_i \bullet inf\ w_{A(e)}(u) \geq s_i \bullet inf\ w_{B(e)}(u),\ s_i \bullet inf\ w_{B(e)}(u) \geq s_i \bullet inf\ w_{A(e)}(u)$

$s_i \bullet sup\ w_{A(e)}(u) \geq s_i \bullet sup\ w_{B(e)}(u),\ s_i \bullet sup\ w_{B(e)}(u) \geq s_i \bullet sup\ w_{A(e)}(u)$

for all $i \in 1, 2, 3, \ldots, m;\ e \in E$ and $u \in \mathcal{U}$.

Definition 3.4

Let $F_A \in$ mPIVNSS over $\mathcal{U}$, then empty mPIVNSS can be represented as $F_{\breve{0}}$, and defined as follows

$F_{\breve{0}} = \{e, < u, ([0, 0], [1, 1], [1, 1]),\ ([0, 0], [1, 1], [1, 1]), \ldots, ([0, 0], [1, 1], [1, 1]) > : e \in E, u \in \mathcal{U}\}$.

Definition 3.5

Let $F_A \in$ mPIVNSS over $\mathcal{U}$, then universal mPIVNSS can be represented as $F_{\breve{E}}$, and defined as follows

$F_{\breve{E}} = \{e, < u, ([1, 1], [0, 0], [0, 0]),\ ([1, 1], [0, 0], [0, 0]), \ldots, ([1, 1], [0, 0], [0, 0]) > : e \in E, u \in \mathcal{U}\}$.

Example 2 Assume $\mathcal{U} = \{u_1,\ u_2\}$ be a universe of discourse and $E = \{x_1,\ x_2,\ x_3,\ x_4\}$ be a set of attributes. The tabular representation of $F_{\breve{0}}$ and $F_{\breve{E}}$ given as follows in table 1 and table 2 respectively.

Table 1: Tablur representation of mPIVNSS $F_{\breve{0}}$

$\mathcal{U}$	u_1	u_2	$\cdots$	u_n
x_1	([0, 0], [1, 1], [1, 1])	([0, 0], [1, 1], [1, 1])	$\cdots$	([0, 0], [1, 1], [1, 1])

x_2	$([0,0],[1,1],[1,1])$	$([0,0],[1,1],[1,1])$	$\cdots$	$([0,0],[1,1],[1,1])$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_n	$([0,0],[1,1],[1,1])$	$([0,0],[1,1],[1,1])$	$\cdots$	$([0,0],[1,1],[1,1])$

Table 2: Tablur representation of mPIVNSS $F_{\breve{E}}$

$\mathcal{U}$	u_1	u_2	$\cdots$	u_n
x_1	$([1,1],[0,0],[0,0])$	$([1,1],[0,0],[0,0])$	$\cdots$	$([1,1],[0,0],[0,0])$
x_2	$([1,1],[0,0],[0,0])$	$([1,1],[0,0],[0,0])$	$\cdots$	$([1,1],[0,0],[0,0])$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_n	$([1,1],[0,0],[0,0])$	$([1,1],[0,0],[0,0])$	$\cdots$	$([1,1],[0,0],[0,0])$

Definition 3.6

Let $F_A \in$ mPIVNSS over $\mathcal{U}$, then the complement of mPIVNSS is defined as follows

$F_A^c(e) = \{e, < u, [s_i \bullet inf\ w_{A(e)}(u), s_i \bullet sup\ w_{A(e)}(u)], [(1,1,\dots,1) - s_i \bullet sup\ v_{A(e)}(u), (1,1,\dots,1) - s_i \bullet inf\ v_{A(e)}(u)], [s_i \bullet inf\ u_{A(e)}(u), s_i \bullet sup\ u_{A(e)}(u)] > : u \in \mathcal{U}\}$, for all $i \in 1, 2, 3, \dots, m$; $e \in E$ and $u \in \mathcal{U}$.

Example 3 Assume $\mathcal{U} = \{u_1, u_2\}$ be a universe of discourse and $E = \{x_1, x_2, x_3, x_4\}$ be a set of attributes and $A = \{x_1, x_2\} \subseteq E$. Consider $F_A \in$ 3-PIVNSS over $\mathcal{U}$ can be represented as follows

$$F_A = \left\{\begin{matrix} (x_1, \{\langle u_1, ([.6,.8],[.4,0.6],[.1,.4]), ([.4,.7],[.3,.4],[.2,.6]), ([.5,.7],[.6,.9],[1,1]), \\ (u_2, ([.3,.6],[.5,0.7],[.1,.5]), ([.3,.8],[.2,.6],[.1,.5]), ([.4,1],[.5,.9],[.4,.6]))\rangle, \\ (x_2, \{\langle u_1, ([.4,.7],[.3,.7],[.3,.5]), ([0,.3],[.2,.5],[.3,.7]), ([.4,.9],[.2,.6],[.5,.7])), \\ (u_2, ([.2,.9],[.1,.5],[.7,.8]), ([.6,.9],[.3,.5],[1,1]), ([.5,.9],[.3,.7],[.1,.8]))\rangle \end{matrix}\right\}$$

Then,

$$F_A^c(x) = \left\{\begin{matrix} (x_1, \{\langle u_1, ([.1,.4],[.4,0.6],[.6,.8]), [.2,.6](,[.6,.7],[.4,.7]), ([1,1],[.1,.4],[.5,.7]), \\ (u_2, ([.1,.5],[.3,0.5],[.3,.6]), ([.1,.5],[.4,.8],[.3,.8]), ([.4,.6],[.1,.5],[.4,1]))\rangle, \\ (x_2, \{\langle u_1, ([.3,.5],[.3,.7],[.4,.7]), ([.3,.7],[.5,.8],[0,.3]), ([.5,.7],[.4,.8],[.4,.9])), \\ (u_2, ([.7,.8],[.5,.9],[.2,.9]), ([1,1],[.5,.7],[.6,.9]), ([.1,.8],[.3,.7],[.5,.9]))\rangle \end{matrix}\right\}$$

Proposition 3.7

If $F_A \in$ mPIVNSS, then

1. $(F_A^c)^c = F_A$
2. $(F_{\breve{0}})^c = F_{\breve{E}}$
3. $(F_{\breve{E}})^c = F_{\breve{0}}$

Proof 1 Let

$$F_A(e) = \left\{\begin{matrix} < u, [s_i \bullet inf\ u_{A(e)}(u), s_i \bullet sup\ u_{A(e)}(u)], \\ [s_i \bullet inf\ v_{A(e)}(u), s_i \bullet sup\ v_{A(e)}(u)], \\ [s_i \bullet inf\ w_{A(e)}(u), s_i \bullet sup\ w_{A(e)}(u)] > : u \in \mathcal{U}, e \in E \end{matrix}\right\}.$$

Then by using definition 3.6, we get

$$F_A^c(e) = \left\{ \begin{array}{c} < u, [s_i \bullet inf\, w_{A(e)}(u), s_i \bullet sup\, w_{A(e)}(u)], \\ [(1,1,\dots,1) - s_i \bullet sup\, v_{A(e)}(u), (1,1,\dots,1) - s_i \bullet inf\, v_{A(e)}(u)], \\ [s_i \bullet inf\, u_{A(e)}(u), s_i \bullet sup\, u_{A(e)}(u)] >: u \in \mathcal{U}, e \in E \end{array} \right\}$$

Again by using definition 3.6

$(F_A^c(e))^c=$

$$\left\{ \begin{array}{c} < u, [s_i \bullet inf\, u_{A(e)}(u), s_i \bullet sup\, u_{A(e)}(u)], \\ [(1,1,\dots,1) - (1,1,\dots,1) - s_i \bullet inf\, v_{A(e)}(u), (1,1,\dots,1) - (1,1,\dots,1) - s_i \bullet sup\, v_{A(e)}(u)], \\ [s_i \bullet inf\, w_{A(e)}(u), s_i \bullet sup\, w_{A(e)}(u)] >: u \in \mathcal{U}, e \in E \end{array} \right\}$$

$$(F_A^c(e))^c= \left\{ \begin{array}{c} < u, [s_i \bullet inf\, u_{A(e)}(u), s_i \bullet sup\, u_{A(e)}(u)], \\ [s_i \bullet inf\, v_{A(e)}(u), s_i \bullet sup\, v_{A(e)}(u)], \\ [s_i \bullet inf\, w_{A(e)}(u), s_i \bullet sup\, w_{A(e)}(u)] >: u \in \mathcal{U}, e \in E \end{array} \right\}$$

$(F_A^c(e))^c= F_A(e)$.

Similarly, we can prove 2 and 3.

Definition 3.8

Let $F_{A(e)}$ and $G_{B(e)}$ ∈ mPIVNSS over $\mathcal{U}$, then

$F_{A(e)} \cup G_{B(e)}=$

$$\left\{ \begin{array}{c} (e, < u, [max\{s_i \bullet inf\, u_{A(e)}(u), s_i \bullet inf\, u_{B(e)}(u)\}, max\{s_i \bullet sup\, u_{A(e)}(u), s_i \bullet sup\, u_{B(e)}(u)\}], \\ [min\{s_i \bullet inf\, v_{A(e)}(u), s_i \bullet inf\, v_{B(e)}(u)\}, min\{s_i \bullet sup\, v_{A(e)}(u), s_i \bullet sup\, v_{B(e)}(u)\}], \\ [min\{s_i \bullet inf\, w_{A(e)}(u), s_i \bullet inf\, w_{B(e)}(u)\}, min\{s_i \bullet sup\, w_{A(e)}(u), s_i \bullet sup\, w_{B(e)}(u)\}] >: u \in \mathcal{U}, e \in E) \end{array} \right\}$$

Example 4 Assume $\mathcal{U} = \{u_1, u_2\}$ be a universe of discourse and $E = \{x_1, x_2, x_3, x_4\}$ be a set of attribuites and $A = B = \{x_1, x_2\} \subseteq E$. Consider $F_{A(e)}$ and $G_{B(e)}$ ∈ 3-PIVNSS over $\mathcal{U}$ can be represented as follows

$$F_{A(x)} = \left\{ \begin{array}{c} (x_1, \{\langle u_1, ([.5,.8],[.2,0.5],[.1,.2]), ([.3,.5],[.1,.3],[.2,.4]), ([.6,.9],[.7,.8],[.8,1]), \\ (u_2, ([.2,.4],[.3,0.4],[.1,.3]), ([.2,.5],[.1,.6],[.1,.3]), ([.8,1],[.6,.9],[.6,.7]))\rangle, \\ (x_2, \{\langle u_1, ([.3,.6],[.1,.6],[.3,.4]), ([0,.2],[.1,.4],[.3,.5]), ([.5,.9],[.3,.8],[.5,.8])), \\ (u_2, ([.2,.5],[.2,.3],[.5,.6]), ([.3,.5],[.1,.5],[.5,.8]), ([.6,.9],[.5,.8],[.6,.9]))\rangle \end{array} \right\}$$

and

$$G_{B(x)} = \left\{ \begin{array}{c} (x_1, \{\langle u_1, ([.4,.8],[.3,0.6],[.2,.5]), ([.2,.7],[.3,.4],[.4,.6]), ([.7,.8],[.4,.9],[.5,1]), \\ (u_2, ([.1,.6],[.5,0.7],[.1,.2]), ([.3,.4],[.2,.5],[.2,.5]), ([.5,.9],[.7,.8],[.4,.6]))\rangle, \\ (x_2, \{\langle u_1, ([.2,.7],[.3,.5],[.2,.6]), ([.1,.3],[.2,.5],[.2,.7]), ([.4,.9],[.4,.7],[.5,.8])), \\ (u_2, ([.1,.6],[.1,.5],[.4,.8]), ([.3,.6],[.3,.4],[1,1]), ([.5,.9],[.3,.7],[.1,.8]))\rangle \end{array} \right\}.$$

Then

$$F_{A(x)} \cup G_{B(x)}= \left\{ \begin{array}{c} (x_1, \{\langle u_1, ([.5,.8],[.2,0.5],[.1,.2]), ([.3,.7],[.1,.3],[.2,.4]), ([.7,.9],[.4,.8],[.5,1]), \\ (u_2, ([.2,.6],[.3,0.4],[.1,.2]), ([.3,.5],[.1,.5],[.1,.3]), ([.8,1],[.6,.8],[.4,.6]))\rangle, \\ (x_2, \{\langle u_1, ([.3,.7],[.1,.5],[.2,.4]), ([.1,.3],[.1,.4],[.2,.5]), ([.5,.9],[.3,.7],[.5,.8])), \\ (u_2, ([.2,.6],[.1,.3],[.4,.6]), ([.3,.6],[.1,.4],[.5,.8]), ([.6,.9],[.3,.7],[.1,.8]))\rangle \end{array} \right\}$$

Proposition 3.9

Let $\mathcal{F}_{\breve{A}}$, $\mathcal{G}_{\breve{B}}$, $\mathcal{H}_{\breve{C}}$ ∈ mPIVNSS over $\mathcal{U}$. Then

1. $\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{A}} = \mathcal{F}_{\breve{A}}$
2. $\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{0}} = \mathcal{F}_{\breve{A}}$
3. $\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{E}} = \mathcal{F}_{\breve{E}}$
4. $\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}} = \mathcal{G}_{\breve{B}} \cup \mathcal{F}_{\breve{A}}$

5. $(\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}}) \cup \mathcal{H}_{\breve{C}} = \mathcal{F}_{\breve{A}} \cup (\mathcal{G}_{\breve{B}} \cup \mathcal{H}_{\breve{C}})$

Proof 1. As we know that

$$F_{A}(e) = \left\{\begin{matrix} (e, < u, [s_i \bullet inf\ \mathcal{u}_{A(e)}(u), s_i \bullet sup\ \mathcal{u}_{A(e)}(u)], \\ [s_i \bullet inf\ \mathcal{v}_{A(e)}(u), s_i \bullet sup\ \mathcal{v}_{A(e)}(u)], \\ [s_i \bullet inf\ \mathcal{w}_{A(e)}(u), s_i \bullet sup\ \mathcal{w}_{A(e)}(u)] >): u \in \mathcal{U}, e \in E \end{matrix}\right\}$$

be an mPIVNSS, then by using definition 3.8, we have

$$\mathcal{F}_{\widetilde{A(e)}} \cup \mathcal{F}_{\widetilde{A(e)}} =$$

$$\left\{\begin{matrix} (e, < u, [max\{s_i \bullet inf\ \mathcal{u}_{A(e)}(u), s_i \bullet inf\ \mathcal{u}_{A(e)}(u)\}, max\{s_i \bullet sup\ \mathcal{u}_{A(e)}(u), s_i \bullet sup\ \mathcal{u}_{A(e)}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{v}_{A(e)}(u), s_i \bullet inf\ \mathcal{v}_{A(e)}(u)\}, min\{s_i \bullet sup\ \mathcal{v}_{A(e)}(u), s_i \bullet sup\ \mathcal{v}_{A(e)}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{w}_{A(e)}(u), s_i \bullet inf\ \mathcal{w}_{A(e)}(u)\}, min\{s_i \bullet sup\ \mathcal{w}_{A(e)}(u), s_i \bullet sup\ \mathcal{w}_{A(e)}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$= \left\{\begin{matrix} (e, < u, [s_i \bullet inf\ \mathcal{u}_{A(e)}(u), s_i \bullet sup\ \mathcal{u}_{A(e)}(u)], \\ [s_i \bullet inf\ \mathcal{v}_{A(e)}(u), s_i \bullet sup\ \mathcal{v}_{A(e)}(u)], \\ [s_i \bullet inf\ \mathcal{w}_{A(e)}(u), s_i \bullet sup\ \mathcal{w}_{A(e)}(u)] >): u \in \mathcal{U}, e \in E \end{matrix}\right\} = \mathcal{F}_{\widetilde{A(e)}}$$

Proof 2. $\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{0}} = \mathcal{F}_{\breve{A}}$

$$\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{0}} =$$

$$\left\{\begin{matrix} (e, < u, [max\{s_i \bullet inf\ \mathcal{u}_{A(e)}(u), s_i \bullet inf\ \mathcal{u}_{\breve{0}}(u)\}, max\{s_i \bullet sup\ \mathcal{u}_{A(e)}(u), s_i \bullet sup\ \mathcal{u}_{\breve{0}}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{v}_{A(e)}(u), s_i \bullet inf\ \mathcal{v}_{\breve{0}}(u)\}, min\{s_i \bullet sup\ \mathcal{v}_{A(e)}(u), s_i \bullet sup\ \mathcal{v}_{\breve{0}}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{w}_{A(e)}(u), s_i \bullet inf\ \mathcal{w}_{\breve{0}}(u)\}, min\{s_i \bullet sup\ \mathcal{w}_{A(e)}(u), s_i \bullet sup\ \mathcal{w}_{\breve{0}}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$= \left\{\begin{matrix} (e, < u, [s_i \bullet inf\ \mathcal{u}_{A(e)}(u), s_i \bullet sup\ \mathcal{u}_{A(e)}(u)], \\ [s_i \bullet inf\ \mathcal{v}_{A(e)}(u), s_i \bullet sup\ \mathcal{v}_{A(e)}(u)], \\ [s_i \bullet inf\ \mathcal{w}_{A(e)}(u), s_i \bullet sup\ \mathcal{w}_{A(e)}(u)] >): u \in \mathcal{U}, e \in E \end{matrix}\right\} = \mathcal{F}_{\breve{A}}$$

Proof 3. $\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{E}} = \mathcal{F}_{\breve{E}}$

$$\mathcal{F}_{\breve{A}} \cup \mathcal{F}_{\breve{E}} =$$

$$\left\{\begin{matrix} (e, < u, [max\{s_i \bullet inf\ \mathcal{u}_{A(e)}(u), s_i \bullet inf\ \mathcal{u}_{\breve{0}}(u)\}, max\{s_i \bullet sup\ \mathcal{u}_{A(e)}(u), s_i \bullet sup\ \mathcal{u}_{\breve{0}}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{v}_{A(e)}(u), s_i \bullet inf\ \mathcal{v}_{\breve{0}}(u)\}, min\{s_i \bullet sup\ \mathcal{v}_{A(e)}(u), s_i \bullet sup\ \mathcal{v}_{\breve{0}}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{w}_{A(e)}(u), s_i \bullet inf\ \mathcal{w}_{\breve{0}}(u)\}, min\{s_i \bullet sup\ \mathcal{w}_{A(e)}(u), s_i \bullet sup\ \mathcal{w}_{\breve{0}}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$= \left\{\begin{matrix} (e, < u, [s_i \bullet inf\ \mathcal{u}_{\breve{E}(e)}(u), s_i \bullet sup\ \mathcal{u}_{\breve{E}(e)}(u)], \\ [s_i \bullet inf\ \mathcal{v}_{\breve{E}(e)}(u), s_i \bullet sup\ \mathcal{v}_{\breve{E}(e)}(u)], \\ [s_i \bullet inf\ \mathcal{w}_{\breve{E}(e)}(u), s_i \bullet sup\ \mathcal{w}_{\breve{E}(e)}(u)] >): u \in \mathcal{U}, e \in E \end{matrix}\right\} = \mathcal{F}_{\breve{E}}.$$

Proof 4. $\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}} = \mathcal{G}_{\breve{B}} \cup \mathcal{F}_{\breve{A}}$

$$\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}} =$$

$$\left\{\begin{matrix} (e, < u, [max\{s_i \bullet inf\ \mathcal{u}_{A(e)}(u), s_i \bullet inf\ \mathcal{u}_{B(e)}(u)\}, max\{s_i \bullet sup\ \mathcal{u}_{A(e)}(u), s_i \bullet sup\ \mathcal{u}_{B(e)}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{v}_{A(e)}(u), s_i \bullet inf\ \mathcal{v}_{B(e)}(u)\}, min\{s_i \bullet sup\ \mathcal{v}_{A(e)}(u), s_i \bullet sup\ \mathcal{v}_{B(e)}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{w}_{A(e)}(u), s_i \bullet inf\ \mathcal{w}_{B(e)}(u)\}, min\{s_i \bullet sup\ \mathcal{w}_{A(e)}(u), s_i \bullet sup\ \mathcal{w}_{B(e)}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$\mathcal{G}_{\breve{B}} \cup \mathcal{F}_{\breve{A}} =$$

$$\left\{\begin{matrix} (e, < u, [max\{s_i \bullet inf\ \mathcal{u}_{B(e)}(u), s_i \bullet inf\ \mathcal{u}_{A(e)}(u)\}, max\{s_i \bullet sup\ \mathcal{u}_{B(e)}(u), s_i \bullet sup\ \mathcal{u}_{A(e)}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{v}_{B(e)}(u), s_i \bullet inf\ \mathcal{v}_{A(e)}(u)\}, min\{s_i \bullet sup\ \mathcal{v}_{B(e)}(u), s_i \bullet sup\ \mathcal{v}_{A(e)}(u)\}], \\ [min\{s_i \bullet inf\ \mathcal{w}_{B(e)}(u), s_i \bullet inf\ \mathcal{w}_{A(e)}(u)\}, min\{s_i \bullet sup\ \mathcal{w}_{B(e)}(u), s_i \bullet sup\ \mathcal{w}_{A(e)}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$\mathcal{G}_{\breve{B}} \cup \mathcal{F}_{\breve{A}} =$

$$\left\{\begin{matrix} (e, < u, [max\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\ u_{B(e)}(u)\}, max\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\}], \\ [min\{s_i \bullet inf\ v_{A(e)}(u), s_i \bullet inf\ v_{B(e)}(u)\}, min\{s_i \bullet sup\ v_{A(e)}(u), s_i \bullet sup\ v_{B(e)}(u)\}], \\ [min\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{B(e)}(u)\}, min\{s_i \bullet sup\ w_{A(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\}] >: u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

So, $\mathcal{F}_{\breve{A}} \cup \mathcal{G}_{\breve{B}} = \mathcal{G}_{\breve{B}} \cup \mathcal{F}_{\breve{A}}$.

Proof 5. Similar to assertion 4.

Definition 3.10

Let $F_{A(e)}$ and $G_{B(e)} \in$ mPIVNSS over $\mathcal{U}$, then

$F_{A(e)} \cap G_{B(e)} =$

$$\left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\ u_{B(e)}(u)\}, min\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\}], \\ [max\{s_i \bullet inf\ v_{A(e)}(u), s_i \bullet inf\ v_{B(e)}(u)\}, max\{s_i \bullet sup\ v_{A(e)}(u), s_i \bullet sup\ v_{B(e)}(u)\}], \\ [max\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{B(e)}(u)\}, max\{s_i \bullet sup\ w_{A(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\}] >: u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

Proposition 3.11

Let $\mathcal{F}_{\breve{A}}$, $\mathcal{G}_{\breve{B}}$, $\mathcal{H}_{\breve{C}} \in$ mPIVNSS over $\mathcal{U}$. Then

1. $\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{A}} = \mathcal{F}_{\breve{A}}$
2. $\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{0}} = \mathcal{F}_{\breve{A}}$
3. $\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{E}} = \mathcal{F}_{\breve{A}}$
4. $\mathcal{F}_{\breve{A}} \cap \mathcal{G}_{\breve{B}} = \mathcal{G}_{\breve{B}} \cap \mathcal{F}_{\breve{A}}$
5. $(\mathcal{F}_{\breve{A}} \cap \mathcal{G}_{\breve{B}}) \cap \mathcal{H}_{\breve{C}} = \mathcal{F}_{\breve{A}} \cap (\mathcal{G}_{\breve{B}} \cap \mathcal{H}_{\breve{C}})$

Proof 1. As we know that

$$\mathcal{F}_{\widetilde{A(e)}} = \left\{\begin{matrix} (e, < u, [s_i \bullet inf\ u_{A(e)}(u), s_i \bullet sup\ u_{A(e)}(u)], \\ [s_i \bullet inf\ v_{A(e)}(u), s_i \bullet sup\ v_{A(e)}(u)], \\ [s_i \bullet inf\ w_{A(e)}(u), s_i \bullet sup\ w_{A(e)}(u)] >): u \in \mathcal{U}, e \in E \end{matrix}\right\}$$ be an mPIVNSS, then by using definition 3.8, we have

$\mathcal{F}_{\widetilde{A(e)}} \cap \mathcal{F}_{\widetilde{A(e)}} =$

$$\left\{\begin{matrix} (e, < u, [max\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\ u_{A(e)}(u)\}, max\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{A(e)}(u)\}], \\ [min\{s_i \bullet inf\ v_{A(e)}(u), s_i \bullet inf\ v_{A(e)}(u)\}, min\{s_i \bullet sup\ v_{A(e)}(u), s_i \bullet sup\ v_{A(e)}(u)\}], \\ [min\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{A(e)}(u)\}, min\{s_i \bullet sup\ w_{A(e)}(u), s_i \bullet sup\ w_{A(e)}(u)\}] >: u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$= \left\{\begin{matrix} (e, < u, [s_i \bullet inf\ u_{A(e)}(u), s_i \bullet sup\ u_{A(e)}(u)], \\ [s_i \bullet inf\ v_{A(e)}(u), s_i \bullet sup\ v_{A(e)}(u)], \\ [s_i \bullet inf\ w_{A(e)}(u), s_i \bullet sup\ w_{A(e)}(u)] >): u \in \mathcal{U}, e \in E \end{matrix}\right\} = \mathcal{F}_{\widetilde{A(e)}}$$

Proof 2. $\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{0}} = \mathcal{F}_{\breve{A}}$

$\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{0}} =$

$$\left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\ u_{B(e)}(u)\}, min\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\}], \\ [max\{s_i \bullet inf\ v_{A(e)}(u), s_i \bullet inf\ v_{B(e)}(u)\}, max\{s_i \bullet sup\ v_{A(e)}(u), s_i \bullet sup\ v_{B(e)}(u)\}], \\ [max\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{B(e)}(u)\}, max\{s_i \bullet sup\ w_{A(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\}] >: u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{0}} = \left\{\begin{matrix} (e, < u, [s_i \bullet inf\ u_{A(e)}(u), s_i \bullet sup\ u_{A(e)}(u)], \\ [s_i \bullet inf\ v_{A(e)}(u), s_i \bullet sup\ v_{A(e)}(u)], \\ [s_i \bullet inf\ w_{A(e)}(u), s_i \bullet sup\ w_{A(e)}(u)] >): u \in \mathcal{U}, e \in E \end{matrix}\right\} = \mathcal{F}_{\widetilde{A(e)}}$$

Proof 3. $\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{E}} = \mathcal{F}_{\breve{A}}$

$$\mathcal{F}_{\breve{A}} \cap \mathcal{F}_{\breve{E}} = \left\{\begin{matrix} (e, < u, \left[min\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\ u_{\breve{E}}(u)\}, min\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{\breve{E})}(u)\}\right], \\ \left[max\{s_i \bullet inf\ v_{A(e)}(u), s_i \bullet inf\ v_{\breve{E}}(u)\}, max\{s_i \bullet sup\ v_{A(e)}(u), s_i \bullet sup\ v_{\breve{E}}(u)\}\right], \\ \left[max\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{\breve{E}}(u)\}, max\{s_i \bullet sup\ w_{A(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\}\right] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$\left\{\begin{matrix} (e, < u, \left[min\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\ u_{B(e)}(u)\}, min\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\}\right], \\ \left[max\{s_i \bullet inf\ v_{A(e)}(u), s_i \bullet inf\ v_{B(e)}(u)\}, max\{s_i \bullet sup\ v_{A(e)}(u), s_i \bullet sup\ v_{B(e)}(u)\}\right], \\ \left[max\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{B(e)}(u)\}, max\{s_i \bullet sup\ w_{A(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\}\right] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$= \left\{\begin{matrix} (e, < u, \left[s_i \bullet inf\ u_{A(e)}(u), s_i \bullet sup\ u_{A(e)}(u)\right], \\ \left[s_i \bullet inf\ v_{A(e)}(u), s_i \bullet sup\ v_{A(e)}(u)\right], \\ \left[s_i \bullet inf\ w_{A(e)}(u), s_i \bullet sup\ w_{A(e)}(u)\right] >): u \in \mathcal{U}, e \in E \end{matrix}\right\} = \mathcal{F}_{\breve{A}}$$

Proof 4. 5. Similar to assertion 3.

Proposition 3.12

Let F_A and $G_B \in$ mPIVNSS over $\mathcal{U}$, then

1. $(F_{A(e)} \cup G_{B(e)})^C = {F_{A(e)}}^C \cap {G_{B(e)}}^C$
2. $(F_{A(e)} \cap G_{B(e)})^C = {F_{A(e)}}^C \cup {G_{B(e)}}^C$

Proof 1 As we know that

$$F_A(e) = \left\{\begin{matrix} (e, < u, \left[s_i \bullet inf\ u_{A(e)}(u), s_i \bullet sup\ u_{A(e)}(u)\right], \\ \left[s_i \bullet inf\ v_{A(e)}(u), s_i \bullet sup\ v_{A(e)}(u)\right], \\ \left[s_i \bullet inf\ w_{A(e)}(u), s_i \bullet sup\ w_{A(e)}(u)\right] >): u \in \mathcal{U}, e \in E \end{matrix}\right\} \text{ and}$$

$$G_B(e) = \left\{\begin{matrix} (e, < u, \left[s_i \bullet inf\ u_{B(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\right], \\ \left[s_i \bullet inf\ v_{B(e)}(u), s_i \bullet sup\ v_{B(e)}(u)\right], \\ \left[s_i \bullet inf\ w_{B(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\right] >): u \in \mathcal{U}, e \in E \end{matrix}\right\}$$

By using definition 3.8, we get

$$F_{A(e)} \cup G_{B(e)} = \left\{\begin{matrix} (e, < u, \left[max\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\ u_{B(e)}(u)\}, max\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\}\right], \\ \left[min\{s_i \bullet inf\ v_{A(e)}(u), s_i \bullet inf\ v_{B(e)}(u)\}, min\{s_i \bullet sup\ v_{A(e)}(u), s_i \bullet sup\ v_{B(e)}(u)\}\right], \\ \left[min\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{B(e)}(u)\}, min\{s_i \bullet sup\ w_{A(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\}\right] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

Now by using definition 3.6, we get

$$\left(F_{A(e)} \cup G_{B(e)}\right)^c = \left\{\begin{matrix} (e, < u, \left[min\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{B(e)}(u)\}, min\{s_i \bullet sup\ w_{A(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\}\right], \\ \left[(1,1,\dots,1) - min\{s_i \bullet sup\ v_{A(e)}(u), s_i \bullet sup\ v_{B(e)}(u)\}, (1,1,\dots,1) - min\{s_i \bullet inf\ v_{A(e)}(u), s_i \bullet inf\ v_{B(e)}(u)\}\right], \\ \left[max\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\ u_{B(e)}(u)\}, max\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\}\right] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

Now

$${F_{A(e)}}^C = \left\{\begin{matrix} (e, < u, \left[s_i \bullet inf\ w_{A(e)}(u), s_i \bullet sup\ w_{A(e)}(u)\right], \\ \left[(1,1,\dots,1) - s_i \bullet sup\ v_{A(e)}(u), (1,1,\dots,1) - s_i \bullet inf\ v_{A(e)}(u)\right], \\ \left[s_i \bullet inf\ u_{A(e)}(u), s_i \bullet sup\ u_{A(e)}(u)\right] >): u \in \mathcal{U}, e \in E \end{matrix}\right\}$$

$${G_{B(e)}}^C = \left\{\begin{matrix} (e, < u, \left[s_i \bullet inf\ w_{B(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\right], \\ \left[(1,1,\dots,1) - s_i \bullet sup\ v_{B(e)}(u), (1,1,\dots,1) - s_i \bullet inf\ v_{B(e)}(u)\right], \\ \left[s_i \bullet inf\ u_{B(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\right] >): u \in \mathcal{U}, e \in E \end{matrix}\right\}$$

By using definition 3.10

$$F_{A(e)}{}^{C} \cap G_{B(e)}{}^{C} =$$

$$\left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{B(e)}(u)\}, min\{s_i \bullet sup\ w_{A(e)}(u),\ s_i \bullet up\ w_{B(e)}(u)\}], \\ [min\ \{(1,1,\dots,1) - s_i \bullet sup\ v_{A(e)}(u), (1,1,\dots,1) - s_i \bullet sup\ v_{B(e)}(u)\}, min\ \{(1,1,\dots,1) - s_i \bullet inf\ v_{A(e)}(u), (1,1,\dots,1) - s_i \bullet inf\ v_{B(e)}(u)\}] \\ [max\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\, u_{B(e)}(u)\}, max\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$F_{A(e)}{}^{C} \cap G_{B(e)}{}^{C} =$$

$$\left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ w_{A(e)}(u), s_i \bullet inf\ w_{B(e)}(u)\}, min\{s_i \bullet sup\ w_{A(e)}(u), s_i \bullet sup\ w_{B(e)}(u)\}], \\ [(1,1,\dots,1) - min\{s_i \bullet sup\ v_{A(e)}(u), s_i \bullet sup\ v_{B(e)}(u)\}, (1,1,\dots,1) - min\{s_i \bullet inf\ v_{A(e)}(u), s_i \bullet inf\ v_{B(e)}(u)\}], \\ [max\{s_i \bullet inf\ u_{A(e)}(u), s_i \bullet inf\ u_{B(e)}(u)\}, max\{s_i \bullet sup\ u_{A(e)}(u), s_i \bullet sup\ u_{B(e)}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

Hence

$(F_{A(e)} \cup G_{B(e)})^{C} = F_{A(e)}{}^{C} \cap G_{B(e)}{}^{C}$.

Proof 4, 5. Similar to assertion 1.

Proposition 3.13

Let $\mathcal{F}_{\breve{A}(e)}$, $\mathcal{G}_{\widetilde{B(e)}}$, $\mathcal{H}_{\breve{C}(e)} \in$ mPIVNSS over $\mathcal{U}$. Then

1. $\mathcal{F}_{\breve{A}(e)} \cup (\mathcal{G}_{\widetilde{B(e)}} \cap \mathcal{H}_{\breve{C}(e)}) = (\mathcal{F}_{\widetilde{A(e)}} \cup \mathcal{G}_{\widetilde{B(e)}}) \cap (\mathcal{F}_{\widetilde{A(e)}} \cup \mathcal{H}_{\widetilde{C(e)}})$
2. $\mathcal{F}_{\widetilde{A(e)}} \cap (\mathcal{G}_{\widetilde{B(e)}} \cup \mathcal{H}_{\breve{C}(e)}) = (\mathcal{F}_{\breve{A}(e)} \cap \mathcal{G}_{\widetilde{B(e)}}) \cup (\mathcal{F}_{\widetilde{A(e)}} \cap \mathcal{H}_{\breve{C}(e)})$
3. $\mathcal{F}_{\breve{A}(e)} \cup (\mathcal{F}_{\breve{A}(e)} \cap \mathcal{G}_{\widetilde{B(e)}}) = \mathcal{F}_{\widetilde{A(e)}}$
4. $\mathcal{F}_{\widetilde{A(e)}} \cap (\mathcal{F}_{\widetilde{A(e)}} \cup \mathcal{G}_{\widetilde{B(e)}}) = \mathcal{F}_{\widetilde{A(e)}}$

Proof 1 As we know that

$$\mathcal{G}_{\widetilde{B(e)}} \cap \mathcal{H}_{\widetilde{C(e)}} =$$

$$\left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ u_{\widetilde{B(e)}}(u), s_i \bullet inf\ u_{\widetilde{C(e)}}(u)\}, min\{s_i \bullet sup\ u_{\widetilde{B(e)}}(u), s_i \bullet sup\ u_{\widetilde{C(e)}}(u)\}], \\ [max\{s_i \bullet inf\ v_{\widetilde{B(e)}}(u), s_i \bullet inf\ v_{\widetilde{C(e)}}(u)\}, max\{s_i \bullet sup\ v_{\widetilde{B(e)}}(u), s_i \bullet sup\ v_{\widetilde{C(e)}}(u)\}], \\ [max\{s_i \bullet inf\ w_{\widetilde{B(e)}}(u), s_i \bullet inf\ w_{\widetilde{C(e)}}(u)\}, max\{s_i \bullet sup\ w_{\widetilde{B(e)}}(u), s_i \bullet sup\ w_{\widetilde{C(e)}}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$\mathcal{F}_{\breve{A}(e)} \cup (\mathcal{G}_{\widetilde{B(e)}} \cap \mathcal{H}_{\widetilde{C(e)}}) =$$

$$\left\{\begin{matrix} (e, < u, [max\{s_i \bullet inf\ u_{\widetilde{A(e)}}(u), min\{s_i \bullet inf\ u_{\widetilde{B(e)}}(u), s_i \bullet inf\ u_{\widetilde{C(e)}}(u)\}\}, max\{s_i \bullet sup\ u_{\widetilde{A(e)}}(u), min\{s_i \bullet sup\ u_{\widetilde{B(e)}}(u), s_i \bullet sup\ u_{\widetilde{C(e)}}(u)\}\}], \\ [min\{s_i \bullet inf\ v_{\widetilde{A(e)}}(u), max\{s_i \bullet inf\ v_{\widetilde{B(e)}}(u), s_i \bullet inf\ v_{\widetilde{C(e)}}(u)\}\}, min\{s_i \bullet sup\ v_{\widetilde{A(e)}}(u), max\{s_i \bullet sup\ v_{\widetilde{B(e)}}(u), s_i \bullet sup\ v_{\widetilde{C(e)}}(u)\}\}], \\ [min\{s_i \bullet inf\ w_{\widetilde{A(e)}}(u), max\{s_i \bullet inf\ w_{\widetilde{B(e)}}(u), s_i \bullet inf\ w_{\widetilde{C(e)}}(u)\}\}, min\{s_i \bullet sup\ w_{\widetilde{A(e)}}(u), max\{s_i \bullet sup\ w_{\widetilde{B(e)}}(u), s_i \bullet sup\ w_{\widetilde{C(e)}}(u)\}\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$\mathcal{F}_{\widetilde{A(e)}} \cap \mathcal{G}_{\widetilde{B(e)}} =$$

$$\left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ u_{\widetilde{A(e)}}(u), s_i \bullet inf\ u_{\widetilde{B(e)}}(u)\}, min\{s_i \bullet sup\ u_{\widetilde{A(e)}}(u), s_i \bullet sup\ u_{\widetilde{B(e)}}(u)\}], \\ [max\{s_i \bullet inf\ v_{\widetilde{A(e)}}(u), s_i \bullet inf\ v_{\widetilde{B(e)}}(u)\}, max\{s_i \bullet sup\ v_{\widetilde{A(e)}}(u), s_i \bullet sup\ v_{\widetilde{B(e)}}(u)\}], \\ [max\{s_i \bullet inf\ w_{\widetilde{A(e)}}(u), s_i \bullet inf\ w_{\widetilde{B(e)}}(u)\}, max\{s_i \bullet sup\ w_{\widetilde{A(e)}}(u), s_i \bullet sup\ w_{\widetilde{B(e)}}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$$\mathcal{F}_{\widetilde{A(e)}} \cap \mathcal{H}_{\widetilde{C(e)}} =$$

$$\left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ u_{\widetilde{A(e)}}(u), s_i \bullet inf\ u_{\widetilde{C(e)}}(u)\}, min\{s_i \bullet sup\ u_{\widetilde{A(e)}}(u), s_i \bullet sup\ u_{\widetilde{C(e)}}(u)\}], \\ [max\{s_i \bullet inf\ v_{\widetilde{A(e)}}(u), s_i \bullet inf\ v_{\widetilde{C(e)}}(u)\}, max\{s_i \bullet sup\ v_{\widetilde{A(e)}}(u), s_i \bullet sup\ v_{\widetilde{C(e)}}(u)\}], \\ [max\{s_i \bullet inf\ w_{\widetilde{A(e)}}(u), s_i \bullet inf\ w_{\widetilde{C(e)}}(u)\}, max\{s_i \bullet sup\ w_{\widetilde{A(e)}}(u), s_i \bullet sup\ w_{\widetilde{C(e)}}(u)\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$(\mathcal{F}_{\widetilde{A(e)}} \cap \mathcal{G}_{\widetilde{B(e)}}) \cup (\mathcal{F}_{\widetilde{A(e)}} \cap \mathcal{H}_{\widetilde{C(e)}}) =$

$$\left\{\begin{matrix} (e, < u, \begin{bmatrix} max\{min\{s_i \bullet inf\ u_{\widetilde{A(e)}}(u), s_i \bullet inf\ u_{\widetilde{B(e)}}(u)\}, min\{s_i \bullet inf\ u_{\widetilde{B(e)}}(u), s_i \bullet inf\ u_{\widetilde{C(e)}}(u)\}\}, \\ max\{min\{s_i \bullet sup\ u_{\widetilde{A(e)}}(u), s_i \bullet sup\ u_{\widetilde{B(e)}}(u)\}, min\{s_i \bullet sup\ u_{\widetilde{B(e)}}(u), s_i \bullet sup\ u_{\widetilde{C(e)}}(u)\}\}\end{bmatrix}, \\ \begin{bmatrix} min\{max\{s_i \bullet inf\ v_{\widetilde{A(e)}}(u), inf\ v_{\widetilde{B(e)}}(u)\}, max\{s_i \bullet inf\ v_{\widetilde{B(e)}}(u), s_i \bullet inf\ v_{\widetilde{C(e)}}(u)\}\}, \\ min\{max\{s_i \bullet sup\ v_{\widetilde{A(e)}}(u), s_i \bullet sup\ v_{\widetilde{B(e)}}(u)\}, max\{s_i \bullet sup\ v_{\widetilde{B(e)}}(u), s_i \bullet sup\ v_{\widetilde{C(e)}}(u)\}\}\end{bmatrix}, \\ \begin{bmatrix} min\{max\{s_i \bullet inf\ w_{\widetilde{A(e)}}(u), s_i \bullet inf\ w_{\widetilde{B(e)}}(u)\}, max\{s_i \bullet inf\ w_{\widetilde{B(e)}}(u), s_i \bullet inf\ w_{\widetilde{C(e)}}(u)\}\}, \\ min\{max\{s_i \bullet sup\ w_{\widetilde{A(e)}}(u), s_i \bullet sup\ w_{\widetilde{B(e)}}(u)\}, max\{s_i \bullet sup\ w_{\widetilde{B(e)}}(u), s_i \bullet sup\ w_{\widetilde{C(e)}}(u)\}\}\end{bmatrix} > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

$(\mathcal{F}_{\widetilde{A(e)}} \cap \mathcal{G}_{\widetilde{B(e)}}) \cup (\mathcal{F}_{\widetilde{A(e)}} \cap \mathcal{H}_{\widetilde{C(e)}}) =$

$$\left\{\begin{matrix} (e, < u, [max\{s_i \bullet inf\ u_{\widetilde{A(e)}}(u), min\{s_i \bullet inf\ u_{\widetilde{B(e)}}(u), s_i \bullet inf\ u_{\widetilde{C(e)}}(u)\}\}, max\{s_i \bullet sup\ u_{\widetilde{A(e)}}(u), min\{s_i \bullet sup\ u_{\widetilde{B(e)}}(u), s_i \bullet sup\ u_{\widetilde{C(e)}}(u)\}\}], \\ [min\{s_i \bullet inf\ v_{\widetilde{A(e)}}(u), max\{s_i \bullet inf\ v_{\widetilde{B(e)}}(u), s_i \bullet inf\ v_{\widetilde{C(e)}}(u)\}\}, min\{s_i \bullet sup\ v_{\widetilde{A(e)}}(u), max\{s_i \bullet sup\ v_{\widetilde{B(e)}}(u), s_i \bullet sup\ v_{\widetilde{C(e)}}(u)\}\}], \\ [min\{s_i \bullet inf\ w_{\widetilde{A(e)}}(u), max\{s_i \bullet inf\ w_{\widetilde{B(e)}}(u), s_i \bullet inf\ w_{\widetilde{C(e)}}(u)\}\}, min\{s_i \bullet sup\ w_{\widetilde{A(e)}}(u), max\{s_i \bullet sup\ w_{\widetilde{B(e)}}(u), s_i \bullet sup\ w_{\widetilde{C(e)}}(u)\}\}] > : u \in \mathcal{U}, e \in E) \end{matrix}\right\}$$

Hence

$\mathcal{F}_{\breve{A}(e)} \cup (\mathcal{G}_{\widetilde{B(e)}} \cap \mathcal{H}_{\breve{C}(e)}) = (\mathcal{F}_{\widetilde{A(e)}} \cup \mathcal{G}_{\widetilde{B(e)}}) \cap (\mathcal{F}_{\widetilde{A(e)}} \cup \mathcal{H}_{\widetilde{C(e)}})$.

Similarly, we can prove other results.

Definition 3.14

Let F_A, G_B $\in$ mPIVNSS, then their difference defined as follows

$F_A \setminus G_B =$

$$\left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ u_A(u), s_i \bullet inf\ u_B(u)\}, min\{s_i \bullet sup\ u_A(u), s_i \bullet sup\ u_B(u)\}], \\ [max\{s_i \bullet inf\ v_A(u),\ (1,1,\dots,1) - s_i \bullet sup\ v_B(u)\}, max\{s_i \bullet sup\ v_A(u),\ (1,1,\dots,1) - s_i \bullet inf\ v_B(u)\}], \\ [max\{s_i \bullet inf\ w_A(u), s_i \bullet inf\ w_B(u)\}, max\{s_i \bullet sup\ w_A(u), s_i \bullet sup\ w_B(u)\}] > : u \in \mathcal{U}) \end{matrix}\right\}$$

Definition 3.15

Let F_A, G_B $\in$ mPIVNSS, then their addition defined as follows

$F_A + G_B =$

$$\left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ u_A(u) + s_i \bullet inf\ u_B(u), (1,1,\dots,1)\}, min\{s_i \bullet sup\ u_A(u) + s_i \bullet sup\ u_B(u), (1,1,\dots,1)\}], \\ [min\{s_i \bullet inf\ v_A(u) + s_i \bullet inf\ v_B(u), (1,1,\dots,1)\}, min\{s_i \bullet sup\ v_A(u) + s_i \bullet sup\ v_B(u), (1,1,\dots,1)\}], \\ [min\{s_i \bullet inf\ w_A(u) + s_i \bullet inf\ w_B(u), (1,1,\dots,1)\}, min\{s_i \bullet sup\ w_A(u) + s_i \bullet sup\ w_B(u), (1,1,\dots,1)\}] > : u \in \mathcal{U}) \end{matrix}\right\}$$

Definition 3.16

Let F_A $\in$ mPIVNSS, then its scalar multiplication is represented as $F_A.\breve{a}$, where $\breve{a} \in [0, 1]$ and defined as follows

$$F_A.\breve{a} = \left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ u_A(u).\breve{a}, (1,1,\dots,1)\}, min\{s_i \bullet sup\ u_A(u).\breve{a}, (1,1,\dots,1)\}], \\ [min\{s_i \bullet inf\ v_A(u).\breve{a}, (1,1,\dots,1)\}, min\{s_i \bullet sup\ v_A(u).\breve{a}, (1,1,\dots,1)\}], \\ [min\{s_i \bullet inf\ w_A(u).\breve{a}, (1,1,\dots,1)\}, min\{s_i \bullet sup\ w_A(u).\breve{a}, (1,1,\dots,1)\}] > : u \in \mathcal{U}) \end{matrix}\right\}.$$

Definition 3.17

Let F_A $\in$ mPIVNSS, then its scalar division is represented as $F_A/\breve{a}$, where $\breve{a} \in [0, 1]$ and defined as follows

$$F_A/\breve{a} = \left\{\begin{matrix} (e, < u, [min\{s_i \bullet inf\ u_A(u)/\breve{a}, (1,1,\dots,1)\}, min\{s_i \bullet sup\ u_A(u)/\breve{a}, (1,1,\dots,1)\}], \\ [min\{s_i \bullet inf\ v_A(u)/\breve{a}, (1,1,\dots,1)\}, min\{s_i \bullet sup\ v_A(u)/\breve{a}, (1,1,\dots,1)\}], \\ [min\{s_i \bullet inf\ w_A(u)/\breve{a}, (1,1,\dots,1)\}, min\{s_i \bullet sup\ w_A(u)/\breve{a}, (1,1,\dots,1)\}] > : u \in \mathcal{U}) \end{matrix}\right\}.$$

4. Distance and Similarity Measure of Multi-Polar Interval Valued Neutrosophic Soft set

In this section, we introduce the Hamming distance and Euclidean distance between two mPIVNSS and develop the similarity measure by using these distances.

Definition 4.1

$\mathcal{U}$ and E are universal set and set of attributes respectively, assume mPIVNSS($\mathcal{U}$) represents the collection of all multi polar interval-valued neutrosophic soft sets. Suppose $(\Phi_{\mathcal{F}}, E)$ and $(\varphi_{\mathcal{G}}, E) \in$ mPIVNSS and there exist a mapping $\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}: E \to$ mPIVNSS($\mathcal{U}$), then we define the distances between $(\Phi_{\mathcal{F}}, E)$ and $(\varphi_{\mathcal{G}}, E)$ as follows

Hamming distance

$$d_H(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = \frac{1}{2m}\left\{\sum_{i=1}^{m}\sum_{j=1}^{p}\begin{matrix}\left(\left| s_i \bullet u_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet u_{\varphi_{\mathcal{G}}}(u_j)\right|\right) + \left(\left| s_i \bullet v_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet v_{\varphi_{\mathcal{G}}}(u_j)\right|\right) + \\ \left(\left| s_i \bullet w_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet w_{\varphi_{\mathcal{G}}}(u_j)\right|\right)\end{matrix}\right\} \tag{4.1}$$

Where

$$s_i \bullet u_{\Phi_{\mathcal{F}}}(u_j) = \frac{1}{2}\left(s_i \bullet inf\, u_{\Phi_{\mathcal{F}}}(u_j) + s_i \bullet sup\, u_{\Phi_{\mathcal{F}}}(u_j)\right)$$

$$s_i \bullet v_{\Phi_{\mathcal{F}}}(u_j) = \frac{1}{2}\left(s_i \bullet inf\, v_{\Phi_{\mathcal{F}}}(u_j) + s_i \bullet sup\, v_{\Phi_{\mathcal{F}}}(u_j)\right)$$

$$s_i \bullet w_{\Phi_{\mathcal{F}}}(u_j) = \frac{1}{2}\left(s_i \bullet inf\, w_{\Phi_{\mathcal{F}}}(u_j) + s_i \bullet sup\, w_{\Phi_{\mathcal{F}}}(u_j)\right)$$

$$s_i \bullet u_{\varphi_{\mathcal{G}}}(u_j) = \frac{1}{2}\left(s_i \bullet inf\, u_{\varphi_{\mathcal{G}}}(u_j) + s_i \bullet sup\, u_{\varphi_{\mathcal{G}}}(u_j)\right)$$

$$s_i \bullet v_{\varphi_{\mathcal{G}}}(u_j) = \frac{1}{2}\left(s_i \bullet inf\, v_{\varphi_{\mathcal{G}}}(u_j) + s_i \bullet sup\, v_{\varphi_{\mathcal{G}}}(u_j)\right)$$

$$s_i \bullet w_{\varphi_{\mathcal{G}}}(u_j) = \frac{1}{2}\left(s_i \bullet inf\, w_{\varphi_{\mathcal{G}}}(u_j) + s_i \bullet sup\, w_{\varphi_{\mathcal{G}}}(u_j)\right)$$

Normalized Hamming distance

$$d_{NH}(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = \frac{1}{2mp}\left\{\sum_{i=1}^{m}\sum_{j=1}^{p}\begin{matrix}\left(\left| s_i \bullet u_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet u_{\varphi_{\mathcal{G}}}(u_j)\right|\right) + \left(\left| s_i \bullet v_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet v_{\varphi_{\mathcal{G}}}(u_j)\right|\right) + \\ \left(\left| s_i \bullet w_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet w_{\varphi_{\mathcal{G}}}(u_j)\right|\right)\end{matrix}\right\} \tag{4.2}$$

Euclidean distance

$$d_E(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = \left(\frac{1}{2m}\left\{\sum_{i=1}^{m}\sum_{j=1}^{p}\begin{matrix}\left(\left| s_i \bullet u_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet u_{\varphi_{\mathcal{G}}}(u_j)\right|\right)^2 + \left(\left| s_i \bullet v_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet v_{\varphi_{\mathcal{G}}}(u_j)\right|\right)^2 + \\ \left(\left| s_i \bullet w_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet w_{\varphi_{\mathcal{G}}}(u_j)\right|\right)^2\end{matrix}\right\}\right)^{\frac{1}{2}} \tag{4.3}$$

Normalized Euclidean distance

$$d_{NE}(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = \left(\frac{1}{2mp}\left\{\sum_{i=1}^{m}\sum_{j=1}^{p}\begin{matrix}\left(\left| s_i \bullet u_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet u_{\varphi_{\mathcal{G}}}(u_j)\right|\right)^2 + \left(\left| s_i \bullet v_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet v_{\varphi_{\mathcal{G}}}(u_j)\right|\right)^2 + \\ \left(\left| s_i \bullet w_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet w_{\varphi_{\mathcal{G}}}(u_j)\right|\right)^2\end{matrix}\right\}\right)^{\frac{1}{2}} \tag{4.4}$$

Weighted distance

$d^w(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) =$

$$\left(\frac{1}{2m}\left\{\sum_{i=1}^{m}\sum_{j=1}^{p} w_i \left\{\begin{matrix}\left(|\ s_i \bullet u_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet u_{\varphi_{\mathcal{G}}}(u_j)|\ \right)^r + \left(|\ s_i \bullet v_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet v_{\varphi_{\mathcal{G}}}(u_j)|\ \right)^r + \\ \left(|\ s_i \bullet w_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet w_{\varphi_{\mathcal{G}}}(u_j)|\ \right)^r\end{matrix}\right\}\right\}\right)^{\frac{1}{r}} \quad (4.5)$$

Where $r > 0$ and $w = (w_1, w_2, w_3, \ldots, w_n)^T$ be a weight vector of e_i (i = 1, 2, 3, …, n). If r = 1 and r = 2, then equation 4.5 becomes the weighted hamming and weighted euclidean distances respectively.

Definition 4.2

$\mathcal{U}$ and E are universal set and set of attributes respectively and $(\Phi_{\mathcal{F}}, E)$, $(\varphi_{\mathcal{G}}, E)$ are two mIVNSS($\mathcal{U}$). Then similarity measure based on definition 4.1 between $(\Phi_{\mathcal{F}}, E)$ and $(\varphi_{\mathcal{G}}, E)$ defined as follows

$$S(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) = \frac{1}{1 + d(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}})} \quad (4.6)$$

Another similarity measure between $(\Phi_{\mathcal{F}}, E)$ and $(\varphi_{\mathcal{G}}, E)$ defined as

$$S(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) = e^{-\beta d(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}})} \quad (4.7)$$

Where β is a steepness measure and a positive real number.

Definition 4.3

$\mathcal{U}$ and E are universal set and set of attributes respectively and $(\Phi_{\mathcal{F}}, E)$, $(\varphi_{\mathcal{G}}, E)$ are two mIVNSS($\mathcal{U}$). Then the following distances between $(\Phi_{\mathcal{F}}, E)$ and $(\varphi_{\mathcal{G}}, E)$ defined as follows

$$d(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = \left(\frac{1}{2m}\left\{\sum_{i=1}^{m}\sum_{j=1}^{p} \begin{matrix}\left(|\ s_i \bullet u_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet u_{\varphi_{\mathcal{G}}}(u_j)|\ \right)^r + \left(|\ s_i \bullet v_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet v_{\varphi_{\mathcal{G}}}(u_j)|\ \right)^r + \\ \left(|\ s_i \bullet w_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet w_{\varphi_{\mathcal{G}}}(u_j)|\ \right)^r\end{matrix}\right\}\right)^{\frac{1}{r}}$$

(4.8)

And

$d(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) =$

$$\left(\frac{1}{2mp}\left\{\sum_{i=1}^{m}\sum_{j=1}^{p} \begin{matrix}\left(|\ s_i \bullet u_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet u_{\varphi_{\mathcal{G}}}(u_j)|\ \right)^r + \left(|\ s_i \bullet v_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet v_{\varphi_{\mathcal{G}}}(u_j)|\ \right)^r + \\ \left(|\ s_i \bullet w_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet w_{\varphi_{\mathcal{G}}}(u_j)|\ \right)^r\end{matrix}\right\}\right)^{\frac{1}{r}} \quad (4.9)$$

Where $r > 0$, equations 4.8 and 4.9 reduced to 4.1 and 4.2 respectively, if r = 1. Similarly, if r = 2 then equations 4.8 and 4.9 reduced to 4.3 and 4.4 respectively.

Definition 4.4

Similarity measure between two mIVNSS $(\Phi_{\mathcal{F}}, E)$ and $(\varphi_{\mathcal{G}}, E)$ based on the weighted distance of $(\Phi_{\mathcal{F}}, E)$ and $(\varphi_{\mathcal{G}}, E)$ defined as follows

$$S(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) = \frac{1}{1 + d^w(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}})} \quad (4.10)$$

Definition 4.5

Let $\Phi_{\mathcal{F}}$ and $\varphi_{\mathcal{G}}$ are mPIVNSS over the universal set, then $\Phi_{\mathcal{F}}$ and $\varphi_{\mathcal{G}}$ are said to be α – similar if and only if $S_{mPIVNSS}(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) \geq \alpha$ for $\alpha \in (0, 1)$. If $S_{mPIVNSS}(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) > \frac{1}{2}$, then we can say that $\Phi_{\mathcal{F}}$ and $\varphi_{\mathcal{G}}$ are significantly similar.

5. Applications of Similarity Measures of mPIVNSS in Medical Diagnoses

In this section, we proposed the algorithm for mPIVNSS by using developed similarity measures. We also used the proposed methods for medical diagnoses.

5.1. Application of Similarity Measure in Medical Diagnoses

We develop the algorithm of mPIVNSS for similarity measure and used the developed similarity measure for medical diagnoses by using the proposed algorithm.

5.1.1. Algorithm for Similarity Measure of mPIVNSS

Step 1. Pick out the set containing parameters.

Step 2. Construct the mPIVNSS according to experts.

Step 3. Construct mPIVNSS $\varphi_{\mathcal{G}}^{t}$ for the evaluation of different decision-makers, where t = 1, 2,…,m.

Step 4. Find the distance between two mPIVNSS by using the distance formula.

$$d_{mPIVNSS}^{H}(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = \frac{1}{2m}\left\{\sum_{i=1}^{m}\sum_{j=1}^{p} \begin{matrix} \left(\left| s_i \bullet u_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet u_{\varphi_{\mathcal{G}}}(u_j)\right|\right) + \left(\left| s_i \bullet v_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet v_{\varphi_{\mathcal{G}}}(u_j)\right|\right) + \\ \left(\left| s_i \bullet w_{\Phi_{\mathcal{F}}}(u_j) - s_i \bullet w_{\varphi_{\mathcal{G}}}(u_j)\right|\right) \end{matrix}\right\}$$

Step 5. Compute the similarity measure between two mPIVNSS by utilizing the following formula

$$S_{mPIVNSS}(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) = \frac{1}{1+d(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}})}$$

Step 6. Analyze the result.

5.2. Problem Formulation and Application of Similarity Measure of mPIVNSS For Disease Diagnoses

The general proposed algorithm can be used in diagnosis complications, then we are giving one numerical example containing way out those diagnosis problems in the general lighted of scientific discipline. This planned algorithm may be obtained from immoderate medical disease diagnosis complications. We consider typhoid disease as a diagnosis problem, so whether a well-advised patient has typhoid or not, as many containing the overall signs and symptoms of typhoid are going to be compatible as well as other diseases such as malaria. For a verbal description of the disease, we tend dispensed similarity measures along the mPIVNSS structure to attain an insured person as well as high-fidelity consequences. The general m-polar anatomical structure offers us a record of medical experts rating for the extraordinary disease.

5.2.1. Application of Similarity Measure

Now we assume the universal set as follows $\mathcal{U}$ = {u_1 = typhoid, u_2 = not typhoid} and E be a set of parameters which consist of symptoms of typhoid disease such as E = {x_1 = flu, x_2 = body pain, x_3 = headache}. Assume $\mathcal{F}$ and $\mathcal{G} \subseteq E$, then we construct the 3-PIVNSS of $\mathcal{F}$ and $\mathcal{G}$ such as $\Phi_{\mathcal{F}}(x)$ and $\varphi_{\mathcal{G}}(x)$ according to experts given as follows.

Table 3: 3-PIVNSS of $\mathcal{F}_{\breve{A}}$ according to experts

$\boldsymbol{\Phi_{\mathcal{F}}(x)}$	$\boldsymbol{x_1}$	$\boldsymbol{x_2}$
$\boldsymbol{u_1}$	$\begin{pmatrix}([.5,.8],[.2,.5],[.1,.2]),([.3,.5],[.1,.3],[.2,.4]),\\ ([.6,.9],[.7,.8],[.8,1])\end{pmatrix}$	$([.2,.4],[.3,0.4],[.1,.3]),([.2,.5],[.1,.6],[.1,.3]),$ $([.8,1],[.6,.9],[.6,.7])$

u_2	([.3,.6],[.1,.6],[.3,.4]),([0,.2],[.1,.4],[.3,.5]),([.5,.9],[.3,.8],[.5,.8])	([.2,.5],[.2,.3],[.5,.6]),([.3,.5],[.1,.5],[.5,.8]),([.6,.9],[.5,.8],[.6,.9])

Table 4: 3-PIVNSS of $\mathcal{G}_{\breve{B}}$ according to experts

$\varphi_{\mathcal{G}}(x)$	x_1	x_2
u_1	([.4,.8],[.3,0.6],[.2,.5]),([.2,.7],[.3,.4],[.4,.6]),([.7,.8],[.4,.9],[.5,1])	([.1,.6],[.5,0.7],[.1,.2]),([.3,.4],[.2,.5],[.2,.5]),([.5,.9],[.7,.8],[.4,.6])
u_2	([.2,.7],[.3,.5],[.2,.6]),([.1,.3],[.2,.5],[.2,.7]),([.4,.9],[.4,.7],[.5,.8])	([.1,.6],[.1,.5],[.4,.8]),([.3,.6],[.3,.4],[1,1]),([.5,.9],[.3,.7],[.1,.8])

Now we compute distances between $\Phi_{\mathcal{F}}(x)$ and $\varphi_{\mathcal{G}}(x)$ by using definition 4.1 given as follows.

$d^{H}_{3-PIVNSS}(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = 0.55$

$d^{NH}_{3-PIVNSS}(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = 0.275$

$d^{E}_{3-PIVNSS}(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = 0.31111$

$d^{NE}_{3-PIVNSS}(\Phi_{\mathcal{F}}(e), \varphi_{\mathcal{G}}(e)) = 0.22$

Now by using the above-calculated distances we will find the similarity measure between $\Phi_{\mathcal{F}}(e)$ as well as $\varphi_{\mathcal{G}}(e)$ given as follows

$S^{H}_{3-PIVNSS}(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) = 0.6452 > 0.5$

$S^{NH}_{3-PIVNSS}(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) = 0.7843 > 0.5$

$S^{E}_{3-PIVNSS}(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) = 0.7627 > 0.5$

$S^{NE}_{3-PIVNSS}(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) = 0.8197 > 0.5$

According to the above calculation analyze that $S_{3-PIVNSS}(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}) \geq 0.5$, so 3-PIVNSS of $\Phi_{\mathcal{F}}$ and $\varphi_{\mathcal{G}}$ are significantly similar which shows that the patient suffering from typhoid.

6. Conclusion

In this article, we studied IVNSS and proposed the idea of mPIVNSS with some basic operations and properties. We use attributes and numerical examples to develop some basic operators. By using Hamming distance and Euclidean distance and their characteristics, a distance-based mPIVNSS similarity measure was also developed in this research. By using the presented distance-based similarity measure, a decision-making method has been developed for mPIVNSS. Finally, the developed technique has been used in medical diagnosis. In the future, the concept of mIVPNSS will be extended to neutrosophic fuzzy soft sets, interval-valued neutrosophic fuzzy soft sets, m-polar neutrosophic fuzzy soft sets, m-polar interval neutrosophic fuzzy soft sets, etc., and will be used to solve different real-life Problems, such as medical diagnosis, decision making, etc.

References

1. Zadeh, L.A. Fuzzy Sets. *Information and Control*. **1965,** *8*, 338–353.
2. Atanassov, K. Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems.* **1986**, *20*, 87–96.
3. Molodtsov, D. Soft Set Theory First Results. *Computers & Mathematics with Applications.* **1999**, *37*, 19–31.
4. Zulqarnain, R.M.; Xin, X.L,; Saqlain.; Khan, W.A. TOPSIS Method Based on the Correlation Coefficient of Interval-Valued Intuitionistic Fuzzy Soft Sets and Aggregation Operators with Their Application in Decision-Making, Journal of Mathematics, Volume 2021, Article ID 6656858, 16 pages, https://doi.org/10.1155/2021/6656858.

5. Maji, P.K.; Biswas, R.; Roy, A.R. Soft set theory. *Computers and Mathematics with Applications.* **2003,** *45(4–5)*, 555–562.
6. Maji, P.K.; Roy, A.R.; Biswas, R. An Application of Soft Sets in A Decision Making Problem. *Computers and Mathematics with Applications*. **2002,** *44*, 1077–1083.
7. Smarandache, F. Neutrosophic set – a generalization of intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics*, **2005**, *24(3)*, 287–297.
8. Maji, P.K.; Roy, A.R.; Biswas, R. An Application of Soft Sets in A Decision Making Problem. *Computers and Mathematics with Applications*. **2002,** *44*, 1077–1083.
9. Ali, M.I.; Feng, F.; Liu, X.; Keun, W.; Shabir, M. On some new operations in soft set theory. *Computers and Mathematics with Applications*. **2009**, *57(9)*, 1547–1553.
10. Sezgin, A.; Atagun, A.O. On operations of soft sets. *Computers and Mathematics with Applications*. **2011**, *61(5)*, 1457–1467.
11. Çağman, N.; Enginoğlu, S. Soft matrix theory, and its decision making. *Computers and Mathematics with Applications.* **2010,** *59(10)*, 3308–3314.
12. Çağman, N.; Enginoğlu, S. Soft set theory and uni – int decision making. *European Journal of Operational Research*. **2010,** *207*, 848–855.
13. Atag, O.; Ayg, E. Difference Operations of Soft Matrices with Applications in Decision Making *Punjab University Journal of Mathematics*. **2019,** *51(3)*, 1–21.
14. Zulqarnain, R.M,; Xin, X.L,; Siddique, I;, Asghar Khan, W.; Yousif, M. A. TOPSIS Method Based on Correlation Coefficient under Pythagorean Fuzzy Soft Environment and Its Application towards Green Supply Chain Management. Sustainability 2021, 13, 1642. https://doi.org/10.3390/su13041642.
15. Zulqarnain, R.M,; Xin, X.L,; Garg H.; Siddique, I;, Asghar Khan, W, Aggregation operators of pythagorean fuzzy soft sets with their application for green supplier chain management, Journal of Intelligent and Fuzzy Systems, (2021) 40 (3), 5545-5563. DOI: 10.3233/JIFS-202781.
16. Maji, P.K. Neutrosophic soft set. *Annals of Fuzzy Mathematics and Informatics*. **2013,** *5(1)*, 157–168.
17. Karaaslan, F. Possibility neutrosophic soft sets and PNS-decision making method. *Applied Soft Computing Journal*. **2016**, *54*, 403–414.
18. Broumi, S. Generalized Neutrosophic Soft Set. *International Journal of Computer Science, Engineering and Information Technology*. **2013,** *3(2)*, 17–30.
19. Deli, I.; Şubaş, Y. A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. *Int. J. Mach. Learn. & Cyber.* **2017**, *8*, 1309–1322.
20. Wang, H.; Smarandache, F.; Zhang, Y. Single valued neutrosophic sets. *Int. J. Gen. Syst*, **2013**, *42*, 386–394.
21. Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent and Fuzzy Systems.* **2014,** *26*, 2459–2466.
22. Anjan, M.; Rakhal, D, Neutrosophic Bipolar Vague Soft Set and Its Application to decision Making Problems Neutrosophic sets and sysyems, 2020 (32), 410-425.
23. Zulqarnain, R.M.; Xin, X.L.; Saqlain, M.; Smarandache, F.; Ahamad, M.I, An integrated model of Neutrosophic TOPSIS with application in Multi-Criteria Decision-Making Problem. Neutrosophic Sets and Systems, 40,118-133, (2021).
24. Zulqarnain, R. M, Xin, X. L., Saeed, M., Smarandache, F., Ahmad, N. Generalized Neutrosophic TOPSIS to Solve Multi-Criteria Decision-Making Problems, Neutrosophic Sets and Systems, 38, 276-292, (2020).
25. Hashmi, M.R.; Riaz, M.; Smarandache, F. m-Polar Neutrosophic Topology with Applications to Multi criteria Decision-Making in Medical Diagnosis and Clustering Analysis. *Int. J. Fuzzy Syst*, **2019,** Dec (2019).
26. Garg, H. An improved cosine similarity measure for intuitionistic fuzzy sets and their applications to decision-making process. *Hacettepe Journal of Mathematics and Statistics*. 2018, 47(6), 1578-1594.
27. Garg, H.; Kumar, K. An advanced study on the similarity measures of intuitionistic fuzzy sets based on the set pair analysis theory and their application in decision making. *Soft Computing*. 2018, https://doi.org/10.1007/s00500-018-3202-1.
28. Nguyen, X.T.; Nguyen, V.D.; Nguyen, V.H.; Garg, H. Exponential similarity measures for Pythagorean fuzzy sets and their applications to pattern recognition and decision-making process. *Complex & Intelligent Systems*, 2019, 5, 217–228.
29. Peng, X.; Garg, H. Multiparametric similarity measures on Pythagorean fuzzy sets with applications to pattern recognition. *Applied Intelligence*. 2019, https://doi.org/10.1007/s10489-019-01445-0.

30. Saeed, M.; Saqlain, M.; Mehmood, A.; Naseer, K.; Yaqoob, S. Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making. *Neutrosophic Sets and Systems*. 2020, 33, 183-207.
31. F. Smarandache, 2018. Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set. Neutrosophic Sets and Systems, 22 (2018), 168-170.
32. Zulqarnain, R. M., Xin, X. L., Saeed, M. Extension of TOPSIS method under intuitionistic fuzzy hypersoft environment based on correlation coefficient and aggregation operators to solve decision making problem, AIMS Mathematics, 6 (3), 2732-2755, (2021).
33. Zulqarnain, R. M., Xin, X. L., Ali, B., Broumi, S., Abdal, S., Ahamad, M. I. Decision-Making Approach Based on Correlation Coefficient with its Properties Under Interval-Valued Neutrosophic hypersoft set environment. Neutrosophic Sets and Systems, 40,12-28, (2021).
34. Gayen, S. Smarandache, F., Jha, S., Singh, M. K., Broumi, S., Kumar, R. (2020). Introduction to Plithogenic Subgroup. In Neutrosophic Graph Theory and Algorithms (pp. 213-259). IGI Global.
35. Zulqarnain, R. M., Xin, X. L., Saqlain, M., Smarandache, F. (2020), Generalized Aggregate Operators on Neutrosophic Hypersoft Set, Neutrosophic Sets and Systems, 36, 271-281.
36. Zulqarnain, R. M., Siddique, I., Ali, R., Jarad, R., Samad, A., Abdeljawad, T. Neutrosophic Hypersoft Matrices with Application to Solve Multiattributive Decision-Making Problems, Volume 2021, Article ID 5589874, 1-17, https://doi.org/10.1155/2021/5589874.
37. Zulqarnain, R. M., Xin, X. L., Saqlain, M., Saeed, M., Smarandache, F., Ahamad, M. I. Some Fundamental Operations on Interval Valued Neutrosophic Hypersoft Set with Their Properties. Neutrosophic Sets and Systems, 40,134-148, (2021).
38. Zulqarnain, R. M., Siddique, I., Ali, R., Pamucar, D., Marinkovic, D., Bozanic, D. Robust Aggregation Operators for Intuitionistic Fuzzy Hyper-soft Set With Their Application to Solve MCDM Problem, Entropy 2021, 23, 688. https://doi.org/10.3390/e23060688.
39. Abbas, M., Murtaza, G., Smarandache, F. Basic operations on hypersoft sets and hypersoft point, Neutrosophic Sets and Systems, Vol. 35, 2020, 407-421.
40. Zulqarnain, R. M., Saddique, I., Jarad, F., Ali, R., Abdeljawad, T. Development of TOPSIS Technique Under Pythagorean Fuzzy Hypersoft Environment Based on Correlation Coefficient and Its Application Towards the Selection of Antivirus Mask in COVID-19 Pandemic. Complexity, Volume 2021, Article ID 6634991, 27 pages, https://doi.org/10.1155/2021/6634991.
41. Martin, N., Smarandache, F., Introduction to Combined Plithogenic Hypersoft Sets, Neutrosophic Sets and Systems, Vol. 35, 2020, 503-510.
42. Zulqarnain, R. M., Xin, X. L., Saeed, M. A Development of Pythagorean fuzzy hypersoft set with basic operations and decision-making approach based on the correlation coefficient, Theory and Application of Hypersoft Set, Publisher: Pons Publishing House Brussels, 2021, 85-106.
43. Samad, A., Zulqarnain, R. M., Sermutlu, E., Ali, R., Siddique, I., Jarad, F., Abdeljawad, T. Selection of an Effective Hand Sanitizer to Reduce COVID-19 Effects and Extension of TOPSIS Technique Based on Correlation Coefficient under Neutrosophic Hypersoft Set, Complexity, 2021, Article ID 5531830, 1-22. https://doi.org/10.1155/2021/5531830.
44. Zulqarnain, R. M., Siddique, I., Ali, R., Jarad, F., Iampan, A. Multi Criteria Decision Making Approach For Pythagorean Fuzzy Hypersoft Sets Interaction Aggregation Operators, Mathematical Problems in Engineering, Volume 2021, https://doi.org/10.1155/2021/9964492.
45. Deli, I.; Interval-valued neutrosophic soft sets and its decision making. *Int. J. Mach. Learn. & Cyber*. DOI 10.1007/s13042-015-0461-3.
46. Aiwua, Z.; Jianguoa, D.; Hongjun, G. Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregation operator, Journal of Intelligent & Fuzzy Systems., 29 (2015), 2697–2706.

Received: Aug 17, 2021. Accepted: Dec 3, 2021