





A multi-criteria fuzzy neutrosophic decision-making model for solving the supply chain network problem

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Abstract: Supply Chain is a multi-objective decision-making problem with multiple conflicting objective functions related to each supply chain operation and its corresponding sub-criteria. The main focus of this paper is the development of a model that takes into account some important components of real-world supply chain planning. To do so, we proposed a supply chain model that involves multiple suppliers, multiple plants, multiple warehouses, and multiple distributors firms. This approach is designed to tackle a complex multi-site composite supply chain issue under uncertainty as a fuzzy multi-objective model with the primary objective to optimize the transportation cost and delivery time simultaneously. We have used neutrosophical set theory to tackle the ambiguity related to supply chain by using truth, indeterminacy and falsity membership functions and, finally neutrosophical compromise programming approach has been used for obtaining the desired solution. In order to demonstrate the efficiency of the developed models, an industrial design problems has been given. The findings reported is compared to other well-known approaches.

Keywords: Supply Chain; Multi-objective Optimization; Neutrosophic Set.

1. Introduction

Supply Chain (SC) network optimization plays a crucial role in assessing the performance of the whole SC. The challenge with the SC layout consists of determining when and how to distribute equipment (plants, factories, distribution centres) and how to transfer material (raw materials, components, finished products) through the network of organizations (suppliers, producers, sellers, retailers and customers) to maximize overall efficiency (Nurjanni et al. [1]). SC is a network of factories processing raw materials, converting them into intermediate products and then finished products, and supplying the products via a delivery chain to customers. SC's fundamental goal is to "optimize chain efficiency and provide as much benefit as possible with as little expense as possible". In other words, it seeks to unite all the representatives in the SC to work together within the organization as a way to optimize efficiency in the SC and provide the maximum value to all relevant parties. If a company buys raw materials for use in the manufacture of a product, it then sells them to customers, which means that the organization has an SC, which it must manage afterwards. Companies face difficulties in seeking solutions to satisfy ever-increasing consumer demands and stay successful in the markets while maintaining expenses controlled. SC includes handling of a number of tasks related to the arranging, scheduling and monitoring of the flow of supplies, components and products; maintaining inventories of acquired components and packaging issues; reasonable and cost-effective storage of products; and, ultimately, delivering them to the consumer (Khan et al. [2]). Effective governance of SC needs continuous enhancement at both the level at customer support and the internal operational efficiencies of the SC firms. In the most simplistic point, customer support involves reliably adequate order fill levels, strong on-time fulfilment levels and a relatively small number of goods returned by consumers for whatever reason. Internal productivity for SC companies ensures that such entities get an acceptable rate of return for their product and other resource expenditures (Hugos [3]). Mathematical programming frameworks have been commonly used to evaluate and improve SC efficiency, and it could play a significant role in the creation of alternatives to complex SC design.

The neutrosophic set is considered as a generalization of the intuitionistic fuzzy set. While fuzzy sets use true and false for express relationship, neutrosophic sets uses three different types of membership functions (Smarandache [4]). The neutrosophical set has three membership functions, i.e., maximizing truth (belonging), indeterminacy (partly belonging) and reducing falsity (nonbelonging) effectively. The neutrosophical programming approach was developed and widely utilized in real-life applications based on the neutrosophical set. Gamal et al. [5] used neutrosophic set theory in supplier selection to overcome the situation when the decision makers might have restricted knowledge or different opinions, and to specify deterministic valuation values to comparison judgments. Later on, Abdel-Baset et al. [6] proposed an advanced type of neutrosophic technique, called type 2 neutrosophic numbers for the supplier selection problem.

Motivated by different studies in supply chain and neutrosophic programming, which is being a new research area with the potential to capture the decision-makers truth, indeterminacy and falsity goals, we have formulated the mathematical model of supply chain under neutrosophic environment. The objective of this study is to offer SC with a more realistic context for achieving better results in the context of uncertainty. In addition, the neutrosophical compromise programming approach does not just focus on maximizing and minimizing the satisfaction and dissatisfaction of the decision makers, but also on optimizing the degree of satisfaction related to indeterminacy. Moreover, the developed approach is also compared with simple additive, weighted additive and pre-emptive goal programming approaches, to show the efficacy of the proposed methodology.

This paper consists of six sections: the current segment presenting an introduction to the study problem. Section 2 describes relevant work on this topic. Section 3 explains the structure of the SC model. The technique of the solution to solving the problem is discussed in Section 4. Section 5 describes the implementation of the theoretical model to a case study, and Section 6 concludes with the analysis and future directions of research in this area.

2. Literature Review

The literature review undertaken in the framework of this study allowed us to find out a gap in SC optimization. To the extent of our understanding, there is a limited number of research work discussing neutrosophicity utilizing a multiobjective optimization to tackle trade-offs between overall transportation cost and total delivery time in SC. The literature review discussing the issues of transportation and distribution planning constructed as a single and multiobjective model and solved using a complicated approach to optimization.

Badhotiya et al. [7] tackle the issue of distribution, manufacturing and delivery planning for a two-echelon SC, composed of several producers that supplying to different sales locations and formulated it as a multi-objective model. Further ambiguity and imprecision were regarded in the problem, and a fuzzy multi-objective optimization technique was applied that simultaneously optimizes three objectives; total cost, total delivery time, and backorder amount. Rabbani et al. [8] considered a closed-loop SC that involved a logistics supplier for a producer, a dealer and a third party. Three tri-level leader-follower Stackelberg game models have been introduced to explore how a producer can do remanufacturing or pay a product license charge for retailers and partner with them in remanufacturing. Modak and Kelle [9] identified the double-channel SC with contingent

stochastic consumer demand under price and distribution period, and the findings indicated that market volatility influences the optimal price and lead time. Sharahi et al. [10] dealt with the issue of location-allocation and delivery of output in an SC of three echelons. Type-II fuzzy sets theory were used to model uncertainty in supply, operation, and demand. Gholamian et al. [11] proposed a mathematical model for production planning by considering the majority of SC expenses parameters, such as cost of shipping, cost of inventory holding, cost of shortage, cost of processing and associated human costs under uncertainty of demand, and formulated it using a complex multi-objective model of optimization. Kristianto et al. [12] suggested a two-stage model with the goal of improving product distribution and transportation when adjustments have disrupted the SC network as a consequence of a catastrophe or market shift. They implemented the methodology of decomposition to transform the problem into the shortest problem of the fuzzy path. Bilgen [13] tackled the problem of fuzzy centralized manufacturing and delivery plans underneath a packaged products company's SC network. Vagueness in the objective function and capacity restrictions is replicated by Zimmermann's [14] linear membership function approach. Three separate aggregation operators were introduced to transform the Fuzzy model into a crisp one.

Current neutrosophic literature shows that a limited number of authors have taken an interest in this framework, and this is expected to be a significant new area of research in the future. Kar et al. [15] proposed a neutrosophic optimization technique for a shortage-free inventory model where the cost of output is inversely proportional to the set-up costs and the volume of supply. A neutrosophical fuzzy programming method (NFPA) focused on the neutrosophic decision was suggested by Ahmad et al. [16] to solve the proposed SC design problem. The developed SC network has been built for various multi-product raw materials/parts, and multi-echelons together with single time horizons. To identify the activities contributing to improving the economic and environmental performance, Abdel-Baset et al. [17] tested green SC activities using the robust rating with neutrosophic set theory. The feasibility of the new approach is measured using the two different types of case studies, i.e., Egypt's petroleum sector and China's manufacturing company. As a technique to solve multi-criteria decision-making in green supplier selection problems, Liang et al. [18] suggested single-valued trapezoidal neutrosophic choice relations. In the neutrosophical framework, Thamaraiselvi and Santhi [19] developed the mathematical representation of a transportation problem. Abdel-Baset et al. [20] addressed the complexities of the issue, increasing awareness among healthcare sector experts, and assessing smart medical devices according to specific assessment requirements. In the decisionmaking process, neutrosophics with TOPSIS methodology was implemented to cope with the vagueness, and ambiguity, by taking into consideration the decision conditions in the evidence gathered by the decision-makers. Liang et al. [21] established a novel fuzzy-based method for assessing B2C e-commerce websites and defined interrelationships and prioritized orders within parameters through integrating single-valued neutrosophic trapezoidal numbers with DEMATEL methodology. Some recent works related to the use neutrosophic includes, Abdel-Basset et al. [22] suggested a novel hybrid methodology for the selection of the offshore wind power plant location integrating the two distinct forms of MCDM approaches in the neutrosophic environment. Also, by use of MCDM model, Abdel-Basset et al. [23] has conducted a comprehensive sustainability assessment of the hydrogen generation possibilities.

Practical alignment of transportation and distribution planning in SC frequently requires trade-offs with multiple conflicting priorities that need to be balanced by the decision-maker at the same time. Owing to many reasons such as variability in human behavior, shifting environmental circumstances, and unavailability or inappropriate knowledge, these objective roles are sometimes fuzzy or uncertain. This study introduces a complex multi-objective programming framework to address the SC problem including multiple locations and different time periods, then illustrates the same on a real-life manufacturing problem to validate the accuracy of the developed model. The benefit of implementing fuzzy set theory is that it helps the decision-maker to calculate an imprecise expectation.

3. Mathematical Model

According to Nurjanni et al. [1], SCM is "A set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs while satisfying service level requirements." Charles et al. [24] presented the demand-and-supply-rooted concept of ambiguity with the constrained multiobjective optimization framework and established a fuzzy goal programming approach to solve it. To achieve the desired solution, the proposed model was solved through three separate approaches, including simple additive goal programming, weighted goal programming, and pre-emptive goal programming approaches respectively. Gupta et al. [25] presented an effective goal programming methodology to solve the SC problem in order to concurrently reduce overall shipping costs and total production period related to inventory volumes, initial stock available at each source, as well as customer demand and usable storage capacity at each destination, and restrictions on total expenditure in an uncertain environment. Gupta et al. [26] presented the problem of the SC network as a bi-level programming problem in which the primary goal is to decide the optimum order allocation of goods where the requirements of the consumer and the availability for the items are elastic. Motivated by such studies in SC, we have formulated the multi-objective SC model and the following notations have been used for the model formulation which are listed below:

The nomenclature for the notations and terms used in the design of the model is as follows:

Indices

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i – Multiple suppliers indices, (i=1, 2, ...,I);
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j – Multiple plants indices, (j=1, 2, ...,J);

k – Multiple warehouses indices, (k=1, 2, ...,K);

l – Multiple distributors indices, (l=1, 2, ...,L);

t — Objective function indices, (t=1, 2, ...,T);

Parameters

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SCS_i – Supply capacity of the i^{th} suppliers (in '000),
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 PCP_i – Potential capacity of the j^{th} plants (in'000),

 PCW_k - Potential capacity of the k^{th} warehouses (in '000),

 ADR_{l} – Annual demand from the l^{th} distributors (in '000),

 CSP_{ij} — Cost of shipping one unit from the supply suppliers i to the plant j, (in '000),

 CPW_{ik} – Cost of producing and shipping one unit from the plant j to the warehouse k, (in '000),

 CPR_{il} – Cost of producing and shipping one unit from the plant j to the distributors l, (in '000),

 CWR_{kl} – Cost of shipping one unit from the warehouse k to the distributors l, (in '000),

 TPW_{jk} – Delivery time of shipping one unit from the plant j to the warehouse k (in Hrs),

 TPR_{il} – Delivery time of shipping one unit from the plant j to the distributors l (in Hrs),

 TWR_{kl} – Delivery time of shipping one unit from the warehouse k to the distributors l (in Hrs),

Decision variables

 W_{ii} – Quantity shipped from the supply suppliers i to the plant j

 X_{jk} – Quantity shipped from the plant j to the warehouse k

 Y_{il} – Quantity shipped from the plant j to the distributors i

Z_{kl} – Quantity shipped from the warehouse k to the distributors i

The mathematical model of multi-objective SC problem formulated in the case of a deterministic situation by using the notations mentioned above as:

The 1^{st} objective function, which helps in the optimization of the SC shipping costs, is given by: Minimize

$$F_{1} = \sum_{i=1}^{I} \sum_{j=1}^{J} CSP_{ij}W_{ij} + \sum_{j=1}^{J} \sum_{k=1}^{K} CPW_{jk}X_{jk} + \sum_{j=1}^{J} \sum_{l=1}^{L} CPR_{jl}Y_{jl} + \sum_{k=1}^{K} \sum_{l=1}^{L} CWR_{kl}Z_{kl}$$

$$(1)$$

The 2^{nd} objective function, which helps in the optimization of the SC delivery time, is given by: Minimize

$$F_{2} = \sum_{j=1}^{J} \sum_{k=1}^{K} TPW_{jk} X_{jk} + \sum_{j=1}^{J} \sum_{l=1}^{L} TPR_{jl} Y_{jl} + \sum_{k=1}^{K} \sum_{l=1}^{L} TWR_{kl} Z_{kl}$$
 (2)

Subject to

Constraint I is related to the overall volume of the product to be delivered from the supplier to the plant.

$$\sum_{i=1}^{J} W_{ij} \le SCS_i \tag{3}$$

Constraint II is concerned with the quantity produced at the plant, which cannot surpass its efficiency.

$$\sum_{l=1}^{L} Y_{jl} + \sum_{k=1}^{K} X_{jk} \le PCP_{j} \tag{4}$$

Constraint III is concerned with the volume to be delivered via the various warehouses that cannot surpass its efficiency.

$$\sum_{l=1}^{L} Z_{kl} \le PCW_k \tag{5}$$

Constraint IV is concerned about the volume to be delivered to the distributors, which will meet the demand of the consumer.

$$\sum_{k=1}^{K} Z_{kl} + \sum_{j=1}^{J} Y_{jl} \le ADR_{l} \tag{6}$$

Constraint V is concerned with the total quantity delivered to the warehouse and distributors from the plant, which cannot surpass the quantity of the obtained materials.

$$\sum_{i=1}^{I} W_{ij} \ge \sum_{k=1}^{K} X_{jk} + \sum_{l=1}^{L} Y_{jl} \tag{7}$$

Constraint VI is concerned with the volume delivered to the distributors from the warehouse, which cannot surpass their capacity.

$$\sum_{j=1}^{J} X_{jk} \ge \sum_{l=1}^{L} Z_{kl} \tag{8}$$

with non-negative restriction:

$$W_{ii} \geq 0, \forall i, j$$

$$X_{ik} \geq 0, \forall j, k$$

$$Y_{il} \geq 0, \forall j, l$$

$$Z_{kl} \geq 0, \forall k, l$$

The multi-objective optimization model of SC can be mathematically formulated as follows by combining all the objective functions and constraints, are combined:

Model 1

$$\begin{aligned} & \text{Minimize } F_1 = \sum\nolimits_{i=1}^{I} \sum\nolimits_{j=1}^{J} CSP_{ij}W_{ij} + \sum\nolimits_{j=1}^{J} \sum\nolimits_{k=1}^{K} CPW_{jk}X_{jk} + \sum\nolimits_{j=1}^{J} \sum\nolimits_{l=1}^{L} CPR_{jl}Y_{jl} + \sum\nolimits_{k=1}^{K} \sum\nolimits_{l=1}^{L} CWR_{kl}Z_{kl} \\ & \text{Minimize } F_2 = \sum\nolimits_{j=1}^{J} \sum\nolimits_{k=1}^{K} TPW_{jk}X_{jk} + \sum\nolimits_{j=1}^{J} \sum\nolimits_{l=1}^{L} TPR_{jl}Y_{jl} + \sum\nolimits_{k=1}^{K} \sum\nolimits_{l=1}^{L} TWR_{kl}Z_{kl} \end{aligned}$$

Subject to

$$\begin{split} & \sum\nolimits_{j=1}^{J} W_{ij} \leq SCS_{i} \\ & \sum\nolimits_{l=1}^{L} Y_{jl} + \sum\nolimits_{k=1}^{K} X_{jk} \leq PCP_{j} \\ & \sum\nolimits_{l=1}^{L} Z_{kl} \leq PCW_{k} \\ & \sum\nolimits_{k=1}^{K} Z_{kl} + \sum\nolimits_{j=1}^{J} Y_{jl} \leq ADR_{l} \\ & \sum\nolimits_{i=1}^{I} W_{ij} \geq \sum\nolimits_{k=1}^{K} X_{jk} + \sum\nolimits_{l=1}^{L} Y_{jl} \\ & \sum\nolimits_{j=1}^{J} X_{jk} \geq \sum\nolimits_{l=1}^{L} Z_{kl} \\ & W_{ij} \geq 0, \forall i, j \\ & X_{jk} \geq 0, \forall j, k \\ & Y_{jl} \geq 0, \forall j, l \\ & Z_{kl} \geq 0, \forall k, l \end{split}$$

3.1 Uncertain Model

The model formulated above has been developed when the decision-maker knows the exact value of each parameter being considered. Due to sudden increases in prices of raw materials, higher gasoline costs, higher deployment sites, fluctuating consumer behavior, rivalry amongst customer service policies of various firms, environmental factors, inability to supply requested goods in a timely manner, political and government decisions on specific taxes on purchase, development, delivery end-of-use stock management are the most influential factors creating uncertainty in SC. In the past many methods were suggested to cope with the environment of ambiguity. Zadeh's [27] fuzzy sets (FS) just allow membership function and can't accommodate certain vagueness parameters. In order to address this knowledge deficit, Atanassov [28] proposed an expansion to fuzzy sets called intuitionistic fuzzy sets (IFS). Though IFS theory can accommodate missing knowledge for specific real-world problems, it cannot solve all forms of ambiguity such as contradictory and indeterminate proof. Therefore, the neutrosophic set (NS) was developed by Smarandache [29] as a comprehensive composition that generalizes classical theory of all forms of FS. NS can handle indefinite, vague and conflicting information where the indeterminacy is explicitly quantified, and can separately identify the three forms of membership functions. Furthermore, with such assumptions of uncertainty, Model 1 with uncertain parameters could be reformulated as:

Model 2

$$\begin{split} & \text{Minimize } F_1 = \sum\nolimits_{i=1}^{I} \sum\nolimits_{j=1}^{J} \tilde{CSP_{ij}} \, W_{ij} + \sum\nolimits_{j=1}^{J} \sum\nolimits_{k=1}^{K} \tilde{CPW}_{jk} \, X_{jk} + \sum\nolimits_{j=1}^{J} \sum\nolimits_{l=1}^{L} \tilde{CPR}_{jl} \, Y_{jl} + \sum\nolimits_{k=1}^{K} \sum\nolimits_{l=1}^{L} \tilde{CWR}_{kl} \, Z_{kl} \\ & \text{Minimize } F_2 = \sum\nolimits_{j=1}^{J} \sum\nolimits_{k=1}^{K} \tilde{TPW}_{jk} \, X_{jk} + \sum\nolimits_{j=1}^{J} \sum\nolimits_{l=1}^{L} \tilde{TPR}_{jl} \, Y_{jl} + \sum\nolimits_{k=1}^{K} \sum\nolimits_{l=1}^{L} \tilde{TWR}_{kl} \, Z_{kl} \\ & \text{Subject to} \end{split}$$

$$\begin{split} & \sum_{j=1}^{J} W_{ij} \leq \tilde{SCS}_{i} \\ & \sum_{l=1}^{L} Y_{jl} + \sum_{k=1}^{K} X_{jk} \leq PCP_{j} \\ & \sum_{l=1}^{L} Z_{kl} \leq PCW_{k} \\ & \sum_{k=1}^{K} Z_{kl} + \sum_{j=1}^{J} Y_{jl} \leq \tilde{ADR}_{l} \\ & \sum_{i=1}^{I} W_{ij} \geq \sum_{k=1}^{K} X_{jk} + \sum_{l=1}^{L} Y_{jl} \\ & \sum_{j=1}^{J} X_{jk} \geq \sum_{l=1}^{L} Z_{kl} \\ & W_{ij} \geq 0, \forall i, j \\ & X_{jk} \geq 0, \forall j, k \\ & Y_{jl} \geq 0, \forall j, l \\ & Z_{kl} \geq 0, \forall k, l \end{split}$$

where, the uncertain parameters $\tilde{CSP}, \tilde{CPW}, \tilde{CPR}, \tilde{CWR}, TPW, TPR, TWR, SCS$ and \tilde{ADR} are assumed to hold the neutrosophic sets assumptions (detail see Liang et al. [18]). Let us assumed that $\delta_{\tilde{CSP}}$, $\varphi_{\tilde{CSP}}$, $\varphi_{\tilde{C$

single-value triangular neutrosophic number $\tilde{CSP} = \left((CSP_1, CSP_2, CSP_3); \delta_{\tilde{CSP}}, \varphi_{\tilde{CSP}}, \gamma_{\tilde{CSP}} \right)$ is a

special neutrosophic set on the real line set \Re , whose truth-membership, indeterminacy-membership, and falsity-membership functions are given as follows:

$$\mu_{\tilde{CSP}}(CSP) = \begin{cases} \delta_{\tilde{CSP}} \frac{(CSP - CSP_1)}{(CSP_2 - CSP_1)}, & CSP_1 \leq CSP \leq CSP_2 \\ \delta_{\tilde{CSP}}, & CSP = CSP_2 \\ \delta_{\tilde{CSP}} \frac{(CSP_3 - CSP)}{(CSP_3 - CSP_2)}, & CSP_2 \leq CSP \leq CSP_3 \\ 0, & otherwise \end{cases}$$

$$(9)$$

$$\theta_{\tilde{CSP}}(CSP) = \begin{cases} \frac{(CSP_2 - CSP + \varphi_{\tilde{CSP}}(CSP - CSP_1))}{(CSP_2 - CSP_1)}, & CSP_1 \leq CSP \leq CSP_2 \\ \varphi_{\tilde{CSP}}, & CSP = CSP_2 \\ \frac{(CSP - CSP_2 + \varphi_{\tilde{CSP}}(CSP_3 - CSP))}{(CSP_3 - CSP_2)}, & CSP_2 \leq CSP \leq CSP_3 \\ 0, & otherwise \end{cases}$$

$$(10)$$

$$\psi_{\tilde{CSP}}(CSP) = \begin{cases} \frac{(CSP_2 - CSP + \gamma_{\tilde{CSP}}(CSP - CSP_1))}{(CSP_2 - CSP_1)}, & CSP_1 \leq CSP \leq CSP_2 \\ \gamma_{\tilde{CSP}}, & CSP = CSP_2 \\ \frac{(CSP - CSP_2 + \gamma_{\tilde{CSP}}(CSP_3 - CSP))}{(CSP_3 - CSP_2)}, & CSP_2 \leq CSP \leq CSP_3 \\ 0, & otherwise \end{cases}$$
where $\delta_{\tilde{CSP}}, \varphi_{\tilde{CSP}}, \gamma_{\tilde{CSP}}$ denote the maximum truth-membership degree, minimum regionary membership, degree, and minimum falcity membership, degree, respectively. A

where $\delta_{\tilde{CSP}}$, $\varphi_{\tilde{CSP}}$, $\gamma_{\tilde{CSP}}$ denote the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree, respectively. A single-valued triangular neutrosophic number $\tilde{CSP} = \left((CSP_1, CSP_2, CSP_3); \delta_{\tilde{CSP}}, \varphi_{\tilde{CSP}}, \gamma_{\tilde{CSP}}\right)$ may express an ill-defined quantity about CSP, which is approximately equal to CSP. Then, the score function for the \tilde{CSP} is obtained by using the equation (12), which is given below:

$$S(\tilde{CSP}) = \frac{1}{16}(CSP_1 + CSP_2 + CSP_3) \times (2 + \delta_{\tilde{CSP}} - \varphi_{\tilde{CSP}} - \gamma_{\tilde{CSP}})$$
(12)

The same holds for other uncertain parameters.

3.2 Neutrosophic Compromise Programming

An approach to solving the multi-optimization problem has been implemented based on the NS principle. The neutrosophical compromise goal programming solution is based on the principle of NS, which consists of optimization of three membership functions such as optimizing the degree of truth and indeterminacy and decreasing the extent of falsity membership. Firstly, the bounds for each objective function have been defined to construct the three different types of membership functions for the formulated multi-objective SC problem. The upper U_t , $\forall t$ and lower L_t , $\forall t$ values for the neutrosophical problem for case minimization have therefore been determined as:

$$\begin{aligned} \boldsymbol{U}_{t}^{T} &= \boldsymbol{U}_{t}, \ \boldsymbol{L}_{t}^{T} &= \boldsymbol{L}_{t}, \ \forall t & \text{for truth membership} \\ \boldsymbol{U}_{t}^{I} &= \boldsymbol{L}_{t}^{T} + \boldsymbol{q}_{t}^{T}(\boldsymbol{U}_{t}^{T} - \boldsymbol{L}_{t}^{T}), \boldsymbol{L}_{t}^{I} &= \boldsymbol{L}_{t}^{T}, \ \forall t & \text{for Indeterminacy membership} \\ \boldsymbol{U}_{t}^{F} &= \boldsymbol{U}_{t}^{T}, \ \boldsymbol{L}_{t}^{F} &= \boldsymbol{L}_{t}^{T} + \boldsymbol{q}_{t}(\boldsymbol{U}_{t}^{T} - \boldsymbol{L}_{t}^{T}), \ \forall t & \text{for falsity membership} \end{aligned}$$

Where q_t and q_t are sensitivity variables for falsity and indeterminacy membership functions shall be selected by the decision-maker, and based on these sensitivity variables, the three different types of membership function for the neutrosophical problem can be constructed as follows:

$$\mu_{t}^{T} = \begin{cases} 1, & F_{t} \leq L_{t}^{T} \\ \frac{U_{t}^{T} - F_{t}}{U_{t}^{T} - L_{t}^{T}}, & L_{t}^{T} \leq F_{t} \leq U_{t}^{T} \\ 0, & F_{t} \geq U_{t}^{T} \end{cases}$$

$$\sigma_{t}^{I} = \begin{cases} 1, & F_{t} \leq L_{t}^{I} \\ \frac{U_{t}^{I} - F_{t}}{U_{t}^{I} - L_{t}^{I}}, & L_{t}^{I} \leq F_{t} \leq U_{t}^{I} \\ 0, & F_{t} \geq U_{t}^{I} \end{cases}$$

$$\upsilon_{t}^{F} = \begin{cases} 0, & F_{t} \leq L_{t}^{F} \\ \frac{F_{t} - L_{t}^{F}}{U_{t}^{F} - L_{t}^{F}}, & L_{t}^{F} \leq F_{t} \leq U_{t}^{F} \\ 1, & F_{t} \geq U_{t}^{F} \end{cases}$$

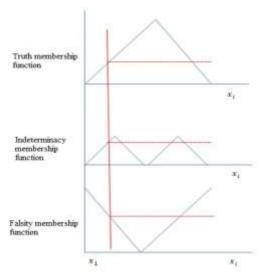


Fig. 1. Neutrosophication Process

Where, we trying to maximize the Truth (μ_t^T) and Indeterminacy (σ_t^I) membership functions; and also trying to minimize the falsity (υ_t^F) membership functions. Following the optimization process introduced by (Bellman and Zadeh, [30]; Rizk-Allah et al., [31]; Das et al. [32]; Khan et al. [33]), the multi-objective SCN neutrosophical optimization model can be formulated as follows: **Model 2(a)**

Max min
$$\mu_t^T$$
Max min σ_t^I
Min max v_t^F

Subject to
$$\sum_{j=1}^J W_{ij} \leq \tilde{SCS}_i$$

$$\sum_{l=1}^L Y_{jl} + \sum_{k=1}^K X_{jk} \leq PCP_j$$

$$\sum_{l=1}^L Z_{kl} \leq PCW_k$$

$$\sum_{k=1}^K Z_{kl} + \sum_{j=1}^J Y_{jl} \leq \tilde{ADR}_l$$

$$\sum_{i=1}^I W_{ij} \geq \sum_{k=1}^K X_{jk} + \sum_{l=1}^L Y_{jl}$$

$$\sum_{j=1}^J X_{jk} \geq \sum_{l=1}^L Z_{kl}$$

$$W_{ij} \geq 0, \forall i, j$$

$$X_{jk} \geq 0, \forall j, k$$

$$Y_{jl} \geq 0, \forall j, l$$

$$Z_{kl} \geq 0, \forall k, l$$

It is not easy to solve the above model (2a) with the presence of three objective functions, therefore with the help of auxiliary parameters, the model (2a) can be transformed into a single objective model, given below:

Model 2(b)

Maximize
$$\sum_{t=1}^{2} (\mu_{t} + \sigma_{t} - \upsilon_{t})$$
Subject to
$$\mu_{t}^{T} \geq \mu_{t}, \ \forall \ t$$

$$\sigma_{t}^{T} \geq \sigma_{t}, \ \forall \ t$$

$$\upsilon_{t}^{T} \leq \upsilon_{t}, \ \forall \ t$$

$$\mu_{t} \geq \sigma_{t}, \ \forall \ t$$

$$\mu_{t} \geq \upsilon_{t}, \ \forall \ t$$

$$\mu_{t} + \sigma_{t} + \upsilon_{t} \leq 3, \ \forall \ t$$
constraints of model 2(a)

The above Model 2(b) has been used to get the compromise solution of the formulated problem.

4. Numerical Illustration

In view of demonstrating the method established, we considered the fictional scenario of modeling and optimizing a SC network situation, with some imprecise data being considered on it, described by neutrosophical triangular fuzzy numbers. We assumed a network consisting of multiple numbers of suppliers, multiple numbers of production plants, multiple numbers of warehouses and multiple numbers of distributors, in various regional areas or places. Five suppliers are assumed to distribute the raw resources to four manufacturing plants. The delivery network consists of six warehouses where, before being shipped out to eight distributors, goods are temporarily positioned and processed, and eventually, items are shipped out to many consumers. The imprecise information in Tables 1-8 are listed below:

Table 1. Uncertain Transportation Cost from the Supplier to the Manufacturing Plant.

Supplier	•	Manufacturing Plant						
	P_1	P_2	P_3	P_4				
S_1	((196,199,202);	((89,93,97);	((146,148,150);	((194,196,198);				
	0.3,0.4,0.5)	0.6,0.7,0.8)	0.2,0.3,0.4)	0.1,0.2,0.3)				
S_2	((294,306,312);	((146,148,150);	((194,196,198);	(((196,199,202);				
	0.6,0.8,0.9)	0.2,0.3,0.4)	0.1,0.2,0.3)	0.3,0.4,0.5)				
S_3	((491,499,507);	((119,121,123);	((204,206,208);	((202,205,208);				
	0.1,0.2,0.3)	0.4,0.5,0.6)	0.1,0.2,0.3)	0.3,0.4.0.5)				
S_4	((389,394,399);	((296,300,304);	((239,244,249);	((296,300,304);				
	0.7,0.8,0.9)	0.5,0.6,0.7)	0.7,0.8,0.9)	0.5,0.6,0.7)				
S_5	((591,599,607);	((689,691,693);	((296,300,304);	((339,341,345);				
	0.3,0.4.0.5)	0.3,0.4,0.5)	0.5,0.6,0.7)	0.4,0.5,0.6)				

Table 2. Uncertain Transportation Cost from the Plant to the Distributor.

Plant		Distributor						
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
P_1	((296,300 ,304); 0.5,0.6,0. 7)	((429,432,43 6); 0.4,0.5,0.6)	((341,344,3 47); 0.6,0.8,0.9)	((429,432,4 36); 0.4,0.5,0.6)	((204,206,2 08); 0.1,0.2,0.3)	((339,341,3 45); 0.4,0.5,0.6)	((391,394,3 96); 0.6,0.7,0.8)	((469,471,4 73); 0.1,0.2,0.3)
P_2	((339,341 ,345); 0.4,0.5,0. 6)	((491,494,49 6); 0.2,0.3,0.4)	((294,300,3 06); 0.2,0.3,0.4)	((371,374,3 78); 0.5,0.6,0.7)	((269,272,2 75); 0.7,0.8,0.9)	((371,374,3 78); 0.5,0.6,0.7)	((469,471,4 73); 0.1,0.2,0.3)	((431,435,4 39); 0.7,0.8,0.9)

P_3	((431,435 ,439); 0.7,0.8,0. 9)	((469,472,47 5); 0.1,0.2,0.3)	((341,343,3 46); 0.7,0.8,0.9)	((339,341,3 45); 0.4,0.5,0.6)	((296,298,3 00); 0.1,0.2,0.3)	((369,371,3 24); 0.3,0.4,0.5)	((431,435,4 39); 0.7,0.8,0.9)	((469,471,4 73); 0.1,0.2,0.3)
P_4	((489,492 ,495); 0.5,0.6,0. 7)	((431,435,43 8); 0.3,0.4.0.5)	((319,321,3 24); 0.5,0.6,0.7)	((391,394,3 96); 0.6,0.7,0.8)	((319,321,3 23) 0.3,0.4,0.5)	((386,388,4 00); 0.3,0.4,0.5)	((319,321,3 23) 0.3,0.4,0.5)	((431,435,4 39); 0.7,0.8,0.9)

Table 3. Uncertain Transportation Cost from the Manufacturing Plant to the Warehouse.

Plant		-	War	ehouses		
	W_1	W_2	W_3	W_4	W_5	W_6
P_1	((296,300,304);	((144,148,152);	((196,199,202);	((196,199,202);	((121,123,125);	((296,300,304);
	0.5,0.6,0.7)	0.2,0.3,0.4)	0.3,0.4,0.5)	0.3,0.4,0.5)	0.3,0.4.0.5)	0.5,0.6,0.7)
P_2	((389,392,395);	((121,123,125);	((219,221,225);	((241,244,247);	((269,271,273);	((311,313,317);
	0.2,0.3,0.4)	0.3,0.4.0.5)	0.4,0.5,0.6)	0.6,0.7,0.8)	0.3,0.4,0.5)	0.4,0.5,0.6)
P_3	((541,545,548);	((144,148,152);	((196,199,202);	((296,300,304);	((241,244,247);	((296,300,304);
	0.7,0.8,0.9)	0.2,0.3,0.4)	0.3,0.4,0.5)	0.5,0.6,0.7)	0.6,0.7,0.8)	0.5,0.6,0.7)
P_4	((639,641,643);	((341,344,347);	((296,300,304);	((121,123,125);	((294,296,298);	((301,303,307);
	0.6,0.7,0.8)	0.6,0.8,0.9)	0.5,0.6,0.7)	0.3,0.4.0.5)	0.4,0.5,0.6)	0.4,0.5,0.6)

Table 4. Uncertain Transportation Cost from the Warehouses to the Distributor.

	Table 4. Uncertain Transportation Cost from the Warehouses to the Distributor.								
Ware				Di	stributor				
house	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	
	((146,148,1	((179,182,1	((161,163,1	((169,171,1	((169,171,1	((194,196,1	((181,184,1	((164,166,16	
W_1	50);	83);	65);	73);	73);	98);	87);	8);	
	0.3,0.4,0.5)	0.1,0.2,0.3)	0.5,0.6,0.7)	0.3,0.4,0.5)	0.3,0.4,0.5)	0.1,0.2,0.3)	0.1,0.2,0.3)	0.1,0.2,0.3)	
	((109,111,1	((191,193,1	((164,166,1	((166,168,1	((179,181,1	((181,184,1	((179,181,1	((172,174,17	
W_2	13);	95);	68);	70);	84);	87);	84);	6);	
	0.2,0.3,0.4)	0.5,0.6,0.7)	0.1,0.2,0.3)	0.3,0.4,0.5)	0.2,0.4,0.5)	0.1,0.2,0.3)	0.2,0.4,0.5)	0.7,0.8,0.9)	
	((121,124,1	((189,191,1	((131,134,1	((176,179,1	((179,181,1	((181,184,1	((181,184,1	((169,171,17	
W_3	28);	94);	37);	83);	84);	87);	87);	3);	
	0.5,0.6,0.7)	0.3,0.4,0.5)	0.5,0.6,0.7)	0.1,0.2,0.3)	0.2,0.4,0.5)	0.1,0.2,0.3)	0.1,0.2,0.3)	0.3,0.4,0.5)	
	((126,129,1	((169,171,1	((136,139,1	((181,184,1	((189,191,1	((171,175,1	((179,181,1	((171,174,17	
W_4	32);	73);	42);	87);	94);	80);	84);	6);	
	0.7,0.8,0.9)	0.3,0.4,0.5)	0.1,0.2,0.3)	0.1,0.2,0.3)	0.3,0.4,0.5)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.7,0.8,0.9)	
	((136,139,1	((169,171,1	((146,148,1	((179,181,1	((191,194,1	((159,161,1	((191,194,1	((169,171,17	
W_5	42);	73);	50);	84);	98);	64);	96);	3);	
	0.1,0.2,0.3)	0.2,0.4,0.5)	0.3,0.4,0.5)	0.2,0.4,0.5)	0.7,0.8,0.9)	0.2,0.4,0.5)	0.3,0.4,0.5)	0.3,0.4,0.5)	
	((169,171,1	((151,153,1	((146,148,1	((191,193,1	((194,196,1	((181,184,1	((189,191,1	((164,166,16	
W_6	73);	55);	50);	94);	98);	87);	94);	8);	
	0.3,0.4,0.5)	0.7,0.8,0.9)	0.3,0.4,0.5)	0.3,0.4,0.5)	0.1,0.2,0.3)	0.1,0.2,0.3)	0.3,0.4,0.5)	0.1,0.2,0.3)	

Table 5. Uncertain Delivery Time of Item from the Plant to the Distributor.

Plant		Distributor						
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
P_1	((46,49,52);	((64,68,72);	((51,56,59);	((59,61,64);	((36,40,44);	((46,49,52);	((71,74,78);	((74,88,94);
	0.1,0.2,0.3)	0.1,0.2,0.3)	0.3,0.4,0.5)	0.1,0.2,0.3)	0.5,0.6,0.7)	0.1,0.2,0.3)	0.2,0.4,0.5)	0.4,0.5,0.6)
P_2	((29,33,37);	((56,58,60);	((39,41,45);	((40,44,48);	((19,21,24);	((41,46,50);	((64,76,1,3)	((91,93,95);
	0.7,0.8,0.9)	0.2,0.4,0.5)	0.1,0.2,0.3)	0.7,0.8,0.9)	0.4,0.5,0.6)	0.3,0.4,0.5)	0.7,0.8,0.9)	0.1,0.2,0.3)

P_3	((71,74,78);	((64,76,1,3)	((71,74,78);	((76,80,84);	((56,60,62);	((66,70,74);	((71,74,78);	((89,93,96);
	0.2,0.4,0.5)	0.7,0.8,0.9)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.3,0.4,0.5)	0.1,0.2,0.3)	0.2,0.4,0.5)	0.3,0.4,0.5)
P_4	((89,93,96);	((91,93,95);	((74,78,82);	((81,83,85);	((54,58,60);	((66,74,78);	((74,88,94);	((81,83,85);
	0.3,0.4,0.5)	0.1,0.2,0.3)	0.7,0.8,0.9)	0.7,0.8,0.9)	0.5,0.6,0.7)	0.5,0.6,0.7)	0.4,0.5,0.6)	0.7,0.8,0.9)

Table 6. Uncertain Delivery Time of Item from the Manufacturing Plant to the Warehouse.

		Warehouses							
Plant	W_1	W_2	W_3	W_4	W_5	W_6			
P_1	((26,34,,42);	((14,26,34);	((16,24,32);	((9,16,23);	((26,29,34);	((24,31,38);			
	0.7,0.8,0.9)	0.2,0.4,0.5)	0.1,0.2,0.3)	0.7,0.8,0.9)	0.2,0.4,0.5)	0.7,0.8,0.9)			
P_2	((34,46,54);	((16,24,32);	((19,31,35);	((26,34,,42);	((24,31,35);	((36,39,44);			
	0.1,0.2,0.3)	0.1,0.2,0.3)	0.2,0.4,0.5)	0.7,0.8,0.9)	0.1,0.2,0.3)	0.2,0.4,0.5)			
P_3	((51,59,64);	((54,66,72);	((51,59,64);	((54,66,72);	((56,64,72);	((66,74,78);			
	0.2,0.4,0.5)	0.7,0.8,0.9)	0.2,0.4,0.5)	0.7,0.8,0.9)	0.5,0.6,0.7)	0.5,0.6,0.7)			
P_4	((76,80,84);	((54,66,72);	((29,33,37);	((51,59,64);	((64,68,72);	((71,74,77);			
	0.2,0.4,0.5)	0.7,0.8,0.9)	0.7,0.8,0.9)	0.2,0.4,0.5)	0.1,0.2,0.3)	0.2,0.3,0.4)			

Table 7. Uncertain Delivery Time of Item from the Warehouse to the Distributor

Warehouses			·	Dis	stributor			
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
W_1	((16,24,28);	((14,26,34);	((21,29,35);	((26,29,34);	((21,29,35);	((19,22,25);	((36,40,44);	((29,33,37);
	0.7,0.8,0.9)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.7,0.8,0.9)	0.5,0.6,0.7)	0.7,0.8,0.9)
W_2	((19,22,25);	((16,24,28);	((19,31,35);	((21,24,28);	((26,29,34);	((26,29,34);	(29,35,38);	((21,24,28);
	0.7,0.8,0.9)	0.7,0.8,0.9)	0.2,0.4,0.5)	0.1,0.2,0.3)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.1,0.2,0.3)
W_3	((21,29,35);	((14,26,34);	((21,29,35);	((29,33,37);	((31,34,38);	((34,46,54);	((41,44,48);	((34,46,54);
	0.2,0.4,0.5)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.7,0.8,0.9)	0.7,0.8,0.9)	0.1,0.2,0.3)	0.3,0.4,0.5)	0.1,0.2,0.3)
W_4	((14,26,34);	((24,31,35);	((19,22,25);	((24,31,35);	((24,31,35);	((26,29,34);	((19,31,35);	((21,29,35);
	0.2,0.4,0.5)	0.1,0.2,0.3)	0.7,0.8,0.9)	0.1,0.2,0.3)	0.1,0.2,0.3)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.2,0.4,0.5)
W_5	((16,24,28);	((19,22,25);	((16,24,28);	((16,24,28);	((34,46,54);	((34,46,54);	((36,40,44);	((41,44,48);
	0.7,0.8,0.9)	0.7,0.8,0.9))	0.7,0.8,0.9)	0.7,0.8,0.9)	0.1,0.2,0.3)	0.1,0.2,0.3)	0.5,0.6,0.7)	0.3,0.4,0.5)
W_6	((16,24,28);	((19,31,35);	((14,26,34);	((14,26,34);	((29,33,37);	((31,34,36);	((41,44,48);	((64,68,72);
	0.7,0.8,0.9)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.2,0.4,0.5)	0.7,0.8,0.9)	0.2,0.4,0.5)	0.3,0.4,0.5)	0.1,0.2,0.3)

Table 8. Right hand side parameters

Fuzzy demand	Fuzzy supply	Fixed capacity of plant	Fixed capacity of warehouse
((180,190,200);	((90,95,100);	470	150
0.7,0.8,0.9)	0.2,0.4,0.5)		
((480,490,500);	((50,55,60);	300	180
0.1,0.2,0.3)	0.3,0.4,0.5)		
((200,210,220);	((85,90,95);	330	160
0.2,0.4,0.5)	0.1,0.2,0.3)		
((205,215,225);	((65,70,75);	320	200
0.3,0.4,0.5)	0.4,0.5,0.6)		
((290,300,310);	((60,65,70);		180
0.4,0.5,0.6)	0.7,0.8,0.9)		
	((105,110,115);		220
	0.4,0.5,0.6)		
	((110,115,120);		
	0.5,0.6,0.7)		
	((80,85,90);		
	0.3,0.4,0.5)		

By using all the information given in the table from 1 to 8, the multi-objective SC problem has been formulated. With the presence of uncertainty, the model cannot be solved directly; therefore, the crisp model has been obtained by using the equation (12). Before solving the formulated non-linear multi-objective SC model, the feasibility of the formulated is determined by using the LINGO software (LINGO software is a comprehensive tool designed to make building and solving Linear and Nonlinear (convex and non-convex) programming problem) by determining the lower and upper bound of both the objective functions. LINGO software includes identification of the infeasibility and unboundness of the formulated linear and non-linear model. The Solver Status box of LINGO software details the model classification (linear, non-linear or other), state of the current solution (whether local or global optimum, feasible or infeasible, etc.), the value of the objective function, the infeasibility of the model (amount constraints are violated by), and the number of iterations required to solve the model.

After checking the feasibility of the model construct, the next task is to solve the formulated multi-objective SC model by using the neutrosophic compromise programming. Neutrosophic compromise programming has the key advantage over the other techniques because it helps the decision-makers to consider three categories of membership functions (truth degree, falsity degree or degree of indeterminacy) and while other techniques employed for solving a multi-objective model only takes one membership function dependent on both upper and lower limits of the objective functions. For solving the formulated problem, decision-maker first solve the multiple objective optimization problem by considering a single objective at a time and ignoring the others objectives with the given set of constraints. The solution thus obtained is consider as the idle solution for each of the objective functions and helps in the determination of aspiration level to each of the objective functions. The bounds for the two objective functions are determined as:

The truth membership functions for the first and second objective functions are constructed as follows.

$$\mu_1^T(F_1(x)) = \begin{cases} 1 & \text{if } F_1(x) < 278631.5 \\ \frac{394228.8 - F_1(x)}{394228.8 - 278631.5} & \text{if } F_1(x) \in [278631.5, 394228.8] \\ 0 & \text{if } F_1(x) > 394228.8 \end{cases}$$

$$\mu_2^T(F_2(x)) = \begin{cases} 1 & \text{if } F_2(x) < 25273.76 \\ \frac{28307.68 - F_2(x)}{28307.68 - 25273.76} & \text{if } F_2(x) \in [25273.76, 28307.68] \\ 0 & \text{if } F_2(x) > 28307.68 \end{cases}$$

The Indeterminacy membership functions for the first and second objective functions are constructed as follows.

$$\sigma_1^I(F_1(x)) = \begin{cases} 1 & \text{if } F_1(x) < 278631.5 \\ \frac{382669.07 - F_1(x)}{382669.07 - 278631.5} & \text{if } F_1(x) \in [278631.5,382669.07] \\ 0 & \text{if } F_1(x) > 382669.07 \end{cases}$$

$$\sigma_2^I(F_2(x)) = \begin{cases} 1 & \text{if } F_2(x) < 25273.76 \\ \frac{28004.288 - F_2(x)}{28004.288 - 25273.76} & \text{if } F_2(x) \in [25273.76,28004.288] \\ 0 & \text{if } F_2(x) > 28004.288 \end{cases}$$

The falsity membership functions for the first and second objective functions are constructed as follows.

$$\upsilon_{1}^{F}(F_{1}(x)) = \begin{cases} 0 & \text{if } F_{1}(x) < 290191.23 \\ \frac{F_{1}(x) - 290191.23}{394228.8 - 290191.23} & \text{if } F_{1}(x) \in [290191.23, 394228.8] \\ 1 & \text{if } F_{1}(x) > 394228.8 \end{cases}$$

$$\upsilon_{2}^{F}(F_{2}(x)) = \begin{cases} 0 & \text{if } F_{2}(x) < 25577.152 \\ \frac{F_{2}(x) - 25577.152}{28307.68 - 25577.152} & \text{if } F_{2}(x) \in [25577.152, 28307.68] \\ 1 & \text{if } F_{2}(x) > 28307.68 \end{cases}$$

After combining all the membership function together, the compromise solution for the multi-objective SC neutrosophic model is obtained as:

$$\begin{split} F_1 &= 304305.60, F_2 = 25742.69, \mu_1^T = 0.7779007, \sigma_1^I = 0.7532231, \nu_1^F = 0.1356658, \\ \mu_2^T &= 0.8454378, \sigma_2^I = 0.8282642, \nu_2^F = 0.06062465, W_{11} = 135, W_{14} = 47, W_{22} = 275, \\ W_{24} &= 9, W_{32} = 198, X_{14} = 135, X_{22} = 177, X_{44} = 56, Y_{21} = 93, Y_{24} = 31, Y_{25} = 63, \\ Y_{26} &= 109, Z_{22} = 54, Z_{23} = 88, Z_{24} = 35, Z_{47} = 110, Z_{48} = 81 \end{split}$$

After using the neutrosophic compromise programming, the total minimum transportation cost incurred from various multiple sources to different distributors through multiple plants and warehouses is 304305:60; furthermore, the minimum delivery time taken from various multiple sources to different distributors through multiple plants and warehouses is 25742:69. The final finished goods quantity to be shipped from various multiple plants to various warehouses is 368 units; the quantity to be shipped from various multiple plants to various distributors is 296 units; the quantity to be shipped from various multiple warehouses to various distributors is 368 units. We have also compared the proposed work of neutrosophic compromise programming with other well-known techniques used to solve the multi-objective model. The used approach of neutrosophic compromise programming is based on three different types of membership functions, i.e., the degree of truth and indeterminacy and the extent of falsity membership that provides more flexibility in decision making process. To show the efficacy of the proposed work, the formulated model has been solved by using three different approaches namely, simple additive approach, simple weighted additive approach, and pre-emptive goal programming approach. The obtained result has been presented in below Fig. 2, shows the supremacy of the proposed work over other methods.

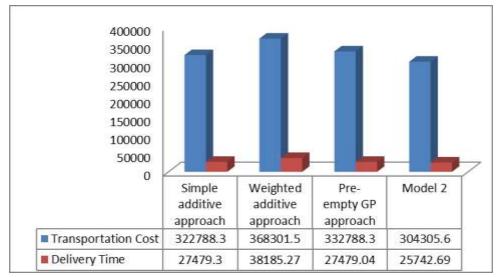


Fig. 2 Result Comparison

After obtaining the deterministic form of each of neutrosophical triangular fuzzy number by using the equation number (12), and also after constructing the membership function of each of the objective functions (using lower and upper bound), different approaches namely, simple additive approach, simple weighted additive approach, and pre-emptive goal programming approach has been used over model (2a) for getting the compromise solution. Simple additive approach (Tiwari et al., [34]) is a method used to solve the problem of multi-attribute decision making. The basic concept simple additive approach is to find the sum of each alternative's performance rating on all attributes; simple weighted additive approach (Chou et al.[35]) is the method used in solving the problem of multi-attribute decision making The basic concept weighted additive approach is to find the sum of the weighted performance rating for each alternative on all attributes; and pre-emptive goal programming approach (Biswas and Pal [36]) is a hierarchy of priority levels for the goals, so the primary importance is to receive first-priority attention, secondary importance receives second-priority attention, and so forth (if there are more than two priority levels. The results indicated that, these approaches failed to optimize the objective function completely, but through neutrosophical compromise programming approach we are able to optimize the each objective functions efficiently that is very important for supply chain.

Conclusion

There are numerous causes of uncertainty, which can arise from the demand side, production side, manufacturing cycle, and scheduling and distribution processes, constantly endanger the quality and efficacy of the SC. Uncertainty can result in shortages with bottlenecks, and can also impact the SC's overall efficiency. Therefore, it is important to find the means of managing it. The well-known methods such as probability, fuzzy set, and multi-choices theory are not sufficient in certain real-world circumstances to cope with such conditions in which indeterminacy is involved. The main aim of this paper is to implement the novel neutrosophical compromise programming approach, that together optimizes the degrees of truth, indeterminacy and falsity of objectivity functions. The efficiency of the proposed work is also studied where the suggested approach produces improved results in compare to simple additive approach, simple weighted additive approach and a pre-emptive goal programming approach. This result demonstrates the efficiency or dominance on current strategies that the neutrosophic technique's is quite adequate, explanatory, and a good representative of real-life situations. Therefore, it is expected that the approach developed would open up new opportunities in the field of multi-criteria problems and can be applied in other realistic field problems, such as scheduling problems, transportation problems, project management, capital utilization planning, traveling salesman problems, etc.

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References

- 1. Nurjanni, K. P., Carvalho, M. S., & Costa, L. (2017). Green supply chain design: A mathematical modeling approach based on a multi-objective optimization model. *International Journal of Production Economics*, 183, 421-432. https://doi.org/10.1016/j.ijpe.2016.08.028
- 2. Khan, S. A., Chaabane, A., & Dweiri, F. T. (2018). Multi-criteria decision-making methods application in supply chain management: A systematic literature. *Multi-criteria methods and techniques applied to supply chain management*, 1.
- 3. Hugos, M. H. (2018). Essentials of supply chain management. John Wiley & Sons.
- 4. Smarandache, F. (2010). Neutrosophic set–a generalization of the intuitionistic fuzzy set. *Journal of Defense Resources Management (JoDRM)*, *1*(1), 107-116.
- 5. Gamal, A., Ismail, M., & Smarandache, F. A Novel Methodology Developing an Integrated ANP: A Neutrosophic Model for Supplier Selection. Infinite Study.
- 6. Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. (2019). An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*, 77, 438-452. https://doi.org/10.1016/j.asoc.2019.01.035
- 7. Badhotiya, G. K., Soni, G., & Mittal, M. L. (2019). Fuzzy multi-objective optimization for multi-site integrated production and distribution planning in two echelon supply chain. *The International Journal of Advanced Manufacturing Technology*, 102(1), 635-645. https://doi.org/10.1007/s00170-018-3204-2
- 8. Rabbani, M., Ahmadzadeh, K., & Farrokhi-Asl, H. (2019). Remanufacturing models under technology licensing with consideration of environmental issues. *Process Integration and Optimization for Sustainability*, 3(3), 383-401. https://doi.org/10.1007/s41660-019-00085-8
- 9. Modak, N. M., & Kelle, P. (2019). Managing a dual-channel supply chain under price and delivery-time dependent stochastic demand. *European Journal of Operational Research*, 272(1), 147-161. https://doi.org/10.1016/j.ejor.2018.05.067
- 10. J.-Sharahi, S., Khalili-Damghani, K., Abtahi, A. R., & Rashidi-Komijan, A. (2018). Type-II fuzzy multi-product, multi-level, multi-period location–allocation, production–distribution problem in supply chains: modelling and optimisation approach. *Fuzzy Information and Engineering*, 10(2), 260-283. https://doi.org/10.1080/16168658.2018.1517978
- 11. Gholamian, N., Mahdavi, I., & Tavakkoli-Moghaddam, R. (2016). Multi-objective multi-product multi-site aggregate production planning in a supply chain under uncertainty: fuzzy multi-objective optimisation. International Journal of Computer Integrated Manufacturing, 29(2), 149-165. https://doi.org/10.1080/0951192X.2014.1002811
- 12. Kristianto, Y., Gunasekaran, A., Helo, P., & Hao, Y. (2014). A model of resilient supply chain network design: A two-stage programming with fuzzy shortest path. *Expert systems with applications*, 41(1), 39-49. https://doi.org/10.1016/j.eswa.2013.07.009
- 13. Bilgen, B. (2010). Application of fuzzy mathematical programming approach to the production allocation and distribution supply chain network problem. *Expert Systems with Applications*, 37(6), 4488-4495. https://doi.org/10.1016/j.eswa.2009.12.062
- 14. Zimmermann, H. J. (1975). Description and optimization of fuzzy systems. *International journal of general System*, 2(1), 209-215. https://doi.org/10.1080/03081077508960870
- 15. Kar, C., Mondal, B., & Roy, T. K. (2018). An inventory model under space constraint in neutrosophic environment: a neutrosophic geometric programming approach. *Neutrosophic Sets and Systems: An International Book Series in Information Science and Engineering, vol.* 21, 2018, 201893.
- 16. Ahmad, F., Adhami, A. Y., & Smarandache, F. (2020). Modified neutrosophic fuzzy optimization model for optimal closed-loop supply chain management under uncertainty. In *Optimization theory based on neutrosophic and plithogenic sets* (pp. 343-403). Academic Press. https://doi.org/10.1016/B978-0-12-819670-0.00015-9
- 17. Abdel-Baset, M., Chang, V., & Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel neutrosophic approach. *Computers in Industry*, 108, 210-220. https://doi.org/10.1016/j.compind.2019.02.013
- 18. Liang, R. X., Wang, J. Q., & Zhang, H. Y. (2018). A multi-criteria decision-making method based on single-valued trapezoidal neutrosophic preference relations with complete weight information. *Neural Computing and Applications*, 30(11), 3383-3398. https://doi.org/10.1007/s00521-017-2925-8

- 19. Thamaraiselvi, A., & Santhi, R. (2016). A new approach for optimization of real life transportation problem in neutrosophic environment. *Mathematical Problems in Engineering*, 2016. https://doi.org/10.1155/2016/5950747
- 20. Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2019). A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, 43(2), 38. https://doi.org/10.1007/s10916-019-1156-1
- 21. Liang, R., Wang, J., & Zhang, H. (2017). Evaluation of e-commerce websites: An integrated approach under a single-valued trapezoidal neutrosophic environment. *Knowledge-Based Systems*, 135, 44-59. https://doi.org/10.1016/j.knosys.2017.08.002
- 22. Abdel-Basset, M., Gamal, A., Chakrabortty, R. K., & Ryan, M. J. (2021). Evaluation of sustainable hydrogen production options using an advanced hybrid MCDM approach: A case study. *International Journal of Hydrogen Energy*, 46(5), 4567-4591. https://doi.org/10.1016/j.ijhydene.2020.10.232
- 23. Abdel-Basset, M., Gamal, A., Chakrabortty, R. K., & Ryan, M. (2021). A new hybrid multi-criteria decision-making approach for location selection of sustainable offshore wind energy stations: A case study. *Journal of Cleaner Production*, 280, 124462. https://doi.org/10.1016/j.jclepro.2020.124462
- 24. Charles, V., Gupta, S., & Ali, I. (2019). A fuzzy goal programming approach for solving multi-objective supply chain network problems with pareto-distributed random variables. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 27(04), 559-593. https://doi.org/10.1142/S0218488519500259
- 25. Gupta, S., Ali, I., & Ahmed, A. (2018). Efficient fuzzy goal programming model for multi-objective production distribution problem. *International journal of applied and computational mathematics*, 4(2), 1-19. https://doi.org/10.1007/s40819-018-0511-0
- 26. Gupta, S., Ali, I., & Ahmed, A. (2018). Multi-objective bi-level supply chain network order allocation problem under fuzziness. *Opsearch*, 55(3), 721-748. https://doi.org/10.1007/s12597-018-0340-2
- 27. Zadeh, L. A. (1996). Fuzzy sets. In Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh (pp. 394-432). https://doi.org/10.1142/9789814261302_0021
- 28. Atanassov, K. T. (1999). Intuitionistic fuzzy sets. In *Intuitionistic fuzzy sets* (pp. 1-137). Physica, Heidelberg. https://doi.org/10.1007/978-3-7908-1870-3_1
- 29. Smarandache, F. (1999). A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic.
- 30. Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management science*, 17(4), B-141. https://doi.org/10.1287/mnsc.17.4.B141
- 31. Rizk-Allah, R. M., Hassanien, A. E., & Elhoseny, M. (2018). A multi-objective transportation model under neutrosophic environment. *Computers & Electrical Engineering*, 69, 705-719. https://doi.org/10.1016/j.compeleceng.2018.02.024
- 32. Das, S. K., & Roy, S. K. (2019). Effect of variable carbon emission in a multi-objective transportation-p-facility location problem under neutrosophic environment. *Computers & Industrial Engineering*, 132, 311-324. https://doi.org/10.1016/j.cie.2019.04.037
- 33. Khan, M. F., Haq, A., Ahmed, A., & Ali, I. (2021). Multiobjective Multi-Product Production Planning Problem Using Intuitionistic and Neutrosophic Fuzzy Programming. IEEE Access, 9, 37466-37486. https://doi.org/10.1109/ACCESS.2021.3063725
- 34. Tiwari, R. N., Dharmar, S., & Rao, J. (1987). Fuzzy goal programming—an additive model. Fuzzy sets and systems, 24(1), 27-34. https://doi.org/10.1016/0165-0114(87)90111-4
- 35. Chou, S. Y., Chang, Y. H., & Shen, C. Y. (2008). A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes. *European Journal of Operational Research*, 189(1), 132-145. https://doi.org/10.1016/j.ejor.2007.05.006
- 36. Biswas, A., & Pal, B. B. (2005). Application of fuzzy goal programming technique to land use planning in agricultural system. *Omega*, 33(5), 391-398. https://doi.org/10.1016/j.omega.2004.07.003

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