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An Algorithm Based on Correlation Coefficient Under Neutrosophic hypersoft set environment with its Application for Decision-Making

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Abstract:

The correlation coefficient among the two parameters plays a significant part in statistics. Further, the exactness in the assessment of correlation depends upon information from the set of discourse. The data collected for various statistical studies is full of ambiguities. In this paper, we discuss some basic concepts which are helpful to build the structure of present research such as soft set, hypersoft set, and neutrosophic hypersoft set (NHSS). The neutrosophic hypersoft set is an extension of the neutrosophic soft set. In it, we establish the idea of correlation and weighted correlation coefficients with some desirable properties under NHSS. We also, propose a new decision-making technique and construct an algorithm based on developed correlation measures. Furthermore, To ensure the applicability of the proposed methods an illustrative example is given.

Keywords: Hypersoft set, NHSS, correlation coefficient, weighted correlation coefficient

1. Introduction

Ambiguity plays a dynamic role in many areas of life (such as modeling, medicine, engineering, etc.). However, people have raised a common question, that is, how do we express and use the concept of uncertainty in mathematical modeling. Many researchers in the world have proposed and recommended different methods of using uncertainty theory. First of all, Zadeh developed the concept of a fuzzy set (FS) [1] to solve problems that contain uncertainty and ambiguity. In some cases, we must carefully consider membership as a non-membership value to correctly represent objects that FS cannot handle. To overcome these difficulties, Atanasov proposed the idea of intuitionistic fuzzy sets (IFS) [2]. Atanassov's intuitionistic fuzzy sets only deal with insufficient data due to membership and non-membership values, but IFS cannot deal with incompatible and imprecise information. Molodtsov [3] proposed a general mathematical tool to deal with uncertain, ambiguous, and uncertain matters, called soft set (SS). Maji et al. [4] extended the concept of SS and developed some operations with properties and used the established concepts for decision-making [5]. By combining the FS and SS Maji et al. [6] established the fuzzy soft set (FSS) and intuitionistic fuzzy soft set (IFSS) and studied their operations and properties [7]. Zulqarnain et al. [8] established the correlation coefficient for interval-valued intuitionistic fuzzy soft set and developed the TOPSIS approach based on their presented correlation measures. Zulqarnain et al. [9, 10] discussed the Pythagorean fuzzy soft sets (PFSS) and established the aggregation operator and TOPSIS technique to solve the MCDM problem.

Maji [11] offered the idea of a neutrosophic soft set (NSS) with necessary operations and properties. The idea of the possibility NSS was developed by Karaaslan [12] and introduced a possibility of neutrosophic soft decision-making method to solve those problems which contain uncertainty based on And-product. Broumi [13] developed the generalized NSS with some operations and properties and used the proposed concept for decision making. To solve MCDM problems with PFSS, Zulqarnain et al. [14] presented the interaction aggregation operators for PFSS. Based on the correlation of IFS, the term CC of SVNSs [15] was introduced. In [16] the idea of simplified NSs introduced with some operational laws and aggregation operators such as weighted arithmetic and weighted geometric average operators. They constructed an MCDM method on the base of proposed aggregation operators. Masooma et al. [17] progressed a new concept through combining the multipolar fuzzy set and neutrosophic set which is known as the multipolar neutrosophic set, they also established various characterization and operations with examples. Zulqarnain et al. [18, 19] utilized the neutrosophic TOPSIS model to solve the MCDM problem and for the selection of suppliers in the production industry.

Correlation performs a significant part in statistics as well as engineering. By way of correlation analysis, the mixture of two variables can be utilized to compute the mutuality of the two variables. Although probabilistic methods have been applied to various practical engineering problems, there are still some obstacles to probabilistic strategies. For example, the probability of this process depends on the large amount of data collected, which is random. However, large complex systems have many fuzzy uncertainties, so it is difficult to obtain accurate probability events. Therefore, due to limited quantitative information, results based on probability theory do not always provide useful information for experts. In addition, in actual applications, sometimes there is not enough data to correctly process standard statistical data. Due to the aforementioned obstacles, results based on probability theory are not always available to experts. Therefore, probabilistic methods are usually insufficient to resolve such inherent uncertainties in the data. Many researchers in the world have proposed and proposed different methods to solve problems that contain uncertainty. To measure the relationship between fuzzy numbers, Yu [20] established the CC of fuzzy numbers.

Recently, Smarandache [21] extended the concept of the SS to hypersoft set (HSS) by replacing the single-parameter function F with a multi-parameter (sub-attribute) function defined on Cartesian products of n different attributes. The established HSS is more flexible than SS and is more suitable for the decision-making environment. He also introduced the further extension of HSS, such as crisp HSS, fuzzy HSS, intuitionistic fuzzy HSS, neutrosophic HSS, and plithogenic HSS. Nowadays, HSS theory and its extensions are developing rapidly. Many researchers have developed different operators and properties based on HSS and its extensions [22-36]. Abdel-Basset [37] uses a plithogenic set theory to resolve uncertain information and evaluate the financial performance of manufacturing. Then, they use VIKOR and TOPSIS methods to find the weight vector of financial ratios using the AHP method to achieve this goal. Abdel-basset et al. [38] recommended an efficient combination of plithogenic aggregation operations as well as quality feature deployment strategies. The advantage of this combination is that it can improve accuracy as well as assess the decision-makers.

The following research is organized as follows: In Section 2, we review some basic definitions used in the following sequels, such as SS, NSS, and NHSS, etc. In Section 3, the idea of CC and WCC is developed with some necessary properties. An algorithm and decision-making method will be developed in section 4. We also used the developed approach to solve decision making problems in an uncertain environment. Finally, the conclusion is made in section 5.

2. Preliminaries

In this section, we recollect some basic definitions which are helpful to build the structure of the following manuscript such as soft set, hypersoft set, and neutrosophic hypersoft set.

Definition 2.1 [3]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathsf{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathsf{A})$ is called a soft set over $\mathcal{U}$ and its mapping is given as

$$\mathcal{F}: \mathsf{A} \to \mathcal{P}(\mathcal{U})$$

It is also defined as:

$$(\mathcal{F}, \mathsf{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}): e \in \mathcal{E}, \mathcal{F}(e) = \emptyset \; if \; e \notin \mathsf{A}\}$$

Definition 2.2 [21]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, \ldots, k_n\}$, $(n \geq 1)$ be a set of attributes and set K_i a set of corresponding sub-attributes of k_i respectively with $K_i \cap K_j = \varphi$ for $n \geq 1$ for each $i, j \in \{1,2,3 \ldots n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathcal{A}} = \{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of multi-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and α, β, and $\gamma \in \mathbb{N}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}})$ is said to be HSS over $\mathcal{U}$ and its mapping is defined as

$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}} \to \mathcal{P}(\mathcal{U})$.

It is also defined as

$(\mathcal{F}, \ddot{\mathsf{A}}) = \{\breve{\alpha}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{\alpha}): \breve{\alpha} \in \ddot{\mathsf{A}}, \mathcal{F}_{\ddot{\mathcal{A}}}(\breve{\alpha}) \in \mathcal{P}(\mathcal{U})\}$

Definition 2.3 [21]

Let $\mathcal{U}$ be a universe of discourse and $\mathcal{P}(\mathcal{U})$ be a power set of $\mathcal{U}$ and $k = \{k_1, k_2, k_3, \ldots, k_n\}$, $(n \geq 1)$ be a set of attributes and set K_i a set of corresponding sub-attributes of k_i respectively with $K_i \cap K_j = \varphi$ for $n \geq 1$ for each $i, j \in \{1,2,3 \ldots n\}$ and $i \neq j$. Assume $K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}} = \{a_{1h} \times a_{2k} \times \cdots \times a_{nl}\}$ be a collection of sub-attributes, where $1 \leq h \leq \alpha$, $1 \leq k \leq \beta$, and $1 \leq l \leq \gamma$, and α, β, and $\gamma \in \mathbb{N}$ and $NS^{\mathcal{U}}$ be a collection of all neutrosophic subsets over $\mathcal{U}$. Then the pair $(\mathcal{F}, K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}})$ is said to be NHSS over $\mathcal{U}$ and its mapping is defined as

$\mathcal{F}: K_1 \times K_2 \times K_3 \times \ldots \times K_n = \ddot{\mathsf{A}} \to NS^{\mathcal{U}}$.

It is also defined as

$(\mathcal{F}, \ddot{\mathsf{A}}) = \{(\breve{\alpha}, \mathcal{F}_{\ddot{\mathsf{A}}}(\breve{\alpha})): \breve{\alpha} \in \ddot{\mathsf{A}}, \mathcal{F}_{\ddot{\mathsf{A}}}(\breve{\alpha}) \in NS^{\mathcal{U}}\}$, where $\mathcal{F}_{\ddot{\mathsf{A}}}(\breve{\alpha}) = \{\langle \delta, \sigma_{\mathcal{F}(\breve{\alpha})}(\delta), \tau_{\mathcal{F}(\breve{\alpha})}(\delta), \gamma_{\mathcal{F}(\breve{\alpha})}(\delta) \rangle: \delta \in \mathcal{U}\}$, where $\sigma_{\mathcal{F}(\breve{\alpha})}(\delta)$, $\tau_{\mathcal{F}(\breve{\alpha})}(\delta)$, and $\gamma_{\mathcal{F}(\breve{\alpha})}(\delta)$ represent the truth, indeterminacy, and falsity grades of the attributes such as $\sigma_{\mathcal{F}(\breve{\alpha})}(\delta), \tau_{\mathcal{F}(\breve{\alpha})}(\delta), \gamma_{\mathcal{F}(\breve{\alpha})}(\delta) \in [0,1]$, and $0 \leq \sigma_{\mathcal{F}(\breve{\alpha})}(\delta) + \tau_{\mathcal{F}(\breve{\alpha})}(\delta) + \gamma_{\mathcal{F}(\breve{\alpha})}(\delta) \leq 3$. Simply a neutrosophic hypersoft number (NHSN) can be expressed as $\mathcal{F} = \{(\sigma_{\mathcal{F}(\breve{\alpha})}(\delta), \tau_{\mathcal{F}(\breve{\alpha})}(\delta), \gamma_{\mathcal{F}(\breve{\alpha})}(\delta))\}$, where $0 \leq \sigma_{\mathcal{F}(\breve{\alpha})}(\delta) + \tau_{\mathcal{F}(\breve{\alpha})}(\delta) + \gamma_{\mathcal{F}(\breve{\alpha})}(\delta) \leq 3$.

Example 2.4

Consider the universe of discourse $\mathcal{U} = \{\delta_1, \delta_2\}$ and $\mathfrak{L} = \{\ell_1 = Teaching\ methdology, \ell_2 = Subjects, \ell_3 = Classes\}$ be a collection of attributes with following their corresponding attribute values are given as teaching methodology $= L_1 = \{a_{11} = project\ base, a_{12} = class\ discussion\}$, Subjects $= L_2 = \{a_{21} = Mathematics, a_{22} = Computer\ Science, a_{23} = Statistics\}$, and Classes $= L_3 = \{a_{31} = Masters, a_{32} = Doctorol\}$. Let $\ddot{\mathsf{A}} = L_1 \times L_2 \times L_3$ be a set of attributes

$\ddot{\mathsf{A}} = L_1 \times L_2 \times L_3 = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}, a_{23}\} \times \{a_{31}, a_{32}\}$

$= \begin{Bmatrix} (a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), (a_{11}, a_{23}, a_{31}), (a_{11}, a_{23}, a_{32}), \\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32}), (a_{12}, a_{23}, a_{31}), (a_{12}, a_{23}, a_{32}), \end{Bmatrix}$

$\ddot{\mathsf{A}} = \{\breve{\alpha}_1, \breve{\alpha}_2, \breve{\alpha}_3, \breve{\alpha}_4, \breve{\alpha}_5, \breve{\alpha}_6, \breve{\alpha}_7, \breve{\alpha}_8, \breve{\alpha}_9, \breve{\alpha}_{10}, \breve{\alpha}_{11}, \breve{\alpha}_{12}\}$

Then the NHSS over $\mathcal{U}$ is given as follows

$(\mathcal{F},\ddot{A}) =$

$$\left\{\begin{array}{l}(\breve{\alpha}_1,(\delta_1,(.6,.3,.8)),(\delta_2,(.9,.3,.5))),(\breve{\alpha}_2,(\delta_1,(.5,.2,.7)),(\delta_2,(.7,.1,.5))),(\breve{\alpha}_3,(\delta_1,(.5,.2,.8)),(\delta_2,(.4,.3,.4))),\\(\breve{\alpha}_4,(\delta_1,(.2,.5,.6)),(\delta_2,(.5,.1,.6))),(\breve{\alpha}_5,(\delta_1,(.8,.4,.3)),(\delta_2,(.2,.3,.5))),(\breve{\alpha}_6,(\delta_1,(.9,.6,.4)),(\delta_2,(.7,.6,.8))),\\(\breve{\alpha}_7,(\delta_1,(.6..5,.3)),(\delta_2,(.4,.2,.8))),(\breve{\alpha}_8,(\delta_1,(.8,.2,.5)),(\delta_2,(.6,.8,.4))),(\breve{\alpha}_9,(\delta_1,(.7,.4,.9)),(\delta_2,(.7..3,.5))),\\(\breve{\alpha}_{10},(\delta_1,(.8,.4,.6)),(\delta_2,(.7,.2,.9))),(\breve{\alpha}_{11},(\delta_1,(.8,.4,.5)),(\delta_2,(.4,.2,.5))),(\breve{\alpha}_5,(\delta_1,(.7,.5,.8)),(\delta_2,(.7,.5,.9)))\end{array}\right\}$$

3. Correlation Coefficient for Neutrosophic Hypersoft Set

In this section, the concept of correlation coefficient and weighted correlation coefficient on NHSS has been proposed with some basic properties.

Definition 3.1

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\bar{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs defined over a universe of discourse $\mathcal{U}$. Then, the informational neutrosophic energies of $(\mathcal{F},\ddot{A})$ and $(\mathcal{G},\ddot{\bar{A}})$ can be described as follows:

$$\varsigma_{NHSS}(\mathcal{F},\ddot{A}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right)^2 + \left(\gamma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right)^2\right) \quad (1)$$

$$\varsigma_{NHSS}(\mathcal{G},\ddot{\bar{A}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right)^2 + \left(\gamma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right)^2\right). \quad (2)$$

Definition 3.2

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\bar{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs defined over a universe of discourse $\mathcal{U}$. Then, the correlation measure between $(\mathcal{F},\ddot{A})$ and $(\mathcal{G},\ddot{\bar{A}})$ can be described as follows:

$$\mathcal{C}_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\bar{A}})) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sigma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i) * \sigma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i) + \tau_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i) * \tau_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i) + \gamma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i) * \gamma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right). \quad (3)$$

Proposition 3.3

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\bar{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs and $\mathcal{C}_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\bar{A}}))$ be a correlation between them, then the following properties hold.

1. $\mathcal{C}_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\bar{A}})) = \varsigma_{NHSS}(\mathcal{F},\ddot{A})$
2. $\mathcal{C}_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\bar{A}})) = \varsigma_{NHSS}(\mathcal{G},\ddot{\bar{A}})$

Proof: The proof is trivial.

Definition 3.4

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\bar{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs, then correlation coefficient between them given as $\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\bar{A}}))$ and expressed as follows:

$$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\bar{A}})) = \frac{\mathcal{C}_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\bar{A}}))}{\sqrt{\varsigma_{NHSS}(\mathcal{F},\ddot{A})} * \sqrt{\varsigma_{NHSS}(\mathcal{G},\ddot{\bar{A}})}} \quad (4)$$

$$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\bar{A}})) = \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sigma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i) * \sigma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i) + \tau_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i) * \tau_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i) + \gamma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i) * \gamma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right)^2 + \left(\gamma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right)^2\right)}\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right)^2\right) + \left(\gamma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right)^2}} \quad (5)$$

Proposition 3.5

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\bar{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{\alpha}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs, then CC between them satisfies the following properties

1. $0 \leq \delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) \leq 1$
2. $\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) = \delta_{NHSS}((\mathcal{G},\ddot{\Lambda}),(\mathcal{F},\ddot{A}))$
3. If $(\mathcal{F},\ddot{A}) = (\mathcal{G},\ddot{\Lambda})$, that is $\forall\ i,\ k,\ \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i),\ \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)$ then $\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) = 1$.

Proof 1. $\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) \geq 0$ is trivial, here we only need to prove that $\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) \leq 1$.
From equation 3, we have

$$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)$$

$$= \sum_{k=1}^{m}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_1) + \tau_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_1) + \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)$$
$$+ \sum_{k=1}^{m}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_2) + \tau_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_2) + \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)$$
$$+ \vdots +$$
$$\sum_{k=1}^{m}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_n) + \tau_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_n) + \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)$$

$$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda})) = \left\{\begin{array}{c}\left(\sigma_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \sigma_{\mathcal{G}(\breve{a}_1)}(\delta_1) + \tau_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \tau_{\mathcal{G}(\breve{a}_1)}(\delta_1) + \gamma_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \gamma_{\mathcal{G}(\breve{a}_1)}(\delta_1)\right)\\ \left(\sigma_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \sigma_{\mathcal{G}(\breve{a}_2)}(\delta_1) + \tau_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \tau_{\mathcal{G}(\breve{a}_2)}(\delta_1) + \gamma_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \gamma_{\mathcal{G}(\breve{a}_2)}(\delta_1)\right)\\ \vdots \\ \left(\sigma_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \sigma_{\mathcal{G}(\breve{a}_m)}(\delta_1) + \tau_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \tau_{\mathcal{G}(\breve{a}_m)}(\delta_1) + \gamma_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \gamma_{\mathcal{G}(\breve{a}_m)}(\delta_1)\right)\end{array}\right\}$$
$$+ \left\{\begin{array}{c}\left(\sigma_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \sigma_{\mathcal{G}(\breve{a}_1)}(\delta_2) + \tau_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \tau_{\mathcal{G}(\breve{a}_1)}(\delta_2) + \gamma_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \gamma_{\mathcal{G}(\breve{a}_1)}(\delta_2)\right) +\\ \left(\sigma_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \sigma_{\mathcal{G}(\breve{a}_2)}(\delta_2) + \tau_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \tau_{\mathcal{G}(\breve{a}_2)}(\delta_2) + \gamma_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \gamma_{\mathcal{G}(\breve{a}_2)}(\delta_2)\right) +\\ \vdots \\ \left(\sigma_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \sigma_{\mathcal{G}(\breve{a}_m)}(\delta_2) + \tau_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \tau_{\mathcal{G}(\breve{a}_m)}(\delta_2) + \gamma_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \gamma_{\mathcal{G}(\breve{a}_m)}(\delta_2)\right)\end{array}\right\}$$
$$+ \vdots +$$
$$\left\{\begin{array}{c}\left(\sigma_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_1)}(\delta_n) + \tau_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_1)}(\delta_n) + \gamma_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_1)}(\delta_n)\right) +\\ \left(\sigma_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_2)}(\delta_n) + \tau_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_2)}(\delta_n) + \gamma_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_2)}(\delta_n)\right) +\\ \vdots \\ \left(\sigma_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_m)}(\delta_n) + \tau_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_m)}(\delta_n) + \gamma_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_m)}(\delta_n)\right)\end{array}\right\}$$

$$= \sum_{k=1}^{m}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right) + \left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right) + \cdots + \left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\right) +$$
$$\sum_{k=1}^{m}\left(\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right) + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right) + \cdots + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\right) +$$
$$\sum_{k=1}^{m}\left(\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right) + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right) + \cdots + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\right)$$

By using Cauchy-Schwarz inequality

$$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda}))^2 \leq$$
$$\sum_{k=1}^{m}\left\{\begin{array}{c}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right) + \left(\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right)\\ + \left(\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right)\end{array}\right\}$$
$$\times \sum_{k=1}^{m}\left\{\begin{array}{c}\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right) + \left(\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right)\\ + \left(\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right)\end{array}\right\}$$

$$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\Lambda}))^2 \leq$$

$$\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)$$
$$\times\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)$$

$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))^2 \le \varsigma_{NHSS}(\mathcal{F},\ddot{A}) \times \varsigma_{NHSS}(\mathcal{G},\ddot{\mathbb{A}})$.

Therefore, $\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))^2 \le \varsigma_{NHSS}(\mathcal{F},\ddot{A}) \times \varsigma_{NHSS}(\mathcal{G},\ddot{\mathbb{A}})$. Hence, by using definition 3.4, we have $\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) \le 1$. So, $0 \le \delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) \le 1$.

Proof 2. The proof is obvious.

Proof 3. From equation 5, we have

$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) =$

$$\frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)+\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2}}$$

As we know that

$\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\ \forall\ i,\ k$. We get

$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) =$

$$\frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}}$$

$\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = 1$

Thus, prove the required result.

Definition 3.6

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs. Then, their correlation coefficient is given as $\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$ and defined as follows:

$$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = \frac{\mathcal{C}_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))}{max\{\varsigma_{NHSS}(\mathcal{F},\ddot{A}),\varsigma_{NHSS}(\mathcal{G},\ddot{\mathbb{A}})\}} \quad (6)$$

$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) =$

$$\frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)}{max\left\{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right),\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\right\}} \quad (7)$$

Proposition 3.7

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i, \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\ddot{\mathbb{A}}) = \left\{\left(\delta_i, \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i), \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i), \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs. Then, CC between them satisfies the following properties

1. $0 \le \delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) \le 1$
2. $\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = \delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$
3. If $(\mathcal{F},\ddot{A}) = (\mathcal{G},\ddot{\mathbb{A}})$, that is $\forall\ i,\ k,\ \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, then $\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = 1$.

Proof 1. $\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) \ge 0$ is trivial, here we only need to prove that $\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) \le 1$. From equation 3, we have

$$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)$$

$$= \sum_{k=1}^{m}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\tau_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\tau_{\mathcal{G}(\breve{a}_k)}(\delta_1)+\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_1)*\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)$$
$$+ \sum_{k=1}^{m}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\tau_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\tau_{\mathcal{G}(\breve{a}_k)}(\delta_2)+\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_2)*\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)$$
$$+$$
$$\vdots$$

$$+\sum_{k=1}^{m}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_n) + \tau_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_n) + \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)$$

$$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))$$

$$= \begin{Bmatrix} \left(\sigma_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \sigma_{\mathcal{G}(\breve{a}_1)}(\delta_1) + \tau_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \tau_{\mathcal{G}(\breve{a}_1)}(\delta_1) + \gamma_{\mathcal{F}(\breve{a}_1)}(\delta_1) * \gamma_{\mathcal{G}(\breve{a}_1)}(\delta_1)\right) \\ \left(\sigma_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \sigma_{\mathcal{G}(\breve{a}_2)}(\delta_1) + \tau_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \tau_{\mathcal{G}(\breve{a}_2)}(\delta_1) + \gamma_{\mathcal{F}(\breve{a}_2)}(\delta_1) * \gamma_{\mathcal{G}(\breve{a}_2)}(\delta_1)\right) \\ \vdots \\ \left(\sigma_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \sigma_{\mathcal{G}(\breve{a}_m)}(\delta_1) + \tau_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \tau_{\mathcal{G}(\breve{a}_m)}(\delta_1) + \gamma_{\mathcal{F}(\breve{a}_m)}(\delta_1) * \gamma_{\mathcal{G}(\breve{a}_m)}(\delta_1)\right) \end{Bmatrix}$$

$$+ \begin{Bmatrix} \left(\sigma_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \sigma_{\mathcal{G}(\breve{a}_1)}(\delta_2) + \tau_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \tau_{\mathcal{G}(\breve{a}_1)}(\delta_2) + \gamma_{\mathcal{F}(\breve{a}_1)}(\delta_2) * \gamma_{\mathcal{G}(\breve{a}_1)}(\delta_2)\right) + \\ \left(\sigma_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \sigma_{\mathcal{G}(\breve{a}_2)}(\delta_2) + \tau_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \tau_{\mathcal{G}(\breve{a}_2)}(\delta_2) + \gamma_{\mathcal{F}(\breve{a}_2)}(\delta_2) * \gamma_{\mathcal{G}(\breve{a}_2)}(\delta_2)\right) + \\ \vdots \\ \left(\sigma_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \sigma_{\mathcal{G}(\breve{a}_m)}(\delta_2) + \tau_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \tau_{\mathcal{G}(\breve{a}_m)}(\delta_2) + \gamma_{\mathcal{F}(\breve{a}_m)}(\delta_2) * \gamma_{\mathcal{G}(\breve{a}_m)}(\delta_2)\right) \end{Bmatrix}$$

$$+ \\ \vdots \\ +$$

$$\begin{Bmatrix} \left(\sigma_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_1)}(\delta_n) + \tau_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_1)}(\delta_n) + \gamma_{\mathcal{F}(\breve{a}_1)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_1)}(\delta_n)\right) + \\ \left(\sigma_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_2)}(\delta_n) + \tau_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_2)}(\delta_n) + \gamma_{\mathcal{F}(\breve{a}_2)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_2)}(\delta_n)\right) + \\ \vdots \\ \left(\sigma_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_m)}(\delta_n) + \tau_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_m)}(\delta_n) + \gamma_{\mathcal{F}(\breve{a}_m)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_m)}(\delta_n)\right) \end{Bmatrix}$$

$$= \sum_{k=1}^{m}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right) + \left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right) + \cdots + \left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\right) +$$
$$\sum_{k=1}^{m}\left(\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right) + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right) + \cdots + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\right) +$$
$$\sum_{k=1}^{m}\left(\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_1) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right) + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_2) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right) + \cdots + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_n) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)\right)$$

By using Cauchy-Schwarz inequality

$$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) \le$$
$$\sum_{k=1}^{m}\begin{Bmatrix}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right) + \left(\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right) \\ + \left(\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_n)\right)^2\right)\end{Bmatrix}$$
$$\times \sum_{k=1}^{m}\begin{Bmatrix}\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right) + \left(\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right) \\ + \left(\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_1)\right)^2 + \left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_2)\right)^2 + \cdots + \left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_n)\right)^2\right)\end{Bmatrix}$$

$$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))^2 \le$$
$$\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)$$
$$\times \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)$$

$$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))^2 \le \varsigma_{NHSS}(\mathcal{F},\ddot{A}) \times \varsigma_{IFHSS}(\mathcal{G},\ddot{\mathbb{A}}).$$

Therefore, $\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}}))^2 \le \varsigma_{NHSS}(\mathcal{F},\ddot{A}) \times \varsigma_{NHSS}(\mathcal{G},\ddot{\mathbb{A}})$. Hence, by using definition 3.6, we have $\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) \le 1$. So, $0 \le \delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) \le 1$.

Proof 2. The proof is obvious.

Proof 3. From equation 7, we have

$$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\ddot{\mathbb{A}})) = \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i) + \gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) * \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)}{max\left\{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right), \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2 + \left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\right\}}$$

As we know that

$\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)$ $\forall$ i, k. We get

$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) =$

$$\frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)}{max\left\{\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right),\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)\right\}}$$

$\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) = 1$

Thus, prove the required result.

Definition 3.8

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i,\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i),\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i),\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\dddot{A}) = \left\{\left(\delta_i,\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i),\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i),\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs. Then, their weighted correlation coefficient is given as $\delta_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))$ and defined as follows:

$$\delta_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) = \frac{\mathcal{C}_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))}{\sqrt{\varsigma_{WNHSS}(\mathcal{G},\dddot{A})}*\sqrt{\varsigma_{WNHSS}(\mathcal{G},\dddot{A})}} \quad (8)$$

$\delta_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) =$

$$\frac{\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)\right)}{\sqrt{\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)}\sqrt{\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)}}$$

(9)

Definition 3.9

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i,\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i),\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i),\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\dddot{A}) = \left\{\left(\delta_i,\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i),\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i),\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs. Then, their weighted correlation coefficient is given as $\delta^1_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))$ and defined as follows:

$$\delta^1_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) = \frac{\mathcal{C}_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))}{max\{\varsigma_{WNHSS}(\mathcal{F},\ddot{A}),\varsigma_{WNHSS}(\mathcal{G},\dddot{A})\}} \quad (10)$$

$\delta^1_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) =$

$$\frac{\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)+\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)*\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)\right)}{max\left\{\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\left(\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right)^2\right)\right),\sum_{k=1}^{m}\dot{\omega}_k\left(\sum_{k=1}^{m}\Omega_k\left(\sum_{i=1}^{n}\gamma_i\left(\left(\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2+\left(\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right)^2\right)\right)\right)\right\}}$$

(11)

If we consider $\Omega = \{\frac{1}{m},\frac{1}{m},\ldots,\frac{1}{m}\}$ and $\gamma = \{\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\}$, then $\delta_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))$ and $\delta^1_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))$ are reduced to $\delta_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))$ and $\delta^1_{NHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A}))$ respectively.

Proposition 3.10

Let $(\mathcal{F},\ddot{A}) = \left\{\left(\delta_i,\sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i),\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i),\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ and $(\mathcal{G},\dddot{A}) = \left\{\left(\delta_i,\sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i),\tau_{\mathcal{G}(\breve{a}_k)}(\delta_i),\gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)\right) \mid \delta_i \in \mathcal{U}\right\}$ be two NHSSs. Then, CC between them satisfies the following properties

1. $0 \leq \delta_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) \leq 1$
2. $\delta_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) = \delta_{WNHSS}((\mathcal{G},\dddot{A}),(\mathcal{F},\ddot{A}))$
3. If $(\mathcal{F},\ddot{A}) = (\mathcal{G},\dddot{A})$, that is $\forall\ i,\ k,\ \sigma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \sigma_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, $\tau_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \tau_{\mathcal{G}(\breve{a}_k)}(\delta_i)$, and $\gamma_{\mathcal{F}(\breve{a}_k)}(\delta_i) = \gamma_{\mathcal{G}(\breve{a}_k)}(\delta_i)$ then $\delta_{WNHSS}((\mathcal{F},\ddot{A}),(\mathcal{G},\dddot{A})) = 1$.

Proof Similar to proposition 3.5.

4. Application of Correlation Coefficient for Decision Making Under NHSS Environment

In this section, we proposed the algorithm based on CC under NHSS and utilize the proposed approach for decision making in real-life problems.

4.1 Algorithm for Correlation Coefficient under NHSS

Step 1. Pick out the set containing sub-attributes of parameters.

Step 2. Construct the NHSS according to experts in form of NHSNs.

Step 3. Find the informational neutrosophic energies of NHSS.

Step 4. Calculate the correlation between NHSSs by using the following formula

$\boldsymbol{\mathcal{C}}_{\boldsymbol{NHSS}}((\boldsymbol{\mathcal{F}},\ddot{\mathrm{A}}),(\boldsymbol{\mathcal{G}},\ddot{\mathbb{A}}))$ =

$$\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sigma_{\mathcal{F}(\check{a}_k)}(\delta_i) * \sigma_{\mathcal{G}(\check{a}_k)}(\delta_i) + \tau_{\mathcal{F}(\check{a}_k)}(\delta_i) * \tau_{\mathcal{G}(\check{a}_k)}(\delta_i) + \gamma_{\mathcal{F}(\check{a}_k)}(\delta_i) * \gamma_{\mathcal{G}(\check{a}_k)}(\delta_i)\right)$$

Step 5. Calculate the CC between NHSSs by using the following formula

$\delta_{NHSS}((\mathcal{F},\ddot{\mathrm{A}}),(\mathcal{G},\ddot{\mathbb{A}}))$ = $\frac{\mathcal{C}_{NHSS}((\mathcal{F},\ddot{\mathrm{A}}),(\mathcal{G},\ddot{\mathbb{A}}))}{\sqrt{\varsigma_{NHSS}(\mathcal{F},\ddot{\mathrm{A}})} * \sqrt{\varsigma_{NHSS}(\mathcal{G},\ddot{\mathbb{A}})}}$

Step 6. Choose the alternative with a maximum value of CC.

Step 7. Analyze the ranking of the alternatives.

A flowchart of the above-presented algorithm can be seen in figure 1.

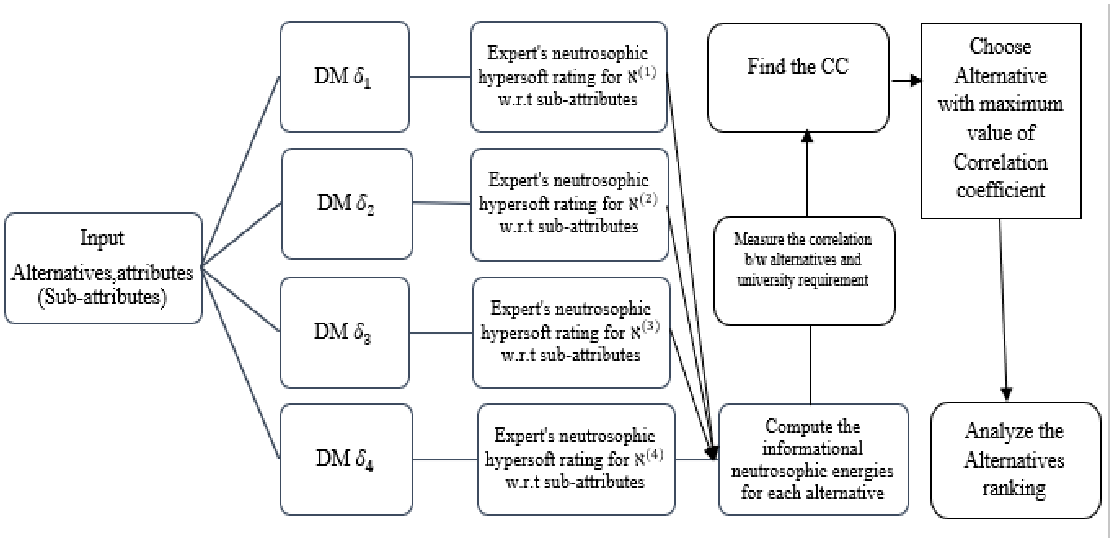

Figure 1: Flowchart for correlation coefficient under NHSS

4.1 Problem Formulation and Application of NHSS For Decision Making

Department of the scientific discipline of some university $\mathcal{U}$ will have one scholarship for the position of post-doctorate. Several scholars apply to get a scholarship but referable probabilistic along with CGPA (cumulative grade points average), simply four scholars call for enrolled for undervaluation such as $\aleph$ = $\{\aleph^1, \aleph^2, \aleph^3, \aleph^4\}$ be a set of selected scholars call for the interview. The president of the university hires a committee of four decision-makers (DM) $\mathcal{U}$ = $\{\delta_1, \delta_2, \delta_3, \delta_4\}$ for the selection post-doctoral scholar. The team of DM decides the criteria (attributes) for the selection of post-doctorate position such as $\mathfrak{L}$ = $\{\ell_1 = \text{Publications}, \ell_2 = \text{Subjects}, \ell_3 = \text{IF}\}$ be a collection of attributes and their corresponding sub-attribute are given as Publications = ℓ_1 = $\{a_{11} = \textit{more than } 10, a_{12} = \textit{ less than } 10\}$, Subjects = ℓ_2 = $\{a_{21} = \textit{Mathematics}, a_{22} = \textit{ Computer Science}\}$, and IF = ℓ_3 = $\{a_{31} = 45, a_{32} = 47\}$. Let $\mathfrak{L}'$ = $\ell_1 \times \ell_2 \times \ell_3$ be a set of sub-attributes

$\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3 = \{a_{11}, a_{12}\} \times \{a_{21}, a_{22}\} \times \{a_{31}, a_{32}\}$

= $\begin{Bmatrix} (a_{11}, a_{21}, a_{31}), (a_{11}, a_{21}, a_{32}), (a_{11}, a_{22}, a_{31}), (a_{11}, a_{22}, a_{32}), \\ (a_{12}, a_{21}, a_{31}), (a_{12}, a_{21}, a_{32}), (a_{12}, a_{22}, a_{31}), (a_{12}, a_{22}, a_{32}) \end{Bmatrix}$, $\mathfrak{L}' = \{\breve{a}_1, \breve{a}_2, \breve{a}_3, \breve{a}_4, \breve{a}_5, \breve{a}_6, \breve{a}_7, \breve{a}_8\}$ be a set of all multi sub-attributes. Each DM will evaluate the ratings of each alternative in the form of NHSNs under the considered multi sub-attributes. The developed method to find the best alternative is as follows.

4.1.1. Application of NHSS For Decision Making

Assume $\aleph = \{\aleph^1, \aleph^2, \aleph^3, \aleph^4\}$ be a set of alternatives who are shortlisted for interview and $\mathfrak{L} = \{\ell_1 = \text{Publications}, \ell_2 = \text{Subjects}, \ell_3 = \text{Qualification}\}$ be a set of parameters for the selection of scholarship positions. Let the corresponding sub-attribute are given as Publications = $\ell_1 = \{a_{11} = more\ than\ 10, a_{12} = less\ than\ 10\}$, Subjects = ℓ_2 = $\{a_{21} = Mathematics, a_{22} = Computer\ Science\}$, and IF = $\ell_3 = \{a_{31} = 45, a_{32} = 47\}$. Let $\mathfrak{L}' = \ell_1 \times \ell_2 \times \ell_3$ be a set of sub-attributes. Development of decision matrix according to the requirement of the scientific discipline department in terms of NHSNs.

Table 1. Decision Matrix of Concerning Department

$\wp$	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
δ_1	(.2,.5,.9)	(.5,.7,.6)	(.5,.6,.9)	(.5,.8,.7)	(.4,.7,.6)	(.8,.6,.3)	(.5,.4,.7)	(.6,.4,.8)
δ_2	(.5,.9,.7)	(.6,.4,.7)	(.5,.8 .2)	(.7,.4 .2)	(.9,.5,.7)	(.4,.7,.9)	(.9,.2,.5)	(.2,.8,.5)
δ_3	(.7,.3,.5)	(.7,.4,.2)	(.8,.2,.6)	(.7,.3,.6)	(.8,.4 .9)	(.7,.5,.8)	(.9,.6,.8)	(.6,.3,.8)
δ_4	(.5,.4,.7)	(.4,.7,.3)	(.6,.3,.8)	(.5,.4,.6)	(.7,.3,.5)	(.8,.3,.2)	(.5,.4,.7)	(.6,.2,.7)

Table 2. Decision Matrix for Alternative $\aleph^{(1)}$

$\aleph^{(1)}$	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
δ_1	(.3,.5,.8)	(.2,.3,.6)	(.5,.1,.3)	(.8,.6,.7)	(.5,.9,.6)	(.8,.2,.6)	(.5,.4,.1)	(.9,.3,.5)
δ_2	(.5,.2,.7)	(.2,.4,.6)	(.3,.8 .4)	(.7,.5 .2)	(.9,.2,.6)	(.5,.2,.4)	(.9,.2,.5)	(.8,.4,.5)
δ_3	(.6,.2,.4)	(.4,.7,.5)	(.5,.1,.6)	(.7,.3,.4)	(.2,.6 .9)	(.9,.3,.5)	(.2,.3,.8)	(.6,.3,.8)
δ_4	(.2,.4,.7)	(.7,.2,.3)	(.6,.3,.8)	(.2,.4,.6)	(.7,.3,.5)	(.9,.3,.6)	(.3,.4,.5)	(.6,.2,.7)

Table 3. Decision Matrix for Alternative $\aleph^{(2)}$

$\aleph^{(2)}$	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
δ_1	(.8,.5,.6)	(.5,.4,.2)	(.4,.3,.6)	(.4,.8,.6)	(.7,.6,.5)	(.4,.1,.3)	(.7,.8,.5)	(.8,.4,.7)
δ_2	(.6,.5,.2)	(.5,.6,.5)	(.9,.5,.8)	(.6,.4,.5)	(.7,.5,.8)	(.7,.5,.7)	(.3,.5,.9)	(.6,.4,.9)
δ_3	(.2,.5,.2)	(.9,.4,.6)	(.2,.5,.4)	(.7,.3,.2)	(.6,.4,.5)	(.3,.5,.7)	(.4,.6,.2)	(.6,.7,.9)
δ_4	(.5,.2,.4)	(.7,.5,.9)	(.6,.3,.4)	(.9,.5,.1)	(.3,.4,.6)	(.6,.5,.2)	(.9,.5,.6)	(.3,.4,.3)

Table 4. Decision Matrix for Alternative $\aleph^{(3)}$

$\aleph^{(3)}$	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
δ_1	(.3,.5,.2)	(.8,.7,.3)	(.7,.2,.9)	(.9,.5,.1)	(.3,.4,.6)	(.1,.5,.2)	(.9,.5,.1)	(.7,.4,.3)
δ_2	(.6,.7,.2)	(.7,.8,.3)	(.2,.4,.6)	(.6,.1,.2)	(.9,.5,.6)	(.7,.2,.3)	(.4,.7,.6)	(.7,.2,.4)
δ_3	(.3,.9,.7)	(.5,.9,.1)	(.7,.3,.2)	(.2,.1,.2)	(.7,.9,.8)	(.7,.2,.1)	(.7,.4,.5)	(.1,.7,.9)

$\boldsymbol{\delta_4}$	(.7,.8,.6)	(.7,.2,.5)	(.7,.3,.2)	(.3,.2,.7)	(.4,.6,.8)	(.5,.6,.2)	(.7,.2,.6)	(.8,.6,.9)

Table 5. Decision Matrix for Alternative $\aleph^{(4)}$

$\aleph^{(4)}$	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
$\boldsymbol{\delta_1}$	(.7,.4,.1)	(.7,.3,.1)	(.7,.4,.6)	(.4,.9,.6)	(.7,.2,.5)	(.7,.3,.2)	(.7,.4,.6)	(.9,.4,.3)
$\boldsymbol{\delta_2}$	(.1,.4,.5)	(.6,.2,.3)	(.7,.4,.3)	(.6,.2,.5)	(.6,.2,.1)	(.5,.4,.7)	(.3,.5,.1)	(.6,.2,.7)
$\boldsymbol{\delta_3}$	(.5,.4,.3)	(.6,.4,.7)	(.6,.2,.1)	(.6,.3,.5)	(.4,.7,.9)	(.2,.7,.4)	(.5,.3,.9)	(.3,.6,.2)
$\boldsymbol{\delta_4}$	(.4,.2,.6)	(.7,.4,.3)	(.5,.4,.9)	(.4,.2,.3)	(.4,.1,.3)	(.4,.5,.2)	(.1,.6,.5)	(.1,.5,.2)

By using Tables 1-5, compute the correlation coefficient between $\delta_{NHSS}(\wp, \aleph^{(1)})$, $\delta_{NHSS}(\wp, \aleph^{(2)})$, $\delta_{NHSS}(\wp, \aleph^{(3)})$, $\delta_{NHSS}(\wp, \aleph^{(4)})$ by using equation 5 given as follows:
$\delta_{NHSS}(\wp, \aleph^{(1)})$ = .99658, $\delta_{NHSS}(\wp, \aleph^{(2)})$ = .99732, $\delta_{NHSS}(\wp, \aleph^{(3)})$ = .99894, and $\delta_{NHSS}(\wp, \aleph^{(4)})$ = .99669. This shows that $\delta_{IVNHSS}(\wp, \aleph^{(3)}) > \delta_{IVNHSS}(\wp, \aleph^{(2)}) > \delta_{IVNHSS}(\wp, \aleph^{(4)}) > \delta_{IVNHSS}(\wp, \aleph^{(1)})$. It can be seen from this ranking alternative $\aleph^{(3)}$ is the most suitable alternative. Therefore $\aleph^{(3)}$ is the best alternative, the ranking of other alternatives given as $\aleph^{(3)} > \aleph^{(2)} > \aleph^{(4)} > \aleph^{(1)}$. Graphical results of alternatives ratings can be seen in figure 2.

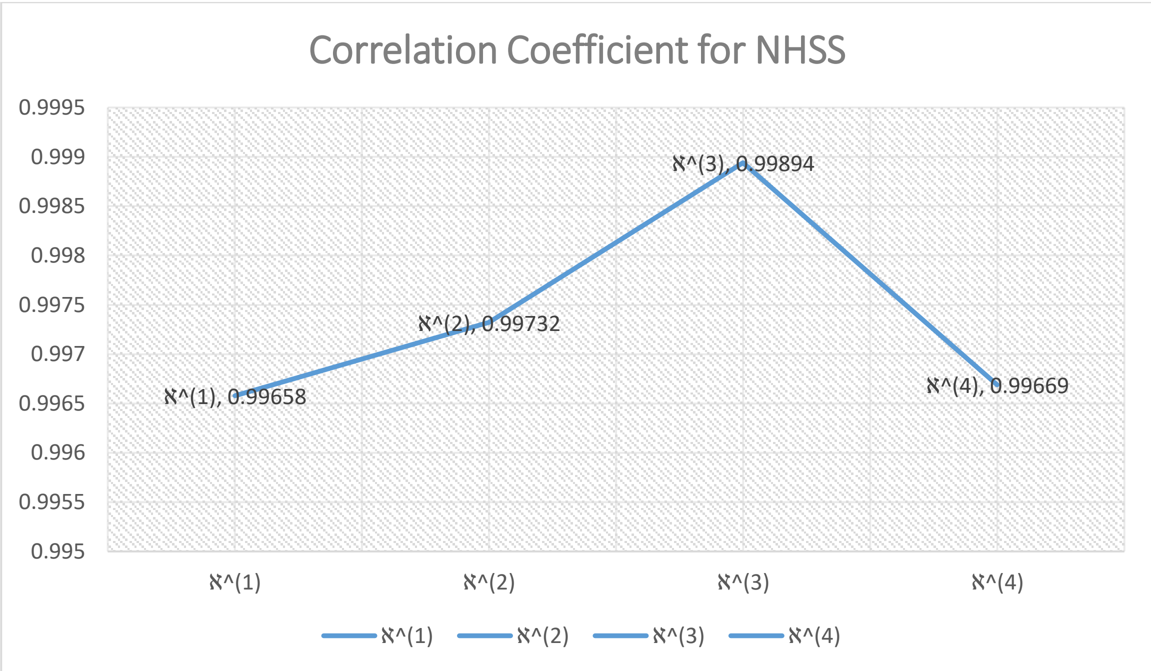

Figure 2: Alternative's rating based on correlation coefficient under NHSS

5. Conclusion

The neutrosophic hypersoft set is a novel concept that is an extension neutrosophic soft set. In this manuscript, we studied some basic concepts which were necessary to build the structure of the paper. We introduced the correlation and weighted correlation coefficient with some necessary properties under the NHSS environment. A decision-making approach has been developed based on the established correlation coefficient and presented an algorithm under NHSS. Finally, a numerical illustration has been described to solve the decision-making problem by using the proposed technique. In the future, anyone can extend the NHSS to interval valued NHSS, aggregation operators, TOPSIS technique based on developed CC.

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