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# Neutrosophic Fuzzy Boundary Value Problem under Generalized Hukuhara Differentiability

**Bassem A. Kamal [1,*], A. A. Salama [2], M.shokry [3], Magdi S. El-Azab [4] and Galal I. El-Baghdady [4]**

[1] Department of Mathematics and Engineering Physics, Faculty of Engineering, Delta University for science and technology, Egypt; belmahmoudy@gmail.com , bassem.ahmed.kamal@std.mans.edu.eg
[2] Department of Mathematics and Computer Science, Faculty of Science, Port Saied University, Egypt; ahmed_salama_2000@sci.psu.edu.eg
[3] Physics and engineering mathematics department, Faculty of engineering, Tanta university, Egypt; mohamed.nail@f-eng.tanta.edu.eg
4 Department of Mathematics and Engineering Physics, Faculty of Engineering, Mansoura University, Egypt; ms_elazab@hotmail.com, enggalalebrahim@mans.edu.eg

* Correspondence: belmahmoudy@gmail.com; Tel.: (020)01002760004

**Abstract:** In this article, the main definitions and differentiation concepts of neutrosophic fuzzy environment will be reviewed. This article will introduce an analytical methodology for solving the second-order linear ordinary differential problem with neutrosophic fuzzy boundary values, this analysis will be under generalized Hukuhara differentiability to show the analytical solutions from a different point of view for the uncertain system, some of these solutions may be decreasing in uncertainty or maybe reflecting the behavior of some real-world systems better. Some applications and numeral examples will be solved to show the behavior of the solution.

**Keywords:** Fuzzy; neutrosophic fuzzy number; Hukuhara differentiability; neutrosophic fuzzy differential equation.

## 1. Introduction

The fuzzy differential equation as a topic has been developed in last years so rapidly and it can be used as a suitable way to model dynamic systems under possible uncertainty. The concept of fuzzy and fuzzy derivative was initiated by Zadeh and Chang [1] then it was followed up by a wide group of researchers to develop many different methods as Dubois, Prade, Puri, Goetschel, and Voxman [2]. After Kaleva [3] formulated the first concept of differential equations in a fuzzy environment, Hukuhara changed the concept of difference and differentiability, and to overcome the difficulty of no solution for BVPs, the generalized Hukuhara differentiability was developed by Stefanini and Bede [4:6].

As time goes, the definition of the possible uncertainty was developed and generalized, which was firstly introduced by Atanassov [7,8] and it is called intuitionistic fuzzy which generalized the definition of the fuzzy environment from depending on the definition of the membership only into definition depends on membership degree and non-membership degree which summed to less than one to highlight the non-belongingness and add too many questions on the degree of hesitation. Wide applications are solved by Mondal and Roy [9,10].

It is fact that the world is always searching for a more generalized definition for the fuzzy environment to model more reliable systems. So it was important to find a more generalized fuzzy set and functions that depend not only on the membership and non-membership but also describe the degree of hesitation, and it was firstly initiated by Smarandache [11:15] who calls it a neutrosophic fuzzy set which is considered as a generalization for fuzzy set and intuitionistic fuzzy set because it adds the concept of indeterminacy. As an extension of the neutrosophic Logic, A. A. Salama introduced the Neutrosophic Crisp Sets Theory as a generalization of crisp sets theory and developed, inserted, and formulated new concepts in the fields of mathematics, statistics, computer science, and information systems through neutrosophic [16:21 ].

In this paper, the second-order homogenous ordinary differential equation via generalized neutrosophic fuzzy numbers as boundaries will be solved under strongly generalized Hukuhara differentiability. And the solution will be applied to numeral problems.

## 2. Definitions and theories

We begin this section by defining the notation and theories we will use in this paper.

**2.1. Definition** [11] neutrosophic memberships

Let $X$ be universe set and a neutrosophic set $A$ on $X$ is defined as $A=\{(T_A(x),I_A(x),F_A(x)): x\epsilon X\}$ represents the degree of membership $T_A(x)$, the degree of indeterministic $I_A(x)$ and the degree of non-membership $F_A(x)$. Such that $0\le T_A(x)+I_A(x)+F_A(x)\le 3$.

**2.2. Definition** [11]$(\alpha,\beta,\gamma)$-cuts

The $(\alpha,\beta,\gamma)$-cuts are fixed values on set $A$ where $A_{\alpha,\beta,\gamma}=\{(T_A(x),I_A(x),F_A(x)): x\epsilon X, T_A(x)\ge \alpha, I_A(x)\le\beta, F_A(x)\le\gamma\}$ which define each of $T_A(x), I_A(x), F_A(x)$ in terms of lower and upper functions of $(\alpha,\beta,\gamma)$-cuts.

**2.3. Definition** [11,12] neutrosophic number

**A neutrosophic set $A$ defined on a universal set of real numbers $R$ is said to be a neutrosophic number**

i) $A$ is normal if $x_a\epsilon R\ T_A(x_a)=1, I_A(x_a)=F_A(x_a)=0$.

ii) $A$ is a convex set on truth function $T_A(x)$where $T_A(\lambda x_1+(1-\lambda)x_2)\ge \min(T_A(x_1),T_A(x_2))$

iii) $A$ is a concave set on indeterministic and falsity functions $I_A(x), F_A(x)$where $I_A(\lambda x_1+(1-\lambda)x_2)\ge \max(I_A(x_1),I_A(x_2))$

$$F_A(\lambda x_1+(1-\lambda)x_2)\ge \max(F_A(x_1),F_A(x_2))$$

**2.4. Definition** Triangular neutrosophic number

Let $A$ be a Generalized triangle neutrosophic number $A_{GTN}=(a,b,c:\omega,\eta,\xi)$

$$T_A(x)=\begin{cases}\frac{x-a}{b-a}\omega & a\le x<b\\ \omega & x=b \quad \text{and zero otherwise}\\ \frac{c-x}{c-b}\omega & b<x\le c\end{cases}$$

$$I_A(x)=\begin{cases}\frac{b-x}{b-a}\eta & a\le x<b\\ \eta & x=b \quad \text{and 1 otherwise}\\ \frac{b-x}{b-c}\eta & b<x\le c\end{cases}$$

$$F_A(x)=\begin{cases}\frac{b-x}{b-a}\xi & a\le x<b\\ \xi & x=b \quad \text{and 1 otherwise}\\ \frac{b-x}{b-c}\xi & b<x\le c\end{cases}$$

And we can represent $(\alpha,\beta,\gamma)$-cuts on the generalized triangle neutrosophic number as

$$A_{\alpha,\beta,\gamma}=\left[\underline{A(\alpha)},\overline{A(\alpha)}\right],\left[\underline{A(\beta)},\overline{A(\beta)}\right],\left[\underline{A(\gamma)},\overline{A(\gamma)}\right]$$

$$\left[\underline{A(\alpha)},\overline{A(\alpha)}\right]=\left[\left(a+\frac{\alpha}{\omega}(b-a)\right),\left(c-\frac{\alpha}{\omega}(c-b)\right)\right]$$

$$\left[\underline{A(\beta)},\overline{A(\beta)}\right]=\left[\left(a+\frac{1-\beta}{1-\eta}(b-a)\right),\left(c-\frac{1-\beta}{1-\eta}(c-b)\right)\right]$$

$$\left[\underline{A(\gamma)},\overline{A(\gamma)}\right]=\left[\left(a+\frac{1-\gamma}{1-\xi}(b-a)\right),\left(c-\frac{1-\gamma}{1-\xi}(c-b)\right)\right]$$

**2.5. Definition** Trapezoidal neutrosophic number

Let $A$ be a generalized Trapezoidal neutrosophic number $A_{GTRN} = (a,b,c,d:\omega,\eta,\xi)$

$$T_A(x)=\begin{cases}\frac{x-a}{b-a}\omega & a\le x\le b\\ \omega & b\le x\le c\\ \frac{d-x}{d-c}\omega & c\le x\le d\end{cases} \text{ and zero otherwise}$$

$$I_A(x)=\begin{cases}\frac{b-x}{b-a}\eta & a\le x\le b\\ \eta & b\le x\le c\\ \frac{c-x}{c-d}\eta & c\le x\le d\end{cases} \text{ and 1 otherwise}$$

$$F_A(x)=\begin{cases}\frac{b-x}{b-a}\xi & a\le x\le b\\ \xi & b\le x\le c\\ \frac{c-x}{c-d}\xi & c\le x\le d\end{cases} \text{ and 1 otherwise}$$

And we can represent $(\alpha,\beta,\gamma)$-cuts on the generalized trapezoidal neutrosophic number as

$$A_{\alpha,\beta,\gamma}=\left[\underline{A(\alpha)},\overline{A(\alpha)}\right],\left[\underline{A(\beta)},\overline{A(\beta)}\right],\left[\underline{A(\gamma)},\overline{A(\gamma)}\right]$$

$$\left[\underline{A(\alpha)},\overline{A(\alpha)}\right]=\left[\left(a+\frac{\alpha}{\omega}(b-a)\right),\left(d-\frac{\alpha}{\omega}(d-c)\right)\right]$$

$$\left[\underline{A(\beta)},\overline{A(\beta)}\right]=\left[\left(a+\frac{1-\beta}{1-\eta}(b-a)\right),\left(d-\frac{1-\beta}{1-\eta}(d-c)\right)\right]$$

$$\left[\underline{A(\gamma)},\overline{A(\gamma)}\right]=\left[\left(a+\frac{1-\gamma}{1-\xi}(b-a)\right),\left(d-\frac{1-\gamma}{1-\xi}(d-c)\right)\right]$$

**2.6. Definition [4,6]**Let $F:(a,b)\to\mathcal{F}(R)$, if the next limits

$$\lim_{h\to 0+}\frac{F(x_0+h)\ominus_H F(x_0)}{h},\qquad \lim_{h\to 0+}\frac{F(x_0)\ominus_H F(x_0-h)}{h}$$

exist and equal some elements $F'_H(x_0)\in\mathcal{F}(R)$, then $F$ is Hukuhara differentiable at $x_0$, and $F'_H(x_0)$ is its derivative at $x_0$.

**Theorem 1 [7,8]** Let $F:(a,b)\to\mathcal{F}(R)$ be a generalized Hukuhara differentiable Function if and only if (a) or (b) are satisfied

(a) $\underline{f'}_\alpha(x)$ is increasing and $\overline{f'}_\alpha(x)$ is decreasing

(b) $\underline{f'}_\alpha(x)$ is decreasing and $\overline{f'}_\alpha(x)$ is increasing

Then,

$$\left[F'_{gH}(x)\right]_{\boldsymbol{\alpha}}=\left[min\left(\underline{f'}_\alpha(x),\overline{f'}_\alpha(x)\right),max\left(\underline{f'}_\alpha(x),\overline{f'}_\alpha(x)\right)\right]$$

This concept is so near to the generalized differentiability, but may be it focuses on the cases of the function and both of these differentiability change the concept of derivatives, so let us show these changes in the next definition of generalized Hukuhara derivatives.

**2.7. Definition [8,9]**The generalized Hukuhara first derivative of a fuzzy parametric function is defined as; $f'(x_0) = \lim_{h\to 0}\frac{f(x_0+h)\ominus_{gh} f(x_0)}{h}$, From the definition, we have two classes:

$(i)$-differentiable at $x_0$ : $[f'(x_0)]_\alpha = [\underline{f'}_\alpha(x_0), \overline{f'}_\alpha(x_0)]$

$(ii)$-differentiable at $x_0$ : $[f'(x_0)]_\alpha = [\overline{f'}_\alpha(x_0), \underline{f'}_\alpha(x_0)]$

**2.8. Definition [8,9]**The generalized Hukuhara second derivative of the fuzzy function is defined as

$$f''(x_0) = \lim_{h\to 0}\frac{f'(x_0+h)\ominus_{gh} f'(x_0)}{h}$$

According to the last definitions, we have the following classes:

$$f'(x_0) \text{ is } (i)\text{-differentiable if: } f''(x_0) = \begin{cases} \left[\underline{f''}_\alpha(x_0), \overline{f''}_\alpha(x_0)\right] \; if\ f\ is\ (i) - differentiable \\ class(1,1) \\ \left[\overline{f''}_\alpha(x_0), \underline{f''}_\alpha(x_0)\right] \; if\ f\ is\ (ii) - differentiable \\ class(2,2) \end{cases}$$

$$f'(x_0) \text{ is } (ii)\text{-differentiable if: } f''(x_0) = \begin{cases} \left[\overline{f''}_\alpha(x_0), \underline{f''}_\alpha(x_0)\right] \; if\ f\ is\ (i) - differentiable \\ class(1,2) \\ \left[\underline{f''}_\alpha(x_0), \overline{f''}_\alpha(x_0)\right] \; if\ f\ is\ (ii) - differentiable \\ class(2,1) \end{cases}$$

**2.9. Definition** The solution$\tilde{y}(t,\alpha,\beta,\gamma)$ of the neutrosophic fuzzy differential equation is strong only if, $\frac{\partial \underline{y}}{\partial\alpha} > 0, \frac{d\bar{y}}{d\alpha} < 0$ but $\frac{\partial \underline{y}}{\partial\beta} < 0, \frac{d\bar{y}}{d\beta} > 0$ and $\frac{\partial \underline{y}}{\partial\gamma} < 0, \frac{d\bar{y}}{d\gamma} > 0$.

## 3 . Methodology

As it is defined before both Hukuhara theory and neutrosophic fuzzy definition, so we can deal with differential problems by applying the generalized Hukuhara differentiability on Neutrosophic Fuzzy differential equations

### 3.1 Neutrosophic second order differential equation under generalized Hukuhara:

Choosing second order differential equation in linear homogenous form with full terms of differentiability in different cases of coefficients signs.

$$\tilde{y}''(t) = \pm p\tilde{y}'(t) \pm q\tilde{y}(t)$$

Neutrosophic boundary conditions

$$\tilde{y}(t_0) = \tilde{a} \quad \tilde{y}(T) = \tilde{b}$$

Using $(\alpha,\beta,\gamma) - cut$ to apply generalized Hukuhara differentiability classes. Where $\tilde{a}\ and\ \tilde{b}$are two generalized trapezoidal neutrosophic numbers.

$$\tilde{a} = (a_1, a_2, a_3, a_4; \omega, \eta, \xi)$$
$$\tilde{b} = (b_1, b_2, b_3, b_4; \omega, \eta, \xi)$$

**3.1.1. Case (1): Positive sign for p and q (+ +)**

The analysis and solution will be introduced in case of positive signs of $p, q$.
According to generalized Hukuhara, the problem will be studied in 4 classes

**Class (1,1)**

$$\bar{y}''(t,\alpha,\beta,\gamma) = p.\bar{y}'(t,\alpha,\beta,\gamma) + q.\bar{y}(t,\alpha,\beta,\gamma),$$

$$\underline{y}''(t,\alpha,\beta,\gamma) = p.\underline{y}'(t,\alpha,\beta,\gamma) + q.\underline{y}(t,\alpha,\beta,\gamma),$$

**Class (1,2)**

$$\underline{y}''(t,\alpha,\beta,\gamma) = p.\bar{y}'(t,\alpha,\beta,\gamma) + q.\bar{y}(t,\alpha,\beta,\gamma),$$

$$\overline{y''}(t,\alpha,\beta,\gamma) = p.\underline{y}'(t,\alpha,\beta,\gamma) + q.\underline{y}(t,\alpha,\beta,\gamma),$$

**Class (2,1)**

$$\bar{y}''(t,\alpha,\beta,\gamma) = p.\bar{y}'(t,\alpha,\beta,\gamma) + q.\underline{y}(t,\alpha,\beta,\gamma),$$

$$\underline{y}''(t,\alpha,\beta,\gamma) = p.\underline{y}'(t,\alpha,\beta,\gamma) + q.\bar{y}(t,\alpha,\beta,\gamma),$$

**Class (2, 2)**

$$\bar{y}''(t,\alpha,\beta,\gamma) = p.\underline{y}'(t,\alpha,\beta,\gamma) + q.\bar{y}(t,\alpha,\beta,\gamma)$$

$$\underline{y}''(t,\alpha,\beta,\gamma) = p.\bar{y}'(t,\alpha,\beta,\gamma) + q.\underline{y}(t,\alpha,\beta,\gamma)$$

to find the solution of each class as a function in the general values of coefficients p and q, the methodology of getting the analytical solution of this system will be described in class(1,1) in detail as an example.

**Class (1,1)**

$$\bar{y}''(t,\boldsymbol{\alpha},\beta,\gamma) = p.\bar{y}'(t,\boldsymbol{\alpha},\beta,\gamma) + q.\bar{y}(t,\boldsymbol{\alpha},\beta,\gamma), \quad \bar{y}(t_0,\boldsymbol{\alpha},\beta,\gamma) = \bar{a}, \bar{y}(T,\boldsymbol{\alpha},\beta,\gamma) = \bar{b}$$

$$\underline{y}''(t,\boldsymbol{\alpha},\beta,\gamma) = p.\underline{y}'(t,\boldsymbol{\alpha},\beta,\gamma) + q.\underline{y}(t,\boldsymbol{\alpha},\beta,\gamma), \quad \underline{y}(t_0,\boldsymbol{\alpha},\beta,\gamma) = \underline{a}, \underline{y}(T,\boldsymbol{\alpha},\beta,\gamma) = \underline{b}$$

**Let**

$$\underline{y} = x, \quad \bar{y} = z$$

$$z'' = pz' + qz \qquad z' = v$$

$$x'' = px' + qx \qquad x' = u$$

$$\begin{bmatrix} x \\ u \\ z \\ v \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ q & p & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & q & p \end{bmatrix} \begin{bmatrix} x \\ u \\ z \\ v \end{bmatrix}$$

From this system, the eign values of the matrix will appear in the solution, so these definitions will be used

$$d_1 = p + \sqrt{p^2 + 4q} \qquad d_2 = p + \sqrt{p^2 - 4q} \qquad d_3 = p - \sqrt{p^2 + 4q} \qquad d_4 = p - \sqrt{p^2 - 4q}$$

General solution:

$$x = \underline{y}(t,\boldsymbol{\alpha},\beta,\gamma) = c_2 e^{\frac{(d_1)t}{2}} + c_4 e^{\frac{(d_3)t}{2}}$$

$$z = \bar{y}(t,\boldsymbol{\alpha},\beta,\gamma) = c_1 e^{\frac{(d_1)t}{2}} + c_3 e^{\frac{(d_3)t}{2}}$$

By using the boundary condition to get $c_1, c_2, c_3 and\ c_4$ :

At $t = t_0$

$$\underline{y}(t_0,\boldsymbol{\alpha},\beta,\gamma) = \underline{a} = c_2 e^{\frac{(d_1)t_0}{2}} + c_4 e^{\frac{(d_3)t_0}{2}}$$

$$\bar{y}(t_0,\boldsymbol{\alpha},\beta,\gamma) = \bar{a} = c_1 e^{\frac{(d_1)t_0}{2}} + c_3 e^{\frac{(d_3)t_0}{2}}$$

At $t = T$

$$\underline{y}(T,\boldsymbol{\alpha},\beta,\gamma) = \underline{b} = c_2 e^{\frac{(d_1)T}{2}} + c_4 e^{\frac{(d_3)T}{2}}$$

$$\bar{y}(T,\boldsymbol{\alpha},\beta,\gamma) = \bar{b} = c_1 e^{\frac{(d_1)T}{2}} + c_3 e^{\frac{(d_3)T}{2}}$$

$$\begin{bmatrix}\underline{a}\\ \overline{a}\\ \underline{b}\\ \overline{b}\end{bmatrix}=\begin{bmatrix}0 & e^{\frac{(d_1)t_0}{2}} & 0 & e^{\frac{(d_3)t_0}{2}}\\ e^{\frac{(d_1)t_0}{2}} & 0 & e^{\frac{(d_3)t_0}{2}} & 0\\ 0 & e^{\frac{(d_1)T}{2}} & 0 & e^{\frac{(d_3)T}{2}}\\ e^{\frac{(d_1)T}{2}} & 0 & e^{\frac{(d_3)T}{2}} & 0\end{bmatrix}\begin{bmatrix}c_1\\ c_2\\ c_3\\ c_4\end{bmatrix}$$

The solution of C's:

$$c_1=\frac{-(\overline{a}e^{\frac{(d_3)T}{2}}-\overline{b}e^{\frac{(d_3)t_0}{2}})}{(e^{\frac{(d_1)T}{2}}e^{\frac{(d_3)t_0}{2}}-e^{\frac{(d_3)T}{2}}e^{\frac{(d_1)t_0}{2}})},\quad c_2=\frac{-(\underline{a}e^{\frac{(d_3)T}{2}}-\underline{b}e^{\frac{(d_3)t_0}{2}})}{(e^{\frac{(d_1)T}{2}}e^{\frac{(d_3)t_0}{2}}-e^{\frac{(d_3)T}{2}}e^{\frac{(d_1)t_0}{2}})},$$

$$c_3=\frac{(\overline{a}e^{\frac{(d_1)T}{2}}-\overline{b}e^{\frac{(d_1)t_0}{2}})}{(e^{\frac{(d_1)T}{2}}e^{\frac{(d_3)t_0}{2}}-e^{\frac{(d_3)T}{2}}e^{\frac{(d_1)t_0}{2}})},\quad c_4=\frac{(\underline{a}e^{\frac{(d_1)T}{2}}-\underline{b}e^{\frac{(d_1)t_0}{2}})}{(e^{\frac{(d_1)T}{2}}e^{\frac{(d_3)t_0}{2}}-e^{\frac{(d_3)T}{2}}e^{\frac{(d_1)t_0}{2}})}$$

By using the value of $c, \underline{a}, \overline{a}, \underline{b}$ $and \overline{b}$ then get $\underline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$ and $\overline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$

Then, the other classes can be obtained by the same method.

**3.1.2. Case (2): Positive sign for (+p) but the negative sign for (-q)**

Also, any case of negative sign of the coefficient is needed to be shown to realize the effect of the negative sign on the generalized Hukuhara differentiability definition for the system.

$$\boldsymbol{\widetilde{y''}(t)=p\widetilde{y'}(t)-q\widetilde{y}(t)}$$

According to the Hukuhara definition, it is known that the negative sign turns the lower term into the upper term and vice versa.

**Class (1, 1)**

$$\overline{y''}(t,\boldsymbol{\alpha},\beta,\gamma)=p.\overline{y'}(t,\boldsymbol{\alpha},\beta,\gamma)-q.\underline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$$

$$\underline{y''}(t,\boldsymbol{\alpha},\beta,\gamma)=p.\underline{y'}(t,\boldsymbol{\alpha},\beta,\gamma)-q.\overline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$$

**Class (1, 2)**

$$\overline{y}''(t,\boldsymbol{\alpha},\beta,\gamma)=p.\underline{y'}(t,\boldsymbol{\alpha},\beta,\gamma)-q.\overline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$$

$$\underline{y''}(t,\boldsymbol{\alpha},\beta,\gamma)=p.\overline{y'}(t,\boldsymbol{\alpha},\beta,\gamma)-q.\underline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$$

**Class (2, 1)**

$$\overline{y}''(t,\boldsymbol{\alpha},\beta,\gamma)=p.\overline{y'}(t,\boldsymbol{\alpha},\beta,\gamma)-q.\overline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$$

$$\boldsymbol{\underline{y''}(t,\alpha,\beta,\gamma)=p.\underline{y'}(t,\alpha,\beta,\gamma)-q.\underline{y}(t,\alpha,\beta,\gamma)}$$

**Class (2, 2)**

$$\underline{y''}(t,\boldsymbol{\alpha},\beta,\gamma)=p.\overline{y'}(t,\boldsymbol{\alpha},\beta,\gamma)-q.\overline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$$

$$\overline{y''}(t,\boldsymbol{\alpha},\beta,\gamma)=p.\underline{y'}(t,\boldsymbol{\alpha},\beta,\gamma)-q.\underline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$$

Also to find the analytical solution of each class, the same methodology will be used and it will be described in solving first class as an example

**Class (1, 1)**

$$\overline{y''}(t,\boldsymbol{\alpha},\beta,\gamma)=p.\overline{y'}(t,\boldsymbol{\alpha},\beta,\gamma)-q.\underline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$$

$$\underline{y''}(t,\boldsymbol{\alpha},\beta,\gamma)=p.\underline{y'}(t,\boldsymbol{\alpha},\beta,\gamma)-q.\overline{y}(t,\boldsymbol{\alpha},\beta,\gamma)$$

Let $\underline{y}=x$, $\overline{y}=z$

$$x''=px'-qz$$

$$x' = u$$

$$z'' = pz' - qx$$

$$z' = v$$

$$\begin{bmatrix} x \\ u \\ z \\ v \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & p & -q & 0 \\ 0 & 0 & 0 & 1 \\ -q & 0 & 0 & p \end{bmatrix} \begin{bmatrix} x \\ u \\ z \\ v \end{bmatrix}$$

The solution

$$\boldsymbol{x = \underline{y}(t,\alpha) = c_1 e^{\frac{t}{2}d_2} + c_2 e^{\frac{t}{2}d_1} + c_3 e^{\frac{t}{2}d_4} + c_4 e^{\frac{t}{2}d_3}}$$

$$\boldsymbol{z = \overline{y}(t,\alpha) = c_1 e^{\frac{t}{2}d_2} - c_2 e^{\frac{t}{2}d_1} + c_3 e^{\frac{t}{2}d_4} - c_4 e^{\frac{t}{2}d_3}}$$

After studying all classes of all cases of different signs of coefficients, a table of collected general solutions of all cases will be introduced to help in solving applications by substituting

Table 1. General solutions of N.F Differential equations

| | Class(1,1) | Class(1,2) | Class(2,1) | Class(2,2) |
|---|---|---|---|---|
| +p, +q | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ q & p & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & q & p \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & q & p \\ 0 & 0 & 0 & 1 \\ q & p & 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & p & q & 0 \\ 0 & 0 & 0 & 1 \\ q & 0 & 0 & p \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ q & 0 & 0 & p \\ 0 & 0 & 0 & 1 \\ 0 & p & q & 0 \end{bmatrix}$ |
| G. S | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_2 e^{\frac{(d_1)t}{2}} + c_4 e^{\frac{(d_3)t}{2}}$ <br> $\overline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{(d_1)t}{2}} + c_3 e^{\frac{(d_3)t}{2}}$ | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{-t}{2}d_2} + c_2 e^{\frac{-t}{2}d_4}$ <br> $+ c_3 e^{\frac{t}{2}d_3} + c_4 e^{\frac{t}{2}d_1}$ <br> $\overline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{-t}{2}d_2}\left(1 - d_4(p/q)\right)$ <br> $+ c_2 e^{\frac{-t}{2}d_4}\left(1 - d_2(p/q)\right)$ <br> $+ c_3 e^{\frac{t}{2}d_3} + c_4 e^{\frac{t}{2}d_1}$ | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{t}{2}d_2} + c_2 e^{\frac{t}{2}d_1} + c_3 e^{\frac{t}{2}d_4}$ <br> $+ c_4 e^{\frac{t}{2}d_3}$ <br> $\overline{y}(t,\alpha,\beta,\gamma)$ <br> $= -c_1 e^{\frac{t}{2}d_2} + c_2 e^{\frac{t}{2}d_1}$ <br> $- c_3 e^{\frac{t}{2}d_4} + c_4 e^{\frac{t}{2}d_3}$ | $\underline{y}(t,\alpha)$ <br> $= c_1 e^{\frac{-t}{2}d_3} + c_2 e^{\frac{t}{2}d_3}$ <br> $+ c_3 e^{\frac{-t}{2}d_1} + c_4 e^{\frac{t}{2}d_1}$ <br> $\overline{y}(t,\alpha,\beta,\gamma)$ <br> $= -c_1 e^{\frac{-t}{2}d_3} + c_2 e^{\frac{t}{2}d_3}$ <br> $- c_3 e^{\frac{-t}{2}d_1} + c_4 e^{\frac{t}{2}d_1}$ |
| -p -q | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -q & -p \\ 0 & 0 & 0 & 1 \\ -q & -p & 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -q & -p & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -q & -p \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -q & 0 & 0 & -p \\ 0 & 0 & 0 & 1 \\ 0 & -p & -q & 0 \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -p & -q & 0 \\ 0 & 0 & 0 & 1 \\ -q & 0 & 0 & -p \end{bmatrix}$ |
| G S | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{-t}{2}d_2} + c_3 e^{\frac{-t}{2}d_4}$ <br> $+ c_4 e^{\frac{t}{2}d_3} + c_2 e^{\frac{t}{2}d_1}$ <br> $\overline{y}(t,\alpha,\beta,\gamma) =$ <br> $-c_1 e^{\frac{-t}{2}d_2}\left(1 - d_4(p/q)\right) -$ <br> $c_3 e^{\frac{-t}{2}d_4}\left(1 - d_2(p/q)\right)$ <br> $- c_4 e^{\frac{t}{2}d_3} - c_2 e^{\frac{t}{2}d_1}$ | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_2 e^{\frac{(-d_4)t}{2}} + c_4 e^{\frac{(-d_2)t}{2}}$ <br> $\overline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{(-d_4)t}{2}} + c_3 e^{\frac{(-d_2)t}{2}}$ | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{t}{2}d_2} + c_2 e^{\frac{-t}{2}d_4}$ <br> $+ c_3 e^{\frac{-t}{2}d_2} + c_4 e^{\frac{t}{2}d_4}$ <br> $\overline{y}(t,\alpha,\beta,\gamma)$ <br> $= -c_1 e^{\frac{t}{2}d_2} + c_2 e^{\frac{-t}{2}d_4}$ <br> $+ c_3 e^{\frac{-t}{2}d_2} - c_4 e^{\frac{t}{2}d_4}$ | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{-t}{2}d_2} + c_2 e^{\frac{-t}{2}d_1}$ <br> $+ c_3 e^{\frac{-t}{2}d_4} + c_4 e^{\frac{-t}{2}d_3}$ <br> $\overline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{-t}{2}d_2} - c_2 e^{\frac{-t}{2}d_1}$ <br> $+ c_3 e^{\frac{-t}{2}d_4} - c_4 e^{\frac{-t}{2}d_3}$ |
| +p -q | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & p & -q & 0 \\ 0 & 0 & 0 & 1 \\ -q & 0 & 0 & p \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -q & 0 & 0 & p \\ 0 & 0 & 0 & 1 \\ 0 & p & -q & 0 \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -q & p & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -q & p \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -q & p \\ 0 & 0 & 0 & 1 \\ -q & p & 0 & 0 \end{bmatrix}$ |
| G S | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{t}{2}d_2} + c_2 e^{\frac{t}{2}d_1} + c_3 e^{\frac{t}{2}d_4}$ <br> $+ c_4 e^{\frac{t}{2}d_3}$ | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{-t}{2}d_2} + c_2 e^{\frac{t}{2}d_2}$ <br> $+ c_3 e^{\frac{-t}{2}d_4} + c_4 e^{\frac{t}{2}d_4}$ | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_2 e^{\frac{(d_2)t}{2}} + c_4 e^{\frac{(d_4)t}{2}}$ <br> $\overline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{(d_2)t}{2}} + c_3 e^{\frac{(d_4)t}{2}}$ | $\underline{y}(t,\alpha,\beta,\gamma)$ <br> $= c_1 e^{\frac{t}{2}d_2} + c_2 e^{\frac{-t}{2}d_1}$ <br> $+ c_3 e^{\frac{t}{2}d_4} + c_4 e^{\frac{-t}{2}d_3}$ |

| | | | | |
|---|---|---|---|---|
| | $\overline{y}(t,\alpha,\beta,\gamma) = c_1 e^{\frac{t}{2}d_2} - c_2 e^{\frac{t}{2}d_1} + c_3 e^{\frac{t}{2}d_4} - c_4 e^{\frac{t}{2}d_3}$ | $\overline{y}(t,\alpha,\beta,\gamma) = -c_1 e^{\frac{-t}{2}d_2} + c_2 e^{\frac{t}{2}d_2} - c_3 e^{\frac{-t}{2}d_4} + c_4 e^{\frac{t}{2}d_4}$ | | $\overline{y}(t,\alpha,\beta,\gamma) = -c_1 e^{\frac{t}{2}d_2}\left(1 - d_4\left({p}/{q}\right)\right) - c_3 e^{\frac{t}{2}d_4}\left(1 - d_2\left({p}/{q}\right)\right) - c_2 e^{\frac{-t}{2}d_1} - c_4 e^{\frac{-t}{2}d_3}$ |
| -p +q | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ q & 0 & 0 & -p \\ 0 & 0 & 0 & 1 \\ 0 & -p & q & 0 \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -p & q & 0 \\ 0 & 0 & 0 & 1 \\ q & 0 & 0 & -p \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & q & -p \\ 0 & 0 & 0 & 1 \\ q & -p & 0 & 0 \end{bmatrix}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ q & -p & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & q & -p \end{bmatrix}$ |
| G S | $\underline{y}(t,\alpha,\beta,\gamma) = c_1 e^{\frac{-t}{2}d_1} + c_2 e^{\frac{t}{2}d_1} + c_3 e^{\frac{-t}{2}d_3} + c_4 e^{\frac{t}{2}d_3}$ <br> $\overline{y}(t,\alpha,\beta,\gamma) = c_1 e^{\frac{-t}{2}d_1} - c_2 e^{\frac{t}{2}d_1} + c_3 e^{\frac{-t}{2}d_3} - c_4 e^{\frac{t}{2}d_3}$ | $\underline{y}(t,\alpha,\beta,\gamma) = c_1 e^{\frac{-t}{2}d_4} + c_2 e^{\frac{-t}{2}d_3} + c_3 e^{\frac{-t}{2}d_2} + c_4 e^{\frac{-t}{2}d_1}$ <br> $\overline{y}(t,\alpha,\beta,\gamma) = -c_1 e^{\frac{-t}{2}d_4} + c_2 e^{\frac{-t}{2}d_3} - c_3 e^{\frac{-t}{2}d_2} + c_4 e^{\frac{-t}{2}d_1}$ | $\underline{y}(t,\alpha,\beta,\gamma) = c_1 e^{\frac{t}{2}d_2} + c_2 e^{\frac{t}{2}d_4} + c_3 e^{\frac{-t}{2}d_3} + c_4 e^{\frac{-t}{2}d_1}$ <br> $\overline{y}(t,\alpha,\beta,\gamma) = c_1 e^{\frac{t}{2}d_2}\left(1 - d_4\left({p}/{q}\right)\right) + c_2 e^{\frac{t}{2}d_4}\left(1 - d_2\left({p}/{q}\right)\right) + c_3 e^{\frac{-t}{2}d_3} + c_4 e^{\frac{-t}{2}d_1}$ | $\underline{y}(t,\alpha,\beta,\gamma) = c_2 e^{\frac{(-d_1)t}{2}} + c_4 e^{\frac{(-d_3)t}{2}}$ <br> $\overline{y}(t,\alpha,\beta,\gamma) = c_1 e^{\frac{(-d_1)t}{2}} + c_3 e^{\frac{(-d_3)t}{2}}$ |

## 4. Applications

### 4.1. Example 1

$$\widetilde{y''}(t) = 5\widetilde{y'}(t) + 4\tilde{y}(t),$$
$$\tilde{y}(t_0 = 0) = \tilde{a} = (0.8,1,1.4;0.8,0.2,0.3)$$
$$\tilde{y}(T = 1) = \tilde{b} = (2.6,3,3.1,;0.8,0.2,0.3)$$

**Solution:**

- **First step: Analyzing the neutrosophic boundary points**

  According to Definition 4 the generalized NF triangle point, then

$$\left[\underline{a(\alpha)}, \overline{a(\alpha)}\right] = \left[\left(0.8 + \frac{\alpha}{4}\right), \left(1.4 - \frac{\alpha}{2}\right)\right]$$

$$\left[\underline{a(\beta)}, \overline{a(\beta)}\right] = \left[0.8 + \frac{(1-\beta)}{4}, 1.4 - \frac{(1-\beta)}{2}\right]$$

$$\left[\underline{a(\gamma)}, \overline{a(\gamma)}\right] = \left[\left(0.8 + \frac{2(1-\gamma)}{7}\right), \left(1.4 - \frac{4(1-\gamma)}{7}\right)\right]$$

$$\left[\underline{b(\alpha)}, \overline{b(\alpha)}\right] = \left[\left(2.6 + \frac{\alpha}{2}\right), \left(3.1 - \frac{\alpha}{8}\right)\right]$$

$$\left[\underline{b(\beta)}, \overline{b(\beta)}\right] = \left[\left(2.6 + \frac{(1-\beta)}{2}\right), \left(3.1 - \frac{(1-\beta)}{8}\right)\right]$$

$$\left[\underline{b(\gamma)}, \overline{b(\gamma)}\right] = \left[\left(2.6 + \frac{4(1-\gamma)}{7}\right), \left(3.1 - \frac{1-\gamma}{7}\right)\right]$$

- **Second step: Solving the problem according to methodology or use Table 1**

  For positives signs of coefficients and substituting the values of p and q

**Class (1, 1):**

$$\underline{y}(t,\alpha,\beta,\gamma) = c_2 e^{\frac{(5+\sqrt{41})t}{2}} + c_4 e^{\frac{(5-\sqrt{41})t}{2}}$$

$$\overline{y}(t,\alpha,\beta,\gamma) = c_1 e^{\frac{(5+\sqrt{41})t}{2}} + c_3 e^{\frac{(5-\sqrt{41})t}{2}}$$

To find the values of constants for each $\alpha, \beta, \gamma$, then use the boundary points

$$\underline{y}(t_0,\alpha,\beta,\gamma) = c_2 e^{\frac{(5+\sqrt{41})t_0}{2}} + c_4 e^{\frac{(5-\sqrt{41})t_0}{2}} = [0.8 + \frac{\alpha}{4}, 0.8 + \frac{(1-\beta)}{4}, 0.8 + \frac{2(1-\gamma)}{7}]$$

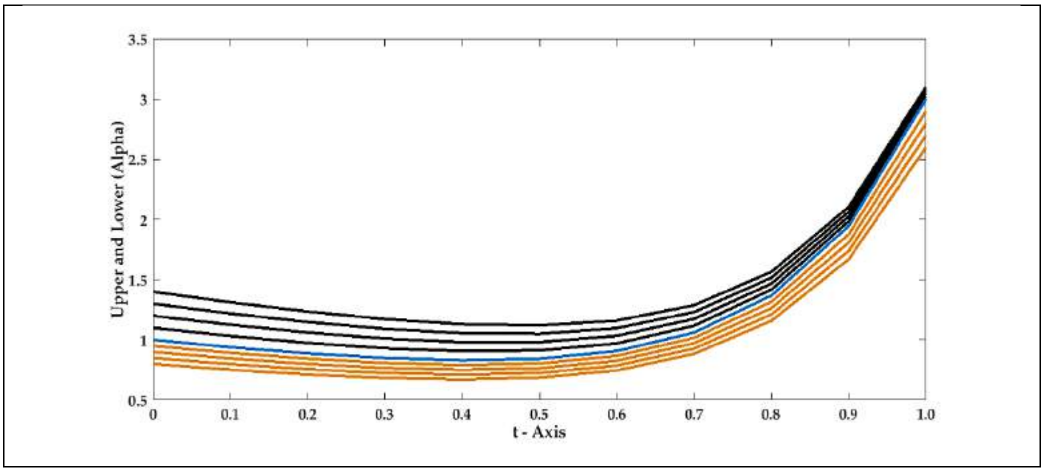
$$\overline{y}(t_0,\alpha,\beta,\gamma) = c_1 e^{\frac{(5+\sqrt{41})t_0}{2}} + c_3 e^{\frac{(5-\sqrt{41})t_0}{2}} = [1.4 - \frac{\alpha}{2}, 1.4 - \frac{(1-\beta)}{2}, 1.4 - \frac{4(1-\gamma)}{7}]$$

$$\underline{y}(T,\alpha,\beta,\gamma) = c_2 e^{\frac{(5+\sqrt{41})T}{2}} + c_4 e^{\frac{(5-\sqrt{41})T}{2}} = [2.6 + \frac{\alpha}{2}, 2.6 + \frac{(1-\beta)}{2}, 2.6 + \frac{4(1-\gamma)}{7}]$$

$$\overline{y}(T,\alpha,\beta,\gamma) = c_1 e^{\frac{(5+\sqrt{41})T}{2}} + c_3 e^{\frac{(5-\sqrt{41})T}{2}} = [3.1 - \frac{\alpha}{8} 3.1 - \frac{(1-\beta)}{8}, 3.1 - \frac{1-\gamma}{7}]$$

*Table (2): The Results of Upper and Lower for* $(\alpha, \beta, \gamma)$ *at* $t = 0.5$

| $\alpha$ | $\underline{y}(0.5,\alpha)$ | $\overline{y}(0.5,\alpha)$ | $\beta$ | $\underline{y}(0.5,\beta)$ | $\overline{y}(0.5,\beta)$ | $\gamma$ | $\underline{y}(0.5,\gamma)$ | $\overline{y}(0.5,\gamma)$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.685682 | 0.713451 | 0.2 | 0.843218 | 0.843218 | 0.3 | 0.843218 | 0.843218 |
| 0.2 | 0.725066 | 0.745893 | 0.4 | 0.803834 | 0.912266 | 0.5 | 0.798207 | 0.922130 |
| 0.4 | 0.764450 | 0.778334 | 0.6 | 0.764450 | 0.981315 | 0.7 | 0.753197 | 1.001043 |
| 0.6 | 0.803834 | 0.810776 | 0.8 | 0.725066 | 1.050363 | 0.9 | 0.708187 | 1.079956 |
| 0.8 | 0.907334 | 0.907334 | 1.0 | 0.685682 | 1.119412 | 1.0 | 0.685682 | 1.119412 |



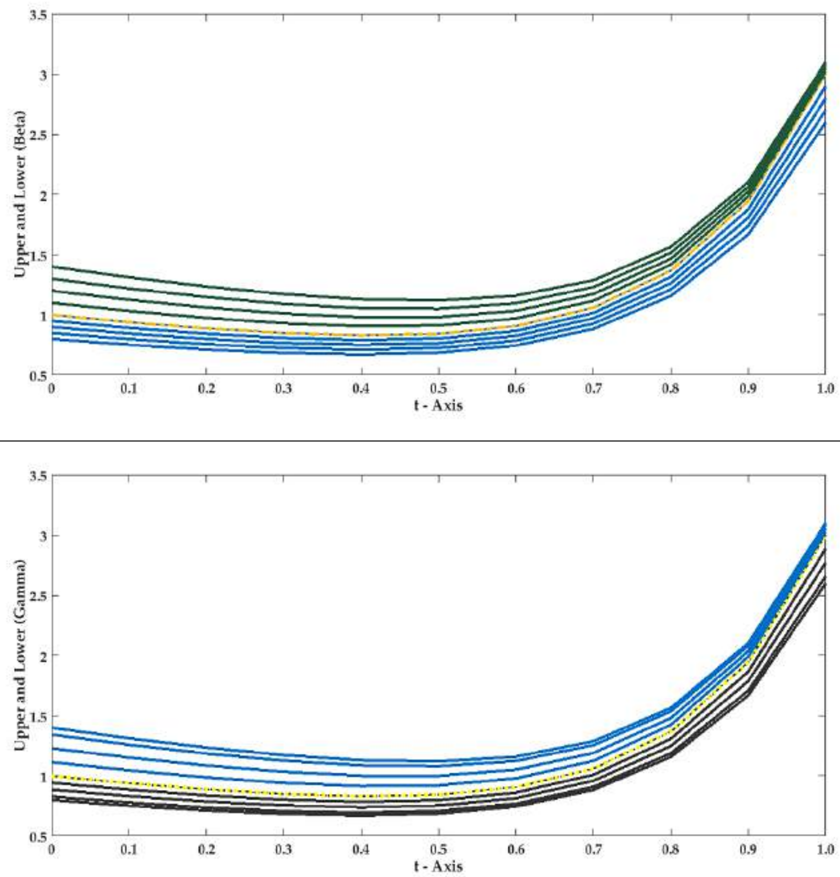



**Fig. 1:** The Results of Upper and Lower for $(\alpha, \beta, \gamma)$ **of Class(1,1).**

**Class (1, 2)**

$$\underline{y}(t, \alpha, \beta, \gamma) = c_1 e^{-4t} + (c_2) e^{-t} + c_3 e^{\frac{(5-\sqrt{41})t}{2}} + c_4 e^{\frac{(5+\sqrt{41})t}{2}},$$

$$\overline{y}(t, \alpha, \beta, \gamma) = c_1 e^{-4t}\left(-\left(3/_2\right)\right) + c_2 e^{-t}(-9) + c_3 e^{\frac{(5-\sqrt{41})t}{2}} + c_4 e^{\frac{(5+\sqrt{41})t}{2}}.$$

To find the values of constants for each $\alpha, \beta, \gamma$, then use the boundary points

$$\underline{y}(t_0, \alpha, \beta, \gamma) = c_1 e^{-4t_0} + (c_2) e^{-t_0} + c_3 e^{\frac{(5-\sqrt{41})t_0}{2}} + c_4 e^{\frac{(5+\sqrt{41})t_0}{2}} = 0.8 + \frac{\alpha}{4}, 0.8 + \frac{(1-\beta)}{4}, 0.8 + \frac{2(1-\gamma)}{7}$$

$$\overline{y}(t_0, \alpha, \beta, \gamma) = -1.5 c_1 e^{-4t_0} - 9 c_2 e^{-t_0} + c_3 e^{\frac{(5-\sqrt{41})t}{2}} + c_4 e^{\frac{(5+\sqrt{41})t}{2}} = 1.4 - \frac{\alpha}{2}, 1.4 - \frac{(1-\beta)}{2}, 1.4 - \frac{4(1-\gamma)}{7}$$

$$\underline{y}(T, \alpha, \beta, \gamma) = c_1 e^{-4T} + (c_2) e^{-T} + c_3 e^{\frac{(5-\sqrt{41})T}{2}} + c_4 e^{\frac{(5+\sqrt{41})T}{2}} = 2.6 + \frac{\alpha}{2}, 2.6 + \frac{(1-\beta)}{2}, 2.6 + \frac{4(1-\gamma)}{7}$$

$$\overline{y}(T, \alpha, \beta, \gamma) = -1.5 c_1 e^{-4T} - 9 c_2 e^{-T} + c_3 e^{\frac{(5-\sqrt{41})T}{2}} + c_4 e^{\frac{(5+\sqrt{41})T}{2}} = 3.1 - \frac{\alpha}{8}, 3.1 - \frac{(1-\beta)}{8}, 3.1 - \frac{1-\gamma}{7}$$

*Table (3): The Results of Upper and Lower for (α,β,γ) at t = 0.5*

| $\alpha$ | $\underline{y}(0.5,\alpha)$ | $\overline{y}(0.5,\alpha)$ | $\beta$ | $\underline{y}(0.5,\beta)$ | $\overline{y}(0.5,\beta)$ | $\gamma$ | $\underline{y}(0.5,\gamma)$ | $\overline{y}(0.5,\gamma)$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.525047 | 1.265411 | 0.2 | 0.843218 | 0.843218 | 0.3 | 0.843218 | 0.843218 |
| 0.2 | 0.604589 | 1.159863 | 0.4 | 0.763675 | 0.948766 | 0.5 | 0.752312 | 0.963844 |
| 0.4 | 0.684132 | 1.054314 | 0.6 | 0.684132 | 1.054314 | 0.7 | 0.661406 | 1.084471 |
| 0.6 | 0.763675 | 0.948766 | 0.8 | 0.604589 | 1.159863 | 0.9 | 0.570500 | 1.205098 |
| 0.8 | 0.907334 | 0.843218 | 1.0 | 0.525047 | 1.265411 | 1.0 | 0.525047 | 1.265411 |

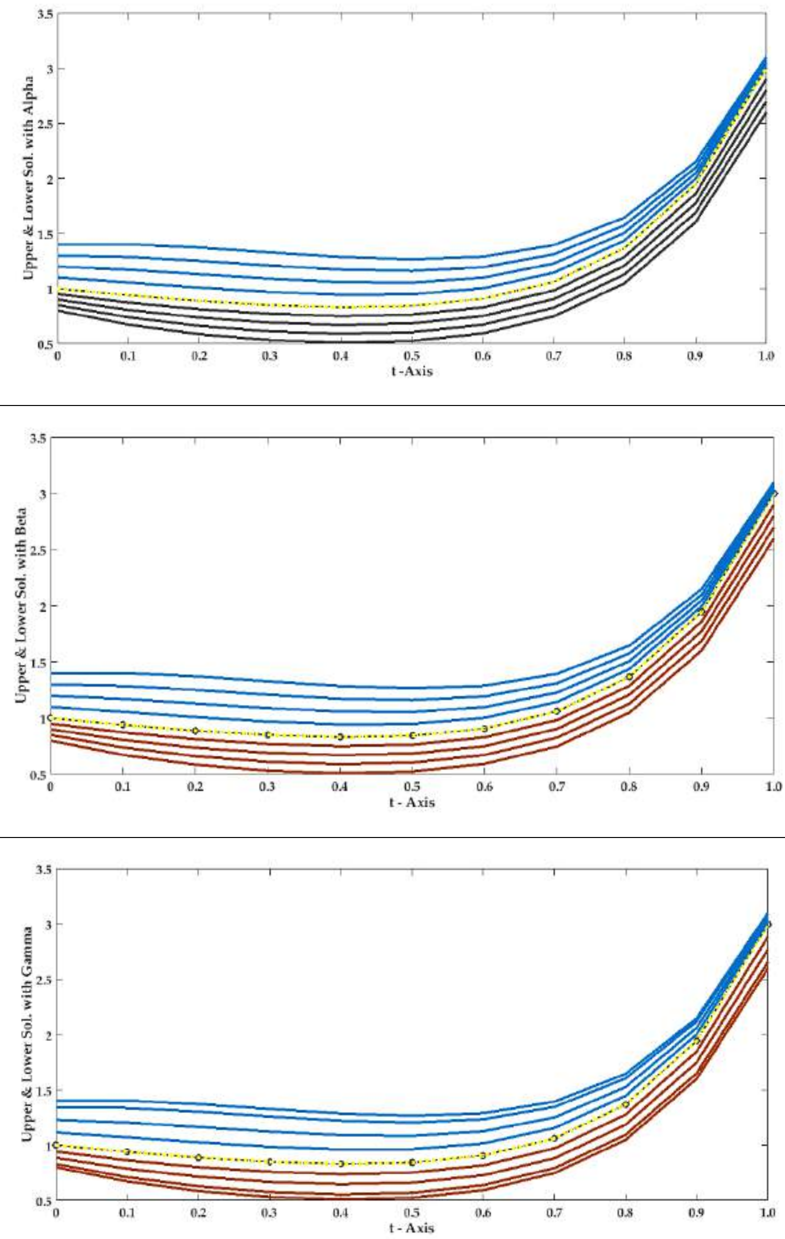



**Fig. 2:** The Results of Upper and Lower for $(\alpha,\beta,\gamma)$ **of Class(1,2).**

**Class (2, 1)**

$$\underline{y}(t,\boldsymbol{\alpha},\beta,\gamma)=\boldsymbol{c_1}e^{4t}+\boldsymbol{c_2}e^{\frac{(5+\sqrt{41})t}{2}}+\boldsymbol{c_3}e^{t}+\boldsymbol{c_4}e^{\frac{(5-\sqrt{41})t}{2}},$$

$$\bar{y}(t,\boldsymbol{\alpha},\beta,\gamma)=-\boldsymbol{c_1}e^{4t}+\boldsymbol{c_2}e^{\frac{(5+\sqrt{41})t}{2}}-\boldsymbol{c_3}e^{t}+\boldsymbol{c_4}e^{\frac{(5-\sqrt{41})t}{2}}$$

To find the values of constants for each $\alpha,\beta,\gamma,$ then use the boundary points

$$\underline{\boldsymbol{y}}(t_0,\boldsymbol{\alpha},\beta,\gamma)=\ \boldsymbol{c_1}e^{4t_0}+\boldsymbol{c_2}e^{\frac{(5+\sqrt{41})t_0}{2}}+\boldsymbol{c_3}e^{t_0}+\boldsymbol{c_4}e^{\frac{(5-\sqrt{41})t_0}{2}}=0.8+\frac{\alpha}{4},0.8+\frac{(1-\beta)}{4},0.8+\frac{2(1-\gamma)}{7}$$

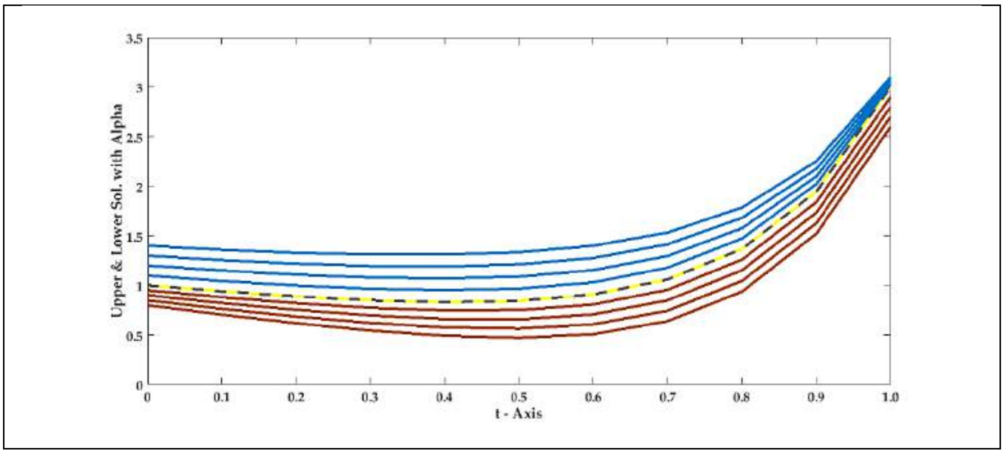
$$\bar{\boldsymbol{y}}(t_0,\boldsymbol{\alpha},\beta,\gamma)=-\boldsymbol{c_1}e^{4t_0}+\boldsymbol{c_2}e^{\frac{(5+\sqrt{41})t_0}{2}}-\boldsymbol{c_3}e^{t_0}+\boldsymbol{c_4}e^{\frac{(5-\sqrt{41})t_0}{2}}=1.4-\frac{\alpha}{2},1.4-\frac{(1-\beta)}{2},1.4-\frac{4(1-\gamma)}{7}$$

$$\underline{\boldsymbol{y}}(T,\boldsymbol{\alpha},\beta,\gamma)=\ \boldsymbol{c_1}e^{4T}+\boldsymbol{c_2}e^{\frac{(5+\sqrt{41})T}{2}}+\boldsymbol{c_3}e^{T}+\boldsymbol{c_4}e^{\frac{(5-\sqrt{41})T}{2}}=2.6+\frac{\alpha}{2},2.6+\frac{(1-\beta)}{2},2.6+\frac{4(1-\gamma)}{7}$$

$$\bar{\boldsymbol{y}}(T,\boldsymbol{\alpha},\beta,\gamma)=\ -\boldsymbol{c_1}e^{4T}+\boldsymbol{c_2}e^{\frac{(5+\sqrt{41})T}{2}}-\boldsymbol{c_3}e^{T}+\boldsymbol{c_4}e^{\frac{(5-\sqrt{41})T}{2}}=3.1-\frac{\alpha}{8},3.1-\frac{(1-\beta)}{8},3.1-\frac{1-\gamma}{7}$$

*Table (4): The Results of Upper and Lower for (α,β,γ) at t = 0.5*

| $\alpha$ | $\underline{\boldsymbol{y}}(0.5,\alpha)$ | $\bar{\boldsymbol{y}}(0.5,\alpha)$ | $\beta$ | $\underline{\boldsymbol{y}}(0.5,\beta)$ | $\bar{\boldsymbol{y}}(0.5,\beta)$ | $\gamma$ | $\underline{\boldsymbol{y}}(0.5,\gamma)$ | $\bar{\boldsymbol{y}}(0.5,\gamma)$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.470499 | 1.334594 | 0.2 | 0.843218 | 0.843218 | 0.3 | 0.843218 | 0.843218 |
| 0.2 | 0.563679 | 1.211750 | 0.4 | 0.750038 | 0.966062 | 0.5 | 0.736727 | 0.983611 |
| 0.4 | 0.656859 | 1.088906 | 0.6 | 0.656859 | 1.088906 | 0.7 | 0.630236 | 1.124004 |
| 0.6 | 0.750038 | 0.966062 | 0.8 | 0.563679 | 1.211750 | 0.9 | 0.523745 | 1.264398 |
| 0.8 | 0.907334 | 0.843218 | 1.0 | 0.470500 | 1.334594 | 1.0 | 0.470499 | 1.334594 |



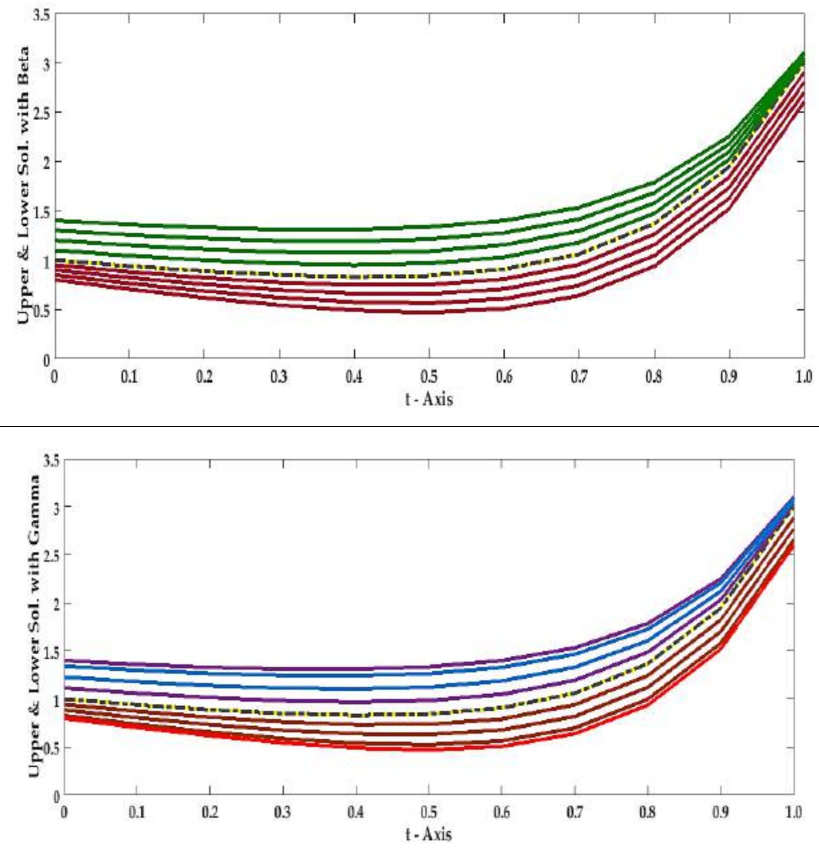



**Fig. 3:** The Results of Upper and Lower for $(\alpha,\beta,\gamma)$ **of Class(2,1).**

**Class (2, 2)**

$$\underline{y}(t,\boldsymbol{\alpha},\beta,\gamma)=\boldsymbol{c_1}e^{\frac{-(5-\sqrt{41})t}{2}}+\boldsymbol{c_2}e^{\frac{(5-\sqrt{41})t}{2}}+\boldsymbol{c_3}e^{\frac{-(5+\sqrt{41})t}{2}}+\boldsymbol{c_4}e^{\frac{(5+\sqrt{41})t}{2}},$$

$$\bar{y}(t,\boldsymbol{\alpha},\beta,\gamma)=-\boldsymbol{c_1}e^{\frac{-(5-\sqrt{41})t}{2}}+\boldsymbol{c_2}e^{\frac{(5-\sqrt{41})t}{2}}-\boldsymbol{c_3}e^{\frac{-(5+\sqrt{41})t}{2}}+\boldsymbol{c_4}e^{\frac{(5+\sqrt{41})t}{2}}$$

To find the values of constants for each $\alpha,\beta,\gamma,$ then use the boundary points

$$\underline{\boldsymbol{y}}(t_0,\boldsymbol{\alpha},\beta,\gamma)=\ \boldsymbol{c_1}e^{\frac{-(5-\sqrt{41})t_0}{2}}+\boldsymbol{c_2}e^{\frac{(5-\sqrt{41})t_0}{2}}+\boldsymbol{c_3}e^{\frac{-(5+\sqrt{41})t_0}{2}}+\boldsymbol{c_4}e^{\frac{(5+\sqrt{41})t_0}{2}}=0.8+\frac{\alpha}{4}\ 0.8+\frac{(1-\beta)}{4},0.8+\frac{2(1-\gamma)}{7}$$

$$\bar{\boldsymbol{y}}(t_0,\boldsymbol{\alpha},\beta,\gamma)=-\boldsymbol{c_1}e^{\frac{-(5-\sqrt{41})t_0}{2}}+\boldsymbol{c_2}e^{\frac{(5-\sqrt{41})t_0}{2}}-\boldsymbol{c_3}e^{\frac{-(5+\sqrt{41})t_0}{2}}+\boldsymbol{c_4}e^{\frac{(5+\sqrt{41})t_0}{2}}=1.4-\frac{\alpha}{2}\ ,1.4-\frac{(1-\beta)}{2},1.4-\frac{4(1-\gamma)}{7},$$

$$\underline{\boldsymbol{y}}(T,\boldsymbol{\alpha},\beta,\gamma)=\ \boldsymbol{c_1}e^{\frac{-(5-\sqrt{41})T}{2}}+\boldsymbol{c_2}e^{\frac{(5-\sqrt{41})T}{2}}+\boldsymbol{c_3}e^{\frac{-(5+\sqrt{41})T}{2}}+\boldsymbol{c_4}e^{\frac{(5+\sqrt{41})T}{2}}=2.6+\frac{\alpha}{2},2.6+\frac{(1-\beta)}{2},2.6+\frac{4(1-\gamma)}{7}$$

$$\bar{\boldsymbol{y}}(T,\boldsymbol{\alpha},\beta,\gamma)=\ -\boldsymbol{c_1}e^{\frac{-(5-\sqrt{41})T}{2}}+\boldsymbol{c_2}e^{\frac{(5-\sqrt{41})T}{2}}-\boldsymbol{c_3}e^{\frac{-(5+\sqrt{41})T}{2}}+\boldsymbol{c_4}e^{\frac{(5+\sqrt{41})T}{2}}=3.1-\frac{\alpha}{8},3.1-\frac{(1-\beta)}{8},3.1-\frac{1-\gamma}{7}$$

*Table (5): The Results of Upper and Lower for (α,β,γ) at t = 0.5*

| $\alpha$ | $\underline{y}(0.5,\alpha)$ | $\overline{y}(0.5,\alpha)$ | $\beta$ | $\underline{y}(0.5,\beta)$ | $\overline{y}(0.5,\beta)$ | $\gamma$ | $\underline{y}(0.5,\gamma)$ | $\overline{y}(0.5,\gamma)$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.716735 | 1.088359 | 0.2 | 0.843218 | 0.843218 | 0.3 | 0.843218 | 0.843218 |
| 0.2 | 0.748356 | 1.027074 | 0.4 | 0.811597 | 0.904503 | 0.5 | 0.807080 | 0.913258 |
| 0.4 | 0.779976 | 0.965788 | 0.6 | 0.779976 | 0.965788 | 0.7 | 0.770942 | 0.983298 |
| 0.6 | 0.811597 | 0.904503 | 0.8 | 0.748356 | 1.027074 | 0.9 | 0.734804 | 1.053339 |
| 0.8 | 0.843218 | 0.843218 | 1.0 | 0.716735 | 1.088359 | 1.0 | 0.716735 | 1.088359 |

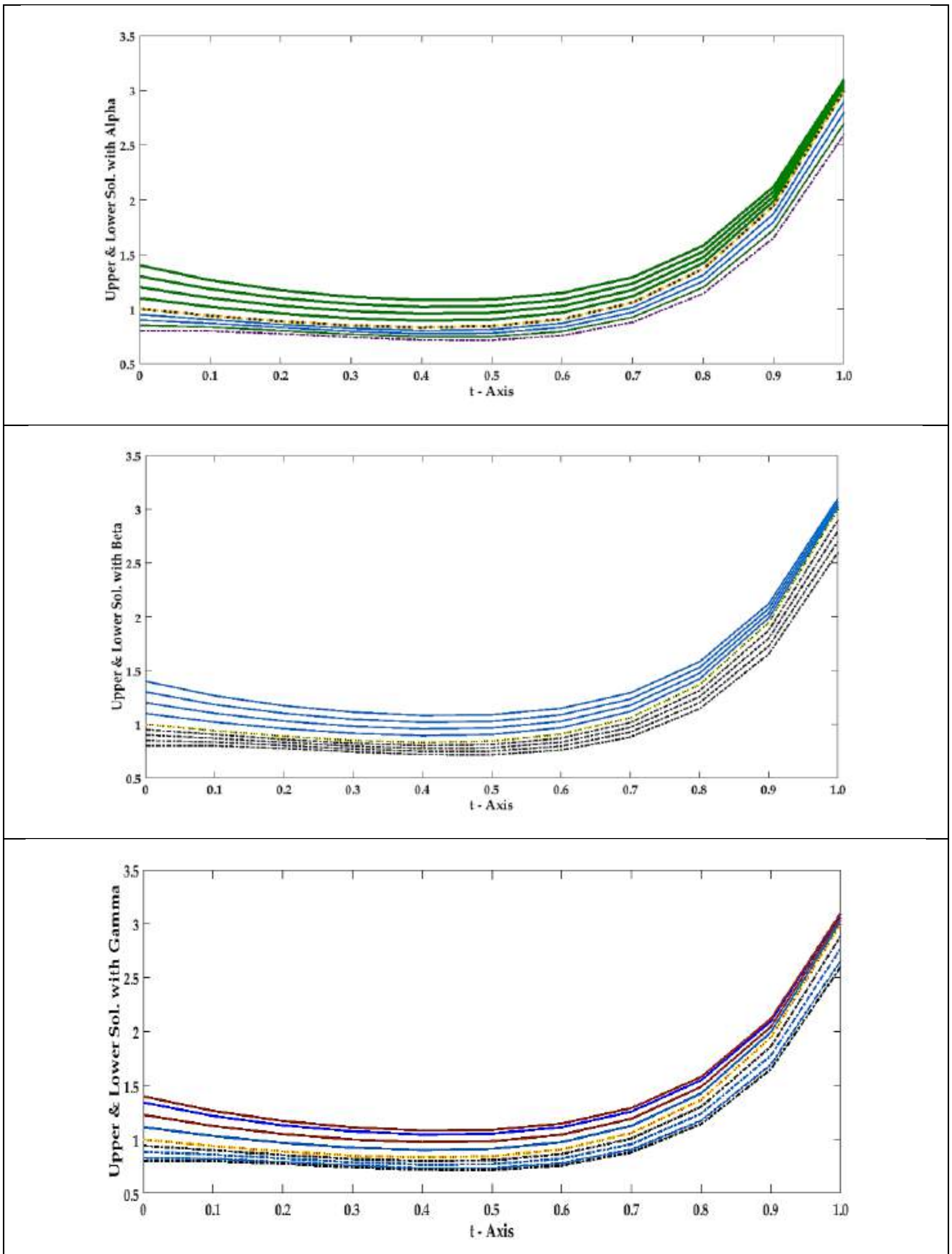



**Fig. 4:** The Results of Upper and Lower for $(\alpha, \beta, \gamma)$ **of Class(2,2).**

In this example, All the last figures which describe the upper and lower solution of each class for each $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$ gave a notification that the solutions bands are the strong solutions because there was no overlapping between the bands, Also by applying the meaning of strong solution from definition 11, the tables show that the solution values at $\boldsymbol{t} = \mathbf{0.5}$ are strong because it is observed that the lower solution is increasing ascending with $\boldsymbol{\alpha}$ but the upper is descending with it, and the lower solution is decreasing with $\boldsymbol{\beta}$ $\boldsymbol{and}$ $\boldsymbol{\gamma}$ but the upper is increasing with them.

**4.2. Example 2**

$$\widetilde{\boldsymbol{y}''}(\boldsymbol{t}) = \mathbf{3}\widetilde{\boldsymbol{y}}'(\boldsymbol{t}) - \mathbf{2}\widetilde{\boldsymbol{y}}(\boldsymbol{t})$$

$$\tilde{y}(t_0 = 0) = \tilde{a} = (0.8,1,1.4; 0.8,0.2,0.3)$$

$$\tilde{y}(T = 1) = \tilde{b} = (2.6,3,3.1,; 0.8,0.2,0.3)$$

**Class (1, 1)**

$$\underline{\boldsymbol{y}}(\boldsymbol{t}, \alpha, \beta, \gamma) = \boldsymbol{c_1}\boldsymbol{e}^{2t} + \boldsymbol{c_2}\boldsymbol{e}^{\frac{(3+\sqrt{17})t}{2}} + \boldsymbol{c_3}\boldsymbol{e}^{t} + \boldsymbol{c_4}\boldsymbol{e}^{\frac{(3-\sqrt{17})t}{2}},$$

$$\overline{\boldsymbol{y}}(\boldsymbol{t}, \alpha, \beta, \gamma) = \boldsymbol{c_1}\boldsymbol{e}^{2t} - \boldsymbol{c_2}\boldsymbol{e}^{\frac{(3+\sqrt{17})t}{2}} + \boldsymbol{c_3}\boldsymbol{e}^{t} - \boldsymbol{c_4}\boldsymbol{e}^{\frac{(3-\sqrt{17})t}{2}}.$$

To find the values of constants for each $\alpha, \beta, \gamma$, then use the boundary points

$$\underline{y}(t_0, \alpha, \beta, \gamma) = c_1 + c_2 + c_3 + c_4 = 0.8 + \frac{\alpha}{4}, 0.8 + \frac{(1-\beta)}{4}, 0.8 + \frac{2(1-\gamma)}{7}$$

$$\overline{y}(t_0, \alpha, \beta, \gamma) = c_1 - c_2 + c_3 - c_4 = 1.4 - \frac{\alpha}{2}, 1.4 - \frac{(1-\beta)}{2}, 1.4 - \frac{4(1-\gamma)}{7}$$

$$\underline{y}(T, \alpha, \beta, \gamma) = c_1 e^2 + c_2 e^{\frac{(3+\sqrt{17})}{2}} + c_3 e^1 + c_4 e^{\frac{(3-\sqrt{17})}{2}} = 2.6 + \frac{\alpha}{2}, 2.6 + \frac{(1-\beta)}{2}, 2.6 + \frac{4(1-\gamma)}{7}$$

$$\overline{y}(T, \alpha, \beta, \gamma) = c_1 e^2 - c_2 e^{\frac{(3+\sqrt{17})}{2}} + c_3 e^1 - c_4 e^{\frac{(3-\sqrt{17})}{2}} = 3.1 - \frac{\alpha}{8}, 3.1 - \frac{(1-\beta)}{8}, 3.1 - \frac{1-\gamma}{7}$$

*Table (6): The Results of Upper and Lower for (α,β,γ) at t = 0.5*

| $\alpha$ | $\underline{y}(0.5, \alpha)$ | $\overline{y}(0.5, \alpha)$ | $\beta$ | $\underline{y}(0.5, \beta)$ | $\overline{y}(0.5, \beta)$ | $\gamma$ | $\underline{y}(0.5, \gamma)$ | $\overline{y}(0.5, \gamma)$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.543156 | 2.019864 | 0.2 | 1.713232 | 1.713232 | 0.3 | 1.713232 | 1.713232 |
| 0.2 | 1.585675 | 1.943206 | 0.4 | 1.670713 | 1.789890 | 0.5 | 1.664639 | 1.800841 |
| 0.4 | 1.628194 | 1.866548 | 0.6 | 1.628194 | 1.866548 | 0.7 | 1.616045 | 1.888450 |
| 0.6 | 1.670713 | 1.789890 | 0.8 | 1.585675 | 1.943206 | 0.9 | 1.567452 | 1.976059 |
| 0.8 | 1.713232 | 1.713232 | 1.0 | 1.543156 | 2.019864 | 1.0 | 1.543156 | 2.019864 |

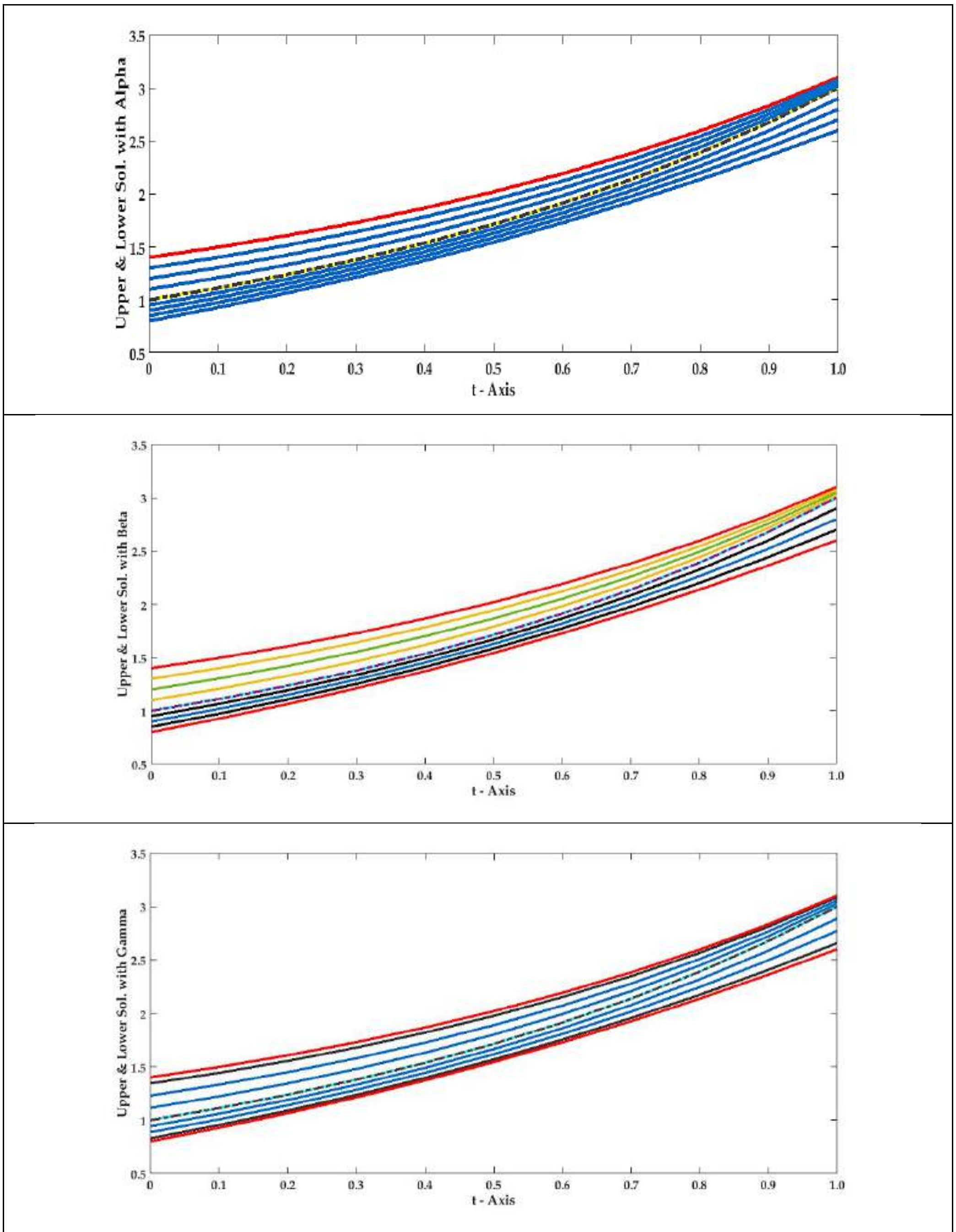



**Fig. 5:** The Results of Upper and Lower for $(\alpha, \beta, \gamma)$ **of Class(1,1).**

**Class (1, 2)**

$$\underline{y}(t, \alpha) = c_1 e^{-2t} + c_2 e^{2t} + c_3 e^{-t} + c_4 e^{t}$$

$$\bar{y}(t, \alpha) = -c_1 e^{-2t} + c_2 e^{2t} - c_3 e^{-t} + c_4 e^{t}$$

To find the values of constants for each $\alpha, \beta, \gamma$, then use the boundary points

$$\underline{y}(t_0, \alpha, \beta, \gamma) = c_1 + c_2 + c_3 + c_4 = 0.8 + \frac{\alpha}{4}, 0.8 + \frac{(1-\beta)}{4}, 0.8 + \frac{2(1-\gamma)}{7}$$

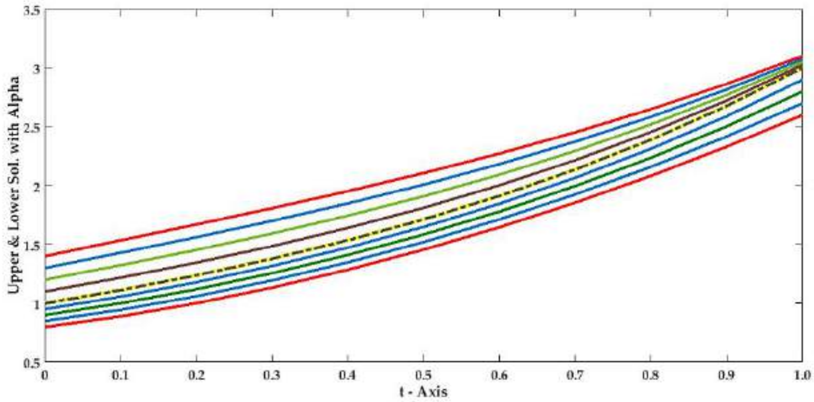
$$\bar{y}(t_0, \alpha, \beta, \gamma) = -c_1 + c_2 - c_3 + c_4 = 1.4 - \frac{\alpha}{2}, 1.4 - \frac{(1-\beta)}{2}, 1.4 - \frac{4(1-\gamma)}{7}$$

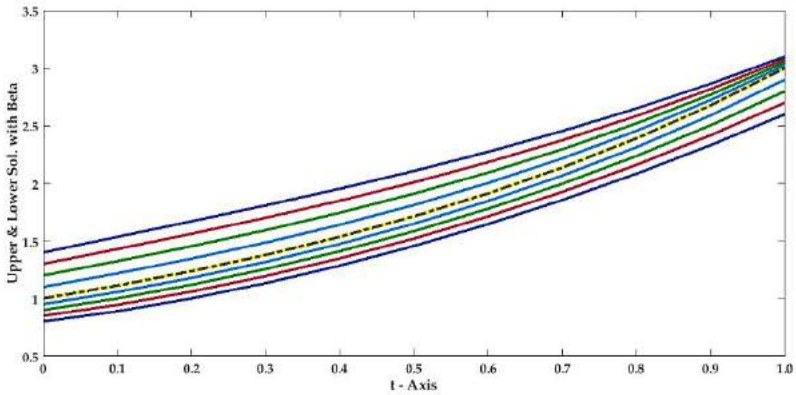
$$\underline{y}(T,\alpha,\beta,\gamma) = c_1e^{-2} + c_2e^{2} + c_3e^{-1} + c_4e^{1} == 2.6 + \frac{\alpha}{2}, 2.6 + \frac{(1-\beta)}{2}, 2.6 + \frac{4(1-\gamma)}{7}$$

$$\bar{y}(T,\alpha,\beta,\gamma) = -c_1e^{-2} + c_2e^{2} - c_3e^{-1} + c_4e^{1} = 3.1 - \frac{\alpha}{8}, 3.1 - \frac{(1-\beta)}{8}, 3.1 - \frac{1-\gamma}{7}$$

*Table (7): The Results of Upper and Lower for (α,β,γ) at t = 0.5*

| $\alpha$ | $\underline{y}(0.5,\alpha)$ | $\bar{y}(0.5,\alpha)$ | $\beta$ | $\underline{y}(0.5,\beta)$ | $\bar{y}(0.5,\beta)$ | $\gamma$ | $\underline{y}(0.5,\gamma)$ | $\bar{y}(0.5,\gamma)$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.456247 | 2.106772 | 0.2 | 1.713232 | 1.713232 | 0.3 | 1.713232 | 1.713232 |
| 0.2 | 1.520493 | 2.008387 | 0.4 | 1.648986 | 1.811617 | 0.5 | 1.639808 | 1.825672 |
| 0.4 | 1.584740 | 1.910002 | 0.6 | 1.584740 | 1.910002 | 0.7 | 1.566383 | 1.938112 |
| 0.6 | 1.648986 | 1.811617 | 0.8 | 1.520493 | 2.008387 | 0.9 | 1.492959 | 2.050552 |
| 0.8 | 1.713232 | 1.713232 | 1.0 | 1.456247 | 2.106772 | 1.0 | 1.456247 | 2.106772 |



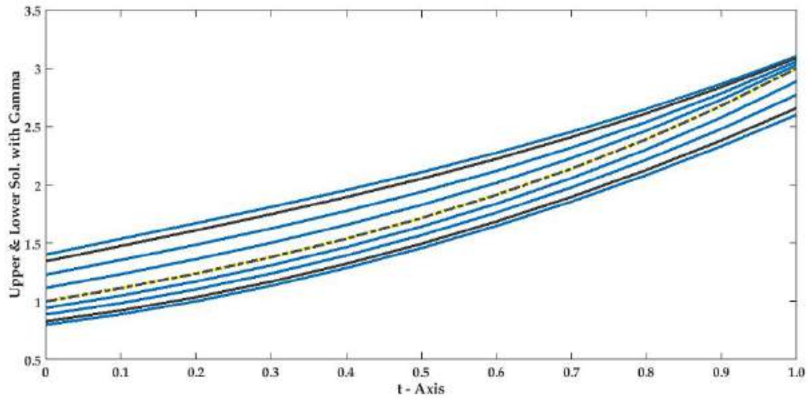



**Fig. 6:** The Results of Upper and Lower for $(\alpha,\beta,\gamma)$ **of Class(1,2).**

**Class (2, 1)**

$$\underline{y}(t,\alpha) = c_2e^{2t} + c_4e^{t}$$

$$\overline{y}(t,\alpha) = c_1e^{2t} + c_3e^{t}$$

To find the values of constants for each $\alpha,\beta,\gamma$, then use the boundary points

$$\underline{\boldsymbol{y}}(t_0,\alpha,\beta,\gamma) = \boldsymbol{c_2} + \boldsymbol{c_4} = 0.8 + \frac{\alpha}{4}, 0.8 + \frac{(1-\beta)}{4}, 0.8 + \frac{2(1-\gamma)}{7}$$

$$\overline{\boldsymbol{y}}(t_0,\alpha,\beta,\gamma) = \boldsymbol{c_1} + \boldsymbol{c_3} = 1.4 - \frac{\alpha}{2}, 1.4 - \frac{(1-\beta)}{2}, 1.4 - \frac{4(1-\gamma)}{7}$$

$$\underline{\boldsymbol{y}}(T,\alpha,\beta,\gamma) = \boldsymbol{c_2}e^2 + \boldsymbol{c_4}e^1 = 2.6 + \frac{\alpha}{2}, 2.6 + \frac{(1-\beta)}{2}, 2.6 + \frac{4(1-\gamma)}{7}$$

$$\overline{\boldsymbol{y}}(T,\alpha,\beta,\gamma) = \boldsymbol{c_1}e^2 + \boldsymbol{c_3}e^1 = 3.1 - \frac{\alpha}{8}, 3.1 - \frac{(1-\beta)}{8}, 3.1 - \frac{1-\gamma}{7}$$

*Table (8): The Results of Upper and Lower for (α,β,γ) at t=0.5*

| $\alpha$ | $\underline{y}(0.5,\alpha)$ | $\overline{y}(0.5,\alpha)$ | $\beta$ | $\underline{y}(0.5,\beta)$ | $\overline{y}(0.5,\beta)$ | $\gamma$ | $\underline{y}(0.5,\gamma)$ | $\overline{y}(0.5,\gamma)$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.416384 | 2.146636 | 0.2 | 1.713232 | 1.713232 | 0.3 | 1.713232 | 1.713232 |
| 0.2 | 1.490596 | 2.038285 | 0.4 | 1.639020 | 1.821583 | 0.5 | 1.628418 | 1.837062 |
| 0.4 | 1.564808 | 1.929934 | 0.6 | 1.564808 | 1.929934 | 0.7 | 1.543604 | 1.960891 |
| 0.6 | 1.639020 | 1.821583 | 0.8 | 1.490596 | 2.038285 | 0.9 | 1.458790 | 2.084721 |
| 0.8 | 1.713232 | 1.713232 | 1.0 | 1.416384 | 2.146636 | 1.0 | 1.416384 | 2.146636 |

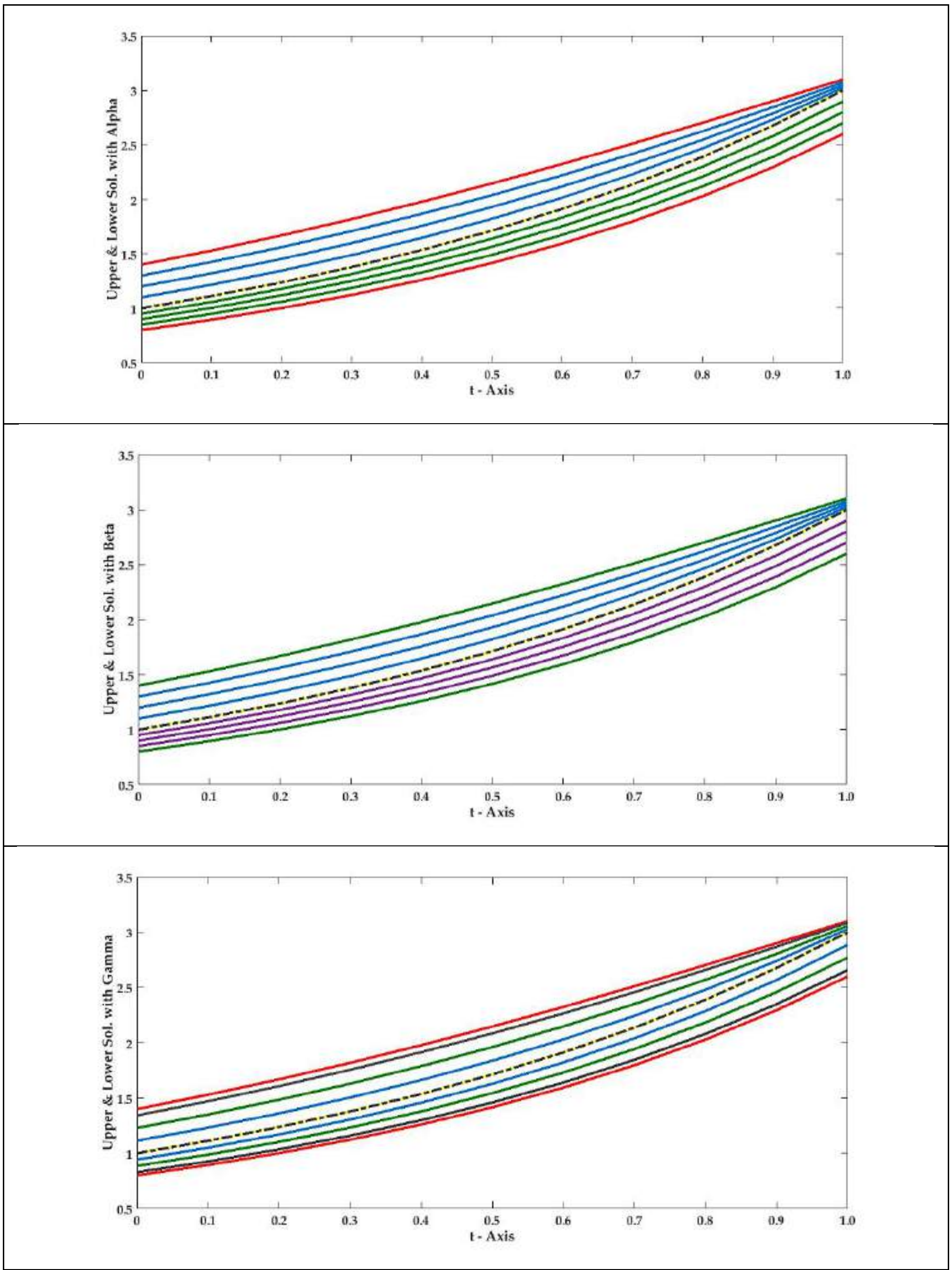



**Fig. 7:** The Results of Upper and Lower for $(\alpha, \beta, \gamma)$ **of Class(2,1).**

**Class (2, 2)**

$$\underline{\boldsymbol{y}}(t, \boldsymbol{\alpha}, \beta, \gamma) = \boldsymbol{c_1} e^{2t} + \boldsymbol{c_2} e^{\frac{-(3+\sqrt{17})t}{2}} + \boldsymbol{c_3} e^{t} + \boldsymbol{c_4} e^{\frac{-(3-\sqrt{17})t}{2}}$$

$$\overline{\boldsymbol{y}}(t, \boldsymbol{\alpha}, \beta, \gamma) = -\boldsymbol{c_1} e^{2t}(\boldsymbol{-2}) - \boldsymbol{c_3} e^{t}(\boldsymbol{-5}) - \boldsymbol{c_2} e^{\frac{-(3+\sqrt{17})t}{2}} - \boldsymbol{c_4} e^{\frac{-(3-\sqrt{17})t}{2}}$$

To find the values of constants for each $\alpha, \beta, \gamma,$ then use the boundary points

$$\underline{\boldsymbol{y}}(t_0, \boldsymbol{\alpha}, \beta, \gamma) = \boldsymbol{c_1} + \boldsymbol{c_2} + \boldsymbol{c_3} + \boldsymbol{c_4} = 0.8 + \frac{\alpha}{4}, 0.8 + \frac{(1-\beta)}{4}, 0.8 + \frac{2(1-\gamma)}{7}$$

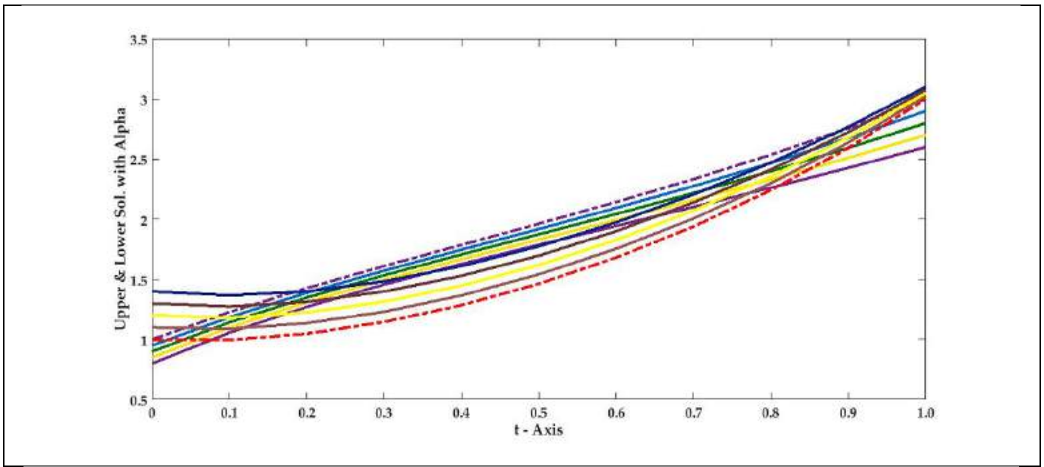
$$\overline{\boldsymbol{y}}(t_0, \boldsymbol{\alpha}, \beta, \gamma) = \boldsymbol{2c_1} + \boldsymbol{5c_3} - \boldsymbol{c_2} - \boldsymbol{c_4} = 1.4 - \frac{\alpha}{2}, 1.4 - \frac{(1-\beta)}{2}, 1.4 - \frac{4(1-\gamma)}{7}$$

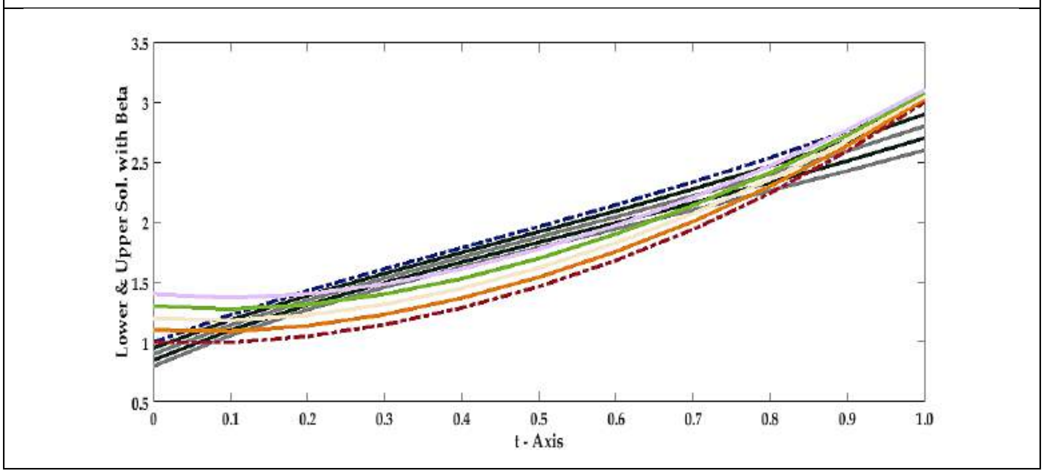
$$\underline{y}(\boldsymbol{T},\boldsymbol{\alpha},\beta,\gamma) = \mathbf{c_1}e^2 + \mathbf{c_2}e^{\frac{-(3+\sqrt{17})}{2}} + \boldsymbol{c_3}e^1 + \boldsymbol{c_4}e^{\frac{-(3-\sqrt{17})}{2}} = 2.6 + \frac{\alpha}{2}, 2.6 + \frac{(1-\beta)}{2}, 2.6 + \frac{4(1-\gamma)}{7}$$

$$\overline{y}(\boldsymbol{T},\boldsymbol{\alpha},\beta,\gamma) = -\boldsymbol{c_1}e^2(-\mathbf{2}) - \boldsymbol{c_3}e^{11}(-\mathbf{5}) - \boldsymbol{c_2}e^{\frac{-(3+\sqrt{17})}{2}} - \boldsymbol{c_4}e^{\frac{-(3-\sqrt{17})}{2}} = 3.1 - \frac{\alpha}{8}, 3.1 - \frac{(1-\beta)}{8}, 3.1 - \frac{1-\gamma}{7}$$

*Table (9): The Results of Upper and Lower for (α,β,γ) at t = 0.5*

| $\alpha$ | $\underline{y}(0.5,\alpha)$ | $\overline{y}(0.5,\alpha)$ | $\beta$ | $\underline{y}(0.5,\beta)$ | $\overline{y}(0.5,\beta)$ | $\gamma$ | $\underline{y}(0.5,\gamma)$ | $\overline{y}(0.5,\gamma)$ |
|---|---|---|---|---|---|---|---|---|
| 0.0 | 1.787831 | 1.775188 | 0.2 | 1.962740 | 1.463724 | 0.3 | 1.962740 | 1.463724 |
| 0.2 | 1.831558 | 1.027074 | 0.4 | 1.919013 | 0.904503 | 0.5 | 1.912766 | 1.552714 |
| 0.4 | 1.875285 | 0.965788 | 0.6 | 1.875285 | 0.965788 | 0.7 | 1.862792 | 1.641703 |
| 0.6 | 1.919013 | 0.904503 | 0.8 | 1.831558 | 1.027074 | 0.9 | 1.812818 | 1.730693 |
| 0.8 | 1.962740 | 1.463724 | 1.0 | 1.787831 | 1.775188 | 1.0 | 1.787831 | 1.775188 |



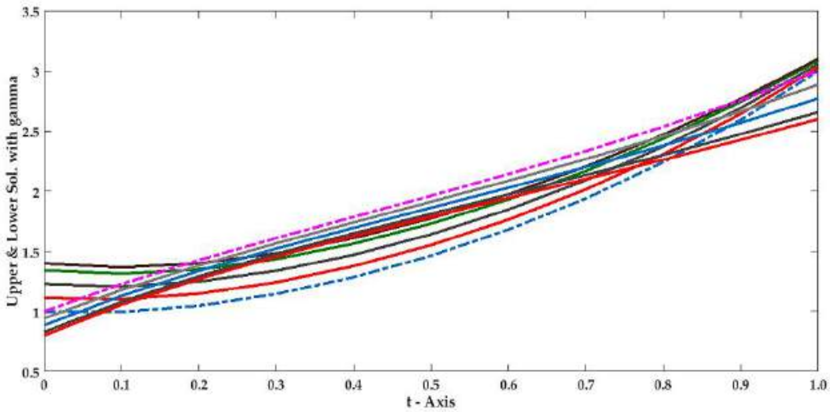



**Fig. 8:** The Results of Upper and Lower for $(\alpha, \beta, \gamma)$ **of Class(2,2).**

In this example, we noticed that classes (1,1), (1,2), and (2,1) have no overlapping between uppers and lowers cuts also it is approved from the tables of these classes have the strong solution, But Class (2,2) in Fig. 8 for $\alpha, \beta$ *and* $\gamma$ have very clear overlapping between uppers and lowers and it can be observed easily from class(2,2) table of solution values at t = 0.5 that $\frac{\partial \underline{y}}{\partial \alpha} < 0, \frac{d\bar{y}}{d\alpha} > 0$ but $\frac{\partial \underline{y}}{\partial \beta} > 0, \frac{d\bar{y}}{d\beta} < 0, \frac{\partial \underline{y}}{\partial \gamma} > 0, \frac{d\bar{y}}{d\gamma} < 0$, $\bar{\boldsymbol{y}}(0.5, \alpha, \beta, \gamma) < \underline{\boldsymbol{y}}(0.5, \alpha, \beta, \gamma)$ which approve that class(2,2) solution is weak.

## 5. Conclusions

In this paper, after solving the neutrosophic fuzzy boundary value problem by analytical solution we conclude that it is a generalization of the fuzzy boundary value problem solution by determining the degree of membership $\boldsymbol{\alpha}$, the degree of indeterministic $\boldsymbol{\beta}$ and the degree of non-membership $\boldsymbol{\gamma}$ and it is discussed clearly in applications where the solution of each class founded by constructing an algorithm using Matlab and graphical representation is also interpreted.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