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On Neutrosophic Γ-Semirings

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Abstract. In this paper, we introduce and study the concept of Neutrosophic Γ-semiring and study various properties. Also, we prove that there is a one-to-one correspondence between Neutrosophic Γ-semirings and sub Γ-semirings of a Γ-semiring. Further, we prove that the set of all neutrosophic Γ-semirings is a De-Morgan algebra. Moreover, we establish that the homomorphic image and inverse image of a Neutrosophic Γ-semiring is also a Neutrosophic Γ-semiring.

Keywords: Γ-semiring, fuzzy set, neutrosophic set, neutrosophic Γ-semiring.

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1. **Introduction**

In 1965, Zadeh, L.A. [14] introduced the concept of fuzzy sets. In 1986, Atanassov, K. [4] proposed intuitionistic fuzzy set theory as an extension of the fuzzy set theory. Next, in 1998, Smarandache F. [13] introduced the notion of neutrosophic sets, which are a common generalization of fuzzy sets and intuitionistic sets.

Recently, Smarandache F. [11,12] defined the NeutroAlgebraic structures and AntiAlgebraic structures. Al-Tahan, M. et al. defined the neutrosophic quadruple H_v-rings, neutrosophic quadruple H_v-subrings, and neutrosophic quadruple homomorphism and studied their various properties [3]. Muzaffar, A. et al. summarized the previous work carried out in the field of neutrosophic logic, set, measure, and also classification techniques in neutrosophy and the relevant research work has been discussed and they investigated some various of applications in the field of neutrosophy [7]. Neutrosophic quadruple algebraic structures and hyperstructures are discussed in [1,2,6]. Further, Rezaei, A. et al. introduced the notions of neutrosemihypergroup and antisemihypergroup and investigated some of their properties [10].

In 1996, Rao, M.K. [8] introduced the concept of Γ-semiring as a generalization of semiring as well as Γ-ring (also see [9]). It is known that, the notion of Γ-semirings is an extension of the ternary semirings. Then Bhargavi, Y. et al. studied on fuzzy Γ-semirings and investigated some of their properties [5].

In this paper, we introduce and study the concept of neutrosophic Γ-semiring and study various properties. Further, we prove that the set of all neutrosophic Γ-semirings is a De-Morgan algebra. Also, we establish that the homomorphic image and inverse image of a neutrosophic Γ-semiring is also a neutrosophic Γ-semiring.

2. Preliminaries

We recall the basic notions and definitions regrading Γ-semirings used in the paper.

Definition 2.1. ([9]) Let E and Γ be two additive commutative semigroups. Then E is called Γ-semiring if there exists a mapping $E \times \Gamma \times E \to E$ image to be denoted by $e\alpha f$ if it satisfies the following conditions: for all $e, f, g \in E; \alpha, \beta \in \Gamma$.
(ΓSR1) $e\alpha(f+g) = e\alpha f + e\alpha g$,
(ΓSR2) $(e+f)\alpha g = e\alpha g + f\alpha g$,
(ΓSR3) $e(\alpha+\beta)f = e\alpha f + e\beta f$,
(ΓSR4) $e\alpha(f\beta g) = (e\alpha f)\beta g$.

Definition 2.2. ([9]) A nonempty subset φ of a Γ-semiring E is said to be a sub Γ-semiring of E if $(F,+)$ is a sub semigroup of $(E,+)$ and $e\alpha f \in F$, for all $e, f \in F; \alpha \in \Gamma$.

Definition 2.3. ([9]) Let E and φ be two Γ-semirings. Then $\varphi : E \to F$ is called a homomorphism if
1. $\varphi(e+f) = \varphi(e) + \varphi(f)$,
2. $\varphi(e\gamma f) = \varphi(e)\gamma\varphi(f)$, for all $e, f \in E; \gamma \in \Gamma$.

Definition 2.4. ([13]) Let E be a space of points (objects), with a generic element in E denoted by E. A neutrosophic set ψ in E is characterized by a truth-membership function $\psi_T(e)$, an indeterminacy-membership function $\psi_I(e)$ and a falsity-membership function $\psi_F(e)$. Then, a simple valued neutrosophic set A can be denoted by

$$\psi = \{< e, \psi_T(e), \psi_I(e), \psi_F(e) >: e \in E\},$$

where $\psi_T(e), \psi_I(e), \psi_F(e) \in [0,1]$ for each point E in E. Therefore, the sum of $\psi_T(e), \psi_I(e), \psi_F(e)$ satisfies the condition $0 \leq \psi_T(e) + \psi_I(e) + \psi_F(e) \leq 3$.

For convenience, simple valued neutrosophic set is abbreviated to neutrosophic set later.

Definition 2.5. ([13]) Let $\psi = (\psi_T, \psi_I, \psi_F)$ and $\phi = (\phi_T, \phi_I, \phi_F)$ be two neutrosophic sets of a universe of discourse E.

The complement of ψ is denoted by ψ^c or ψ' and is defined as

$$\psi^c_T(e) = \psi_F(e),\ \psi^c_I(e) = 1 - \psi_I(e),\ \psi^c_F(e) = \psi_T(e).$$

The intersection of ψ and ϕ is defined as $\psi \cap \phi = ((\psi \cap \phi)_T, (\psi \cap \phi)_I, (\psi \cap \phi)_F)$, where $(\psi \cap \phi)_T(e) = \min\{\psi_T(e), \phi_T(e)\}$, $(\psi \cap \phi)_I(e) = \max\{\psi_I(e), \phi_I(e)\}$ and $(\psi \cap \phi)_F(e) = \max\{\psi_F(e), \phi_F(e)\}$.

The union of ψ and ϕ is defined as $\psi \cup \phi = ((\psi \cup \phi)_T, (\psi \cup \phi)_I, (\psi \cup \phi)_F)$, where $(\psi \cup \phi)_T(e) = \max\{\psi_T(e), \phi_T(e)\}$, $(\psi \cup \phi)_I(e) = \min\{\psi_I(e), \phi_I(e)\}$ and $(\psi \cup \phi)_F(e) = \min\{\psi_F(e), \phi_F(e)\}$.

A neutrosophic set ψ is contained in another neutrosophic set ϕ, defined as follows:

$$\psi \subseteq \phi \text{ if and only if } \psi_T(e) \leq \phi_T(e),\ \psi_I(e) \geq \phi_I(e) \,\text{and}\, \psi_F(e) \geq \phi_F(e), \text{ for all}\, e \in E.$$

Definition 2.6. ([13]) Let $\psi = (\psi_T, \psi_I, \psi_F)$ be a neutrosophic set of a universe of discourse E. For $\alpha, \beta, \gamma \in [0, 1]$ with $0 \leq \alpha + \beta + \gamma \leq 3$, the (α, β, γ)- cut or neutrosophic cut of ψ is the crisp subset of E is given by

$$\psi_{(\alpha,\beta,\gamma)} = \{e \in E : \psi_T(e) \geq \alpha, \psi_I(e) \leq \beta, \psi_F(e) \leq \gamma\}.$$

Definition 2.7. ([13]) Let φ be a mapping from a set E into a set F. Let ψ be a neutrosophic set in E. Then the image $\varphi(\psi)$ of ψ is the neutrosophic set in F defined by:

$$(\varphi(\psi_T))(f) = \begin{cases} \sup\limits_{z \in \varphi^{-1}(f)} \psi_T(z) & \text{if } \varphi^{-1}(f) \neq \phi \\ 0 & otherwise \end{cases},$$

$$(\varphi(\psi_I))(f) = \begin{cases} \inf\limits_{z \in \varphi^{-1}(f)} \psi_I(z) & \text{if } \varphi^{-1}(f) \neq \phi \\ 1 & otherwise \end{cases}$$

and

$$(\varphi(\psi_F))(f) = \begin{cases} \inf\limits_{z \in \varphi^{-1}(f)} \psi_F(z) & \text{if } \varphi^{-1}(f) \neq \phi \\ 1 & otherwise \end{cases},$$

for all $f \in F$, where $\varphi^{-1}(f) = \{e : \varphi(e) = f\}$.

Let ϕ be a neutrosophic set in F. Then the inverse image of $\varphi^{-1}(\phi)$ of ϕ is the neutrosophic set in E by $\varphi^{-1}(\phi)(e) = \phi(\varphi(e))$, for all $e \in E$.

Definition 2.8. ([5]) A fuzzy set μ in a Γ-semiring E is called fuzzy Γ-semiring if it satisfies the following properties: for all $e, f \in E; \gamma \in \Gamma$

$(FI1)$ $\mu(e + f) \geq \min\{\mu(e), \mu(f)\}$,

$(FI2)$ $\mu(e\gamma f) \geq \min\{\mu(e), \mu(f)\}$.

3. On Neutrosophic Γ-semirings

This section presents some important properties of neutrosophic Γ-semirings and characterize neutrosophic Γ-semirings to the crisp Γ-semirings, and we prove that the set of all neutrosophic Γ-semirings is a De-Morgan algebra.

Throughout this section E stands for a Γ-semiring unless otherwise mentioned.
Now, we introduce the following.

Definition 3.1. A neutrosophic set $A = (\psi_T, \psi_I, \psi_F)$ in a Γ-semiring E is called a Neutrosophic Γ-semiring if it satisfies the following properties: for all $e, f \in E; \gamma \in \Gamma$
$(N\Gamma SR1)$ $\psi_T(e+f) \geq \min\{\psi_T(e), \psi_T(f)\}$,
$(N\Gamma SR2)$ $\psi_I(e+f) \leq \max\{\psi_I(e), \psi_I(f)\}$,
$(N\Gamma SR3)$ $\psi_F(e+f) \leq \max\{\psi_F(e), \psi_F(f)\}$,
$(N\Gamma SR4)$ $\psi_T(e\gamma f) \geq \min\{\psi_T(e), \psi_T(f)\}$,
$(N\Gamma SR5)$ $\psi_I(e\gamma f) \leq \max\{\psi_I(e), \psi_I(f)\}$,
$(N\Gamma SR6)$ $\psi_F(e\gamma f) \leq \max\{\psi_F(e), \psi_F(f)\}$.

Example 3.2: Let E be the set of negative integers and Γ be the set of negative even integers. Then E, Γ are additive commutative semigroups. Define the mapping $E \times \Gamma \times E \to E$ by $e\alpha f$ usual product of e, α, f, for all $e, f \in E$; $\alpha \in \Gamma$. Then E is a Γ-semiring. Let $\psi = (\psi_T, \psi_I, \psi_F)$, where $\psi_T : E \to [0,1]$, $\psi_I : E \to [0,1]$ and $\psi_F : E \to [0,1]$ defined by:

$$\psi_T(e) = \begin{cases} 0.6 & \text{if } e = -1 \\ 0.7 & \text{if } e = -2 \\ 0.9 & \text{if } e < -2 \end{cases},$$

$$\psi_I(e) = \begin{cases} 0.5 & \text{if } e = -1 \\ 0.3 & \text{if } e = -2 \\ 0.2 & \text{if } e < -2 \end{cases}$$

and

$$\psi_F(e) = \begin{cases} 0.4 & \text{if } e = -1 \\ 0.2 & \text{if } e = -2 \\ 0.1 & \text{if } e < -2 \end{cases}.$$

Thus ψ is a Neutrosophic Γ-semiring of E.

Example 3.2. Let E be the set of real numbers and Γ be the set of positive numbers. Then E, Γ are additive commutative semigroups. Define the mapping $E \times \Gamma \times E \to E$ by $e\alpha f$ usual product of e, α, f, for all $e, f \in E; \alpha \in \Gamma$. Then E is a Γ-semiring. Let $\psi = (\psi_T, \psi_I, \psi_F)$, where

$\psi_T : E \to [0,1]$, $\psi_I : E \to [0,1]$ and $\psi_F : E \to [0,1]$ defined by:

$$\psi_T(e) = \begin{cases} 0.9 & \text{if } e = 0 \\ 0.7 & \text{if } e \textit{ is positive} \\ 0.6 & \text{if } e \textit{ is negative} \end{cases},$$

$$\psi_I(e) = \begin{cases} 0.2 & \text{if } e = 0 \\ 0.3 & \text{if } e \textit{ is positive} \\ 0.5 & \text{if } e \textit{ is negative} \end{cases}$$

and

$$\psi_F(e) = \begin{cases} 0.1 & \text{if } e = 0 \\ 0.2 & \text{if } e \textit{ is positive} \\ 0.4 & \text{if } e \textit{ is negative} \end{cases}.$$

Thus ψ is a Neutrosophic Γ-semiring of E.

Theorem 3.3. *A neutrosophic set $\psi = (\psi_T, \psi_I, \psi_F)$ is a neutrosophic Γ-semiring of E if and only if ψ_T, $1 - \psi_I$ and $1 - \psi_F$ are fuzzy Γ-semirings of E.*

Proof. Suppose $\psi = (\psi_T, \psi_I, \psi_F)$ is a neutrosophic Γ-semiring of E. Let $e, f \in E$; $\gamma \in \Gamma$.Then
(i) $\psi_T(e + f) \geq \min\{\psi_T(e), \psi_T(f)\}$,
(ii) $\psi_I(e + f) \leq \max\{\psi_I(e), \psi_I(f)\}$, i.e., $1 - \psi_I(e + f) \geq \min\{1 - \psi_I(e), 1 - \psi_I(f)\}$,
(iii) $\psi_F(e + f) \leq \max\{\psi_F(e), \psi_F(f)\}$, i.e., $1 - \psi_F(e + f) \geq \min\{1 - \psi_F(e), 1 - \psi_F(f)\}$,
(iv) $\psi_T(e\gamma f) \geq \min\{\psi_T(e), \psi_T(f)\}$,
(v) $\psi_I(e\gamma f) \leq \max\{\psi_I(e), \psi_I(f)\}$, i.e., $1 - \psi_I(e\gamma f) \geq \min\{1 - \psi_I(e), 1 - \psi_I(f)\}$,
(vi) $\psi_F(e\gamma f) \leq \max\{\psi_F(e), \psi_F(f)\}$, i.e., $1 - \psi_F(e\gamma f) \geq \min\{1 - \psi_F(e), 1 - \psi_F(f)\}$.
Thus, ψ_T, $1 - \psi_I$ and $1 - \psi_F$ are fuzzy Γ-semiring of E. The converse part is obvious from the definition. □

Theorem 3.4. *A neutrosophic set $psi = (\psi_T, \psi_I, \psi_F)$ of E is neutrosophic Γ- semiring of E if and only if for all $\alpha, \beta, \gamma \in [0,1]$, the (α, β, γ)-cut $\psi_{(\alpha,\beta,\gamma)}$ is a sub Γ-semiring of E.*

Proof. Suppose $\psi = (\psi_T, \psi_I, \psi_F)$ of E is a neutrosophic Γ-semiring. Let $e, f \in \psi_{(\alpha,\beta,\gamma)}$; $\eta \in \Gamma$. Then $\psi_T(e), \psi_T(f) \geq \alpha$, $\psi_I(e), \psi_I(f) \leq \beta$, $\psi_F(e), \psi_F(f) \leq \gamma$. Since A is neutrosophic Γ-semiring, we have:
(i) $\psi_T(e + f) \geq \min\{\psi_T(e), \psi_T(f)\} \geq \alpha$,
(ii) $\psi_I(e + f) \leq \max\{\psi_I(e), \psi_I(f)\} \leq \beta$,
(iii) $\psi_F(e + f) \leq \max\{\psi_F(e), \psi_F(f)\} \leq \gamma$,
which implies $e + f \in \psi_{(\alpha,\beta,\gamma)}$.
Also, since
(iv) $\psi_T(e\eta f) \geq \min\{\psi_T(e), \psi_T(f)\} \geq \alpha$,

(v) $\psi_I(e\eta f) \leq \max\{\psi_I(e), \psi_I(f)\} \leq \beta$,
(vi) $\psi_F(e\eta f) \leq \max\{\psi_F(e), \psi_F(f)\} \leq \gamma$,
which implies $e\eta f \in \psi_{(\alpha,\beta,\gamma)}$.
Thus, $\psi_{(\alpha,\beta,\gamma)}$ is a sub Γ-semiring of E.

Conversely, suppose $\psi_{(\alpha,\beta,\gamma)}$ is a sub Γ-semiring of E. Let $e, f \in E$; $\eta \in \Gamma$. Let $\psi_T(e) > \alpha 1,\ \psi_I(e) < \beta\psi_1,\ \psi_F(e) < \gamma\psi_1$ and $\psi_T(f) > \alpha 2,\ \psi_I(f) < \beta\psi_2,\ \psi_F(f) < \gamma_2$.
Put $\alpha = \min\{\alpha 1, \alpha 2\},\ \beta = \max\{\beta\psi_1, \beta\psi_2\}$ *and* $\gamma = \max\{\gamma\psi_1, \gamma\psi_2\}$. Then $e, f \in \psi_{(\alpha,\beta,\gamma)}$, and so $e + f \in \psi_{(\alpha,\beta,\gamma)}$ and $e\eta f \in \psi_{(\alpha,\beta,\gamma)}$. Hence $\psi_T(e + f) \geq \alpha = \min\{\psi_T(e), \psi_T(f)\}$, $\psi_I(e+f) \leq \beta = \max\{\psi_I(e), \psi_I(f)\}$, $\psi_F(e+f) \leq \gamma = \max\{\psi_F(e), \psi_F(f)\}$ and $\psi_T(e\eta f) \geq \alpha = \min\{\psi_T(e), \psi_T(f)\}$, $\psi_I(e\eta f) \leq \beta = \max\{\psi_I(e), \psi_I(f)\}$, $\psi_F(e\eta f) \leq \gamma = \max\{\psi_F(e), \psi_F(f)\}$.
Thus, ψ is a neutrosophic Γ-semiring of E. □

Theorem 3.5. *Let $\psi = (\psi_T, \psi_I, \psi_F)$ be a neutrosophic set of E. The two neutrosophic cuts $\psi_{(\alpha_1,\beta_1,\gamma_1)}$ and $\psi_{(\alpha_2,\beta_2,\gamma_2)}$ of E are equal, where α_1, α_2, α_3, β_1, β_2, β_3, γ_1, γ_2, $\gamma_3 \in [0, 1]$ with $\alpha_1 < \alpha_2$, $\beta_1 > \beta_2$, $\gamma_1 > \gamma_2$ if and only if there is no $e \in E$ such that $\alpha_1 \leq \psi_T(e) < \alpha_2,\ \beta_1 \geq \psi_I(e) > \beta_2,\ \gamma_1 \geq \psi_F(e) > \gamma_2$.*

Proof. Suppose $\psi_{(\alpha 1,\beta_1,\gamma_1)}$ and $\psi_{(\alpha_2,\beta_2,\gamma_2)}$ of E are equal. Suppose if possible there exists $e \in E$ such that $\alpha_1 \leq \psi_T(e) < \alpha_2,\ \beta_1 \geq \psi_I(e) > \beta_2,\ \gamma_1 \geq \psi_F(e) > \gamma_2$. Then $e \in \psi_{(\alpha_1,\beta_1,\gamma_1)} = \psi_{(\alpha_2,\beta_2,\gamma_2)}$, and so $\psi_T(e) \geq \alpha_2,\ \psi_I(e) \leq \beta_2,\ \geq \psi_F(e) \leq \gamma_2$. Which is a contradiction. Hence there exists no $e \in E$ such that $\alpha_1 \leq \psi_T(e) < \alpha_2,\ \beta_1 \geq \psi_I(e) > \beta_2,\ \gamma_1 \geq \psi_F(e) > \gamma_2$.

Conversely, suppose that there exists no $e \in E$ such that $\alpha_1 \leq \psi_T(e) < \alpha_2,\ \beta_1 \geq \psi_I(e) > \beta_2,\ \gamma_1 \geq \psi_F(e) > \gamma_2$. Suppose if possible $\psi_{(\alpha_1,\beta_1,\gamma_1)} \neq \psi_{(\alpha_2,\beta_2,\gamma_2)}$. Then there exists $e \in \psi_{(\alpha_1,\beta_1,\gamma_1)}$ and $e \notin \psi_{(\alpha_2,\beta_2,\gamma_2)}$, i.e., $\psi_T(e) \geq \alpha_1,\ \psi_I(e) \leq \beta_1,\ \psi_F(e) \leq \gamma_1$ and $\psi_T(e) < \alpha_2,\ \psi_I(e) > \beta_2,\ \psi_F(e) > \gamma_2$. So, there exists $e \in E$ such that $\alpha_1 \leq \psi_T(e) < \alpha_2,\ \beta_1 \geq \psi_I(e) > \beta_2,\ \gamma_1 \geq \psi_F(e) > \gamma_2$. Which is a contradiction. Thus, $\psi_{(\alpha_1,\beta_1,\gamma_1)} = \psi_{(\alpha_2,\beta_2,\gamma_2)}$. □

Theorem 3.6. *If $\psi = (\psi_T, \psi_I, \psi_F)$ and $\phi = (\phi_T, \phi_I, \phi_F)$ are two neutrosophic Γ-semirings of E, then $\psi \cap \phi$ is a neutrosophic Γ-semiring of E.*

Proof. Let $e, f \in E$; $\eta \in \Gamma$. Then

$$\begin{aligned}(\psi \cap \phi)_T(e + f) &= \min\{\psi_T(e + f), \phi_T(e + f)\}\\ &\geq \min\{\min\{\psi_T(e), \psi_T(f)\}, \min\{\phi_T(e), \phi_T(f)\}\}\\ &\geq \min\{\min\{\psi_T(e), \phi_T(e)\}, \min\{\psi_T(f), \phi_T(f)\}\}\\ &= \min\{(\psi \cap \phi)_T(e), (\psi \cap \phi)_T(f)\},\end{aligned}$$

$$\begin{aligned}(\psi\cap\phi)_I(e+f) &= \max\{\psi_I(e+f),\phi_I(e+f)\}\\ &\le \max\{\max\{\psi_I(e),\psi_I(f)\},\max\{\phi_I(e),\phi_I(f)\}\}\\ &\le \max\{\max\{\psi_I(e),\phi_I(e)\},\max\{\psi_I(f),\phi_I(f)\}\}\\ &= \max\{(\psi\cap\phi)_I(e),(\psi\cap\phi)_I(f)\}\end{aligned}$$

and

$$\begin{aligned}(\psi\cap\phi)_F(e+f) &= \max\{\psi_F(e+f),\phi_F(e+f)\}\\ &\le \max\{\max\{\psi_F(e),\psi_F(f)\},\max\{\phi_F(e),\phi_F(f)\}\}\\ &\le \max\{\max\{\psi_F(e),\phi_F(e)\},\max\{\psi_F(f),\phi_F(f)\}\}\\ &= \max\{(\psi\cap\phi)_F(e),(\psi\cap\phi)_F(f)\}.\end{aligned}$$

Also, we get

$$\begin{aligned}(\psi\cap\phi)_T(e\eta f) &= \min\{\psi_T(e\eta f),\phi_T(e\eta f)\}\\ &\ge \min\{\min\{\psi_T(e),\psi_T(f)\},\min\{\phi_T(e),\phi_T(f)\}\}\\ &\ge \min\{\min\{\psi_T(e),\phi_T(e)\},\min\{\psi_T(f),\phi_T(f)\}\}\\ &= \min\{(\psi\cap\phi)_T(e),(\psi\cap\phi)_T(f)\},\end{aligned}$$

$$\begin{aligned}(\psi\cap\phi)_I(e\eta f) &= \max\{\psi_I(e\eta f),\phi_I(e\eta f)\}\\ &\le \max\{\max\{\psi_I(e),\psi_I(f)\},\max\{\phi_I(e),\phi_I(f)\}\}\\ &\le \max\{\max\{\psi_I(e),\phi_I(e)\},\max\{\psi_I(f),\phi_I(f)\}\}\\ &= \max\{(\psi\cap\phi)_I(e),(\psi\cap\phi)_I(f)\}\end{aligned}$$

and

$$\begin{aligned}(\psi\cap\phi)_F(e\eta f) &= \max\{\psi_F(e\eta f),\phi_F(e\eta f)\}\\ &\le \max\{\max\{\psi_F(e),\psi_F(f)\},\max\{\phi_F(e),\phi_F(f)\}\}\\ &\le \max\{\max\{\psi_F(e),\phi_F(e)\},\max\{\psi_F(f),\phi_F(f)\}\}\\ &= \max\{(\psi\cap\phi)_F(e),(\psi\cap\phi)_F(f)\}.\end{aligned}$$

Thus, $\psi\cap\phi$ is a neutrosophic Γ-semiring of E. □

Corollary 3.7. *The intersection of arbitrary family of neutrosophic* Γ*-semirings is a neutrosophic* Γ*-semiring.*

The following example shows that the union of two neutrosophic Γ-semirings may not be a neutrosophic Γ-semiring, in general.

Example 3.8. consider the additive abelian group $Z_4 = \{0, 1, 2, 3\}$ and the subgroup $\Gamma = \{0, 2\}$. Define $Z_4 \times \Gamma \times Z_4 \to Z_4$ by $e\alpha f$ usual product of $e, \alpha, f, \ \forall\ e, f \in \ Z_4;\ \alpha \in \Gamma$.
Then Z_4 is a Γ-semiring.
Let $\psi = (\psi_T, \psi_I, \psi_F)$, where $\psi_T : Z_4 \to [0, 1]$, $\psi_I : Z_4 \to [0, 1]$ and $\psi_F : Z_4 \to [0, 1]$ defined by:

$$\psi_T(e) = \begin{cases} 0.8 & \text{if } \ e = 0; \\ 0.6 & \text{if } \ e = 1; \\ 0.4 & \text{otherwise} \end{cases}$$

$$\psi_I(e) = \begin{cases} 0.2 & \text{if } \ e = 0; \\ 0.4 & \text{if } \ e = 1; \\ 0.5 & \text{otherwise} \end{cases}$$

$$\psi_F(e) = \begin{cases} 0.2 & \text{if } \ e = 0; \\ 0.3 & \text{if } \ e = 1; \\ 0.5 & \text{otherwise} \end{cases}$$

Let $\phi = (\phi_T, \phi_I, \phi_F)$, where $\phi_T : Z_4 \to [0, 1]$, $\phi_I : Z_4 \to [0, 1]$ and $\phi_F : Z_4 \to [0, 1]$ defined by:

$$\phi_T(e) = \begin{cases} 0.6 & \text{if } \ e = 0; \\ 0.5 & \text{if } \ e = 2; \\ 0.2 & \text{otherwise} \end{cases}$$

$$\phi_F(e) = \begin{cases} 0.2 & \text{if } \ e = 0; \\ 0.3 & \text{if } \ e = 2; \\ 0.4 & \text{otherwise} \end{cases}$$

$$\phi_F(e) = \begin{cases} 0.3 & \text{if } \ e = 0; \\ 0.4 & \text{if } \ e = 2; \\ 0.5 & \text{otherwise} \end{cases}$$

Thus, ψ and ϕ are neutrosophic Γ-semirings of Z_4, but $\psi \cup \phi$ is not a neutrosophic Γ-semiring of Z_4.

In particular we have the following:

Theorem 3.9. *If $\psi = (\psi_T, \psi_I, \psi_F)$ and $\phi = (\phi_T, \phi_I, \phi_F)$ are two neutrosophic Γ-semirings of E, then $\psi \cup \phi$ is a neutrosophic Γ-semiring of E only if $\psi \subseteq \phi$ or $\phi \subseteq \psi$.*

Proof. Assume that $e, f \in E$; $\eta \in \Gamma$. Suppose $A \subseteq B$. Then

$$\begin{aligned}(\psi \cup \phi)_T(e+f) &= \max\{\psi_T(e+f), \phi_T(e+f)\} \\ &= \phi_T(e+f) \\ &\geq \min\{\phi_T(e), \phi_T(f)\} \\ &= \min\{\max\{\psi_T(e), \phi_T(e)\}, \max\{\psi_T(f), \phi_T(f)\}\} \\ &= \min\{(\psi \cup \phi)_T(e), (\psi \cup \phi)_T(f)\},\end{aligned}$$

$$\begin{aligned}(\psi \cup \phi)_I(e+f) &= \min\{\psi_I(e+f), \phi_I(e+f)\} \\ &= \phi_I(e+f) \\ &\leq \max\{\phi_I(e), \phi_I(f)\} \\ &= \max\{\min\{\psi_I(e), \phi_I(e)\}, \min\{\psi_I(f), \phi_I(f)\}\} \\ &= \max\{(\psi \cup \phi)_I(e), (\psi \cup \phi)_I(f)\}\end{aligned}$$

and

$$\begin{aligned}(\psi \cup \phi)_F(e+f) &= \min\{\psi_F(e+f), \phi_F(e+f)\} \\ &= \phi_F(e+f) \\ &\leq \max\{\phi_F(e), \phi_F(f)\} \\ &= \max\{\min\{\psi_F(e), \phi_F(e)\}, \min\{\psi_F(f), \phi_F(f)\}\} \\ &= \max\{(\psi \cup \phi)_F(e), (\psi \cup \phi)_F(f)\}.\end{aligned}$$

Also, we have

$$\begin{aligned}(\psi \cup \phi)_T(e\eta f) &= \max\{\psi_T(e\eta f), \phi_T(e\eta f)\} \\ &= \phi_T(e\eta f) \\ &\geq \min\{\phi_T(e), \phi_T(f)\} \\ &= \min\{\max\{\psi_T(e), \phi_T(e)\}, \max\{\psi_T(f), \phi_T(f)\}\} \\ &= \min\{(\psi \cup \phi)_T(e), (\psi \cup \phi)_T(f)\},\end{aligned}$$

$$\begin{aligned}(\psi \cup \phi)_I(x\eta y) &= \min\{\psi_I(x\eta y), \phi_I(x\eta y)\} \\ &= \phi_I(e+f) \\ &\leq \max\{\phi_I(e), \phi_I(f)\} \\ &= \max\{\min\{\psi_I(e), \phi_I(e)\}, \min\{\psi_I(f), \phi_I(f)\}\} \\ &= \max\{(\psi \cup \phi)_I(e), (\psi \cup \phi)_I(f)\}\end{aligned}$$

and

$$\begin{aligned}(\psi \cup \phi)_F(e\eta f) &= \min\{\psi_F(e\eta f), \phi_F(e\eta f)\} \\ &= \phi_F(e\eta f) \\ &\leq \max\{\phi_F(e), \phi_F(f)\} \\ &\leq \max\{\min\{\psi_F(e), \phi_F(e)\}, \min\{\psi_F(f), \phi_F(f)\}\} \\ &= \max\{(\psi \cup \phi)_F(e), (\psi \cup \phi)_F(f)\}.\end{aligned}$$

Similarly, we can prove if $\phi \subseteq \psi$. Thus, $\psi \cup \phi$ is a neutrosophic Γ-semiring of E. □

Lemma 3.10. *Let $A(E)$ be the set of all neutrosophic Γ-semirings of E. Then $(A(E), \subseteq)$ is a poset.*

Proof. Let $A, B, C \in A(E)$.
1. Always $A \subseteq A$, for all $A \in A(E)$. So, $\subseteq$ is reflexive.
2. Let $A \subseteq B$ and $B \subseteq A$
$\Rightarrow A = B$.
So, $\subseteq$ is anti symmetric.
3.Let $A \subseteq B$ and $B \subseteq C$
$\Rightarrow A \subseteq C$.
So, $\subseteq$ is transitive.
Thus $\subseteq$ is partial ordering and hence $(A(E), \subseteq)$ is a poset. □

Theorem 3.11. $(A(E), \cup, \cap, ', 0, 1)$ *is a De-Morgan Algebra.*

Proof. We will show that
1. $(A(E), \cup, \cap, ', 0, 1)$ is a bounded distributive lattice
2. $(\psi')' = \psi$, $(\psi \cup \phi)' = \psi' \cap \phi'$ and $(\psi \cap \phi)' = \psi' \cup \psi'$, for all $\psi, \phi \in A(E)$.
Let $\psi = (\psi_T, \psi_I, \psi_F)$, $\phi = (\phi_T, \phi_I, \phi_F)$, $\sigma = (\sigma T, \sigma I, \sigma F) \in A(E)$.

1. Since $0 \leq \psi_T(e) \leq 1$, $0 \leq \psi_I(e) \leq 1$ and $0 \leq \psi_F(e) \leq 1$, for all $x \in R$. So, $A(E)$ is bounded.

Idempotency:

$$\psi \cap \psi = (\psi_T, \psi_I, \psi_F) \cap (\psi_T, \psi_I, \psi_F) = (\psi_T, \psi_I, \psi_F) = \psi,$$
$$\psi \cup \psi = (\psi_T, \psi_I, \psi_F) \cup (\psi_T, \psi_I, \psi_F) = (\psi_T, \psi_I, \psi_F) = \psi.$$

Commutativity:

$$\begin{aligned}
\psi \cap \phi &= (\psi_T, \psi_I, \psi_F) \cap (\phi_T, \phi_I, \phi_F) \\
&= (\min\{\psi_T, \phi_T\}, \max\{\psi_I, \phi_I\}, \max\{\psi_F, \phi_F\}) \\
&= (\min\{\phi_T, \psi_T\}, \max\{\phi_I, \psi_I\}, \max\{\phi_F, \psi_F\}) \\
&= (\phi_T, \phi_I, \phi_F) \cap (\psi_T, \psi_I, \psi_F) \\
&= \phi \cap \psi, \\
\psi \cup \phi &= (\psi_T, \psi_I, \psi_F) \cup (\phi_T, \phi_I, \phi_F) \\
&= (\max\{\psi_T, \phi_T\}, \min\{\psi_I, \phi_I\}, \min\{\psi_F, \phi_F\}) \\
&= (\max\{\phi_T, \psi_T\}, \min\{\phi_I, \psi_I\}, \min\{\phi_F, \psi_F\}) \\
&= (\phi_T, \phi_I, \phi_F) \cup (\psi_T, \psi_I, \psi_F) \\
&= \phi \cup \psi.
\end{aligned}$$

Associativity:

$$\begin{aligned}
\psi \cap (\phi \cap \sigma) &= (\psi_T, \psi_I, \psi_F) \cap ((\phi_T, \phi_I, \phi_F) \cap (\sigma_T, \sigma_I, \sigma_F)) \\
&= (\min\{\psi_T, \min\{\phi_T, \sigma_T\}\}, \max\{\psi_I, \max\{\phi_I, \sigma_I\}\}, \max\{\psi_F, \max\{\phi_F, \sigma_F\}\}) \\
&= (\min\{\min\{\psi_T, \phi_T\}, \sigma_T\}, \max\{\max\{\psi_I, \phi_I\}, \sigma_I\}, \max\{\max\{\psi_F, \phi_F\}, \sigma_F\}) \\
&= (\psi \cap \phi) \cap \sigma, \\
\psi \cup (\phi \cup \sigma) &= (\psi_T, \psi_I, \psi_F) \cup ((\phi_T, \phi_I, \phi_F) \cup (\sigma_T, \sigma_I, \sigma_F)) \\
&= (\max\{\psi_T, \max\{\phi_T, \sigma_T\}\}, \min\{\psi_I, \min\{\phi_I, \sigma_I\}\}, \min\{\psi_F, \min\{\phi_F, \sigma_F\}\}) \\
&= (\max\{\max\{\psi_T, \phi_T\}, \sigma_T\}, \min\{\min\{\psi_I, \phi_I\}, \sigma_I\}, \min\{\min\{\psi_F, \phi_F\}, \sigma_F\}) \\
&= (\psi \cup \phi) \cup \sigma.
\end{aligned}$$

Absorption:

$$\begin{aligned}
\psi \cap (\psi \cup \phi) &= (\min\{\psi_T, \max\{\psi_T, \phi_T\}\}, \max\{\psi_I, \min\{\psi_I, \phi_I\}\}, \max\{\psi_F, \min\{\psi_F, \phi_F\}\}) \\
&= (\psi_T, \psi_I, \psi_F), \\
&= \psi, \\
\psi \cup (\psi \cap \phi) &= (\max\{\psi_T, \min\{\psi_T, \phi_T\}\}, \min\{\psi_I, \max\{\psi_I, \phi_I\}\}, \min\{\psi_F, \max\{\psi_F, \phi_F\}\}) \\
&= (\psi_T, \psi_I, \psi_F) \\
&= \psi.
\end{aligned}$$

<u>**Distributivity**</u>:

$$\begin{aligned}
\psi \cap (\phi \cup \sigma) &= (\min\{\psi_T, \max\{\phi_T, \sigma_T\}\}, \max\{\psi_I, \min\{\phi_I, \sigma_I\}\}, \max\{\psi_F, \min\{\phi_F, \sigma_F\}\}) \\
&= (\max\{\min\{\psi_T, \phi_T\}, \min\{\psi_T, \sigma_T\}\}, \min\{\max\{\psi_I, \phi_I\}, \max\{\psi_I, \sigma_I\}\}, \\
&\quad \min\{\max\{\psi_F, \phi_F\}, \max\{\psi_F, \sigma_F\}\}) \\
&= (\psi \cap \phi) \cup (\psi \cap \sigma), \\
\psi \cup (\phi \cap \sigma) &= (\max\{\psi_T, \min\{\phi_T, \sigma_T\}\}, \min\{\psi_I, \max\{\phi_I, \sigma_I\}\}, \min\{\psi_F, \max\{\phi_F, \sigma_F\}\}) \\
&= (\min\{\max\{\psi_T, \phi_T\}, \max\{\psi_T, \sigma_T\}\}, \max\{\min\{\psi_I, \phi_I\}, \min\{\psi_I, \sigma_I\}\}, \\
&\quad \max\{\min\{\psi_F, \phi_F\}, \min\{\psi_F, \sigma_F\}\}) \\
&= (\psi \cup \phi) \cap (\psi \cup \sigma).
\end{aligned}$$

Thus, $(A(E), \cup, \cap, ', 0, 1)$ is a bounded distributive lattice.

2. Now, we show that $(\psi')' = \psi$, $(\psi \cup \phi)' = \psi' \cap \phi'$ and $(\psi \cap \phi)' = \psi' \cup \phi'$.

$$\begin{aligned}
(\psi \cap \phi)' &= (\min\{\psi_T, \phi_T\}, \max\{\psi_I, \phi_I\}, \max\{\psi_F, \phi_F\})' \\
&= (\max\{\psi_F, \phi_F\}, \min\{1 - \psi_I, 1 - \phi_I\}, \min\{\psi_T, \phi_T\}) \\
&= \psi' \cup \phi'.
\end{aligned}$$

Therefore $(\psi \cap \phi)' = \psi' \cup \phi'$. Similarly, we can show that $(\psi \cup \phi)' = \psi' \cap \phi'$. Also, we have $\psi' = (\psi_F, 1 - \psi_I, \psi_T)$, and so $(\psi')' = \psi$. Thus, $(A(E), \cup, \cap, ', 0, 1)$ is a De-Morgan algebra. □

4. Hommorphic image and Pre-image of Neutrosophic Γ-semirings

Theorem 4.1. *Let φ be a homomorphism from a Γ-semiring E onto a Γ-semiring F and let ϕ be a neutrosophic Γ-semiring of F. Then the pre-image $\varphi^{-1}(\phi)$ of ϕ is a neutrosophic Γ-semiring of E.*

Proof. Assume that $e, f \in E$; $\eta \in \Gamma$. Then

$$\begin{aligned}
(\varphi^{-1}(\phi_T))(e + f) &= \phi_T(\varphi(e + f)) \\
&= \phi_T(\varphi(e) + \varphi(f)) \\
&\geq \min\{\phi_T(\varphi(e)), \phi_T(\varphi(f))\} \\
&= \min\{\varphi^{-1}(\phi_T)(e), \varphi^{-1}(\phi_T)(f)\},
\end{aligned}$$

$$\begin{aligned}(\varphi^{-1}(\phi_I))(e+f) &= \phi_I(\varphi(e+f))\\ &= \phi_I(\varphi(e)+\varphi(f))\\ &\leq \max\{\phi_I(\varphi(e)),\phi_I(\varphi(f))\}\\ &= \max\{\varphi^{-1}(\phi_I)(e),\varphi^{-1}(\phi_I)(f)\},\end{aligned}$$

$$\begin{aligned}(\varphi^{-1}(\phi_F))(e+f) &= \phi_F(\varphi(e+f))\\ &= \phi_F(\varphi(e)+\varphi(f))\\ &\leq \max\{\phi_F(\varphi(e)),\phi_F(\varphi(f))\}\\ &= \max\{\varphi^{-1}(\phi_F)(e),\varphi^{-1}(\phi_F)(f)\}\end{aligned}$$

and

$$\begin{aligned}(\varphi^{-1}(\phi_T))(x\eta y) &= \phi_T(\varphi(e\eta f))\\ &= \phi_T(\varphi(e)\eta\varphi(f))\\ &\geq \min\{\phi_T(f(e)),\phi_T(f(f))\}\\ &= \min\{(\varphi^{-1}\phi_T(e),(\varphi^{-1}(\phi_T)(f)\},\end{aligned}$$

$$\begin{aligned}(\varphi^{-1}(\phi_I))(e\eta f) &= \phi_I(\varphi(e\eta f))\\ &= \phi_I(\varphi(e)\eta\varphi(f))\\ &\leq \max\{\phi_I(\varphi(e)),\phi_I(\varphi(f))\}\\ &= \max\{\varphi^{-1}(\phi_I)(e),\varphi^{-1}(\phi_I)(f)\},\end{aligned}$$

$$\begin{aligned}(\varphi^{-1}(\phi_F))(e\eta f) &= \phi_F(\varphi(e\eta f))\\ &= \phi_F(\varphi(e)\eta\varphi(f))\\ &\leq \max\{\phi_F(\varphi(e)),\phi_F(\varphi(f))\}\\ &= \max\{\varphi^{-1}(\phi_F)(e),\varphi^{-1}(\phi_F)(f)\}.\end{aligned}$$

Thus, $\varphi^{-1}(\phi)$ is a neutrosophic Γ-semiring of E. □

Theorem 4.2. *Let φ be a homomorphism from a Γ-semiring E onto a Γ-semiring F. Let ψ be a neutrosophic Γ-semiring of E. Then the homomorphic image $\varphi(\psi)$ of ψ is a neutrosophic Γ-semiring of F.*

Proof. Let $p, q \in F$; $\gamma \in \Gamma$. If either $\varphi^{-1}(p)$ or $\varphi^{-1}(q)$ is empty, then the result is trivially satisfied.

Suppose $\varphi^{-1}(p)$ and $\varphi^{-1}(q)$ are non-empty. Since $p, q \in F$, then there exist $e, f \in E$ such that $e = f(p), f = f(q)$. Then

$$\begin{aligned}(\varphi(\psi_T))(p+q) &= \sup_{z\in\varphi^{-1}(p+q)} \psi_T(z)\\ &= \sup\{\psi_T(e+f) : e, f \in E, e = \varphi(p), f = \varphi(q)\}\\ &\geq \sup\{\min\{\psi_T(e), \psi_T(f)\} : \ e, f \in E, e = \varphi(p), f = \varphi(q)\}\\ &= \min\{\sup\{\psi_T(e) : e \in E, x = \varphi(p)\}, \sup\{\psi_T(f) : f \in E, f = \varphi(q)\}\\ &= \min\{(\varphi(\psi_T))(p), (\varphi(\psi_T))(q)\},\end{aligned}$$

$$\begin{aligned}(\varphi(\psi_I))(p+q) &= \inf_{z\in\varphi^{-1}(p+q)} \psi_I(z)\\ &= \inf\{\psi_I(e+f) : \ e, f \in E, x = \varphi(p), f = \varphi(q)\}\\ &\leq \inf\{\max\{\psi_I(e), \psi_I(f)\} : \ e, f \in E, x = \varphi(p), f = \varphi(q)\}\\ &= \max\{\inf\{\psi_I(e) : e \in E, e = \varphi(p)\}, \inf\{\psi_I(f) : f \in E, f = \varphi(q)\}\\ &= \max\{(\varphi(\psi_I))(p), (\varphi(\psi_I))(q)\},\end{aligned}$$

$$\begin{aligned}(\varphi(\psi_F))(p+q) &= \inf_{z\in\varphi^{-1}(p+q)} \psi_F(z)\\ &= \inf\{\psi_F(e+f) : e, f \in E, e = \varphi(p), f = \varphi(q)\}\\ &\leq \inf\{\max\{\psi_F(e), \psi_F(f)\} : \ e, f \in E, e = \varphi(p), f = \varphi(q)\}\\ &= \max\{\inf\{\psi_F(e) : e \in E, e = \varphi(p)\}, \inf\{\psi_F(f) : f \in E, f = \varphi(q)\}\\ &= \max\{(\varphi(\psi_F))(p), (\varphi(\psi_F))(q)\}\end{aligned}$$

and

$$\begin{aligned}(\varphi(\psi_T))(p\gamma q) &= \sup_{z\in\varphi^{-1}(p\gamma q)} \psi_T(z)\\ &= \sup\{\psi_T(e\gamma f) : e, f \in E, e = \varphi(p), f = \varphi(q)\}\\ &\geq \sup\{\min\{\psi_T(e), \psi_T(f)\} : \ e, f \in E, e = \varphi(p), f = \varphi(q)\}\\ &= \min\{\sup\{\psi_T(e) : e \in E, e = \varphi(p)\}, \sup\{\psi_T(f) : f \in E, f = \varphi(q)\}\\ &= \min\{(\varphi(\psi_T))(p), (\varphi(\psi_T))(q)\},\end{aligned}$$

$$
\begin{aligned}
(\varphi(\psi_I))(p\gamma q) &= \inf_{z\in\varphi^{-1}(p\gamma q)} \psi_I(z) \\
&= \inf\{\psi_I(e\gamma f) : e, f \in E, e = \varphi(p), f = \varphi(q)\} \\
&\leq \inf\{\max\{\psi_I(e), \psi_I(f)\} : \ e, f \in E, e = \varphi(p), f = \varphi(q)\} \\
&= \max\{\inf\{\psi_I(e) : e \in E, e = \varphi(p)\}, \inf\{\psi_I(f) : f \in E, f = \varphi(q)\} \\
&= \max\{(\varphi(\psi_I))(p), (\varphi(\psi_I))(q)\},
\end{aligned}
$$

and

$$
\begin{aligned}
(\varphi(\psi_F))(p\gamma q) &= \inf_{z\in\varphi^{-1}(p\gamma q)} \psi_F(z) \\
&= \inf\{\psi_F(e\gamma f) : e, f \in E, e = \varphi(p), f = \varphi(q)\} \\
&\leq \inf\{\max\{\psi_F(e), \psi_F(f)\} : \ e, f \in E, e = \varphi(p), f = \varphi(q)\} \\
&= \max\{\inf\{\psi_F(e) : e \in E, e = \varphi(p)\}, \inf\{\psi_F(f) : f \in E, f = \varphi(q)\} \\
&= \max\{(\varphi(\psi_F))(p), (\varphi(\psi_F))(q)\}.
\end{aligned}
$$

Thus, $\varphi(\psi)$ is neutrosophic Γ-semiring of F. □

5. **Conclusions and future works**

In this paper, we introduce the notion of a neutrosophic Γ-semiring and characterized the neutrosophic Γ-semiring in terms of crisp Γ-semirings and obtained some properties. In continuity of this paper, we study neutrosophic ideals, neutrosophic bi-ideals, neutrosophic quasi ideals, neutrosophic interior ideals of Γ-semiring.

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References

1. Agboola, A.A.A., Davvaz, B., and Smarandache, F., Neutrosophic quadruple algebraic hyperstructures, Ann. Fuzzy Math. Inform. vol. 14, 2017, pp. 29–42.
2. Akinleye, S.A., Smarandache, F., and Agboola, A.A.A., On neutrosophic quadruple algebraic structures, Neutrosophic Sets and Systems, vol. 12, 2016, pp. 122–126.
3. Al-Tahan, M., and Davvaz, B., On Some Properties of Neutrosophic QuadrupleH_v-rings, Neutrosophic Sets and Systems, vol. 36, 2020, pp. 256–270.
4. Atanassov, K. Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20, no. 1, 1986, pp. 87–96.
5. Bhargavi, Y., and Eswarlal, T., Fuzzy Γ-semirings, International Journal of Pure and Applied Mathematics, vol. 98, no. 3, 2015, pp. 339–349.

6. Jun, Y.B., Song, S.Z., Smarandache, F., and Bordbar, H., Neutrosophic Quadruple BCK/BCI-Algebras, Axioms, vol. 7, no. 41, 2018. doi:10.3390/axioms7020041
7. Muzaffar, A., Nafis, M.T., and Sohail, S.S., Neutrosophy Logic and its Classification: An Overview, Neutrosophic Sets and Systems, vol. 35, 2020, pp. 239–251.
8. Rao, M.K., Γ-semiring II, Southeast Asian Bulletin of Mathematics, vol. 21, no. 3, 1997, pp. 281–287.
9. Rao, M.K., Γ-semiring I, Southeast Asian Bulletin of Mathematics, vol. 19, 1996, pp. 49–54.
10. Rezaei, A., Smarandache, F., and Mirvakili, S., Applications of (Neutro/Anti) Sophications to Semihypergroups, Journal of Mathematics, vol. 2021, Article ID 6649349, 7 pages.
11. Smarandache, F., Generalizations and Alternatives of Classical Algebraic Structures to NeutroAlgebraic Structures and AntiAlgebraic Structures, Journal of Fuzzy Extension and Applications, vol. 1, no. 2, 2020, pp. 85–87. DOI: 10.22105/jfea.2020.248816.1008
12. Smarandache, F., Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures (revisited), Neutrosophic Sets and Systems, vol. 31, 2020, pp. 1–16.
13. Smarandache, F., Neutrosophy, Neutrosophic Probability, Set, and Logic, Rehoboth, NM: American Research Press, 1998.
14. Zadeh, L.A., Fuzzy Sets, Information and Control, vol. 8, 1965, pp. 338–353.

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