Neutrosophic Sets and Systems, Vol. 46, 2021
University of New Mexico

Quadripartitioned Single Valued Neutrosophic Pythagorean Dombi Aggregate Operators in MCDM Problems

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Abstract: The quadripartitioned single-valued neutrosophic set (QSVNS) is developed to understand the concept of indeterminacy more clearly. It takes care of the diverse approaches while dealing with uncertainty under single-valued neutrosophic environment. To make the QSVNS more functional and logical, the notion of quadripartioned single-valued neutrosophic Pythagorean set (QSVNPS) is introduced. In QSVNPS, the components T, C, U, F are dependent in such a manner that $T+F \leq 1$, $C+U \leq 1$, and $T^2+C^2+U^2+F^2 \leq 2$. So, the QSVNPS is a powerful framework for modeling the imprecise human knowledge in a specific manner. To calculate the arithmetic operations, we consider the quadripartitioned single-valued neutrosophic Pythagorean numbers (QSVNPNs) associated with QSVNPSs. The main advantage of using QSVNPNs is that it allows the decision-makers to carry out the calculation on uncertain parameters. The present paper aims to study Dombi operators and to establish some new Dombi weight aggregate operators and develop some properties under QSVNPN environment for solving multi-criteria decision-making(MCDM) problems that we encounter in our day-to-day life process. Then we define the score and accuracy functions for ranking the QSVNPNs to choose the best-preferred alternative that goes through under a set of certain criteria. A model for MCDM problems based on Dombi operators under QSVNPNs has been introduced. To check the feasibility of the new approach, a numerical example is demonstrated that shows the effectiveness of the proposed model for multi-criteria decision-making. Finally, a comparative analysis between the rankings, obtained by using the proposed model, of the given set of alternatives under a certain set of criteria gives the optimal choice.

Keywords: Pythagorean fuzzy set; Quadripartitioned neutrosophic Pythagorean set; Dombi operator; MCDM.

1. Introduction

The introduction of the fuzzy set(FS)[1] proposed by Zadeh is a conceptual framework to be formed by replacing the two-valued characteristic function with the fuzzy membership function to define the imprecise information that we encounter in physical world phenomenon. It has the rich potential to address ambiguous issues. Due to the novelty of the FS, it has a wide range of applicability in information communication, pattern recognition, artificial intelligence, operation research, medical diagnosis, computer science, game theory, economics, environmental science, engineering, robotics, etc. In FS, every object of the universe is characterized by a membership function and the degree of membership is ranging between 0 and 1. Some contributed works related to fuzzy sets are proposed in the literature given in [2-6]. Later on, after critical investigation, it has been identified by the researchers that, the concept of hesitancy that is natural in human thinking cannot be described by FS due to its inherent difficulty. So, there is an information gap in FS and to eradicate such gap a new mathematical structure called intuitionistic fuzzy set (IFS) [7, 8] is introduced by adding a non-membership degree to the FS. In IFS, every object of the universe has a membership and a non-membership degree, and their sum cannot exceed 1. The hesitancy membership can be obtained by subtracting the sum of the membership and the non-membership degree from 1. Thus, the IFS provides incomplete information to the decision-maker and it can be viewed as an extension of FS. IFS can be reduced to an FS when its non-membership degree is 0. Some recent works on IFS are proposed in the literature given in [9-12].

However, researchers find the existence of an environment which cannot be addressed by FS and IFS. For example, suppose under a certain environment, the membership and the non-membership degrees of an object provided to the decision-maker are 0.5 and 0.7. Then, their sum is $0.5+0.7=1.2>1$. So, this type of phenomenon cannot be defined by FS and IFS. We need another powerful tool that can easily solve this problem. For the demand of the situation, the Pythagorean fuzzy set (PFS) [13] is introduced where the sum of the squares of the membership and the non-membership degree cannot exceed 1. With the help of PFS, the above problem can be easily defined, as $0.5^2+0.7^2=0.74<1$. Therefore, the Pythagorean environment is capable to accommodate both the fuzzy and the intuitionistic fuzzy environment to solve diverse problems. In PFS, with the help of Pythagorean fuzzy membership and non-membership function, we can enhance the applicability of FS and IFS. Different authors use PFS from different angles and develop some new operators which are capable to solve real decision-making problems. Some significant works related to PFSs are studied in the proposed literature given in [14-18].

There is another aspect of extending the notion of IFS and it is due to the unavailability of indeterminacy membership in IFS. The concept of indeterminacy is found relevant in human thinking. For example, in a certain class, a teacher asks a true-false type question to a group of 30 students in that class. The response of the students recorded as 20 says true, 5 say false and according to the remaining 5 students, it is neither true nor false. This gives a clear idea about how indeterminacy exists in our communication information. In 2005, Smarandache[19]introduced neutrosophic set(NS) as a generalization of the crisp set, FS, IFS, paraconsistent set, PFS, etc. Later on, for scientific and technical application, Wang et al. [20] introduced a single-valued neutrosophic set (SVNS) to develop the operators used in NS. In SVNS, each object of the universe is characterized by the three membership functions called truth-membership, indeterminacy-membership, and falsity-membership function in such a way that the sum of the three membership values cannot exceed 3. So, the SVNS is enabled to take care of the issues that contain uncertainty that contains indeterminacy. In [21], Jansi et al. introduced the correlation measures for Pythagorean neutrosophic sets (PNSs) with truth and falsity as dependent components. Furthermore, sometimes while working with NS, there is a doubt in the mind of the researchers that the indeterminacy to an element occurs due to the belongingness or non-belongingness. Such issues were presented by Chatterjee et al. [22] by introducing a quadripartitioned single-valued neutrosophic set (QSVNS). The QSVNS is a more generalized framework than SVNS and the motivation behind adopting this idea is due to Smarandache's four-valued neutrosophic logic and Belnap's four-valued logic. In QSNS, the indeterminacy component is being divided into two parts, namely, contradiction and unknown. By combining QSVNS and PFS, Radha et al. [23] introduced a new model known as a quadripartitioned single-valued neutrosophic Pythagorean set (QSVNPS). In QSVNPS, truth and falsity make one pair of a dependent component on the other hand contradiction and an unknown or ignorance make another pair of dependent components. Therefore, it looks quite logical to apply Pythagorean property on QSVNS.

The relationship of different types of sets in the context of the proposed study is exhibited in the form of an arrow diagram given as:

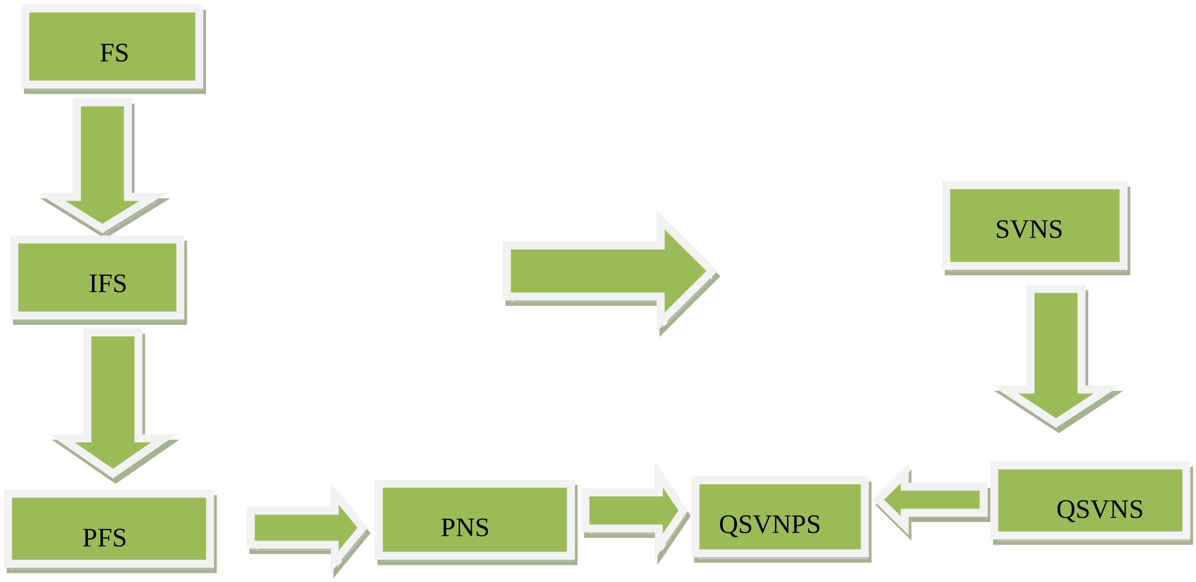

Fig 1. Arrow diagram to represent the relationship of different types of sets

Dombi operators have an excellent potential with operational parameters and due to these operational parameters, it is flexible to operate. In 1982, Dombi[24] introduced triangular t-norm and t-conorm operators. Roychoudhury et al. [25] generalize the Dombi class, intuitionistic fuzzy Dombi aggregation operator and their application to MADM proposed in [26], Dombi prioritized weighted aggregation operator on single-valued neutrosophic set for MADM is given by Wei et al. [27], Jana et al.[28]presented the bipolar fuzzy Dombi aggregation operators, Ashraf et al. [29] introduced the spherical fuzzy Dombi aggregation operator and their application in GDM, Garg et al. [30] initiated the neutrality operations based Pythagorean fuzzy aggregation operators and its application to MAGDM, Qiyas et al. [31] defined linguistic picture fuzzy aggregation operator. Furthermore, the Pythagorean Dombi fuzzy aggregation operator presented by Akram et al. [32], Khan et al. [33]

introduced the Pythagorean fuzzy Dombi aggregation operators and their application in decision support system, Jana et al. [34] described the Pythagorean fuzzy Dombi aggregation operators and their application in MADM, Akram et al. [35] extended the Dombi aggregation operator for DM under m-polar fuzzy information, bipolar neutrosophic Dombi aggregation operators with application in MADM problems are introduced by Mahmood et al. [36], etc.

Over the last decades, decision-making (DM) is an effective scientific approach for making decisions by assessing a set of alternatives and achieve the best results. There are various DM approaches or strategies that help to choose the optimal choice. In real-life scenarios, uncertainty plays an important role in decision-making and it captures considerable attention in various research areas. For getting more information about decision-making we discuss the following: correlation coefficient based TOPSIS method under interval-valued intuitionistic fuzzy soft environment and their aggregate operators for DM are defined in [37]. In [38], Zulqarnain et al. presented a correlation coefficient based TOPSIS method under Pythagorean fuzzy soft environment and apply it in green supply chain management. DM approach under interval-valued neutrosophic hypersoft set defined in [39]. Development of TOPSIS method under Pythagorean fuzzy hypersoft set for the selection of antivirus mask is given in [40]. In [41], an algorithm is introduced by using the generalized multipolar neutrosophic soft set for medical diagnosis DM problems. An extension of the TOPSIS technique based on the correlation coefficient under the neutrosophic hypersoft set is proposed for the selection of effective hand sanitizer to reduce the covid-19 effects as defined in [42]. Another extension of the TOPSIS method under intuitionistic fuzzy hypersoft environment is to solve the DM problem in [43]. Using the matrix representation of the neutrosophic hypersoft set, the MADM problems are solved in [44]. Some other popular DM approaches under different environments were studied in [45-51].

Motivated by the above discussion, we introduce QSVNPNs and studied various operational laws and properties on them. In addition, based on QSVNPNs, we define score function, accuracy function, and the operators QSVNPWA and QSVPWG. Furthermore, based on Dombi t-norm and t-co-norm operators, we introduce two new aggregate operators for the MCDM problem. Some properties based on these two new operators are also investigated in the study. A new model has been proposed by using the new aggregate operators. Also, we execute the model by taking a suitable example.

1.1 Motivation

The Dombi aggregate operators under quadripartitioned single-valued neutrosophic Pythagorean numbers environment has not yet been studied till date. This gives us the motivation to present the proposed study.

The rest of the paper is organized in the form: In section 2, we review some basic definitions that are useful for the subsequent sections. In section 3, we establish some Dombi operators on QSVNPNs. In section 4, we propose the Dombi weighted aggregation operators under QSVNPNs environment. Further, in section 5, we initiate an algorithm-based model for MCDM using QSVNP Information. A practical application based on the proposed model is discussed in section 6. Conclusion and the future scope are presented in section 7.

2. Preliminaries

In this section, we recall some basic definitions that are fundamental to the proposed topic.

Definition 2.1 [13-15] A PFS Ω over the set of the universe Γ is an object of the form $\Omega=\left\{\left\langle s,\mu_\Omega(s),\gamma_\Omega(s)\right\rangle : s\in\Gamma\right\}$, where $\mu_\Omega:\Gamma\to[0,1]$ and $\gamma_\Omega:\Gamma\to[0,1]$ are respectively the membership and the non-membership functions with the restriction that $0\le\left(\mu_\Omega(s)\right)^2+\left(\gamma_\Omega(s)\right)^2\le 1$ and the hesitancy is measured by $\Pi_\Omega(s)=\sqrt{1-\left(\mu_\Omega(s)\right)^2-\left(\gamma_\Omega(s)\right)^2}$. We represent the Pythagorean fuzzy number (PFN) as $\Omega=\left\langle\mu_\Omega,\gamma_\Omega\right\rangle$.

Definition 2.2 [20] A SVNS Ω over the set of the universe Γ is an object of the form $\Omega=\left\{\left\langle s,T_\Omega(s),I_\Omega(s),F_\Omega(s)\right\rangle : s\in\Gamma\right\}$, where $T_\Omega:\Gamma\to[0,1]$, $I_\Omega:\Gamma\to[0,1]$ and $F_\Omega:\Gamma\to[0,1]$ are respectively the truth, indeterminacy and falsity membership functions with the restriction $0\le T_\Omega(s)+I_\Omega(s)+F_\Omega(s)\le 3$. The SVNN is represented by $\Omega=\left\langle T_\Omega,I_\Omega,F_\Omega\right\rangle$.

Definition 2.3 [22] A QSVNS Ω over the set of the universe Γ is an object of the form $\Omega=\left\{\left\langle s,T_\Omega(s),C_\Omega(s),U_\Omega(s),F_\Omega(s)\right\rangle : s\in\Gamma\right\}$, where $T_\Omega(s),C_\Omega(s),U_\Omega(s)$, and $F_\Omega(s)$ are respectively the truth, contradiction, unknown and falsity membership values with the condition $0\le T_\Omega(s)+C_\Omega(s)+U_\Omega(s)+F_\Omega(s)\le 4$.

Definition 2.4 [23] A QSVNPS Ω with dependent neutrosophic components over the universe of discourse Γ is an object of the form $\Omega=\left\{\left\langle s,T_\Omega(s),C_\Omega(s),U_\Omega(s),F_\Omega(s)\right\rangle : s\in\Gamma\right\}$ where $T_\Omega(s),C_\Omega(s),U_\Omega(s)$, and $F_\Omega(s)$ are respectively the truth, contradiction, unknown and falsity membership values with the condition $T_\Omega(s)+F_\Omega(s)\le 1, C_\Omega(s)+U_\Omega(s)\le 1$, and $0\le T^2{}_\Omega(s)+C^2{}_\Omega(s)+U^2{}_\Omega(s)+F^2{}_\Omega(s)\le 2$. The QSVNPN is denoted by $\Omega=\left\langle T_\Omega,C_\Omega,U_\Omega,F_\Omega\right\rangle$.

Definition 2.5 For any two QSVNPNs $\Omega_1=\left\langle T_{\Omega_1},C_{\Omega_1},U_{\Omega_1},F_{\Omega_1}\right\rangle$ and $\Omega_2=\left\langle T_{\Omega_2},C_{\Omega_2},U_{\Omega_2},F_{\Omega_2}\right\rangle$

We have the following properties:

1. $\Omega_1 \subseteq \Omega_2$ iff for any $\varepsilon \in \Gamma$,

$T_{\Omega_1}(\varepsilon) \le T_{\Omega_2}(\varepsilon), C_{\Omega_1}(\varepsilon) \le C_{\Omega_2}(\varepsilon)\ and\ U_{\Omega_1}(\varepsilon) \ge U_{\Omega_2}(\varepsilon), F_{\Omega_1}(\varepsilon) \ge F_{\Omega_2}(\varepsilon)$

2. $\Omega_1 = \Omega_2$ iff $\Omega_1 \subseteq \Omega_2$ and $\Omega_2 \subseteq \Omega_1$

3. $\Omega_1 \cup \Omega_2 = \left\langle \max\left(T_{\Omega_1}, T_{\Omega_2}\right), \max\left(C_{\Omega_1}, C_{\Omega_2}\right), \min\left(U_{\Omega_1}, U_{\Omega_2}\right), \min\left(F_{\Omega_1}, F_{\Omega_2}\right)\right\rangle$

4. $\Omega_1 \cap \Omega_2 = \left\langle \min\left(T_{\Omega_1}, T_{\Omega_2}\right), \min\left(C_{\Omega_1}, C_{\Omega_2}\right), \max\left(U_{\Omega_1}, U_{\Omega_2}\right), \max\left(F_{\Omega_1}, F_{\Omega_2}\right)\right\rangle$

5. $\Omega_1^c = \left\langle F_{\Omega_1}, U_{\Omega_1}, C_{\Omega_1}, T_{\Omega_1}\right\rangle$

Definition 2.6 For two QSVNPNs $\Omega_1 = \left\langle T_{\Omega_1}, C_{\Omega_1}, U_{\Omega_1}, F_{\Omega_1}\right\rangle$ and $\Omega_2 = \left\langle T_{\Omega_2}, C_{\Omega_2}, U_{\Omega_2}, F_{\Omega_2}\right\rangle$, the basic operational laws between them are given by:

1. $\Omega_1 \oplus \Omega_2 = \left\langle T_{\Omega_1}, C_{\Omega_1}, U_{\Omega_1}, F_{\Omega_1}\right\rangle \oplus \left\langle T_{\Omega_2}, C_{\Omega_2}, U_{\Omega_2}, F_{\Omega_2}\right\rangle =$
$\left\langle \sqrt{T_{\Omega_1}^2 + T_{\Omega_2}^2 - T_{\Omega_1}^2.T_{\Omega_2}^2}, \sqrt{C_{\Omega_1}^2 + C_{\Omega_2}^2 - C_{\Omega_1}^2.C_{\Omega_2}^2}, U_{\Omega_1}.U_{\Omega_2}, F_{\Omega_1}.F_{\Omega_2}\right\rangle$

2. $\Omega_1 \otimes \Omega_2 = \left\langle T_{\Omega_1}, C_{\Omega_1}, U_{\Omega_1}, F_{\Omega_1}\right\rangle \otimes \left\langle T_{\Omega_2}, C_{\Omega_2}, U_{\Omega_2}, F_{\Omega_2}\right\rangle =$
$\left\langle T_{\Omega_1}.T_{\Omega_2}, C_{\Omega_1}.C_{\Omega_2}, \sqrt{U_{\Omega_1}^2 + U_{\Omega_2}^2 - U_{\Omega_1}^2.U_{\Omega_2}^2}, \sqrt{F_{\Omega_1}^2 + F_{\Omega_2}^2 - F_{\Omega_1}^2.F_{\Omega_2}^2}\right\rangle$

3. For any $\kappa \ge 0$, $\kappa\Omega = \left\langle \sqrt{1-\left(1-T_\Omega^2\right)^\kappa}, \sqrt{1-\left(1-C_\Omega^2\right)^\kappa}, U_\Omega^k, F_\Omega^k\right\rangle$

4. For any $\kappa \ge 0$, $\Omega^\kappa = \left\langle T_\Omega^k, C_\Omega^k, \sqrt{1-\left(1-U_\Omega^2\right)^\kappa}, \sqrt{1-\left(1-F_\Omega^2\right)^\kappa}\right\rangle$

Definition 2.7 For any QSVNPN $\Omega = \left\langle T_\Omega, C_\Omega, U_\Omega, F_\Omega\right\rangle$ the score function $\Theta(\Omega)$ and the accuracy function $\Lambda(\Omega)$ can be defined as

$\Theta(\Omega) = T_\Omega^2 + C_\Omega^2 - U_\Omega^2 - I_\Omega^2$, where $\Theta(\Omega) \in [-1,1]$ and $\Lambda(\Omega) = T_\Omega^2 + C_\Omega^2 + U_\Omega^2 + I_\Omega^2$, where $\Lambda(\Omega) \in [0,1]$

Definition 2.8 Let $\Omega_1 = \left\langle T_{\Omega_1}, C_{\Omega_1}, U_{\Omega_1}, F_{\Omega_1}\right\rangle$ and $\Omega_2 = \left\langle T_{\Omega_2}, C_{\Omega_2}, U_{\Omega_2}, F_{\Omega_2}\right\rangle$ be two QSVNPNs over the common universe of discourse Γ and their corresponding score and accuracy functions are respectively $\Theta(\Omega_1), \Theta(\Omega_2)$ and $\Lambda(\Omega_1), \Lambda(\Omega_2)$. Then we consider the following:

1. If $\Theta(\Omega_1) < \Theta(\Omega_2)$, then $\Omega_1 \prec \Omega_2$

2. If $\Theta(\Omega_1) > \Theta(\Omega_2)$, then $\Omega_1 \succ \Omega_2$

3. If $\Theta(\Omega_1) = \Theta(\Omega_2)$, then we compare their accuracy function as:

(a)If $\Lambda(\Omega_1) < \Lambda(\Omega_2)$, then $\Lambda_1 \prec \Lambda_2$

(b)If $\Lambda(\Omega_1) > \Lambda(\Omega_2)$, then $\Lambda_1 \succ \Lambda_2$

(c) If $\Lambda(\Omega_1) = \Lambda(\Omega_2)$, then $\Lambda_1 \approx \Lambda_2$

Theorem 2.9

For any three QSVNPNs $\Omega_1 = \left\langle T_{\Omega_1}, C_{\Omega_1}, U_{\Omega_1}, F_{\Omega_1} \right\rangle$, $\Omega_2 = \left\langle T_{\Omega_2}, C_{\Omega_2}, U_{\Omega_2}, F_{\Omega_2} \right\rangle$ and Ω_3 = $\left\langle T_{\Omega_3}, C_{\Omega_3}, U_{\Omega_3}, F_{\Omega_3} \right\rangle$ over the universe of discourse Γ and $\kappa_1, \kappa_2 \geq 0$. Then,

1. $\Omega_1 \oplus \Omega_2 = \Omega_2 \oplus \Omega_1$

2. $\Omega_1 \otimes \Omega_2 = \Omega_2 \otimes \Omega_1$

3. $(\Omega_1 \oplus \Omega_2) \oplus \Omega_3 = \Omega_1 \oplus (\Omega_2 \oplus \Omega_3)$

4. $(\Omega_1 \otimes \Omega_2) \otimes \Omega_3 = \Omega_1 \otimes (\Omega_2 \otimes \Omega_3)$

5. $\kappa_1(\Omega_1 \oplus \Omega_2) = \kappa_1\Omega_1 \oplus \kappa_1\Omega_2, \kappa_1 > 0$

6. $(\Omega_1 \otimes \Omega_2)^{\kappa_1} = \Omega_1^{\kappa_1} \otimes \Omega_2^{\kappa_1}, \kappa_1 > 0$

7. $\kappa_1\Omega_1 \oplus \kappa_2\Omega_1 = (\kappa_1 + \kappa_2)\Omega_1, \kappa_1 \text{ and} \kappa_2 > 0$

8. $\Omega_1^{\kappa_1} \otimes \Omega_1^{\kappa_2} = \Omega_1^{\kappa_1+\kappa_2}, \kappa_1 \text{ and} \kappa_2 > 0$

Proof: All proofs are obvious.

Definition 2.10 Let Ω_m = $\left\langle T_{\Omega_m}, C_{\Omega_m}, U_{\Omega_m}, F_{\Omega_m} \right\rangle$ be a collection of QSVPNs in Γ where $m = 1, 2, 3, ..., n$.

Then the quadripartitioned single-valued neutrosophic Pythagorean weighted averaging (QSVNPWA)operator with weight vector $\varpi_m (m = 1, 2, ..., n)$ where $\varpi_m \geq 0$ and $\sum_{m=1}^{n} \varpi_m = 1$ is given by

$$QSVNPWA\left(\Omega_1,\Omega_2,...,\Omega_n\right)=\sum_{m=1}^{n}\Omega_m\varpi_m .$$

Definition 2.11 Let $\Omega_m = \left\langle T_{\Omega_m}, C_{\Omega_m}, U_{\Omega_m}, F_{\Omega_m} \right\rangle$ be a collection of QSVPNs in Γ where $m=1,2,3,....,n$.

Then the quadripartitioned single-valued neutrosophic Pythagorean weighted geometric (QSVNPWG)operator with weight vector $\varpi_m\left(m=1,2,....,n\right)$ where $\varpi_m \geq 0$ and $\sum_{m=1}^{n}\varpi_m = 1$ is given by

$$QSVNPWA\left(\Omega_1,\Omega_2,...,\Omega_n\right)=\prod_{m=1}^{n}\left(\Omega_m\right)^{\varpi_m} .$$

3. Dombi Operations on QSVNPNs

Definition 3.1 [24] Let p and q be any two real numbers where $\left(p,q\right)\in\left(0,1\right)\times\left(0,1\right)$ with $\xi \geq 1$.

Then Dombi's t-norms and t-co-norms are defined as

$$\hat{H}\left(p,q\right)=\frac{1}{1+\left[\left(\frac{1-p}{p}\right)^{\xi}+\left(\frac{1-q}{q}\right)^{\xi}\right]^{1/\xi}}$$ and

$$\hat{G}\left(p,q\right)=1-\frac{1}{1+\left[\left(\frac{p}{1-p}\right)^{\xi}+\left(\frac{q}{1-q}\right)^{\xi}\right]^{1/\xi}}$$ respectively.

Based on definition **3.1**, we define the following Dombi's t-norms and t-co-norms operational laws on QSVNPNs:

Definition 3.2 Let $\Omega_1=\left\langle T_{\Omega_1}, C_{\Omega_1}, U_{\Omega_1}, F_{\Omega_1}\right\rangle$ and $\Omega_2=\left\langle T_{\Omega_2}, C_{\Omega_2}, U_{\Omega_2}, F_{\Omega_2}\right\rangle$ be two QSVNPNs over Γ with $\kappa \geq 0$ and $\xi \geq 1$. Then the Dombi's t-norms and t-co-norms operational laws defined on QSVNPNs are given by:

1. $$\Omega_1 \oplus \Omega_2 = \left\langle \begin{array}{c} \sqrt{1-\frac{1}{1+\left[\left(\frac{T_{\Omega_1}^2}{1-T_{\Omega_1}^2}\right)^{\xi}+\left(\frac{T_{\Omega_2}^2}{1-T_{\Omega_2}^2}\right)^{\xi}\right]^{1/\xi}}}, \sqrt{1-\frac{1}{1+\left[\left(\frac{C_{\Omega_1}^2}{1-C_{\Omega_1}^2}\right)^{\xi}+\left(\frac{C_{\Omega_2}^2}{1-C_{\Omega_2}^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \frac{1}{1+\left[\left(\frac{1-U_{\Omega_1}}{U_{\Omega_1}}\right)^{\xi}+\left(\frac{1-U_{\Omega_2}}{U_{\Omega_2}}\right)^{\xi}\right]^{1/\xi}}, \frac{1}{1+\left[\left(\frac{1-F_{\Omega_1}}{F_{\Omega_1}}\right)^{\xi}+\left(\frac{1-F_{\Omega_2}}{F_{\Omega_2}}\right)^{\xi}\right]^{1/\xi}} \end{array} \right\rangle$$

2.

$$\Omega_1 \otimes \Omega_2 = \left\langle \begin{array}{l} \dfrac{1}{1+\left[\left(\dfrac{1-T_{\Omega_1}}{T_{\Omega_1}}\right)^{\xi}+\left(\dfrac{1-T_{\Omega_2}}{T_{\Omega_2}}\right)^{\xi}\right]^{1/\xi}}, \dfrac{1}{1+\left[\left(\dfrac{1-C_{\Omega_1}}{C_{\Omega_1}}\right)^{\xi}+\left(\dfrac{1-C_{\Omega_2}}{C_{\Omega_2}}\right)^{\xi}\right]^{1/\xi}}, \\ \sqrt{1-\dfrac{1}{1+\left[\left(\dfrac{U_{\Omega_1}^2}{1-U_{\Omega_1}^2}\right)^{\xi}+\left(\dfrac{U_{\Omega_2}^2}{1-U_{\Omega_2}^2}\right)^{\xi}\right]^{1/\xi}}}, \sqrt{1-\dfrac{1}{1+\left[\left(\dfrac{F_{\Omega_1}^2}{1-F_{\Omega_1}^2}\right)^{\xi}+\left(\dfrac{F_{\Omega_2}^2}{1-F_{\Omega_2}^2}\right)^{\xi}\right]^{1/\xi}}} \end{array} \right\rangle$$

3. $$\kappa\Omega = \left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left[\kappa\left(\dfrac{T_{\Omega}^2}{1-T_{\Omega}^2}\right)^{\xi}\right]^{1/\xi}}}, \sqrt{1-\dfrac{1}{1+\left[\kappa\left(\dfrac{C_{\Omega}^2}{1-C_{\Omega}^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \dfrac{1}{1+\left[\kappa\left(\dfrac{1-U_{\Omega}}{U_{\Omega}}\right)^{\xi}\right]^{1/\xi}}, \dfrac{1}{1+\left[\kappa\left(\dfrac{1-F_{\Omega}}{F_{\Omega}}\right)^{\xi}\right]^{1/\xi}} \end{array} \right\rangle$$

4. $$\Omega^{\kappa} = \left\langle \begin{array}{l} \dfrac{1}{1+\left[\kappa\left(\dfrac{1-T_{\Omega}}{T_{\Omega}}\right)^{\xi}\right]^{1/\xi}}, \dfrac{1}{1+\left[\kappa\left(\dfrac{1-C_{\Omega}}{C_{\Omega}}\right)^{\xi}\right]^{1/\xi}}, \\ \sqrt{1-\dfrac{1}{1+\left[\kappa\left(\dfrac{U_{\Omega}^2}{1-U_{\Omega}^2}\right)^{\xi}\right]^{1/\xi}}}, \sqrt{1-\dfrac{1}{1+\left[\kappa\left(\dfrac{F_{\Omega}^2}{1-F_{\Omega}^2}\right)^{\xi}\right]^{1/\xi}}}, \end{array} \right\rangle$$

Example 3.2.1 Let $\Omega_1 = \langle 0.6, 0.4, 0.6, 0.3 \rangle$ and $\Omega_2 = \langle 0.3, 0.6, 0.4, 0.5 \rangle$ be two QSVNPNs over Γ with $\kappa = 0.5$ and $\xi = 1.4$. Then we compute the above Dombi's t-norms and t-co-norms operators based on QSVNPNs as follows:

$$\Omega_1 \oplus \Omega_2 = \langle 0.611, 0.627, 0.372, 0.261 \rangle$$

$$\Omega_1 \otimes \Omega_2 = \langle 0.276, 0.353, 0.627, 0.522 \rangle$$

$$0.5\,\Omega_1 = \langle 0.505, 0.322, 0.711, 0.412 \rangle$$

$$\Omega_1^{0.5} = \langle 0.711, 0.522, 0.505, 0.238 \rangle$$

4. Dombi Weighted Aggregation Operators under QSVNPNs environment

In this section, based on Dombi operational laws on QSVNPNs, two Dombi ordered weighted aggregation operators, namely, quadripartitioned single-valued neutrosophic Pythagorean Dombi ordered weighted arithmetic aggregation(QSVNPDOWAA) operator and quadripartitioned single-valued neutrosophic Pythagorean Dombi ordered weighted geometric aggregation(QSVNPDOWGA) operator are formed. After that, some significant results based on these two operators are investigated.

Definition 4.1 Let $\Omega_m = \left\langle T_{\Omega_m}, C_{\Omega_m}, U_{\Omega_m}, F_{\Omega_m} \right\rangle$ be a collection of QSVNPNs with the weight vector $W = (\varpi_1, \varpi_2, ..., \varpi_m)^t$ where $l = 1, 2, ..., \mathrm{m}$ and $\varpi_l \geq 0$, $\sum_{l=1}^{m} \varpi_l = 1$ .Then the operator QSVNPDOWAA: $\Omega^m \to \Omega$ is defined as

$$QSVNPDOWAA(\Omega_1, \Omega_2, ..., \Omega_m) = \mathop{\oplus}_{l=1}^{m} \varpi_l \, \Omega_{\sigma(l)},$$

where $\left(\Omega_{\sigma(1)}, \Omega_{\sigma(2)}, ..., \Omega_{\sigma(m)}\right)$ is the permutation of $(\Omega_1, \Omega_2, ..., \Omega_m)$ such that $\Omega_{\sigma(l-1)} \geq \Omega_{\sigma(l)}$ for all $l = 1, 2, ..., m$.

Theorem 4.2 If $\Omega_m = \left\langle T_{\Omega_m}, C_{\Omega_m}, U_{\Omega_m}, F_{\Omega_m} \right\rangle$ be a collection of QSVNPNs, then the resulting of these numbers by using QSVNPDOWAA operator defined above is again a QSVNPN and the resulting number can be obtained by using the following formula:

$$QSVNPDOWAA(\Omega_1, \Omega_2, ..., \Omega_m) = \mathop{\oplus}_{l=1}^{m} \varpi_l \, \Omega_{\sigma(l)} =$$

$$\left\langle \begin{array}{l} \sqrt{1 - \dfrac{1}{1 + \left[\sum_{l=1}^{m} \varpi_l \left(\dfrac{T_{\sigma(l)}^2}{1 - T_{\sigma(l)}^2} \right)^{\xi} \right]^{1/\xi}}}, \sqrt{1 - \dfrac{1}{1 + \left[\sum_{l=1}^{m} \varpi_l \left(\dfrac{C_{\sigma(l)}^2}{1 - C_{\sigma(l)}^2} \right)^{\xi} \right]^{1/\xi}}}, \\ \dfrac{1}{1 + \left[\sum_{l=1}^{m} \varpi_l \left(\dfrac{1 - U_{\sigma(l)}}{U_{\sigma(l)}} \right)^{\xi} \right]^{1/\xi}}, \dfrac{1}{1 + \left[\sum_{l=1}^{m} \varpi_l \left(\dfrac{1 - F_{\sigma(l)}}{F_{\sigma(l)}} \right)^{\xi} \right]^{1/\xi}} \end{array} \right\rangle \quad ....(1)$$

Proof: By mathematical induction, we can prove the theorem given as.

Based on the Dombi operational laws of QSVNPNs for m=2, we have

$$QSVNPDOWAA(\Omega_1,\Omega_2)=\varpi_1\Omega_1\oplus\varpi_2\Omega_2=\left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left[\varpi_1\left(\dfrac{T_1^2}{1-T_1^2}\right)^{\xi}+\varpi_2\left(\dfrac{T_2^2}{1-T_2^2}\right)^{\xi}\right]^{1/\xi}}},\ \sqrt{1-\dfrac{1}{1+\left[\varpi_1\left(\dfrac{C_1^2}{1-C_1^2}\right)^{\xi}+\varpi_2\left(\dfrac{C_2^2}{1-C_2^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \dfrac{1}{1+\left[\varpi_1\left(\dfrac{1-U_1}{U_1}\right)^{\xi}+\varpi_2\left(\dfrac{1-U_2}{U_2}\right)^{\xi}\right]^{1/\xi}},\dfrac{1}{1+\left[\varpi_1\left(\dfrac{1-F_1}{F_1}\right)^{\xi}+\varpi_2\left(\dfrac{1-F_2}{F_2}\right)^{\xi}\right]^{1/\xi}} \end{array}\right\rangle$$

$$=\left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{2}\varpi_l\left(\dfrac{T_l^2}{1-T_l^2}\right)^{\xi}\right]^{1/\xi}}},\ \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{2}\varpi_l\left(\dfrac{C_l^2}{1-C_l^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \dfrac{1}{1+\left[\sum_{l=1}^{2}\varpi_l\left(\dfrac{1-U_l}{U_l}\right)^{\xi}\right]^{1/\xi}},\dfrac{1}{1+\left[\sum_{l=1}^{2}\varpi_l\left(\dfrac{1-F_l}{F_l}\right)^{\xi}\right]^{1/\xi}} \end{array}\right\rangle$$

Suppose the equation (1) holds for m=p, where $p\in N$. Then, we have

$$QSVNPDOWAA(\Omega_1,\Omega_2,.....,\Omega_p)=\left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{T_l^2}{1-T_l^2}\right)^{\xi}\right]^{1/\xi}}},\ \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{C_l^2}{1-C_l^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{1-U_l}{U_l}\right)^{\xi}\right]^{1/\xi}},\dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{1-F_l}{F_l}\right)^{\xi}\right]^{1/\xi}} \end{array}\right\rangle$$

Now, we shall have to show that the equation (1) is true for m=p+1 whenever it is already true for m=p

$$QSVNPDOWAA(\Omega_1,\Omega_2,.....,\Omega_p,\Omega_{p+1})$$

$$=\left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{T_l^2}{1-T_l^2}\right)^{\xi}\right]^{1/\xi}}},\ \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{C_l^2}{1-C_l^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{1-U_l}{U_l}\right)^{\xi}\right]^{1/\xi}},\dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{1-F_l}{F_l}\right)^{\xi}\right]^{1/\xi}} \end{array}\right\rangle\oplus\varpi_{p+1}\Omega_{P+1}$$

$$= \left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{T_l^2}{1-T_l^2}\right)^{\xi}\right]^{1/\xi}}}, \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{C_l^2}{1-C_l^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{1-U_l}{U_l}\right)^{\xi}\right]^{1/\xi}}, \dfrac{1}{1+\left[\sum_{l=1}^{p}\varpi_l\left(\dfrac{1-F_l}{F_l}\right)^{\xi}\right]^{1/\xi}} \end{array} \right\rangle \oplus$$

$$\left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left[\varpi_{p+1}\left(\dfrac{T_{p+1}^2}{1-T_{p+1}^2}\right)^{\xi}\right]^{1/\xi}}}, \sqrt{1-\dfrac{1}{1+\left[\varpi_{p+1}\left(\dfrac{C_{p+1}^2}{1-C_{p+1}^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \dfrac{1}{1+\left[\varpi_{p+1}\left(\dfrac{1-U_{p+1}}{U_{p+1}}\right)^{\xi}\right]^{1/\xi}}, \dfrac{1}{1+\left[\varpi_{p+1}\left(\dfrac{1-F_{p+1}}{F_{p+1}}\right)^{\xi}\right]^{1/\xi}} \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{p+1}\varpi_l\left(\dfrac{T_l^2}{1-T_l^2}\right)^{\xi}\right]^{1/\xi}}}, \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{p+1}\varpi_l\left(\dfrac{C_l^2}{1-C_l^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \dfrac{1}{1+\left[\sum_{l=1}^{p+1}\varpi_l\left(\dfrac{1-U_l}{U_l}\right)^{\xi}\right]^{1/\xi}}, \dfrac{1}{1+\left[\sum_{l=1}^{p+1}\varpi_l\left(\dfrac{1-F_l}{F_l}\right)^{\xi}\right]^{1/\xi}} \end{array} \right\rangle$$

Thus, by the principle of mathematical induction, equation (1) holds for any natural number.

Example 4.2.1 Four farmers namely F_1, F_2, F_3 and F_4 want to check the expected fertility of a field for cultivation. The level of fertility of the field can be determined by using QSVNPNs under considering certain criteria by the decision-maker. According to the four farmers, the level of fertility of the soil is specified under the QSVNPN environment is given by $\Omega_1 = \langle 0.4, 0.3, 0.5, 0.3 \rangle$, $\Omega_2 = \langle 0.6, 0.4, 0.2, 0.3 \rangle$, $\Omega_3 = \langle 0.2, 0.3, 0.4, 0.6 \rangle$ and $\Omega_4 = \langle 0.3, 0.4, 0.6, 0.4 \rangle$ respectively and the corresponding weight vector of the farmers is given by $W = \langle 0.4, 0.25, 0.15, 0.2 \rangle$.

First, we determine the score of each $\Omega_l \left(l = 1,2,3,4 \right)$ given by,

$$\Theta(\Omega_1) = 0.4^2 + 0.3^2 - 0.5^2 - 0.3^2 = -0.09$$

Similarly, $\Theta(\Omega_2) = 0.39$, $\Theta(\Omega_3) = -0.39$ and $\Theta(\Omega_4) = -0.27$

Thus, $\Theta(\Omega_2) > \Theta(\Omega_1) > \Theta(\Omega_4) > \Theta(\Omega_3)$

Therefore, $\Omega_{\sigma(1)} = \Omega_2, \Omega_{\sigma(2)} = \Omega_1, \Omega_{\sigma(3)} = \Omega_4, \Omega_{\sigma(4)} = \Omega_3$

Thus, by using the QSVNPDOWAA operator with $\xi = 2$ we have,

$$QSVNPDOWAA(\Omega_1, \Omega_2, \Omega_3, \Omega_4)$$

$$= \left\langle \begin{array}{l} \sqrt{1 - \dfrac{1}{1 + \left[0.25\left(\dfrac{0.6^2}{1-0.6^2}\right)^2 + 0.4\left(\dfrac{0.4^2}{1-0.4^2}\right)^2 + 0.2\left(\dfrac{0.3^2}{1-0.3^2}\right)^2 + 0.15\left(\dfrac{0.2^2}{1-0.2^2}\right)^2\right]^{1/2}}}, \\ \sqrt{1 - \dfrac{1}{1 + \left[0.25\left(\dfrac{0.4^2}{1-0.4^2}\right)^2 + 0.4\left(\dfrac{0.3^2}{1-0.3^2}\right)^2 + 0.2\left(\dfrac{0.4^2}{1-0.4^2}\right)^2 + 0.15\left(\dfrac{0.3^2}{1-0.3^2}\right)^2\right]^{1/2}}}, \\ \sqrt{\begin{array}{l} \dfrac{1}{1 + \left[0.25\left(\dfrac{1-0.2}{0.2}\right)^2 + 0.4\left(\dfrac{1-0.5}{0.5}\right)^2 + 0.2\left(\dfrac{1-0.6}{0.6}\right)^2 + 0.15\left(\dfrac{1-0.4}{0.4}\right)^2\right]^{1/2}} \\ , \\ \dfrac{1}{1 + \left[0.25\left(\dfrac{1-0.3}{0.3}\right)^2 + 0.4\left(\dfrac{1-0.3}{0.3}\right)^2 + 0.2\left(\dfrac{1-0.4}{0.4}\right)^2 + 0.15\left(\dfrac{1-0.6}{0.6}\right)^2\right]^{1/2}} \end{array}} \end{array} \right\rangle$$

$= \langle 0.486, 0.358, 0.312, 0.331 \rangle$

Therefore, $QSVNPDOWAA(\Omega_1, \Omega_2, \Omega_3, \Omega_4) = \langle 0.486, 0.358, 0.312, 0.331 \rangle$

So, the aggregate of four QSVNPNs under Dombi operation is again a QSVNPN. We can generalize it for any finite numbers.

Theorem 4.3 (Idempotency) For any QSVNPN $\Omega = \langle T_\Omega, C_\Omega, U_\Omega, F_\Omega \rangle$ we have

$$QSVNPDOWAA(\Omega, \Omega, \Omega, ..., \Omega) = \Omega.$$

Proof: Assuming $\Omega_l = \Omega \ (l = 1, 2, ..., m)$ and then applying equation (1), we have

$$QSVNPDOWAA\left(\Omega_1,\Omega_2,...,\Omega_m\right)=\overset{m}{\underset{l=1}{\oplus}}\varpi_l\ \Omega_l=$$

$$\left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\dfrac{T_l^2}{1-T_l^2}\right)^{\xi}\right]^{1/\xi}}},\sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\dfrac{C_l^2}{1-C_l^2}\right)^{\xi}\right]^{1/\xi}}}, \\ \dfrac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\dfrac{1-U_l}{U_l}\right)^{\xi}\right]^{1/\xi}},\dfrac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\dfrac{1-F_l}{F_l}\right)^{\xi}\right]^{1/\xi}} \end{array}\right\rangle$$

$$=\left\langle \begin{array}{l} \sqrt{1-\dfrac{1}{1+\left(\dfrac{T_l^2}{1-T_l^2}\right)}},\sqrt{1-\dfrac{1}{1+\left(\dfrac{C_l^2}{1-C_l^2}\right)}}, \\ \dfrac{1}{1+\left[\left(\dfrac{1-U_l}{U_l}\right)\right]},\dfrac{1}{1+\left(\dfrac{1-F_l}{F_l}\right)} \end{array}\right\rangle=\Omega$$

Hence proved.

Theorem 4.4 (Boundedness) Consider the collection of QSVNPNs $\{\Omega_1,\Omega_2,.....,\Omega_m\}$, where $\Omega_l=\left\langle T_{\Omega_l},C_{\Omega_l},U_{\Omega_l},F_{\Omega_l}\right\rangle, l=1,2,...,m$ in such a manner that $\Omega_{\min}=\min\{\Omega_1,\Omega_2,.....,\Omega_m\}$, and $\Omega_{\max}=\max\{\Omega_1,\Omega_2,.....,\Omega_m\}$. Then, $\Omega_{\min}\le QSVNPDOWAA\left(\Omega_1,\Omega_2,.....,\Omega_m\right)\le\Omega_{\max}$.

Proof: Suppose that

$\Omega_{\min}=\min\{\Omega_1,\Omega_2,.....,\Omega_m\}=\langle T_*,C_*,U_*,F_*\rangle$, and

$\Omega_{\max}=\max\{\Omega_1,\Omega_2,.....,\Omega_m\}=\left\langle T^*,C^*,U^*,F^*\right\rangle$.

Then, $T_*=\min\{T_l\}$, $C_*=\min\{C_l\}$, $U_*=\max\{U_l\}$, $F_*=\max\{F_l\}$ and $T^*=\max\{T_l\}$, $C^*=\max\{C_l\}, U^*=\min\{U_l\}, F^*=\min\{F_l\}$

Therefore, we have the following inequalities for the membership, contradictory, ignorance, and falsity membership respectively

$$\sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\dfrac{T_*^2}{1-T_*^2}\right)^{\xi}\right]^{1/\xi}}}\le\sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\dfrac{T_l^2}{1-T_l^2}\right)^{\xi}\right]^{1/\xi}}}\le\sqrt{1-\dfrac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\dfrac{T^{*2}}{1-T^{*2}}\right)^{\xi}\right]^{1/\xi}}}$$

$$\sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{C_*^2}{1-C_*^2}\right)^{\xi}\right]^{1/\xi}}}\leq\sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{C_l^2}{1-C_l^2}\right)^{\xi}\right]^{1/\xi}}}\leq\sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{C^{*2}}{1-C^{*2}}\right)^{\xi}\right]^{1/\xi}}}$$

$$\sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{U^{*2}}{1-U^{*2}}\right)^{\xi}\right]^{1/\xi}}}\leq\sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{U_l^2}{1-U_l^2}\right)^{\xi}\right]^{1/\xi}}}\leq\sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{U_*^2}{1-U_*^2}\right)^{\xi}\right]^{1/\xi}}}$$

$$\sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{C_*^2}{1-C_*^2}\right)^{\xi}\right]^{1/\xi}}}\leq\sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{C_l^2}{1-C_l^2}\right)^{\xi}\right]^{1/\xi}}}\leq\sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{C^{*2}}{1-C^{*2}}\right)^{\xi}\right]^{1/\xi}}}$$

This completes the proof.

Theorem 4.5 (Monotonicity) suppose the two collections of QSVNPNs are $\left\{\Omega'_1,\Omega'_2,.....,\Omega'_m\right\}$ and $\left\{\Omega_1,\Omega_2,.....,\Omega_m\right\}$ where $\Omega'_l=\left\langle T_{\Omega'_l},C_{\Omega'_l},U_{\Omega'_l},F_{\Omega'_l}\right\rangle,\Omega_l=\left\langle T_{\Omega_l},C_{\Omega_l},U_{\Omega_l},F_{\Omega_l}\right\rangle,l=1,2,...,m$

In such a manner that $T_{\Omega'_l}\leq T_{\Omega_l}$, $C_{\Omega'_l}\leq C_{\Omega_l}$, and $U_{\Omega'_l}\geq U_{\Omega_l}$, $F_{\Omega'_l}\geq F_{\Omega_l}$. Then

$$QSVNPDOWAA\left(\Omega'_1,\Omega'_2,.....,\Omega'_m\right)\leq QSVNPDOWAA\left(\Omega_1,\Omega_2,.....,\Omega_m\right).$$

Proof: Suppose $QSVNPDOWAA\left(\Omega'_1,\Omega'_2,.....,\Omega'_m\right)=\left\langle T',C',U',F'\right\rangle$ and

$QSVNPDOWAA\left(\Omega_1,\Omega_2,.....,\Omega_m\right)=\left\langle T,C,U,F\right\rangle$. At first, we shall show that $T'\leq T$.

Since, $T_{\Omega'_l}\leq T_{\Omega_l}\Rightarrow\frac{T^2_{\Omega'_l}}{1-T^2_{\Omega'_l}}\leq\frac{T^2_{\Omega_l}}{1-T^2_{\Omega_l}}$. Using this result we can write,

$$\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{'2}}{1-T_l^{'2}}\right)^{\xi}\right]^{1/\xi}\le\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{2}}{1-T_l^{2}}\right)^{\xi}\right]^{1/\xi}$$

$$or\ 1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{'2}}{1-T_l^{'2}}\right)^{\xi}\right]^{1/\xi}\le 1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{2}}{1-T_l^{2}}\right)^{\xi}\right]^{1/\xi}$$

$$or\ \frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{'2}}{1-T_l^{'2}}\right)^{\xi}\right]^{1/\xi}}\ge\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{2}}{1-T_l^{2}}\right)^{\xi}\right]^{1/\xi}}$$

$$or\ 1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{'2}}{1-T_l^{'2}}\right)^{\xi}\right]^{1/\xi}}\le\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{2}}{1-T_l^{2}}\right)^{\xi}\right]^{1/\xi}}$$

$$or\ \sqrt{1-\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{'2}}{1-T_l^{'2}}\right)^{\xi}\right]^{1/\xi}}}\le\sqrt{\frac{1}{1+\left[\sum_{l=1}^{m}\varpi_l\left(\frac{T_l^{2}}{1-T_l^{2}}\right)^{\xi}\right]^{1/\xi}}}$$

Hence, $T^{'}\le T$. Similarly, we can show that $C^{'}\le C$, $U^{'}\ge U$ and $I^{'}\ge I$.

Theorem 4.6 (Reducibility) suppose $\{\Omega_1,\Omega_2,.....,\Omega_m\}$ be a collection of QSVNPNs in such a manner that $\Omega_l=\left\langle T_{\Omega_l},C_{\Omega_l},U_{\Omega_l},F_{\Omega_l}\right\rangle, l=1,2,...,m$ with the corresponding weight vector $W=(\varpi_1,\varpi_2,....,\varpi_m)^t=\left(\frac{1}{m},\frac{1}{m},......,\frac{1}{m}\right)^t$. Then we can write

$$QSVNPDOWAA(\Omega_1,\Omega_2,.....,\Omega_m)=$$

$$\left\langle\begin{array}{l}\sqrt{1-\frac{1}{1+\left[\frac{1}{m}\sum_{l=1}^{m}\left(\frac{T_l^{2}}{1-T_l^{2}}\right)^{\xi}\right]^{1/\xi}}},\sqrt{1-\frac{1}{1+\left[\frac{1}{m}\sum_{l=1}^{m}\left(\frac{C_l^{2}}{1-C_l^{2}}\right)^{\xi}\right]^{1/\xi}}},\\ \frac{1}{1+\left[\frac{1}{m}\sum_{l=1}^{m}\left(\frac{1-U_l}{U_l}\right)^{\xi}\right]^{1/\xi}},\frac{1}{1+\left[\frac{1}{m}\sum_{l=1}^{m}\left(\frac{1-F_l}{F_l}\right)^{\xi}\right]^{1/\xi}}\end{array}\right\rangle$$

Definition 4.7 Let $\Omega_m = \left\langle T_{\Omega_m}, C_{\Omega_m}, U_{\Omega_m}, F_{\Omega_m} \right\rangle$ be a collection of QSVNPNs with the weight vector $W = (\varpi_1, \varpi_2, \ldots\ldots, \varpi_m)^t$ where $l = 1, 2, \ldots, \mathrm{m}$ and $\varpi_m \geq 0$, $\sum_{l=1}^{m} \varpi_l = 1$ .Then the operator QSVNPDOWGA: $\Omega^m \to \Omega$ is defined as $QSVNPDOWGA(\Omega_1, \Omega_2, \ldots, \Omega_m) = \overset{m}{\underset{l=1}{\otimes}} \varpi_l \; \Omega_{\sigma(l)}$ Where $\left(\Omega_{\sigma(1)}, \Omega_{\sigma(2)}, \ldots\ldots, \Omega_{\sigma(m)}\right)$ is the permutation of $(\Omega_1, \Omega_2, \ldots, \Omega_m)$ such that $\Omega_{\sigma(l-1)} \geq \Omega_{\sigma(l)}$ for all $l = 1, 2, \ldots, m$.

Theorem 4.8 If $\Omega_m = \left\langle T_{\Omega_m}, C_{\Omega_m}, U_{\Omega_m}, F_{\Omega_m} \right\rangle$ be a collection of QSVNPNs, then the resulting of these numbers by using QSVNPDOWGA operator defined above is again a QSVNPN and the aggregate number can be obtained by using the following formula:

$$QSVNPDOWGA(\Omega_1, \Omega_2, \ldots, \Omega_m) = \overset{m}{\underset{l=1}{\otimes}} \varpi_l \; \Omega_{\sigma(l)} =$$

$$\left\langle \begin{array}{c} \dfrac{1}{1 + \left[\sum_{l=1}^{m} \varpi_l \left(\dfrac{1 - T_{\sigma(l)}}{T_{\sigma(l)}} \right)^{\xi} \right]^{1/\xi}}, \dfrac{1}{1 + \left[\sum_{l=1}^{m} \varpi_l \left(\dfrac{1 - C_{\sigma(l)}}{C_{\sigma(l)}} \right)^{\xi} \right]^{1/\xi}}, \\ \sqrt{1 - \dfrac{1}{1 + \left[\sum_{l=1}^{m} \varpi_l \left(\dfrac{U_{\sigma(l)}^2}{1 - U_{\sigma(l)}^2} \right)^{\xi} \right]^{1/\xi}}}, \sqrt{1 - \dfrac{1}{1 + \left[\sum_{l=1}^{m} \varpi_l \left(\dfrac{F_{\sigma(l)}^2}{1 - F_{\sigma(l)}^2} \right)^{\xi} \right]^{1/\xi}}} \end{array} \right\rangle$$

Proof: the proof is similar to the proof of Theorem **4.2**

Example 4.8.1 Revisiting example **4.2.1,** we can obtain the QSVNPDOWGA operator as follows

$$QSVNPDOWGA(\Omega_1, \Omega_2, \Omega_3, \Omega_4) = \overset{4}{\underset{l=1}{\otimes}} \varpi_l \; \Omega_{\sigma(l)} =$$

$$\left\langle \begin{array}{l} \dfrac{1}{1+\left[0.25\left(\dfrac{1-0.6}{0.6}\right)^2+0.4\left(\dfrac{1-0.4}{0.4}\right)^2+0.2\left(\dfrac{1-0.3}{0.3}\right)^2+0.15\left(\dfrac{1-0.2}{0.2}\right)^2\right]^{1/2}}, \\ \dfrac{1}{1+\left[0.25\left(\dfrac{1-0.4}{0.4}\right)^2+0.4\left(\dfrac{1-0.3}{0.3}\right)^2+0.2\left(\dfrac{1-0.4}{0.4}\right)^2+0.15\left(\dfrac{1-0.3}{0.3}\right)^2\right]^{1/2}}, \\ \sqrt{1-\dfrac{1}{1+\left[0.25\left(\dfrac{0.2^2}{1-0.2^2}\right)^2+0.4\left(\dfrac{0.5^2}{1-0.5^2}\right)^2+0.2\left(\dfrac{0.6^2}{1-0.6^2}\right)^2+0.15\left(\dfrac{0.4^2}{1-0.4^2}\right)^2\right]^{1/2}}}, \\ \sqrt{1-\dfrac{1}{1+\left[0.25\left(\dfrac{0.3^2}{1-0.3^2}\right)^2+0.4\left(\dfrac{0.3^2}{1-0.3^2}\right)^2+0.2\left(\dfrac{0.4^2}{1-0.4^2}\right)^2+0.15\left(\dfrac{0.6^2}{1-0.6^2}\right)^2\right]^{1/2}}} \end{array} \right\rangle$$

$$=\langle 0.320, 0.333, 0.502, 0.445\rangle$$

The aggregate is of four QSVNPNs under the QSVNPDOWGA operator is again a QSVNPN.

Theorem 4.9 (Idempotency) for any QSVNPN $\Omega=\langle T_\Omega, C_\Omega, U_\Omega, F_\Omega\rangle$ we have

$$QSVNPDOWGA(\Omega,\Omega,\Omega,.........,\Omega)=\Omega.$$

Theorem 4.10 (Boundedness) Consider the collection of QSVNPNs $\{\Omega_1,\Omega_2,.....,\Omega_m\}$, where $\Omega_l=\langle T_{\Omega_l}, C_{\Omega_l}, U_{\Omega_l}, F_{\Omega_l}\rangle, l=1,2,...,m$ in such a manner that $\Omega_{\min}=\min\{\Omega_1,\Omega_2,.....,\Omega_m\}$, and $\Omega_{\max}=\max\{\Omega_1,\Omega_2,.....,\Omega_m\}$.

Then, $\Omega_{\min}\le QSVNPDOWGA(\Omega_1,\Omega_2,.....,\Omega_m)\le\Omega_{\max}$.

Theorem 4.11 (Monotonicity) suppose the two collection of QSVNPNs are $\{\Omega'_1,\Omega'_2,.....,\Omega'_m\}$ and $\{\Omega_1,\Omega_2,.....,\Omega_m\}$ where $\Omega'_l=\langle T_{\Omega'_l}, C_{\Omega'_l}, U_{\Omega'_l}, F_{\Omega'_l}\rangle, \Omega_l=\langle T_{\Omega_l}, C_{\Omega_l}, U_{\Omega_l}, F_{\Omega_l}\rangle, l=1,2,...,m$

In such a manner that $T_{\Omega'_l}\le T_{\Omega_l}, C_{\Omega'_l}\le C_{\Omega_l}$ and $U_{\Omega'_l}\ge U_{\Omega_l}, F_{\Omega'_l}\ge F_{\Omega_l}$. Then

$$QSVNPDOWGA(\Omega'_1,\Omega'_2,.....,\Omega'_m)\le QSVNPDOWGA(\Omega_1,\Omega_2,.....,\Omega_m).$$

Theorem 4.12 (Reducibility) suppose $\{\Omega_1,\Omega_2,.....,\Omega_m\}$ be a collection of QSVNPNs in such a manner that $\Omega_l=\langle T_{\Omega_l},C_{\Omega_l},U_{\Omega_l},F_{\Omega_l}\rangle, l=1,2,...,m$ with the corresponding weight vector $W=(\varpi_1,\varpi_2,....,\varpi_m)^t=\left(\frac{1}{m},\frac{1}{m},......,\frac{1}{m}\right)^t$. Then we can write

$$QSVNPDOWGA(\Omega_1,\Omega_2,.....,\Omega_m)=\left\langle \frac{1}{1+\left[\frac{1}{m}\sum_{l=1}^{m}\left(\frac{1-T_l}{T_l}\right)^{\xi}\right]^{1/\xi}}, \frac{1}{1+\left[\frac{1}{m}\sum_{l=1}^{m}\left(\frac{1-C_l}{C_l}\right)^{\xi}\right]^{1/\xi}}, \sqrt{1-\frac{1}{1+\left[\frac{1}{m}\sum_{l=1}^{m}\left(\frac{U_l^2}{1-U_l^2}\right)^{\xi}\right]^{1/\xi}}}, \sqrt{1-\frac{1}{1+\left[\frac{1}{m}\sum_{l=1}^{m}\left(\frac{F_l^2}{1-F_l^2}\right)^{\xi}\right]^{1/\xi}}} \right\rangle$$

Theorem 4.13 (Commutativity)

suppose $\{\Omega_1,\Omega_2,.....,\Omega_m\},\Omega_l=\langle T_{\Omega_l},C_{\Omega_l},U_{\Omega_l},F_{\Omega_l}\rangle, l=1,2,...,m$ be a collection of QSVNPNs and $\{\Omega'_1,\Omega'_2,.....,\Omega'_m\},\Omega'_l=\langle T_{\Omega'_l},C_{\Omega'_l},U_{\Omega'_l},F_{\Omega'_l}\rangle, l=1,2,...,m$ be a permutation of $\{\Omega_1,\Omega_2,.....,\Omega_m\}$.

Then $QSVNPDOWGA\,(\Omega'_1,\Omega'_2,.....,\Omega'_m)=QSVNPDOWGA(\Omega_1,\Omega_2,.....,\Omega_m)$.

5. Model for MCDM Using Quadripartitioned Single-Valued Neutrosophic Pythagorean Information

In this section, a model for MCDM by using quadripartitioned single-valued Pythagorean information is proposed. Here the decision-maker gives the information in the form of quadripartitioned single-valued neutrosophic number form.

Let $A=\{a_1,a_2,.....,a_m\}$ denotes the set of attributes or alternatives denoted by a_i for $i=1,2,..,m$ and $C=\{c_1,c_2,.....,c_n\}$ indicates the set of criteria denoted by c_j for $j=1,2,..,\text{n}$.

An expert is engaged to provide his/her evaluation of an alternative a_i on a criterion c_j in the form of QSVNPN. The expert information is recorded in the form of a decision matrix denoted by $D^M=[\Omega_{ik}]_{m\times n}$ where $\Omega_{ij}=\langle T_{ij},C_{ij},U_{ij},F_{ij}\rangle$. Also, $W=(\varpi_1,\varpi_2,....,\varpi_n)$ is the weight vector of the decision-maker where $\sum_{p=1}^{n}\varpi_p = 1$ and $\varpi_p > 0$, . The criteria can be of two types called benefit criteria and cost criteria. If in the decision matrix, there is any cost type criteria then it can be converted into the normalized decision matrix and it is given by:

$$ND^M = [s_{ik}] = \begin{cases} \Omega_{ik} = \langle T_{ik}, C_{ik}, U_{ik}, F_{ik} \rangle, \text{ for benefit criteria} \\ \Omega^c{}_{ik} = \langle F_{ik}, U_{ik}, C_{ik}, T_{ik} \rangle, \text{ for cost criteria} \end{cases}$$

The algorithm for the proposed model is given by:

Algorithm

Input:

Step1: Input the QSVNPNs given by the expert in the form of a decision matrix(DM).

Computations:

Step2: Normalize the decision matrix if it is required.

Step3: Calculate the collective information by using the proposed Dombi operators to evaluate the alternative preference values with associated weights.

Step4: Find the score $\Theta(A_p)$ and accuracy $\Lambda(A_p)$ values of the cumulative preference values.

Output:

Step5: Rank the alternatives and choose the best which has a maximum score value.

We utilize this algorithm in the following practical application.

6. An Application

Suppose Mr. X wants to buy a smartphone and for this, he has six available alternatives denoted by the set $M = \{M_1, M_2, M_3, M_4, M_5, M_6\}$. He wants to select the best alternative based on certain criteria denoted by the set

$C = \{C_1, C_2, C_3, C_4, C_5\}$, where C_1 =price, C_2 =battery capacity, C_3 =storage space, C_4 =camera quality, and C_5 =looks.

But there are certain issues with selecting the best alternative. All criteria have different units as price represents in a dollar, storage space in GB, battery capacity in MHA, camera quality in MP. So, we can't compare criteria with different units. Another issue is the use of linguistic terms. For example, we do not express the looks or appearance of mobile in units as it depends on the choice of the customers. Moreover, criteria are of two types, namely, non-beneficial and beneficial. Non-beneficial are those criteria whose lower value is desirable. For example, price or cost, we desire a product having a lower cost. On the other hand, beneficial criteria are those whose higher value is desirable. For example, we always desire a mobile with higher storage, having a high megapixel camera with excellent looks, and have high-quality battery. Because of this Mr. X engages a decision-maker or an expert. Firstly, the decision-maker assigns the weight vector associated with each criterion is given by $W=(0.1,0.2,0.3,0.3,0.1)$. The performance of all the alternatives based on the given criteria is determined by the decision-maker in the form of a decision matrix using the QSVNP information is given by:

Step1:

$$D^M=\begin{array}{c} \\ M_1\\ M_2\\ M_3\\ M_4\\ M_5\\ M_6 \end{array}\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{pmatrix} \langle 0.3,0.4,0.5,0.2\rangle & \langle 0.3,0.1,0.4,0.4\rangle & \langle 0.3,0.5,0.3,0.1\rangle & \langle 0.2,0.5,0.4,0.3\rangle & \langle 0.3,0.5,0.1,0.6\rangle \\ \langle 0.6,0.7,0.3,0.2\rangle & \langle 0.5,0.3,0.2,0.4\rangle & \langle 0.3,0.4,0.5,0.3\rangle & \langle 0.5,0.7,0.2,0.3\rangle & \langle 0.3,0.2,0.6,0.7\rangle \\ \langle 0.4,0.3,0.5,0.4\rangle & \langle 0.5,0.3,0.4,0.4\rangle & \langle 0.2,0.4,0.3,0.4\rangle & \langle 0.3,0.5,0.3,0.4\rangle & \langle 0.2,0.1,0.4,0.6\rangle \\ \langle 0.3,0.6,0.4,0.3\rangle & \langle 0.2,0.1,0.3,0.4\rangle & \langle 0.6,0.3,0.4,0.3\rangle & \langle 0.1,0.2,0.2,0.4\rangle & \langle 0.5,0.6,0.4,0.3\rangle \\ \langle 0.5,0.5,0.4,0.3\rangle & \langle 0.4,0.3,0.2,0.2\rangle & \langle 0.3,0.3,0.5,0.3\rangle & \langle 0.3,0.5,0.5,0.3\rangle & \langle 0.3,0.2,0.4,0.6\rangle \\ \langle 0.4,0.6,0.2,0.4\rangle & \langle 0.1,0.4,0.4,0.6\rangle & \langle 0.2,0.4,0.3,0.6\rangle & \langle 0.7,0.3,0.5,0.1\rangle & \langle 0.5,0.3,0.1,0.3\rangle \end{pmatrix} \end{array}$$

As the criteria cost is a non-beneficiary criterion, therefore in the second step, we normalize the decision matrix:

Step2:

$$ND^M=\begin{array}{c} \\ M_1\\ M_2\\ M_3\\ M_4\\ M_5\\ M_6 \end{array}\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{pmatrix} \langle 0.2,0.5,0.4,0.3\rangle & \langle 0.3,0.1,0.4,0.4\rangle & \langle 0.3,0.5,0.3,0.1\rangle & \langle 0.2,0.5,0.4,0.3\rangle & \langle 0.3,0.5,0.1,0.6\rangle \\ \langle 0.2,0.3,0.7,0.6\rangle & \langle 0.5,0.3,0.2,0.4\rangle & \langle 0.3,0.4,0.5,0.3\rangle & \langle 0.5,0.7,0.2,0.3\rangle & \langle 0.3,0.2,0.6,0.7\rangle \\ \langle 0.4,0.5,0.3,0.4\rangle & \langle 0.5,0.3,0.4,0.4\rangle & \langle 0.2,0.4,0.3,0.4\rangle & \langle 0.3,0.5,0.3,0.4\rangle & \langle 0.2,0.1,0.4,0.6\rangle \\ \langle 0.3,0.4,0.6,0.3\rangle & \langle 0.2,0.1,0.3,0.4\rangle & \langle 0.6,0.3,0.4,0.3\rangle & \langle 0.1,0.2,0.2,0.4\rangle & \langle 0.5,0.6,0.4,0.3\rangle \\ \langle 0.3,0.4,0.5,0.5\rangle & \langle 0.4,0.3,0.2,0.2\rangle & \langle 0.3,0.3,0.5,0.3\rangle & \langle 0.3,0.5,0.5,0.3\rangle & \langle 0.3,0.2,0.4,0.6\rangle \\ \langle 0.4,0.2,0.6,0.4\rangle & \langle 0.1,0.4,0.4,0.6\rangle & \langle 0.2,0.4,0.3,0.6\rangle & \langle 0.7,0.3,0.5,0.1\rangle & \langle 0.5,0.3,0.1,0.3\rangle \end{pmatrix} \end{array}$$

Step3: Compute the aggregate preference value of each alternative under all the criteria using the Dombi operator with $\xi=3$ defined in **Theorem 4.2,** given as

$$\begin{array}{c} M_1\\ M_2\\ M_3\\ M_4\\ M_5\\ M_6 \end{array}\begin{pmatrix} \langle 0.279,0.486,0.189,0.141\rangle \\ \langle 0.458,0.626,0.238,0.330\rangle \\ \langle 0.411,0.452,0.317,0.407\rangle \\ \langle 0.527,0.458,0.259,0.332\rangle \\ \langle 0.338,0.433,0.294,0.267\rangle \\ \langle 0.626,0.368,0.189,0.142\rangle \end{pmatrix}$$

Step4: Next we calculate the score of each alternative given by:

$\Theta(M_1)=0.258$, $\Theta(M_2)=0.436$, $\Theta(M_3)=0.107$, $\Theta(M_4)=0.310$, $\Theta(M_5)=0.144$ and $\Theta(M_6)=0.471$

Step5: Rank the scores of all the alternatives, we have

$$\Theta(M_6) > \Theta(M_2) > \Theta(M_4) > \Theta(M_1) > \Theta(M_5) > \Theta(M_3)$$

Since the alternative M_6 is the best alternative as it has the highest rank among all. Therefore, Mr. X will buy M_6 mobile. If it is not available in the market then he will prefer the second-highest rank alternative i.e. M_2.

Aliter:

We repeat up to **step2**.

Step3: Compute the aggregate preference value of each alternative under all the criteria using the Dombi operator with $\xi = 3$ defined in **Theorem 4.8,** given as

$$\begin{array}{c} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{array} \begin{pmatrix} \langle 0.237, 0.159, 0.375, 0.460 \rangle \\ \langle 0.304, 0.308, 0.573, 0.568 \rangle \\ \langle 0.243, 0.190, 0.350, 0.473 \rangle \\ \langle 0.139, 0.153, 0.464, 0.369 \rangle \\ \langle 0.311, 0.296, 0.479, 0.468 \rangle \\ \langle 0.154, 0.297, 0.488, 0.556 \rangle \end{pmatrix}$$

Step4: By using **definition 2.7,** the scores of each alternative are calculated as:

$\Theta(M_1) = -0.270$, $\Theta(M_2) = -0.463$, $\Theta(M_3) = -0.251$, $\Theta(M_4) = -0.308$, $\Theta(M_5) = -0.264$ and $\Theta(M_6) = -0.435$

Step5: Ranking of the alternatives is given by:

$$\Theta(M_3) > \Theta(M_5) > \Theta(M_1) > \Theta(M_4) > \Theta(M_6) > \Theta(M_2)$$

Here, M_3 is the best alternative.

To compare the results of the two methods, we consider the following table:

Table-1

Alternatives	Ranking		Absolute Difference	Modified Rank
	QSVNPDOWAA	QSVNPDOWGA		
M_1	4	3	1	5.5
M_2	2	6	4	2.5
M_3	6	1	5	1

M_4	**3**	**4**	**1**	**5.5**
M_5	**5**	**2**	**3**	**4**
M_6	**1**	**5**	**4**	**2.5**

As the two methods give two different optimal choices, therefore there is a hesitation for a decision-maker to choose the best alternative. To sort out such an issue, we consider the table-1, where we have determined the modified rank against each alternative. As M_3 has the highest modified rank, so M_3 is our preferred choice. In the case of a tie, there is more than one alternative under consideration.

Remark: Instead of taking $\xi = 3$ we can choose any value higher than or equal to 1. For each value of ξ we get the same rank of the alternatives.

7. Conclusion

In the present paper, we have studied the notion of QSVNPNs and their various operational laws based on Dombi operators. We also introduced two new Dombi operators, namely QSVNPDOWAA and QSVNPDOWGA in the quadripartitioned single-valued neutrosophic Pythagorean information. Based on these two operators we have studied different properties such as monotonicity, commutativity, reducibility, boundedness, and idempotency. Moreover, we have proposed a model for MCDM problems under the QSVNPN environment. Finally, for the practical application of the proposed model, a real-life based example is given by which we justify the feasibility and rationality of the model and shows that how it is effective in decision-making problem. In the future, the proposed model can be applied for risk management, disease diagnosis, control theory, MADM, game theory, and many other diverse fields for decision-making.

Acknowledgments: The Author is very much grateful to the anonymous reviewers for their valuable comments that help to improve the quality of the paper.

Conflicts of Interest: Declare no conflict of interest regarding the publication of the paper.

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*, 338–353. doi:10.1016/s0019-9958(65)90241-x

[2] Klir, G., & Yuan, B. (1995). Fuzzy sets and fuzzy logic (Vol. 4). *New Jersey: Prentice hall*.

[3] Dubois, D. J. (1980). Fuzzy sets and systems: theory and applications (Vol. 144). *Academic press*.

[4] Zimmermann, H. J., Jones, A., & Kaufman, A. (Eds.). (1986). Fuzzy sets theory and applications. *D. Reidel*.

[5] Kahraman, C. (2008). Multi-criteria decision making methods and fuzzy sets. *Fuzzy Multi-Criteria Decision Making*, 1–18. doi:10.1007/978-0-387-76813-7_1

[6] Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy sets and systems*, *1*, 45-55.

[7] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20,* 87–96. doi:10.1016/s0165-0114(86)80034-3

[8] Yager, R. R. (2009). Some aspects of intuitionistic fuzzy sets. *Fuzzy Optimization and Decision Making, 8,* 67–90. doi:10.1007/s10700-009-9052-7

[9] De, S. K., Biswas, R., & Roy, A. R. (2001). An application of intuitionistic fuzzy sets in medical diagnosis. *Fuzzy sets and Systems*, *117*, 209-213.

[10] Zhang, H. (2014). Linguistic intuitionistic fuzzy sets and application in MAGDM. *Journal of Applied Mathematics, 2014,* 1–11. doi:10.1155/2014/432092

[11] Liang, Z., & Shi, P. (2003). Similarity measures on intuitionistic fuzzy sets. *Pattern recognition letters*, *24*, 2687-2693.

[12] Ye, J. (2011). Cosine similarity measures for intuitionistic fuzzy sets and their applications. *Mathematical and computer modelling*, *53*, 91-97.

[13] Yager, R. (2013). Pythagorean fuzzy subsets. *2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS),* 57-61.

[14] Ejegwa, P. A. (2019). Pythagorean fuzzy set and its application in career placements based on academic performance using max–min–max composition. *Complex & Intelligent Systems.* doi:10.1007/s40747-019-0091-6

[15] Peng, X., & Selvachandran, G. (2019). Pythagorean fuzzy set: state of the art and future directions. *Artificial Intelligence Review*, *52,* 1873-1927.

[16] Garg, H. (2018). Linguistic Pythagorean fuzzy sets and its applications in multiattribute decision-making process. *International Journal of Intelligent Systems*, *33*, 1234-1263.

[17] Zhang, X., & Xu, Z. (2014). Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets. *International Journal of Intelligent Systems*, *29*, 1061-1078.

[18] Garg, H. (2016). A novel correlation coefficients between Pythagorean fuzzy sets and its applications to decision-making processes. *International Journal of Intelligent Systems*, *31*, 1234-1252.

[19] Smarandache, F. (2005). Neutrosophic set-a generalization of the intuitionistic fuzzy set. *International journal of pure and applied mathematics*, *24*, 287.

[20] Wang, H., Smarandache, F., Zhang, Y., & Sunderraman, R. (2010). *Single valued neutrosophic sets*. Infinite study.

[21] Jansi, R., Mohana, . K., & Smarandache, F. (2019). Correlation measure for Pythagorean neutrosophic sets with T and F as dependent neutrosophic components. *Neutrosophic Sets and Systems*, *30*, 202-212.

[22] Chatterjee, R., Majumdar, P., & Samanta, S. K. (2016). On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets. *Journal of Intelligent & Fuzzy Systems*, *30*, 2475-2485.

[23] Radha, R., & Mary, A. S. A. (2020). Quadri partitioned neutrosophic Pythagorean set. *Internation Journal of Research Publication and Review, 2,* 276-281.

[24] Dombi, J. (1982). A general class of fuzzy operators, the demorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators. *Fuzzy Sets and Systems*, *8,* 149–163. doi:10.1016/0165-0114(82)90005-7

[25] Roychowdhury, S., & Wang, B. H. (1994). Composite generalization of Dombi class and a new family of T-operators using additive-product connective generator. *Fuzzy Sets and Systems, 66,* 329–346. doi:10.1016/0165-0114(94)90100-7

[26] Seikh, M. R., & Mandal, U. (2021). Intuitionistic fuzzy Dombi aggregation operators and their application to multiple attribute decision-making. *Granular Computing*, *6*, 473-488.

[27] Wei, G., & Wei, Y. (2018). Some single-valued neutrosophic Dombi prioritized weighted aggregation operators in multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems*, *35*, 2001-2013.

[28] Jana, C., Pal, M., & Wang, J. (2018). Bipolar fuzzy Dombi aggregation operators and its application in multiple-attribute decision-making process. *Journal of Ambient Intelligence and Humanized Computing.* doi:10.1007/s12652-018-1076-9

[29] Ashraf, S., Abdullah, S., & Mahmood, T. (2019). Spherical fuzzy Dombi aggregation operators and their application in group decision making problems. *Journal of Ambient Intelligence and Humanized Computing.* doi:10.1007/s12652-019-01333-y

[30] Garg, H. (2019). Neutrality operations-based Pythagorean fuzzy aggregation operators and its applications to multiple attribute group decision-making process. *Journal of Ambient Intelligence and Humanized Computing.* doi:10.1007/s12652-019-01448-2

[31] Qiyas, M., Abdullah, S., Ashraf, S., & Abdullah, L. (2019). Linguistic picture fuzzy Dombi aggregation operators and their application in multiple attribute group decision making problem. *Mathematics, 7, 764.* doi:10.3390/math7080764

[32] Akram, M., Dudek, W. A., & Dar, J. M. (2019). Pythagorean Dombi fuzzy aggregation operators with application in multicriteria decision-making. *International Journal of Intelligent Systems.* doi:10.1002/int.22183

[33] Khan, A., Ashraf, S., Abdullah, S., Qiyas, M., Luo, J., & Khan, S. (2019). Pythagorean fuzzy Dombi aggregation operators and their application in decision support system. *Symmetry, 11, 383.* doi:10.3390/sym11030383

[34] Jana, C., Senapati, T., & Pal, M. (2019). Pythagorean fuzzy Dombi aggregation operators and its applications in multiple attribute decision-making. *International Journal of Intelligent Systems, 34,* 2019-2038. doi:10.1002/int.22125

[35] Akram, M., Yaqoob, N., Ali, G., & Chammam, W. (2020). Extensions of Dombi aggregation operators for decision making under m-Polar fuzzy information. *Journal of Mathematics, 2020, 1–20.* doi:10.1155/2020/4739567

[36] Mahmood, M. K., Zeng, S., Gulfam, M., Ali, S., & Jin, Y. (2020). Bipolar neutrosophic Dombi aggregation operators with application in multi-attribute decision making problems. *IEEE Access, 8, 156600–156614.* doi:10.1109/access.2020.3019485

[37] Zulqarnain, R. M., Xin, X. L., Saqlain, M., & Khan, W. A. (2021). TOPSIS method based on the correlation coefficient of interval-valued intuitionistic fuzzy soft sets and aggregation operators with their application in decision-making. *Journal of Mathematics*, *2021*. https://doi.org/10.1155/2021/6656858.

[38] Zulqarnain, R. M., Xin, X. L., Siddique, I., Khan, W. A., & Yousif, M. A. (2021). TOPSIS method based on correlation coefficient under pythagorean fuzzy soft environment and its application towards green supply chain management. *Sustainability*, *13*, 1642. https://doi.org/10.3390/su13041642.

[39] Zulqarnain, R. M., Xin, X. L., Ali, B., Broumi, S., Abda, S., & Ahamad, M. I. (2021). Decision-making approach based on correlation coefficient with its properties under interval-valued neutrosophic hypersoft set environment. *Neutrosophic Sets and Systems*, *40*, 12-28.

[40] Zulqarnain, R. M., Siddique, I., Jarad, F., Ali, R., & Abdeljawad, T. (2021). Development of TOPSIS technique under Pythagorean fuzzy hypersoft environment based on correlation coefficient and its application towards the selection of antivirus mask in covid-19 pandemic. *Complexity*, *2021*. https://doi.org/10.1155/2021/6634991.

[41] Zulqarnain, R. M., Garg, H., Siddique, I., Ali, R., Alsubie, A., Hamadneh, N. N., & Khan, I. (2021). Algorithms for a generalized multipolar neutrosophic soft set with information measures to solve medical diagnoses and decision-making problems. *Journal of Mathematics, 2021,* 1-30. https://doi.org/10.1155/2021/6654657.

[42] Samad, A., Zulqarnain, R. M., Sermutlu, E., Ali, R., Siddique, I., Jarad, F., & Abdeljawad, T. (2021). Selection of an effective hand sanitizer to reduce covid-19 effects and extension of TOPSIS technique based on correlation coefficient under neutrosophic hypersoft set. *Complexity*, *2021*, 1-22. https://doi.org/10.1155/2021/5531830.

[43] Zulqarnain, R. M., Xin, X. L., & Saeed, M. (2020). Extension of TOPSIS method under intuitionistic fuzzy hypersoft environment based on correlation coefficient and aggregation operators to solve decision making problem. *AIMS Mathematics*, *6*, 2732-2755.

[44] Zulqarnain, R. M., Siddique, I., Ali, R., Jarad, F., Samad, A., & Abdeljawad, T. (2021). Neutrosophic hypersoft matrices with application to solve multiattributive decision-making problems. *Complexity*, *2021*. https://doi.org/10.1155/2021/5589874.

[45] Zulqarnain, R. M., Xin, X. L., Garg, H., & Ali, R. Interaction aggregation operators to solve multi criteria decision making problem under pythagorean fuzzy soft environment. *Journal of Intelligent & Fuzzy Systems*, (Preprint), 1-21.

[46] Zulqarnain, R. M., Saeed, M., Bagh, A. L. İ., Abdal, S., Saqlain, M., Ahamad, M. I., & Zafar, Z. (2020). Generalized fuzzy TOPSIS to solve multi-criteria decision-making problems. *Journal of New Theory*, 32, 40-50.

[47] Zulqarnain, R. M., Xin, X. L., Saqlain, M., & Smarandache, F. (2020). Generalized aggregate operators on neutrosophic hypersoft set. *Neutrosophic Sets and Systems*, *36*, 271-281.

[48] Zulqarnain, R. M., Xin, X. L., Saeed, M., Smarandache, F. ., & Ahmad, N. (2020). Generalized neutrosophic TOPSIS to solve multi-criteria decision-making problems. *Neutrosophic Sets and Systems*, *38*, 276-292.

[49] Zulqarnain, R. M., Xin, X. L., Garg, H., & Khan, W. A. (2021). Aggregation operators of pythagorean fuzzy soft sets with their application for green supplier chain management. *Journal of Intelligent & Fuzzy Systems*, 40 , 5545-5563. doi: 10.3233/JIFS-202781

[50] Zulqarnain, R. M., Xin, X. L., Saqlain, M., Smarandache, F., & Ahamad, M. I. (2021). An integrated model of neutrosophic TOPSIS with application in multi-criteria decision-making problem. *Neutrosophic Sets and Systems*, *40*, 118-133.

[51] Zulqarnain, R. M., Siddique, I., Ali, R., Pamucar, D., Marinkovic, D., & Bozanic, D. (2021). Robust aggregation operators for intuitionistic fuzzy hypersoft set with their application to solve MCDM problem. *Entropy*, *23*, 688. https://doi.org/10.3390/e23060688

Received: May 4, 2021. Accepted: October 1, 2021